MEASUREMENT OF THE YOUNG'S MODULUS AND SHEAR MODULUS OF IN-PLANE QUASI-ISOTROPIC MEDIUM-DENSITY FIBERBOARD BY FLEXURAL VIBRATION

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The Young's modulus and the in-plane shear modulus of medium-density fiberboard (MDF) were obtained by conducting a flexural vibration test under the free-free condition based on Timoshenko's vibration theory using specimens with various depth/length ratios and performing a subsequent numerical analysis. The results obtained by the experiment and numerical analysis revealed that the Young's modulus was independent of the specimen configuration. In contrast, the in-plane shear modulus was significantly dependent on the specimen configuration and could not be measured properly based on Timoshenko's theory when the specimen had a small depth/length ratio. The numerical analysis also revealed that the Poisson's ratio has a significant influence on the measurement of shear modulus as well as the specimen configuration. A statistical analysis on the results experimentally obtained suggested that the length of the specimen must be less than 7.5 times the depth to measure the in-plane shear modulus appropriately.

Keywords: Flexural vibration test; Finite element analysis (FEA); In-plane shear modulus; Medium-density fiberboard; Young's modulus

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INTRODUCTION

Among the several experimental methods for determining the elastic properties of solid wood and wood-based materials, the vibration method is very effective because the load applied to the specimen is so small that the elastic properties can be measured non-destructively. For wood and wood-based materials there is a significant relationship between elastic moduli and strength properties (Marra et al. 1966; Haines et al. 1996; Ilic 2001). When conducting the vibration test, the strength properties can therefore be predicted using the elastic modulus non-destructively measured.

To measure the shear modulus of these materials, a torsional vibration test is often conducted because it induces a pure shear stress condition in the specimen (Hearmon 1946; Becker 1973; Nakao 1984; Nakao and Okano 1987; Yoshihara 2009; Tonosaki et al. 2010). Nevertheless, it is often difficult to measure the shear modulus of wood-based materials such as plywood, particleboard, and medium-density fiberboard (MDF) precisely by this method. During the torsional vibration testing of these materials, the influence of the out-of-plane shear modulus, which is usually smaller than the in-plane shear modulus, on the resonance frequencies in the torsional vibration mode is significant and must be reduced to measure the in-plane shear modulus precisely. Therefore, the in-plane shear modulus cannot be obtained easily by the torsional vibration test due to the difficulty in reducing the influence of the out-of-plane shear modulus. Recently, the in-plane shear modulus of particleboard and MDF has been often measured by the static methods such as rail-shear and Iosipescu tests instead of the vibration methods (Janowiak and Pellerin 1991; Suzuki et al. 2000; 2002; De Magistris and Salmén 2004). Nevertheless, it is difficult to obtain the in-plane shear modulus non-destructively by the static methods.

To measure the shear modulus of solid wood and plywood, the flexural vibration method based on Timeshenko's vibration theory, which is described below, is frequently employed (Hearmon 1958, 1966; Nakao 1984; Sobue 1986; Nakao and Okano 1987; Chui and Smith 1999; Divós et al. 1998, 2005; Brancheriau and Baillères 1998, 2002, 2003; Brancheriau 2006; Murata and Kanazawa 2007; Tonosaki et al. 2010; Sohi et al. 2011; Yoshihara 2009, 2011). For these materials, the Young's modulus and the shear modulus can be simultaneously and accurately obtained by the flexural vibration test without considering the out-of-plane deformation of the specimen. Nevertheless, there is an obstacle to the use of this method to measure the Young's modulus and shear modulus of particleboard and MDF.

To obtain the Young's modulus and shear modulus by the flexural vibration method, the shear deflection must be relatively large. For solid wood and plywood, the Young's modulus/shear modulus ratio ranges approximately from 5 to 20 (Hearmon 1948). In this range, the shear deflection, which increases with increasing depth/length ratio, is significant even for slender specimens. In contrast, particleboard and MDF are regarded as being in-plane quasi-isotropic materials, so the Young's modulus/shear modulus ratio ranges from 2 to 3 (Hearmon 1948). In these materials, the shear deflection contribution is small, often making it difficult to measure the in-plane shear modulus properly when the specimen is too slender. To measure the Young's modulus and the shear modulus of these materials accurately, the specimen must have a large depth relative to the length. Nevertheless, there are few reports in the literature that examine the applicability of Timoshenko's vibration theory under various test conditions.

When the flexural vibration test becomes a well established method for measuring the moduli of in-plane quasi-isotropic materials such as particleboard and MDF, the Young's modulus and the in-plane shear modulus of these materials can be obtained more precisely. In this work, flexural vibration tests were performed on MDF specimens with various depth/length ratios, and the Young's modulus and the shear modulus were obtained using Timoshenko's vibration theory. The validity of this method was examined by comparisons with finite element (FE) calculations.

FLEXURAL VIBRATION EQUATIONS BASED ON TIMOSHENKO'S THEORY

Figure 1 shows diagrams of the specimen and finite element (FE) model. The differential equation of flexure given by Timoshenko, which considers the shear deflection and rotary inertia, is

$$EI\frac{\partial^4 y}{\partial x^4}+\rho A\frac{\partial^2 y}{\partial t^2}-\rho I\left(1+\frac{sE}{G}\right)\frac{\partial^4 y}{\partial x^2\partial t^2}+\rho^2 I\left(1+\frac{sE}{G}\right)\frac{\partial^4 y}{\partial t^4}=0 \tag{1}$$

where *E* is the Young's modulus, *G* is the shear modulus, *I* is the secondary moment of inertia, *A* is the cross-sectional area, ρ is the density, and *s* is Timoshenko's shear factor (Timoshenko 1921). Pickett (1945) found *s* to be 1.2 for an isotropic specimen with a rectangular cross-section based on a theoretical analysis. Later, however, additional analyses were conducted on the value of *s* (Sutherland and Goodman 1951; Cowper 1966; Hutchinson and Zilmer 1986; Puchegger et al. 2003, 2005), the details of which are described below. Goens (1931) derived the solution of Eq. (1) under the free-free flexural condition. The detail is also described in several previous works (Mead and Joannides 1991; Kubojima et al. 1996; Yoshihara 2011).

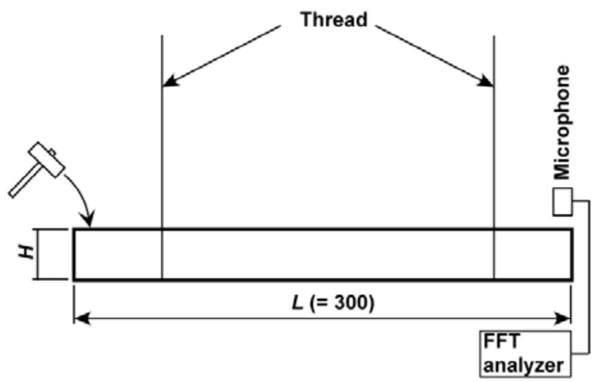

Fig. 1. Diagram of the flexural vibration test of a specimen. Unit = mm. *H* varies from 10 to 60 mm with an interval of 10 mm. Specimen is supported by threads located at the most outer positions of vibration modes.

The flexural vibration method is advantageous in that *E* and *G* can be measured simultaneously and non-destructively. For the in-plane quasi-isotropic MDF, *E*, and *G* correspond to the Young's modulus in the length or width direction of the board and the in-plane shear modulus, respectively. The terms representing the shear deflection in Eq. (1) must have a significant influence on the solution to accurately determine the values of *E* and *G*; therefore, the value of *E/G* must be sufficiently large. For solid wood in which the length and thickness directions coincide with the longitudinal and tangential (or radial) directions, respectively, the value of *E/G* ranges from 5 to 20 (Hearmon 1948). These values are large enough to easily determine the values of *E* and *G* precisely. In contrast, the *E/G* values of isotropic materials range from 2 to 3, making it difficult to obtain the precise value of *G* when the specimen is too slender. To obtain the *G* value precisely, the influence of shear deflection should be significant relative to the resonance frequencies. When the specimen with a small *E/G* value is too slender, however, the influence of shear deflection is obscured and it is difficult to detect the influence of shear deflection on the resonance frequency precisely. To increase the influence of the shear deflection, the depth/length ratio of the specimen must be sufficiently high.

FINITE ELEMENT ANALYSIS (FEA)

Two-dimensional (2D) FEA was performed independently of the flexural vibration test using the FEA program ANSYS 12. Figure 2 shows the homogeneously divided FE mesh of the specimen. In the model, the length (*L*) was 300 mm, the width (*B*) was 9 mm, and the depth (*H*) varied from 10 to 60 mm with an interval of 10 mm. The model consisted of four-noded plane elements. It was confirmed that the mesh size was fine enough and the effect of mesh size could be ignored.

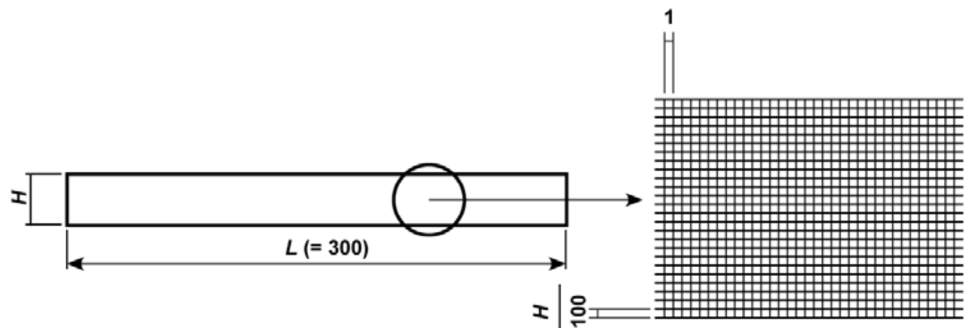

Fig. 2. Finite element model used in the analyses

The model assumed an isotropic material using the actual MDF properties; the density (ρ) and Young's modulus (*E*) used in the FEA were 650 kg/m^3 and 3.0 GPa, respectively. The Poisson's ratio, ν, varied from 0 to 0.5 with an interval of 0.1. The shear modulus was calculated using the values of *E* and ν in the elasticity theory for isotropic materials by

$$G = \frac{E}{2(1+\nu)} \tag{2}$$

Modal analyses were conducted, and the resonance frequencies from the 1st to 4th flexural vibration modes were extracted; *E* and the ratio of the shear modulus to Timeshenko's shear factor (*G*/*s*) were determined from the solution derived by Goens (1931).

Both *E* and *G/s* terms are contained in the solution, and the *G*/*s* values corresponding to each vibration mode were calculated by altering the value of *E*, and the coefficient of variation (COV) among the *G*/*s* values was determined. The *E* value that generates the minimum COV among the *G*/*s* values and the mean value of *G*/*s* can be regarded to be the most feasible. Approximate solutions were obtained using Mathematica 6. The *s* value must be determined previously to calculate *G*. Although the *s* value is conventionally derived as 1.2 based on Pickett's analysis (1945), the value of *s* is discussed below.

EXPERIMENT

Materials

A medium density fiberboard (MDF) with dimensions of 910 mm × 1820 mm × 9 mm was used to obtain the test specimens for this study. The board, with a density of 650 ± 10 kg/m^3 was fabricated in a board mill (Ueno Mokuzai Kogyo Co., Himeji, Japan) using softwood with a typical fiber length of 2 to 4 mm and a urea-formaldehyde (UF) resin. It was stored in a room with a constant temperature of 20 ºC and 65% relative humidity prior to testing.

Initially, 10 specimens with length and depth dimensions of 300 mm × 60 mm, respectively, were cut from the MDF. After conducting the flexural vibration tests described below, the depth of the specimen was decreased, and the succeeding series of flexural vibration tests was conducted using the specimens with decreased depth. The depth (H) of the specimens was decreased from 60 to 10 mm by intervals of 10 mm. The average densities were 652, 651, 650, 650, 649, and 650 kg/m^3 corresponding to the depth from 60 to 10 mm, respectively.

Flexural Vibration Test

The specimen was suspended by threads at the nodal positions of the free-free resonance vibration mode (f_n) and excited in the depth (Y) direction with a hammer (Fig. 1). The supported points were the most outer positions of each vibration mode. It was difficult to measure resonance frequencies above the 5th mode in the vibration test because the amplitude of the vibration was small in the high frequency modes. In addition, it was often difficult to distinguish between the vibrations caused by the flexural mode from those caused by other modes in the high frequency range. Therefore, the 1st- to 4th-mode resonance frequencies were measured in this investigation. The resonance frequencies were analyzed using a Fast Fourier Transform (FFT) analysis program. Similar to the FEA, the E and G/s values were calculated using Mathematica 6.

According to the Euler-Bernoulli vibration theory, the third and fourth terms of Eq. (1) are not considered, and the Young's modulus, E_{EB}, is obtained by,

$$E_{EB} = \frac{4\pi^2 \rho A L^4}{m_n^4 I} f_n \tag{3}$$

where m_n, the coefficient corresponding to the resonance mode, is given by

$$\begin{cases} m_1 = 4.730 \\ m_2 = 7.853 \\ m_n = \dfrac{(2n+1)\pi}{2} \quad (n \geq 3) \end{cases} \tag{4}$$

The value of E_{EB} is influenced by the shear deflection, and it usually decreases as the mode number (n) increases. Therefore, the E_{EB} value should decrease with increasing n to obtain the G value from Timoshenko's vibration theory. Figure 3 shows the dependence of E_{EB} on n. The value of E_{EB} always decreases with increasing n in FEA. Therefore, it is feasible that the G value can be obtained based on Timoshenko's vibration theory. In the actual vibration tests, however, the G value often increases from the 1st to the 3rd mode numbers when the depth of the specimen is 10 mm. This phenomenon suggests that Timoshenko's vibration theory is inapplicable for specimens with the depths of 10 mm when using the 1st and 2nd modes of the resonance frequencies.

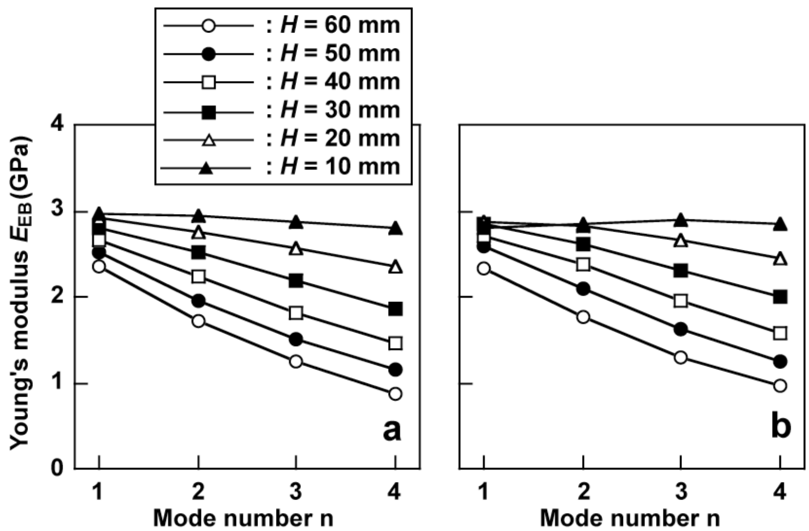

Fig. 3. Dependence of the Young's modulus obtained from Euler-Bernoulli's vibration theory (E_{EB}) on the mode number (n) for (a) FEA and (b) actual vibration tests

The G/s values also cannot be calculated from the data obtained from the low-order modes using the analysis based on Timoshenko's vibration theory. Figure 4 shows the dependence of G/s on the mode number obtained from FEA and actual vibration tests. In FEA, the values of G/s can be obtained over the entire range of mode numbers. In the actual vibration tests, however, the value of G/s is dependent on the mode number when the depth of the specimen is 10 or 20 mm.

For specimens with H = 20 mm, the value of G/s obtained from the 1st resonance mode is significantly smaller than those obtained from the larger modes. For the specimen with H = 10 mm, the G/s values obtained from the 1st and 2nd resonance modes are significantly smaller than those obtained from the 3rd and 4th modes. These phenomena are the result of the small deflection caused by the shearing force in the low-order vibration modes.

In addition, the frequencies of the low-order modes are often influenced by irregularities such as non-homogeneity within the specimen (Mead and Joannides 1991). Therefore, the precise calculation of the shear modulus is not possible using the data obtained from the low-order mode frequencies when the depth/length ratio of specimen is too small.

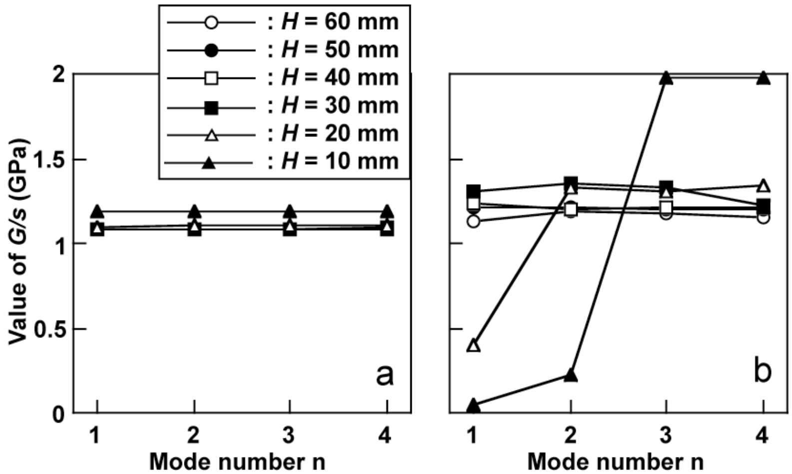

Fig. 4. Dependence of the *G/s* on the mode number (n) for (a) FEA and (b) actual vibration tests. Poisson's ratio used for the FEA = 0.275

From the results shown in Figs. 3 and 4, the data obtained from the 1st mode and 1st and 2nd modes were not used in the analysis of the specimens with the depths of 20 mm and 10 mm, respectively.

Tension Tests

In this study, the in-plane shear modulus was independently obtained by substituting the Young's modulus determined from the flexural vibration test and the Poisson's ratio determined from the tension tests into Eq. (2) and was compared with that obtained from the flexural vibration test. Five specimens with length and width dimensions of 300 mm× 40 mm, respectively, were cut from the MDF used for the flexural vibration tests. Biaxial strain gauges (Tokyo Sokki FCA -2-11, gauge length = 2mm) were bonded at the center of the opposite surfaces of the specimen. The specimen was gripped with 140 mm between the grips, and a tensile load was applied with a crosshead speed of 1 mm/min. The Poisson's ratio ν was obtained by,

$$\nu = -\frac{\Delta\varepsilon_{\mathrm{T}}}{\Delta\varepsilon_{\mathrm{L}}} \tag{5}$$

where $\Delta\varepsilon_{\mathrm{T}}/\Delta\varepsilon_{\mathrm{L}}$ is the initial inclination of the relationship between the strains in the transverse and longitudinal directions of the specimen. Ten specimens were used for the tension tests. The value of Poisson's ratio was obtained as 0.275 ± 0.010. This value was used for the analysis of Timoshenko's shear factor *s*.

RESULTS AND DISCUSSION

Figure 5 shows the independence of Young's modulus relative to the depth/length ratio (*H/L*) obtained by FEA. The FEA results show that the *E* value can be determined precisely in the examined range of *H/L*, even when the Poisson's ratio of the specimen varies.

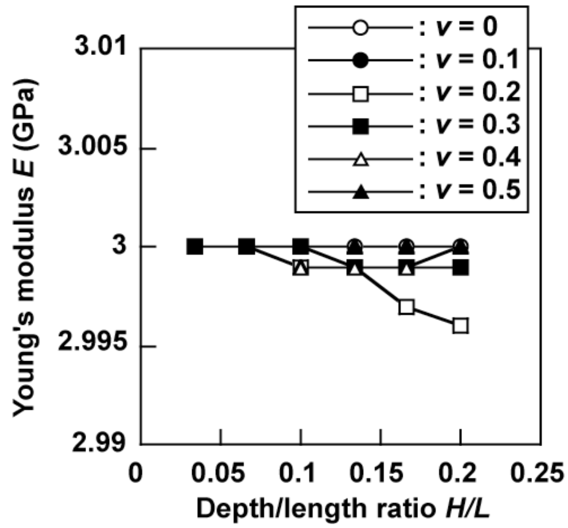

Fig. 5. Relationship between the in-plane Young's modulus (*E*) and depth/length ratio (*H/L*) obtained by FEA for various Poisson's ratios (ν)

Figure 6 shows the dependence of shear modulus *G* on the *H/L* obtained by FEA. Although the *E* value is effectively obtained independently of the *H/L* as shown in Fig. 4,

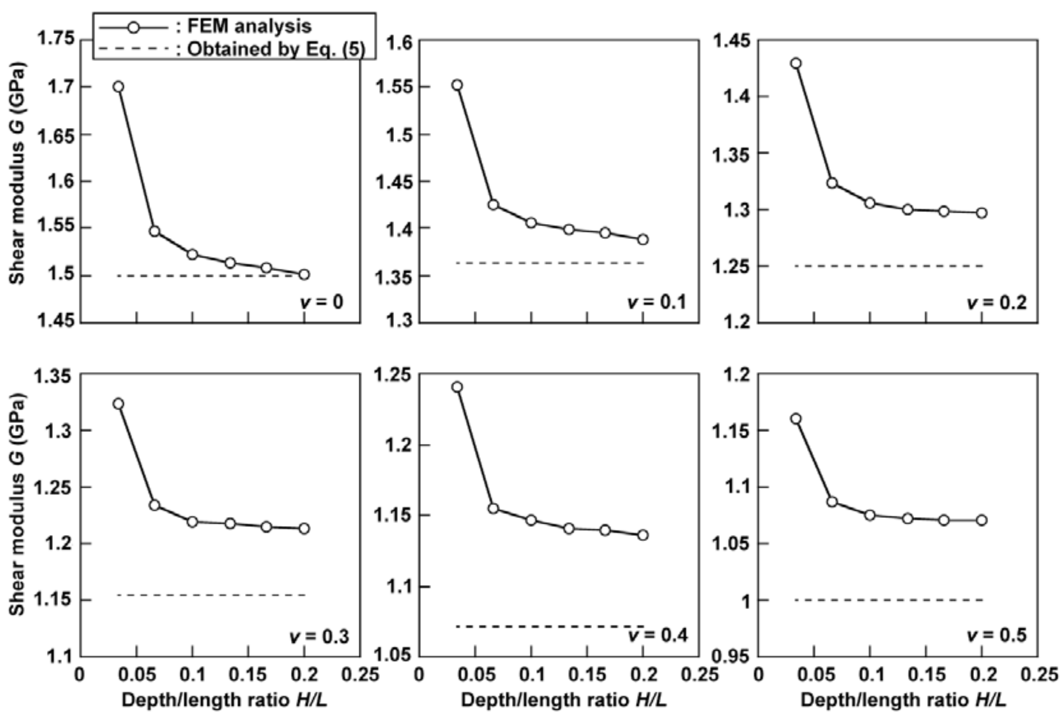

Fig. 6. Dependence of the shear modulus (*G*) on the depth/length ratio (*H/L*) obtained by FEA for various Poisson's ratios (ν). Timoshenko's shear factor (*s*) is 1.2 in the analyses

the *G* value is markedly dependent on the *H/L*. In addition, the *G* value obtained by FEA deviates from that derived by Eq. (5) when the *H/L* decreases and the Poisson's ratio increases. For the *G* value, an *s* of 1.2 was provisionally substituted into *G*/*s* because *s* is close to 1.2 according to several conventional reports on solid wood and plywood (Nakao 1984; Sobue 1986; Nakao and Okano 1987; Chui and Smith 1999; Divós et al. 1998, 2005; Brancheriau and Baillères 1998, 2002, 2003; Brancheriau 2006; Murata and Kanazawa 2007; Tonosaki et al. 2010; Sohi et al. 2011; Yoshihara 2009).

To overcome the deviation between the values of shear moduli obtained by Eq. (2) and FEA, it is feasible to modify Timoshenko's shear factor considering the values of *H/L* and ν. Therefore, the value of *s* was calculated by substituting the *G* value obtained by Eq. (2) into the *G*/*s* obtained by the FEA. Figure 7 shows the dependence of *s* on the *H/L* obtained from FEA by varying the values of ν and indicates that *s* is significantly dependent on the values of *H/L* and ν.

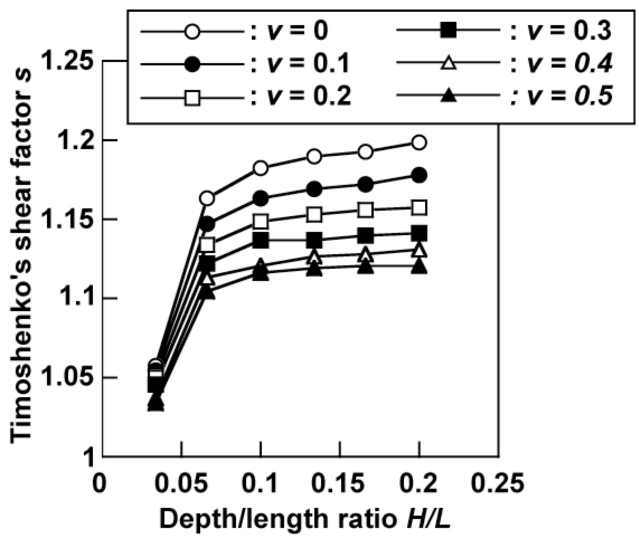

Fig. 7. Dependence of Timoshenko's shear factor (*s*) on the depth/length ratio (*H/L*) obtained by FEA

The influence of Poisson's ratio on *s* has been investigated previously, and various equations for the relationship between *s* and ν have been proposed. Sutherland and Goodman (1951) derived *s* as the solution of

$$\frac{1}{s^3} - \frac{8}{s^2} + \frac{8(2-\nu)}{s(1-\nu)} + \frac{16}{1-\nu} = 0 \qquad (6)$$

Cowper (1966) derived *s* as

$$s = \frac{12 + 11\nu}{10 + 10\nu} \qquad (7)$$

Later, Hutchinson and Zilmer (1986) and Puchegger et al. (2003, 2005) considered the specimen configuration and derived,

$$s = -\frac{\frac{9}{4BH^5}C + \nu\left(1 - \frac{B^2}{H^2}\right)}{2(1+\nu)} \tag{8}$$

where C is given by

$$C = \frac{4}{45}BH^3\left(-12H^2 - 15H^2\nu + 5B^2\nu\right) + \sum_{n=1}^{\infty}\frac{16B^5\nu\left\{n\pi H - B\tanh\left(\frac{n\pi H}{B}\right)\right\}}{n^5\pi^5(1+\nu)} \tag{9}$$

Table 1 shows s values for various H values calculated from Eqs. (6) through (8) and those obtained by FEA. In these computations, the ν obtained by the tension test is used (0.275). The FEA results suggest that the influence of the specimen configuration and the Poisson's ratio is more significant than predicted in the previous works.

Table 1. Timoshenko's Shear Factor (*s*) Used to Compute Shear Modulus (*G*)

	H (mm)					
	10	20	30	40	50	60
FEAs*	1.032	1.125	1.139	1.143	1.145	1.147
Pickett	1.2					
Sutherland and Goodman**	1.173					
Cowper***	1.178					
Hutchinson et al.****	1.152	1.157	1.157	1.157	1.157	1.157

* Obtained using ν = 0.275, which was obtained from the tension tests, in the FE program, and **, ***, and **** calculated by substituting ν = 0.275 into Eqs. (6)-(8), respectively.

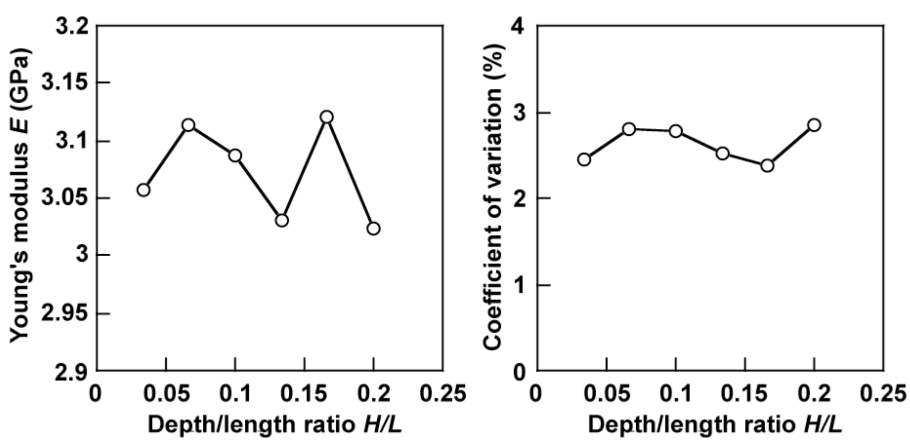

Fig. 8. Young's modulus (*E*) and coefficient of variation obtained by the flexural vibration tests

Figure 8 shows the *E* and COV values obtained experimentally from the flexural vibration tests. Statistical analysis of the difference between the Young's modulus of the specimens with different depth/length ratios showed that the difference was not significant over the entire range of depth/length ratio because every probability value (P-value) obtained by comparing the *E* values corresponding to each *H/L* values was larger than the significance level of 0.05.

Figure 9 shows a comparison of *G* and COV values determined from the flexural vibration tests. Timoshenko's shear factor shown in Table 1 is used to obtain the shear modulus value. The value of *G* increases with decreasing *H/L*. It was expected that the influence of the specimen configuration could be reduced when using the *s* obtained by FEA. From Fig. 8, however, it is difficult to reduce the influence of the specimen configuration even though the *s* obtained by FEA is used. A statistical analysis of the difference between the *G* values calculated from the various *s* values determined that the influence of *s* was not significant. In contrast, a statistical analysis among the *G* values calculated from the various depth/length ratios determined that the *G* values for the specimens with *H/L* of 0.1, 0.067, and 0.033 (corresponding to depth of 30, 20, and 10 mm, respectively) were larger than those with a *H/L* of 0.133, 0.167, and 0.2 (corresponding to depths of 40, 50, and 60 mm, respectively) at a significance level of 0.01. In this *H/L* range, the shear deflection contribution was relatively small, so it is difficult to measure the shear modulus accurately even when the measurement error of resonance frequency is low. In contrast, there were no significant differences among the *G* values for the specimens with depths of 40, 50, and 60 mm. These statistical results suggest that *H* should be larger than 40 mm for specimens with lengths *L* of 300 mm, which corresponds to 7.5 times the depth, to obtain the in-plane shear modulus accurately.

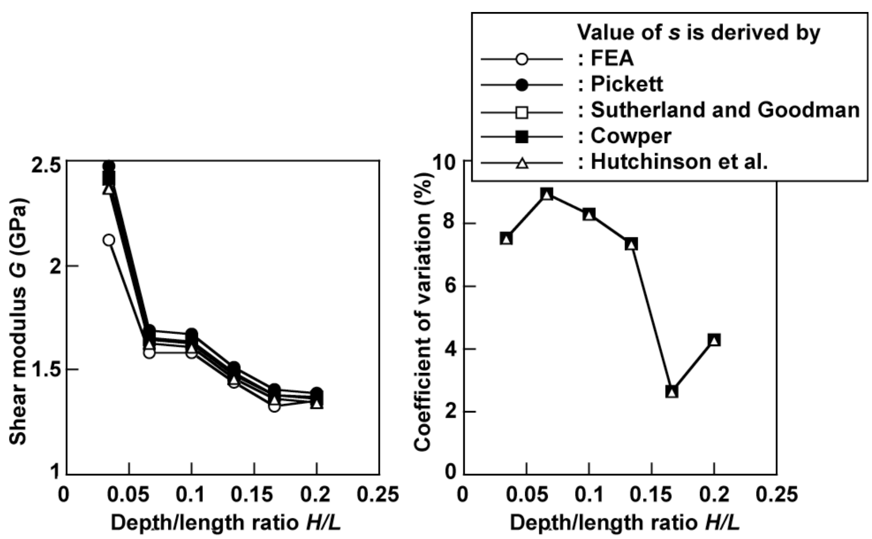

Fig. 9. Comparisons of shear modulus (*G*) and coefficient of variation obtained by the flexural vibration tests. The Timoshenko's shear factors (*s*) shown in Table 1 are used in the calculation.

When the material used is supposed to be entirely isotropic, the *G* value can be obtained by Eq. (2). By substituting the *E* value obtained by the flexural vibration test and ν value static tension tests (= 0.275) into Eq. (2), the *G* value was derived as 1.186 ±

0.033 GPa. This value is significantly smaller than those obtained by the flexural vibration method at the significance level of 0.01. This discrepancy may be because of the slight anisotropy of the MDF used in this experiment.

Several major standards determine the static bending test methods of solid wood and wood-based materials (ASTM D 143-94; ASTM D 198-05; ASTM D 3043-00; BS EN 310-1993; ISO 3349-1975; JIS 2101-2009; JAS 233-2003). In these standards, the span length must be at least 14 times larger than the depth (BS EN 310-1993). Although the flexural vibration test method for wood and wood-based materials is not specified in these standards, it is reasonable that the flexural vibration test of MDF should be conducted using specimens with the configuration these standards require for the static bending test. Under these conditions, however, the in-plane shear modulus may be inaccurate. To measure the in-plane shear modulus of MDF accurately, the flexural vibration test should not be conducted using the slender specimen described above. A statistical analysis reveals that the difference between the G values was not significant when the depth exceeded 40 mm for specimens with lengths of 7.5 times the depth. Therefore, the specimen length should be smaller than 7.5 times the depth to accurately measure the in-plane shear modulus of MDF based on Timoshenko's vibration theory.

The in-plane shear modulus of MDF can be obtained by the flexural vibration tests when using specimens with lengths less than 7.5 times the depth. Although MDF can be regarded as an in-plane quasi-isotropic material, the mechanical properties may vary according to the fabricating processes (Kazemi Najafi et al. 2005; 2007; Wilczyńscki and Kociszewski 2007). Therefore, further research is needed to measure the elastic properties of various kinds of MDFs fabricated under various conditions via vibration tests.

CONCLUSIONS

Medium-density fiberboard (MDF) was tested to determine the Young's modulus and the shear modulus. The free-free flexural vibration test method was analyzed experimentally and numerically. The conclusions are summarized as follows:

1. The experimental and FEA results indicate that valid Young's modulus values can be obtained by the flexural vibration method.
2. The FEA results suggest that the shear modulus cannot be obtained without considering the dependence of Timoshenko's shear factor (s) on the specimen configuration and the Poisson's ratio.
3. The experimental results indicate that the influence of the specimen configuration on s is more significant than that of the Poisson's ratio; and the length of the specimen should be less than 7.5 times the depth.

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