CUTTING FORCES BY PERIPHERAL CUTTING OF LOW DENSITY WOOD SPECIES

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In this paper multifactor non-linear dependencies of cutting forces from several machining parameters for low density wood of *Liriodendron tulipifera Linn*., known as Yellow Poplar, and *Cordia alliodora Ruiz. & Pav*., known as laurel blanco wood or capa prieto, were evaluated from experimental matrices. In the analyzed relations there was evidence for several strong interactions, which have been graphically illustrated and discussed.

Keywords: Cutting Forces, Routing, Milling, Wood, Liriodendron tulipifera, Cordia alliodora

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INTRODUCTION

The problem of cutting forces, especially for routing and milling of low density wood of *Liriodendron tulipifera,* and *Cordia alliodora*, have not been worked out yet. In the literature from the field of wood machining there exists a method for evaluation of the main (tangential) F_c and normal F_n cutting forces, in the form of formulas (1) and (2), for the ten most common European wood species. These formulas employ the relative cutting resistance K and correction coefficients (Afanasev 1961; Amalitskij and Lyubchenko 1977; Bershadskij 1967; Deshevoy 1939; Orlicz 1982). However, large differences, as high as 40% and more between values predicted from equations (1) and (2), relative to the observed forces, suggest that the problem of cutting forces, namely the wood cutting theory, has not yet been worked out completely.

$$F_c = a_p \cdot w_s \cdot K \cdot C_r \cdot C_\delta \cdot C_\rho \cdot C_{ap} \cdot C_{vc} \cdot C_{mc} \cdot C_T \quad (1)$$

$$F_n = C_n \cdot F_c \quad (2)$$

In Eqs. (1) and (2) the terms are defined as follows:

a_p	- Thickness of cutting layer (also known as chip thickness).
w_s	- Cutting width.
$K = f(\varphi_r)$	- Specific cutting resistance [N/mm^2, MPa].
C_r	- Coefficient of wood species, *Pinus silvestris* wood $C_r = 1$.
$C_\delta = f(\delta_f)$	- Coefficient of cutting angle δ_f.
$C_\rho = f(\rho \text{ or } VB)$	- Coefficient of cutting edge dullness ρ, VB.
ρ	- Radius of cutting edge round up.

VB	- Recession of cutting edge.
$C_{ap} = f(a_p)$	- Coeff. of a thickness of a cutting layer (chip thickness) a_p.
$C_{vc} = f(v_c)$	- Coefficient of a cutting speed v_c.
$C_{mc} = f(mc)$	- Coefficient of a moisture content mc.
$C_T = f(T)$	- Coefficient of a temperature T.
$C_n = f(\rho \text{ or } VB)$	- Coefficient of normal force F_n.

In the authors' opinion there are several reasons for the lack of fit. The most important cause is not taking physical and mechanical properties into account in formula (1), instead of an arbitrarily assumed value or range of the correction coefficient C_r. The wood, even of the same wood species, may differ considerably in physical and mechanical properties, resulting in a large dispersion of predicted cutting forces in comparison to observed ones. Another reason is, in the authors' opinion, a general assumption that there is a lack of dependence of the value of one correction coefficient from other cutting parameters, including wood species properties. The average specific cutting resistance K, evaluated without taking into account early and late wood of growth rings, cannot be used to calculate real maximum and minimum cutting resistances. It is already known from the literature that the normal force F_n does not follow the tangential force F_c proportionally to the change in cutting edge wear. An important disadvantage of the method based on equations (1) and (2) in most published works is also a tabular form of the correction coefficients. All assumptions above seem not to be supported by any multifactor experiment, making the method of evaluation of cutting forces based on formula (1) and (2) a rather rough approximation of the problem (Axelsson et al. 1993; Kivimaa 1950; Amalitskij and Lyubchenko 1977).

There are older and newer published works, describing the dependence of main F_c and normal F_n cutting forces on several cutting parameters for different kinds of machining in the form of multinomial or power type functions. However, the limited number of independent variables involved, as well as not having exactly the same and limited range of their variation, makes models difficult to compare (Axelsson et al. 1993). The models available in the literature for most machining methods, including routing and milling, were worked out and collected in the program Wood_Cutting (Porankiewicz 2007).

The present study attempts to evaluate the dependence of main (tangential) F_c and normal (radial) F_n cutting forces from cutting edge dullness VB, average angle φ_r between wood grains and cutting plane, the cutting speed v_c, feed per edge f_z, and moisture content mc during peripheral cutting of low density wood of *Liriodendron tulipifera* and *Cordia alliodora*.

MATERIALS AND METHODS

In the first experiment (Cyra 1997), the cutting forces were measured with use of a measuring system equipped with Hasegawa Tekko Type 4321 load cells, amplifier, and multi-pen recorder, as shown in Fig. 1.

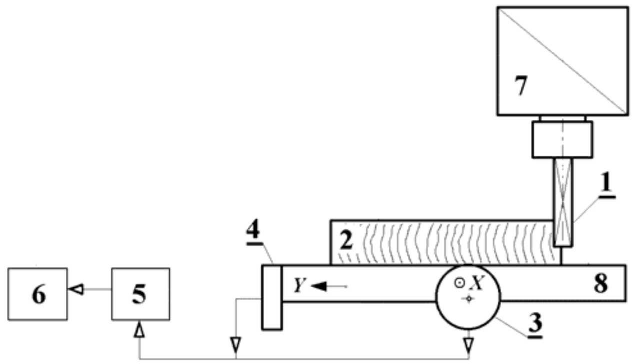

Fig. 1. General scheme of measuring system used in first experiment; 1 - Router bit, 2 – Workpiece, 3,4 – Load cells, 5 – Amplifier, 6 – Multi-pen recorder, 7 – Motor, 8 – Workpiece feed table

In the second experiment (Bermudez 2005; Bermudez et al. 2005) cutting forces were measured with the use of a measuring system equipped with a Hasegawa Tekko Type 4321 load cell, an NEC Type AS1202 amplifier, and a National Instruments Type NI PCI-6034E A/D converter integrated with LabVIEW program software, as shown in Fig. 2. The sampling rate was 100 Hz.

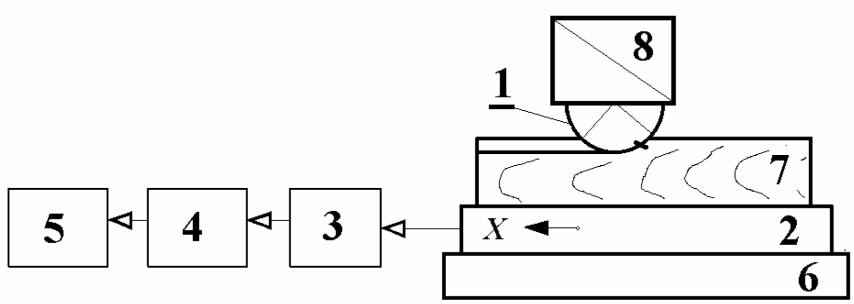

Fig. 2. General scheme of measuring system used in second experiment; 1 – Cutting tool, 2 – Load cell, 3 – Amplifier, 4 – A/D converter, 5 – Computer, 6 – Workpiece feed table, 7 – Workpiece, 8 – Motor

Experiments were performed on a CNC Shoda Fanuc NC-3 vertical router at Shimane University, Matsue, Japan. The X and Y cutting force components for the first experiment and the X component for the second experiment, measured in workpiece feed table coordinates were recalculated to average tangential F_c and normal F_n forces, according to minimum and maximum contact angle, defined by cutting radius r_c and cutting depth g_s. Parameters for the first experiment (Cyra 1997) performed by peripheral up routing (Fig. 3 a) were as follows:

Mechanical and physical properties of wood of *Liriodendron tulipifera*:

Wood density $D = 400$ kg/m^3.
Modulus of rupture by bending $R_b = 69.6$ MPa.
Modulus of rupture by compression parallel to grains $R_c = 38.2$ MPa.
Moisture content $mc = 11$ %.

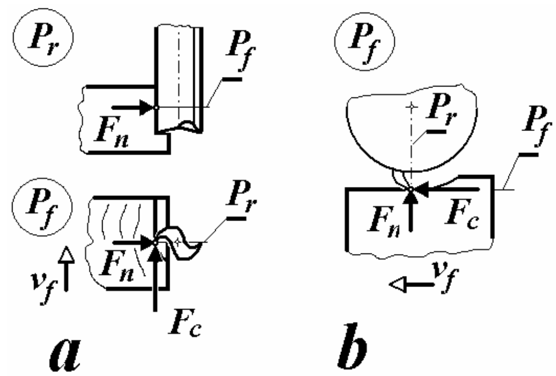

Fig. 3. Scheme of: a – up routing, and, b – up milling, with tangential F_c and normal F_n cutting forces

Machining parameters:

Cutting edge recession measured in bisector of a wedge angle VB_w< 4; 27; 52; 65; 82 µm >.
Contour wedge angle $\beta_f = 37^o$.
By $\beta_f = 37^o$, the range of variation of the ρ is ρ<2; 38 µm >.
Contour rake angle $\gamma_f = 33^o$.
Contour clearance angle $\alpha_f = 20^o$.
Cutting edge inclination angle $\lambda_p = 0^o$.
Cutting radius $r_c = 5$ mm.
Spindle rotational speed $RPM = 5000$ min^{-1}.
Cutting speed $v_c = 30$ m/s.
Feed speed $v_f = 2$ m/min.
Feed per edge $f_z = 0.2$ mm.
Cutting depth $g_s = 2$ mm.
Width of cut $w_s = 10$ mm.
Number of cutting edges z = 2.
Average angle between cutting speed and cutting plain direction and wood grains φ_r and φ_s <23.4; 38.4; 53.4; 68.4; 83.4; 98.4; 113.4; 120.4; 143.4; 158.4; 173.4; 188.4 ^{o} >.
The angle between cutting edge direction and wood grains $\varphi_k = 90^o$.
Growth rings grains orientation towards cutting edge $\varphi_{rt} = 0^o$.
Material of the cutting edge was the cemented carbide K05.

Parameters for the second experiment by peripheral up milling (grooving) with a one side contact (Bermudez 2005; Bermudez et al. 2005; Fig. 3 b) were as follows:

Mechanical and physical properties of wood of *Cordia alliodora* in air dried state:

Wood density $D = 456$ kg/m^3.
Modulus of rupture in bending $R_b = 729.8$ MPa.
Modulus of rupture in compression parallel to grains $R_c = 324.8$ MPa.

Moisture content mc<11.5; 100 %>.

Machining parameters:

Contour wedge angle $\beta_f = 45^o$.
Contour rake angle $\gamma_f = 25^{o,}$
Contour clearance angle $\alpha = 20^o$.
Cutting edge inclination angle $\lambda_p = 0^o$.
Side angle in main plane $\kappa'_r = 1.8^o$.
Side angle in back plane $\tau_p = 4.3^o$.
Cutting radius r_c = 100 mm.
Cutting speed v_c<10; 20; 30; 40 m/s>.
Spindle rotational speed RPM<955; 1910; 2865; 3820 min^{-1}>.
Feed speed v_f< 0.5; 1; 1.5; 2 m/min>.
Cutting depth g_s< 0.5; 1; 2; 3 mm>.
Average angle between cutting speed and wood grain direction φ_r<2; 7^o>.
Width of cut w_s = 1 mm.
Number of cutting edges z = 1.
The angle between cutting edge direction and wood grains $\varphi_k = 90^o$.
The angle between cutting plain direction and wood grains $\varphi_s = 0^o$.
Radial and tangential orientation of growth rings grains towards the cutting edge were not taken into account.
Material of the cutting edge was the high speed steel SKH 51.
Cutting force was analyzed for sharp cutting edge.

In the second experiment, during cutting *Cordia alliodora* wood, the feed velocity v_f was varied in the range of v_f<0.5; 2 m/min>, instead of varying the feed per edge f_z for different cutting speeds v_C. This was achieved by increasing the spindle rotational speed *RPM*, without any increment in the feed velocity. As a result, the feed per edge f_Z did not have the same set of values for the different cutting speeds v_C that were considered.

In order to evaluate the relations $F_c = f(\varphi_r, VB_w)$, $F_n = f(\varphi_r, VB_w)$, and $F_c = f(\varphi_r, f_z, v_c, mc)$, linear models and second order multinomial models, as well as power type functions without and with interactions were analyzed in preliminary calculations. The model should fit experimental matrix by the lowest summation of residuals square *SK*, by the lowest standard deviation *SD,* and by the highest correlation coefficient of predicted and observed values *R*. The experimental matrix can be fitted with more simple models, but this will result in decreasing approximation quality, which means that the *SK* and *SD* values will increase, and *R* will decrease. In this case all predicted values of dependent variable will have higher expected error. Many years of experience by the first author lead us to believe that efforts to fit such data with overly simple models can be expected to hurt the quality of the approximation of the influence of independent variables, especially in the case of variables with small importance, making such a model nonsensical. The proper influence of low importance variables can be only extracted from an experimental matrix when using a more complicating model. The most adequate formulas appeared to be the non-linear, multivariable equations with interactions (3) through (7).

$$F_c = a_1 \cdot e^{a_2 \cdot \sin(\varphi_r + a_3) + a_4 \cdot \sqrt{VB_w} + a_5 \cdot \varphi_r^2 \cdot VB_w + a_6} + a_7 \quad [\mathrm{N}] \tag{3}$$

$$F_n = b_1 \cdot e^{b_2 \cdot \sin(\varphi_r + b_3) + b_4 \cdot \sqrt{VB_w} + b_5 \cdot \varphi_r^2 \cdot VB_w + b_6} + b_7 \quad [\mathrm{N}] \tag{4}$$

$$W_1 = c_2 \cdot \varphi_r + c_3 \cdot f_z + c_4 \cdot v_c + c_5 \cdot \varphi_r / mc + c_6 \cdot \varphi_r / f_z \tag{5}$$

$$W_2 = c_7 \cdot f_z \cdot v_c + c_8 \cdot \varphi_r \cdot v_c + c_9 \cdot v_c \cdot mc + c_{10} \tag{6}$$

$$F_c = c_1 \cdot e^{W_1 + W_2} + c_{11} \quad [\mathrm{N}] \tag{7}$$

For evaluation of estimators from experimental matrixes (containing 60 and 128 measuring points, in case of the first and second experiments, respectively), a special optimization program was applied, based on a least squares method combined with gradient and Monte Carlo methods (Porankiewicz 1988) with further changes. Elimination of the unimportant or low-importance estimators was carried out by use of the coefficient of relative importance, *CRI*, during evaluation of process models (3) through (7). *CRI* was defined by formula (8). It was assumed that *CRI* > 0.1.

$$CRI = (SK - SK_{OK}) / SK \cdot 100\ [\%] \tag{8}$$

In Eq. (8) the new terms are:

SK_{OK} = the summation of residuals square by $c_K = 0$
c_K = k estimator evaluated in statistical model

Calculations were performed at Poznań Networking and Supercomputing Center PCSS on an SGI Origin 3800 computer. For characterization of approximation quality, a summation of residuals square *SK*, standard deviation *SD*, and a square of correlation coefficient of the predicted and observed values R^2 was used.

For comparison of results obtained in the present work with similar data from the literature, the main cutting force F_c was calculated for up routing and up milling, with application of the Wood_Cutting program (Porankiewicz 2007) for *Tilia cordata* low density wood, was used.

RESULTS AND DISCUSSION

For formula (3), describing the relation between the main force F_c and the average angle φ_r between cutting speed direction, wood grains, and cutting edge dullness VB_w for wood of *Liriodendron tulipifera*, the following estimators were evaluated: a_1 = 0.091; a_2 = 0.9662; a_3 = -0.7276; a_4 = 0.2769; a_5 = -0.0007; a_6 = 2.7357; a_7 = 9.0847, by range of variation of independent variables: φ_r <23.4; 188.4°>; VB_w<4; 82 μm>. The quality of the fit to the model (4) is shown by Fig. 4a and the values of the quantifiers: SK = 770.2; R^2 = 0.86; SD = 3.6 N.

The following estimators were evaluated for formula (4), describing the relation between the normal force F_n and the average angle between cutting speed direction and wood grains φ_r, as well as cutting edge dullness VB_w for the wood of *Liriodendron tulipifera*: b_1 = 0.0006; b_2 = 2.8537; b_3 = 0.3148; b_4 = 0.5061; b_5 = 0.0029; b_6 = 3.5853; b_7 = 8.5167, by variation of independent variables: φ_s<23.4; 188.4°>; VB_w<4; 82 μm>. The quality of the model (4) fit is characterized by the quantifiers: SK = 853.8; R^2 = 0.94; SD = 3.8 N, and is also illustrated in Fig. 4b.

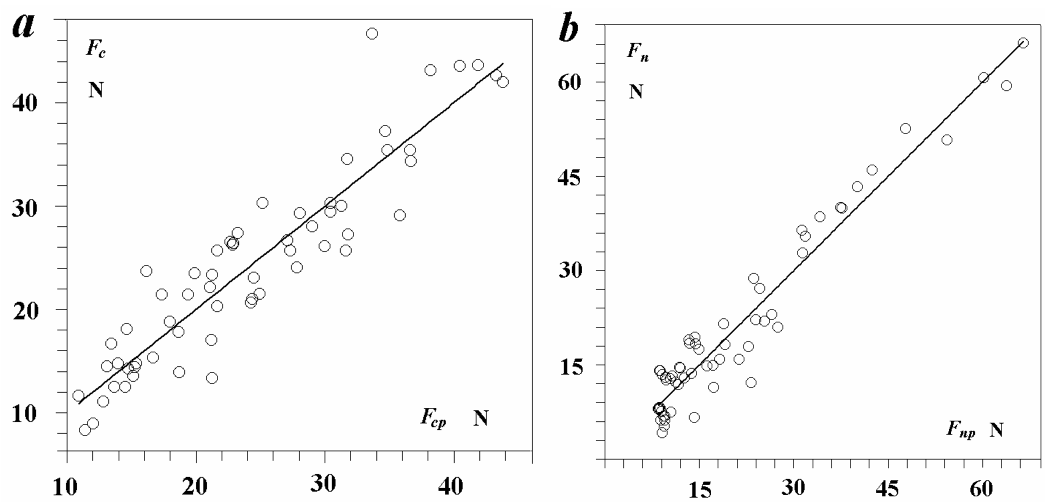

Fig. 4. Plots of main F_c and normal F_n cutting forces observed by routing Liriodendron tulipifera routing against predicted main F_{cp} and normal F_{np} forces according to models: a - (3) and b - (4)

From Fig. 5a it can be seen that for maximum cutting edge dullness VB_w, the F_c and the F_n increased with increasing average angle φ_R, and reached their maximum at average angles φ_r = 117.7° and φ_r = 87.3°, respectively. Such a relation was not evidenced for the sharp cutting edge. In the dependences F_c and $F_n = f(VB_w, \varphi_r)$, a strong interaction $VB_w \cdot \varphi_r$ was evidenced. The F_n reaches it's larger maximum by the φ_r angle as much as 30.4° lower than in case of the F_c. Different shapes of relations $F_c = f(VB_w, \varphi_r)$ and $F_n = f(VB_w, \varphi_r)$, as well as a presence of their maximum beside φ_r = 90° for sharp and dull cutting edge is a phenomenon that is in contradiction with equations (1) and (2).

For almost parallel cutting, by φ_r <30° and φ_r > 160°, enlargement of the cutting edge recession VB_w caused rather small increases of the F_c and F_n values, slightly more for the F_c. This finding also contradicts information from the literature. In the analyzed range of the cutting edge recession VB_w<4; 82 μm>, the ratio between the largest and the lowest value of the F_c was as high as 4.2 in this paper, while only 1.3 according to the literature (Amalitskij and Lyubchenko 1977, Orlicz 1982).

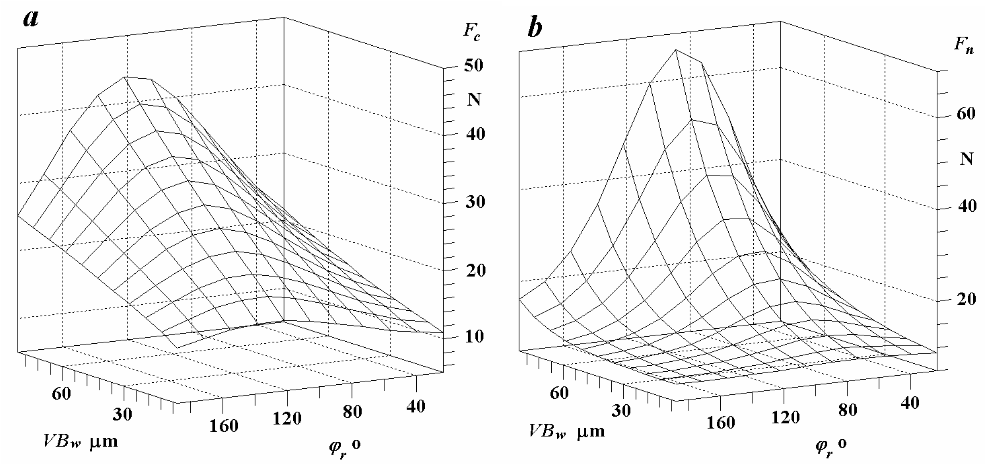

Fig. 5. Dependence between main force F_c and normal force F_n, the grain angle φ_S, and the cutting edge dullness VB_w, according to models: a - (3) and b - (4)

For formulas (5) through (7) describing the relationship between the main force F_c and φ_r, f_z, v_c, and mc, for wood of *Cordia alliodora*, peripheral up-milling, the following estimators were evaluated: c_1 = 1.1364; c_2 = 1.3095; c_3 = 1.2049; c_4 = 1.2561; c_5 = -0.2581; c_6 = $-7.36 \cdot 10^{-2}$; c_7 = 0.3623; c_8 = 1.2681; c_9 = 0.2485; c_{10} = 0.9209; c_{11} = 1.1441; c_{12} = 2.1219; c_{13} = 0.1764; c_{14} = 0.603; c_{15} = 1.6861, by a range of variation of independent variables: φ_r<1.99 ; 6.96^o >; f_z<0.13; 2.09 mm>; v_c<10; 40 m/s>; mc<11.5; 30> %. The quality of the models (5) through (7) fit is characterized by the quantifiers: SK = 47.7; R^2 = 0.93; SD = 0.61 N, and is also illustrated in Fig. 6.

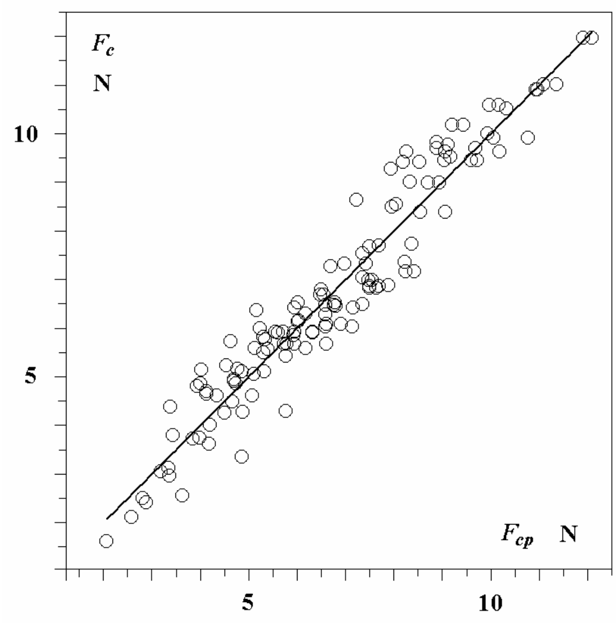

Fig. 6. Plot of main F_c cutting force observed by milling Cordia alliodora against predicted main F_{cp} force according to models (5) through (7)

Graphical illustrations of the relations (5) through (7) are shown in Figs. 7 and 8. From Fig. 7 an increase in the main force F_c was observed with increasing feed per edge f_z and the average angle φ_r between cutting speed direction v_c and wood grain. This relation dropped down for the lowest values of analyzed independent variables. In the range of the feed per edge f_z<0.13; 2,09 mm>, a ratio between the largest and the lowest value of the F_c was in similar range with values given in literature for lime tree wood. Fig. 8 shows that the F_c slightly declined with an increase in *mc* by the highest value of v_c. This is in agreement with the literature. The interactions φ_r / f_z, $f_z \cdot v_c$, and $\varphi_r \cdot v_c$, and the much weaker interactions φ_r / mc and $v_c \cdot mc$ are new findings.

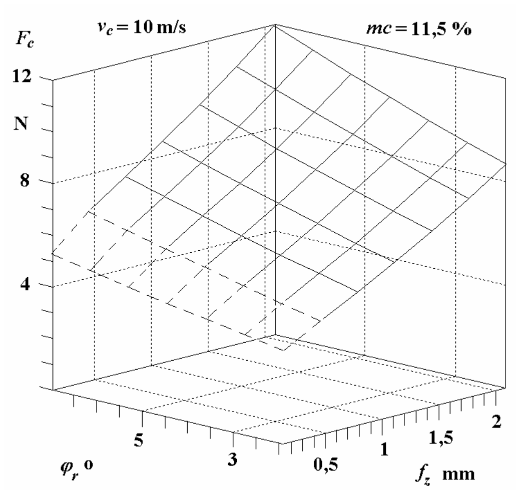

Fig. 7. Dependence of the main force F_c from the average grain angle φ_r and the feed per edge f_z, according to models (5) through (7) by the lowest value of v_c and *mc*; region marked by broken line lay outside experimental matrix

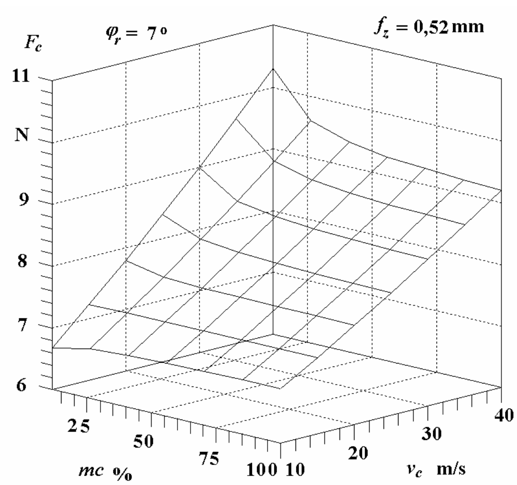

Fig. 8. Dependence of the main force F_c from the cutting speed v_c and the moisture content *mc* according to models (5) through (7)

From Fig. 8 it can be seen that the main force F_c increased with an increase of the v_c over the whole analyzed range, slightly more for the lowest moisture content *mc*, which contradicts information from the literature. Figure 8 also shows that the moisture content *mc* had limited influence on the main force F_c, to the lowest analyzed value. Within the analyzed range of the moisture content *mc*<11.5; 100 %>, the ratio between the largest and the lowest values of the F_c was as high as 1.13 in the present paper, in comparison with 1.25 according to the literature (Kivimaa 1950; Amalitskij and Lyubchenko 1977; Orlicz 1982).

Analysis performed in the present study indicates that for an adequate description of wood cutting forces, including the dependence on machining parameters, more precise formulas have to be applied in place of the relations (1) and (2).

CONCLUSIONS

1. By peripheral up-routing of *Liriodendron tulipifera* wood, for the highest cutting edge dullness VB_w = 82 µm, the main cutting force F_c, strongly increases with increasing average grain angle φ_r towards cutting speed v_c up to $\varphi_r = 117.7^o$. This is followed, for yet higher φ_r values, by rapid decreases in F_c.
2. For the lowest cutting edge dullness VB_w = 4 µm, the main cutting force F_c slightly increases with increasing φ_r, up to $\varphi_r = 117.7^o$ and afterwards for higher φ_r the F_c slowly decreases.
3. The differentiated influence of the cutting edge dullness represented by the VB_w on the main cutting force F_c has it's source in very strong interaction $VB_w \cdot \varphi_r$.
4. By peripheral up-routing of *Liriodendron tulipifera* wood, for the highest cutting edge dullness VB_w = 82 µm, the normal cutting force F_n, strongly increases with increasing average grains angle φ_r towards cutting speed v_c up to $\varphi_r = 87.3^o$ and afterwards for higher φ_r the F_n rapidly decreases.
5. The normal force F_n does not follow the main cutting force F_c. Rather, the F_n reaches it's maximum by the φ_r angle 30.7^o lower than the F_c. force.
6. For the lowest cutting edge dullness VB_w = 4 µm, the normal cutting force F_n slightly increases with an increase of φ_r up to $\varphi_r = 87.3^o$ and afterwards for higher φ_r the F_n slowly decreases.
8. A lack of the maximum cutting forces near average angle $\varphi_r = 90^o$ for a sharp tool by routing of *Liriodendron tulipifera* wood was evidenced.
9. By up-milling of wood of *Cordia alliodora*, the main force F_c strongly increases with increasing rate per edge f_z within the range f_z<0.13; 2.09 mm>.
10. The main force F_c increases with increasing average grain angle φ_r towards cutting speed v_c within the range φ_r<1.99; 6.88^o>, during up-milling of *Cordia alliodora*.
11. During up-milling of *Cordia alliodora*, the main force F_c increases with an increase of cutting speed v_c over the whole analyzed range v_c<10; 40 m/s>.
12. An increase of the moisture content *mc* reduces the main force F_c. This relation is important only at the lowest value of mc and the largest cutting speed *vc.*

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