Application of Mechanical Models to Flax Fiber /Wood Fiber/ Plastic Composites

Yan Cao, Weihong Wang,* Qingwen Wang, and Haigang Wang

Bio-fibers have been used for some time to reinforce thermoplastic composites; such structures are being used in a variety of commercial applications. In this study, wood fiber and flax fiber were used to reinforce high-density polyethylene (HDPE) formed by extrusion. The flexural, tensile, and impact resistance properties of the resulting flax fiber/wood fiber/HDPE (F/W/HDPE) composites were measured and modeled as a function of the volume fraction of flax fiber. Finally, the correctness of the modified model was verified. Based on the measurement data, the volume fraction of flax fiber was shown to play an important role in determining the mechanical properties of these composites. With increasing flax fiber volume fraction, the flexural strength, tensile strength, tensile modulus of elasticity, and impact resistance of the composites generally increased. However, the flexural modulus decreased. Based on the rule of mixtures (ROM) model, two coefficients were introduced and a new curve-fitting model was established based on measurements of macrostructure. Compared with the traditional ROM model, the new model developed in the present study could describe the flexural strength, tensile modulus, and impact strength of F/W/HDPE composites more accurately.

Keywords: Flax fiber; Wood fiber; HDPE; Composite; Mechanical model

*Contact information: Key Laboratory of Bio-based Material Science & Technology of Ministry of Education, Northeast Forestry University, Harbin, 150040, China; *Corresponding author: weihongwang2001@yahoo.com.cn and qwwang@nefu.edu.cn*

INTRODUCTION

Natural-fiber/polymer composites have been increasingly and successfully used in a variety of engineering, aerospace, and automotive applications since the 1960's because of their excellent characteristics such as abundant supply, low cost, good durability, and absence of health hazards (Gibson 2007; Migneault *et al*. 2010). Wood fiber is the most commonly used raw material for reinforcing thermoplastics. Additionally, other non-wood fibers, such as flax, are also being considered as potential sources of material to reinforce polymers.

Flax fiber is an important bast fiber and comprises bundled "ultimate cells," each containing spirally-oriented microfibrils bound together, similar to other natural fibers such as hemp, jute, and sisal (Doan *et al*. 2006). Compared with glass fiber and carbon fiber, which are often used to reinforce polymers, flax fiber has traditionally been associated with a lower cost. However, it has other attributes, including a low density, excellent tensility and toughness, low abrasiveness, the absence of buildup of static electricity, high-temperature durability, rapid heat dissipation, resistance to acids and alkalis, high crystallinity, biodegradability, and ease of recycling and treatment (Facca *et al*. 2006). Most of all, it is produced from a renewable resource. Thus, the use of flax fiber as a reinforcement material has attracted increasing attention from researchers. Flax-fiber-reinforced composites have the potential to provide an alternative to glass-fiber-reinforced plastics in some technological applications (Kaith *et al*. 2008). However, limited publications about flax-reinforced thermoplastic have been found.

Garkhail *et al.* (2000) studied the influence of fiber length and diameter and fiber content on the stiffness, tensile strength, and impact strength of flax/polypropylene (PP) composites. Their results indicated that future research toward significant improvement in the tensile and impact strength of these thermoplastic composites should focus on the optimization of the fiber strength rather than the interfacial bond strength. Van de Velde and Kiekens (2003) studied unidirectional and multidirectional flax/PP composites (containing flax with varying degrees of retting and boiled flax) and the effect of process parameters. Their results indicated that unidirectional composites containing boiled flax showed the best mechanical properties. Arbelaiz *et al.* (2005a,b) treated short flax fiber using chemicals such as maleic anhydride, vinyltrimethoxysilane, and maleic anhydride–polypropylene copolymer. Their results showed that mechanical recycling of flax fiber bundle/PP composites was feasible.

Unlike the case for some other thermo-set resin composites, mathematical models have not been universally used in thermoplastic composites studies. Modniks and Andersons (2010) evaluated the mechanical properties of short-flax-fiber-reinforced PP composites using both measurements and mechanical models reflecting the principal morphological features of the fibers. Their FEM models showed good agreement with the experimentally determined stiffness.

Considering the variability and complexity of the microstructure of fiber-reinforced polymer composites, a mathematical model needs to be as simple as possible to be a useful tool. At the same time, the model needs to be sufficiently accurate to describe and understand the physics of the phenomena associated with the components. Some commonly used models for mechanical properties are the rule of mixtures (ROM), the inverse rule of mixtures (IROM), the Hirsch model, the Cox model, the Halpin–Tsai model, and the Kelly–Tyson model. In addition, the Bowyer–Bader model, the Mori–Tanaka model, the composite cylinder assemblage (CCA) model, the Levin model, the shear-lag model, the Hashin–Rosen model, the Weibull distribution model, and others have also been used in research work (Kelly and Tyson 1965; Hill 1965; Takao *et al*. 1982; Chen and Cheng 1996; Hashin and Rosen 1964; Hashin 1979; Levin 1967; Nairn 1997; Mendels *et al*. 1999). These models have been proposed in order to model the mathematical properties of composite materials in terms of various parameters (Kalaprasad *et al*. 1997).

The ROM is a very simple model that is the result of Voigt's assumption, in which both the matrix and fiber experience the same strain, and it uses a sum of volume-weighted properties of the fibers and matrix to predict the properties of the composite (Migneault *et al*. 2010; Facca *et al*. 2006). The IROM model is based on Reuss's assumption, in which the applied transverse stress is equal in both the fiber and the matrix (Fukuda and Chou 1981; Kalaprasad *et al*. 1997; Facca *et al*. 2006).

$$\text{ROM model: } \begin{cases} \sigma_c = \alpha\sigma_f V_f + \sigma_m V_m \\ E_c = \alpha E_f V_f + E_m V_m \end{cases} ; \text{ IROM model: } \begin{cases} \sigma_c = \dfrac{\sigma_f \sigma_m}{\sigma_f V_m + \sigma_m V_f} \\ E_c = \dfrac{E_f E_m}{E_f V_m + E_m V_f} \end{cases} \quad (1)$$

where V is the volume fraction, σ is the strength, and E is the elastic modulus. The subscripts "f", "m", and "c" denote the fiber, matrix, and composite, respectively. In Eq. 1, the ROM model for E_c is also correct only for continuous fibers. The parameter α is the fiber orientation factor. For synthetic fibers, it has been reported that a three-dimensional random fiber alignment yields an α value of 1/5, planar randomly oriented fibers yield a value of 3/8, and fibers aligned axially with the load yield a value of 1 (Bailon and Dorlot 2000; Gibson 2007). An α value of 3/8 was recently observed for natural fibers (Beckermann and Pickering 2009). For unidirectional continuous fiber composites, under Voigt's assumption of equality in the failure strain in the fibers and matrix, the ROM becomes,

$$\sigma_c = \sigma_f V_f + \sigma_m (1 - V_f) \tag{2}$$

where σ_c and σ_f are the ultimate strengths of the composite and the continuous fibers, respectively, and σ_m is the matrix stress at failure of the composite.

In order to analyze the strength of composite materials containing randomly oriented short SiC fibers, Zhu *et al.* (1994) assumed that the fibers were uniformly distributed and randomly oriented in three dimensions, and applied and developed a modified ROM theory. Compared with previous ROM models, Zhu's modified ROM gave better agreement with experimental results, and it could be used to explain some experimental phenomena. In the same year, Rangaraj and Bhaduri (1994) modified the ROM for prediction of the tensile strength of unidirectional Kevlar/epoxy composites. Furthermore, they pointed out that "A recent modification to the classic ROM to account for fibre-fibre interaction has already resulted in good agreement between measured and predicted values of ultimate tensile strengths at high and low fibre volume fractions for Kevlar/epoxy composites". Doan *et al*. (2006) also modified the ROM theory to study the effect of maleic-anhydride-grafted polypropylene coupling agents on the tensile strength of jute fiber/PP composites. The results showed that the modification fitted well to their experimental data.

The ROM and IROM models are typically treated as upper and lower bounds on modulus, leading to the blend of the two in the Hirsch model (Fukuda and Chou 1981; Kalaprasad *et al*. 1997; Facca *et al*. 2006). The Hirsch model is a combination of a parallel and a series ROM. According to this model, the Young's modulus (or tensile modulus) and the strength are given by,

$$\begin{cases} \sigma_c = \alpha\left(\sigma_f V_f + \sigma_m V_m\right) + \left(1\text{-}\alpha\right)\dfrac{\sigma_f \sigma_m}{\sigma_f V_m + \sigma_m V_f} \\ E_c = \alpha\left(E_f V_f + E_m V_m\right) + \left(1\text{-}\alpha\right)\dfrac{E_f E_m}{E_f V_m + E_m V_f} \end{cases} \tag{3}$$

where α is a parameter that describes the stress transfer between the fibers and the matrix; this is a determining factor in describing the real behavior of short-fiber composites. Kalaprasad *et al*. (1997) reported that agreement between the theoretical and experimental values could only be obtained for longitudinally-oriented composites when the assumed value of α was 0.4.

Literature has shown the application of these models to the single fiber reinforced composites. In order to improve the properties or reduce the cost of composites, hybrid fibers are often used in reinforcement. However, very limited publications present a model for hybrid fibers reinforcing composite. On the other hand, for short fibers, the modulus would be further reduced by the imperfect stress transfer between fiber and matrix. Thus, it is necessary to modify models. The objective of the present work was (1) to study the mechanical properties of flax fiber/wood fiber/HDPE composites, and (2) to establish a mechanical model suitable for describing the resulting composites. This work will benefit the development of hybrid natural-fiber/plastic composites.

EXPERIMENTAL

Materials

Poplar (*Populus alba*) wood fiber was purchased from a local market; it passed through a 30-mesh sieve but was retained on a 60-mesh sieve. Flax fibers, used as a reinforcing fiber in the experiments, were obtained from local sources and then cut into a length of 20 to 30 mm. High-density polyethylene (HDPE) (5000S) granules, used as matrix material, were provided by the Daqing Petrochemical Company, China. This material had a solid density of 954 kg/m^3, a melting flow index of 0.35 g/10 min (measured at 230 °C and 2.16 kg according to ISO 1133), a flexural strength of 24 MPa, and a tensile strength of 29 MPa. Maleated polyethylene (MAPE) of type MAHgPE CMG 9804 was used as a compatibilizer, and wax was used as a process lubricant. These two reagents were produced by Tianjin Bochen Co. Ltd., China.

Methods

In the formula of the composite, part of the wood fiber was replaced by flax fiber. The ratios of wood fiber to flax fiber were chosen as 0:6, 1:5, 2:4, 3:3, 4:2, 5:1, and 6:0, as shown in Table 1. The volume ratios were obtained by dividing the mass ratios by the density ratio. The flax fiber, wood fiber, HDPE, and other additives were mixed with rotation in a high-speed mixer at 120 °C. The mixture was melted and compounded in an SJSH 30 mm conical counterrotating twin-screw extruder, and then fed into an SJ 45 mm conical counterrotating single-screw extruder to extrude samples of composite lumber. A constant screw speed of 100 rpm was chosen to limit fiber breakage. The temperature was maintained at 150 to 175 °C during lumber extrusion. Flax fiber/ wood fiber/HDPE (F/W/HDPE) composite lumber with a cross section of 40 mm × 4 mm was extruded.

Specimens were cut from the above lumber for testing. Rectangular specimens of dimensions 80 mm × 13 mm × 4 mm (length × width × thickness) were used for flexural tests, dumbbell-shaped specimens of (measuring 165 mm in length, 20 mm in width at the ends, 12.7 mm in width in the narrowest portion, and 4 mm in thickness) for tensile tests, and rectangular specimens of dimensions 60 mm × 10 mm × 4 mm for impact resistance tests. The flexural properties were measured according to ASTM D 790-03. The tests were performed at a speed of 1.9 mm/min with a span of 64 mm. The tensile properties were measured according to ASTM D-638. The tensile testing speed was 5 mm/min. Unnotched Izod impact tests were performed at a testing speed of 2.9 m/s on samples with a span of 60 mm. The impact energy was 2 J. All measurements were performed under ambient conditions. Ten replicates were run for each of the seven composite formulations.

Table 1. Formulations of F/W/HDPE Composites

Test	Mass Part of Component				Flax volume fraction in mixture fibers (%)
	HDPE	Flax Fiber	Wood fiber	MAPE	
1	3.6	0	6.0	0.4	0
2	3.6	1.0	5.0	0.4	24.8
3	3.6	2.0	4.0	0.4	45.2
4	3.6	3.0	3.0	0.4	62.3
5	3.6	4.0	2.0	0.4	76.8
6	3.6	5.0	1.0	0.4	89.2
7	3.6	6.0	0	0.4	100

RESULTS AND DISCUSSION

Effects of Flax Fiber Content on Mechanical Properties

The flexural, tensile, and impact resistance properties of the composites were measured, and the mean values and standard deviations of results are listed in Table 2.

Table 2. Mechanical Properties of F/W/HDPE Composites

Test	Flax volume Fraction in Mixture Fibers (%)	Flexure Test		Tensile Test		Impact Test
		Strength σ_F(MPa)	Modulus E_F(GPa)	Strength σ_T(MPa)	Modulus E_T(GPa)	Strength σ_I(KJ/m^2)
1	0	55.17 (0.73)	4.32 (0.12)	31.84 (0.74)	4.11 (0.73)	8.43 (1.25)
2	24.8	55.66 (0.99)	4.31 (0.13)	32.51 (0.90)	4.04 (0.14)	8.85 (1.11)
3	45.2	55.81 (0.94)	3.89 (0.20)	34.03 (1.07)	4.09 (0.44)	9.02 (1.13)
4	62.3	55.89 (0.66)	3.74 (0.10)	-	-	13.00 (1.21)
5	76.8	56.35 (1.09)	3.50 (0.11)	35.23 (0.58)	4.23 (0.34)	13.39 (1.19)
6	89.2	57.03 (1.12)	3.44 (0.13)	36.12 (0.88)	4.33 (0.33)	16.79 (1.22)
7	100	58.07 (0.85)	3.39 (0.21)	36.41 (0.95)	4.68 (0.72)	15.90 (1.20)

Flexural properties

As shown in Fig. 1, Fig. 2, and Table 2, as the volume fraction of flax fiber increased, the flexural strength of the F/W/HDPE composite improved slightly. When the flax fraction in the fiber mixture ranged from 24.8% to 62.3%, the composite had the same flexural strength. When the flax volume fraction was further increased to 89.2–100%, the flexural strength improved by only 3.05%, which is not significant. However, the flax fiber content had more effect on the flexural modulus than on the flexural strength. With increasing flax fiber content, the flexural modulus changed by a greater proportion (21.53%) than did the flexural strength (5.26%).

Flax fiber is soft, and no flexural modulus could be detected. However, wood has excellent flexural strength and modulus of 90 MPa and 12.5 GPa, respectively. When the amount of flax fiber was larger than that of wood fiber, the flexural modulus of reinforced composite depended mainly on flax and decreased.

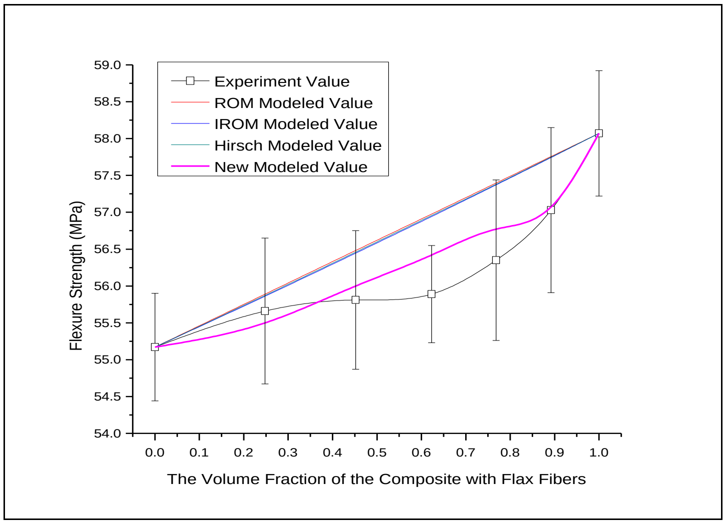

Fig. 1. Measured and modeled (ROM model, IROM model, Hirsch model, and new modified ROM model) flexural strength of F/W/HDPE composites

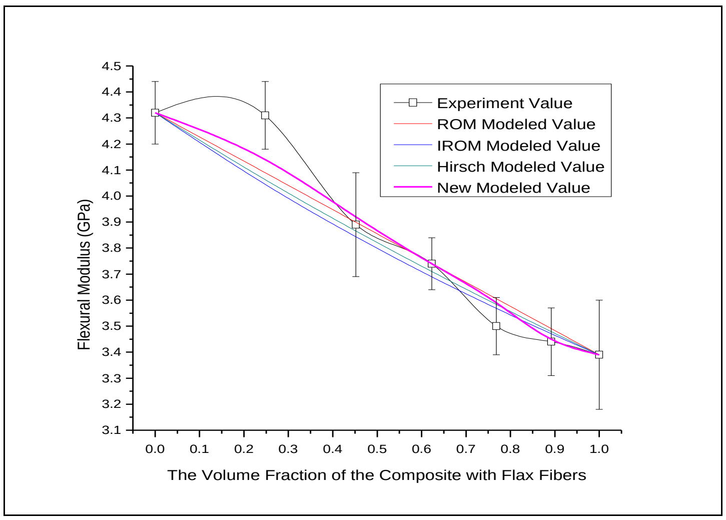

Fig. 2. Measured and modeled (ROM model, IROM model, Hirsch model, and new modified ROM model) flexural modulus of F/W/HDPE composites

Tensile properties

For the tensile strength, the composites showed a significant, almost linear improvement, from 31.84 MPa for the composite without flax fiber to 36.41 MPa for the composite containing 100% flax fiber (Fig. 3). The tensile modulus increased slightly as the flax fiber content was increased to 89.2%. When the flax fiber volume fraction exceeded 89.2%, the tensile modulus of the composite showed an obvious improvement. Compared with the pure wood fiber/HDPE composite, the flax fiber/HDPE composite showed a 13.87% increment in tensile modulus (Fig. 4).

In contrast to the flexural properties, where there was a stable strength and decreased modulus, the tensile properties showed a significant increase in strength and modulus. This is attributed to the excellent tensile strength of flax fiber. Flax fiber possesses a tensile strength of about 650 MPa and tensile modulus of about 20 GPa. By comparison, the average tensile strength and modulus of wood are only about 164 MPa and 2 GPa, respectively.

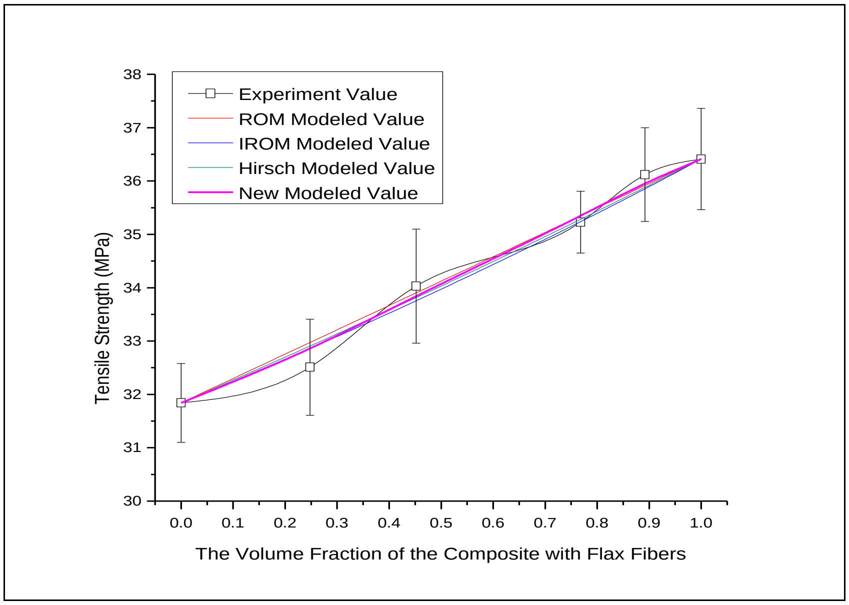

Fig. 3. Measured and modeled (ROM model, IROM model, Hirsch model, and new modified ROM model) tensile strength of F/W/HDPE composites

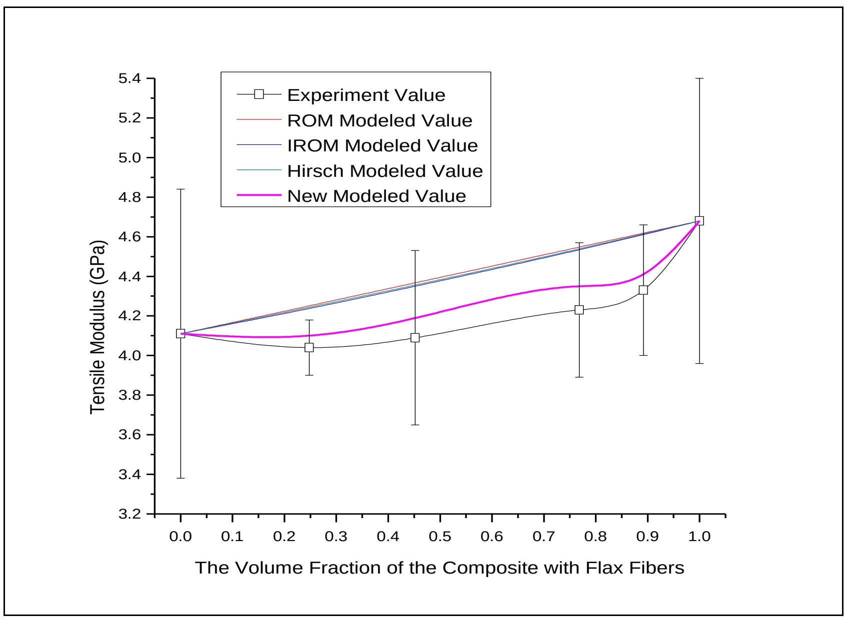

Fig. 4. Measured and modeled (ROM model, IROM model, Hirsch model, and new modified ROM model) tensile modulus of F/W/HDPE composites

Impact resistance properties

It can be seen from Table 2 and Fig. 5 that the impact strength of the composite increased considerably with increasing flax fiber content. The composite that was reinforced solely with flax fibers exhibited an impact strength of 15.90 kJ/m^2, which was 7.47 kJ/m^2 higher than that of the wood-flour-reinforced HDPE composite. This result was expected because flax fiber has a higher yield fracture stress and toughness than wood fiber. The influence of fiber content on the impact property was more significant than that on the flexural and tensile properties.

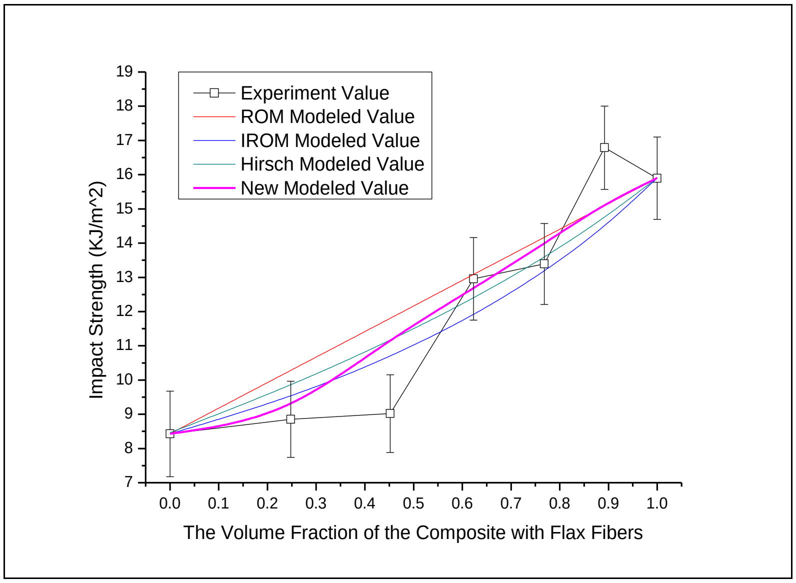

Fig. 5. Measured and modeled (ROM model, IROM model, Hirsch model, and new modified ROM model) impact strength of F/W/HDPE composites

Comparison between Modeled and Measured Values

In manufacturing, the proportions of wood fiber and other components were calculated and measured on the basis of mass for convenience. However, volume fractions are required to apply micromechanical models. The volume fraction was obtained from the following Equation 4, where *M* is the mass fraction, *V* is the volume fraction, and ρ is the density; the subscripts "fc", "wc", and "fwc" denote the composite reinforced only with flax fiber, composite reinforced only with wood fiber, and composite reinforced with flax fiber-wood fiber, respectively:

$$\begin{gathered} M_{fc} + M_{wc} = M_{fwc} = 1 \\ V_{fc} + V_{wc} = V_{fwc} = 1 \\ M_{fc} = \frac{M_{fc}}{M_{fwc}} = \frac{\rho_{fc} V_{fc}}{\rho_{fwc} V_{fwc}} \end{gathered} \tag{4}$$

Based on the characteristics of the material, we developed a new modified ROM model (see Eq. (5) below) to fit the properties of flax fiber/wood fiber dual-reinforced composites. For this model it was assumed that the W/F/HDPE composite consists of two components, wood fiber-reinforced HDPE composite and flax fiber-reinforced HDPE composites, and the two components were mixed in various proportions. All calculations were performed assuming that the properties of the HDPE were unchanged after the formation of the composite (Migneault *et al*. 2010) and that both the matrix and short fiber experience the identical failure strain (Voigt's assumption).

$$\begin{cases} \sigma_{fwc} = \lambda \sigma_{fc} V_{fc} + (1\text{-}\lambda) \sigma_{wc} V_{wc} + \varepsilon \left(\dfrac{\sigma_{fc} + \sigma_{wc}}{2} \right) \\ E_{fwc} = \lambda' E_{fc} V_{fc} + (1\text{-}\lambda') E_{wc} V_{wc} + \varepsilon' \left(\dfrac{E_{fc} + E_{wc}}{2} \right) \end{cases} \quad (5)$$

where σ is the strength and E is the modulus. The volume fraction of flax fiber was taken as 1 when the F/W/HDPE composite contained solely flax fibers, without wood fiber (V_{fc}=1, V_{wc}=0). The volume fraction of flax fiber was taken as 0 when the composite contained only wood fiber and no flax fiber (V_{fc}=0, V_{wc}=1).

In this model, two coefficients λ and ε are introduced. The λ, we call it contribution factor, represents the degree of influences of distinct components on mechanical properties of resulting composites. In other words, λ shows the contribution of flax fiber to the strength of composites, and it is independent of the volume fraction of flax fiber, while $(1 - \lambda)$ shows the contribution of wood fiber to the strength of composites, and it is also independent of the volume fraction of wood fiber. In the same way, λ' shows the contribution of flax fiber to the modulus of composites, while $(1 - \lambda')$ shows the contribution of wood fiber to the modulus of composites. The additional contribution, ε and ε' can be considered as two adjustable parameters to fit the data. The physical meaning of the model is that the mechanical properties of hybrid fibers-reinforced composite goes up and down around the average value of flax fiber-reinforced composite and wood fiber-reinforced composite ($\left(\dfrac{\sigma_{fc} + \sigma_{wc}}{2} \right)$ or $\left(\dfrac{E_{fc} + E_{wc}}{2} \right)$).

Based on measured data under various proportions, λ and ε could be optimized by Origin software (OriginPro 7.5, Fit Polynomial Regression Analysis, NLSF, and Myfitfunction) and listed in Table 3. For flexural property and tensile property, λ and λ' ranged from 0.4217 to 0.5368; ε and ε' ranged from 0.4582 to 0.4959. However, for impact strength of the composite, ε was the lowest (ε = 0.3845) and λ was the highest (λ = 0.7161). This means flax fiber-reinforced composite provided significantly higher impact strength than wood fiber-reinforced composite. The main reason is that flax fiber contributed much more toughness than wood fiber. Impact strength depends more on the proportion of flax fiber. Its amplitude of fluctuations around the average is smaller. Thus, flax fiber gave F/W/HDPE composite a high contribution factor λ and a small adjustable parameter ε in impact strength.

Table 3. Introduced Coefficients λ and ε Values in New Model

Mechanical Properties		λ	ε
Flexure Properties	Flexural Strength	0.5088	0.4908
	Flexural Modulus	0.4217	0.4950
Tensile Properties	Tensile Strength	0.5368	0.4959
	Tensile Modulus	0.5227	0.4582
Impact Property	Impact Strength	0.7161	0.3845

The sum squared error (SSE), also called the "sum of squared error within-subject factors" and the "sum of squared residuals," is the sum of the squared differences between the experimentally determined values $\sigma_{\text{experiment}}$ and the estimated values σ_{model}. The SSE was calculated as,

$$SSE = \sum_{i=1}^{i=7} (\sigma^{i}_{model} - \sigma^{i}_{experiment})^2 \quad (6)$$

where i is the number of flax fiber content levels in the composites studied. It reflects discrete information about the observed values for each specimen. A lower SSE represents a better and more accurate model.

The mechanical properties (flexural strength and modulus, tensile strength and modulus, and impact strength) of the F/W/HDPE composites were calculated using established models (ROM, IROM, and Hirsch models) and the new modified ROM model, and are compared with the actual measured values in Figs. 1 to 5. These results correspond to the SSE values shown in Table 4. For flexural modulus and tensile strength of F/W/HDPE composites, SSE value of traditional ROM models and the new model were similar. This means both the traditional ROM models and the new model are good at fitting to flexural modulus and tensile strength. However, for flexural strength, tensile modulus, and impact strength, SSE value of the new model is obviously smaller than that of the traditional ROM models. It is presumed that the new modified ROM model is more appropriate for describing HDPE composites containing a mixture of flax fiber and wood.

Table 4. SSE Value of ROM, IROM, Hirsch, and New Modified ROM Model for F/W/HDPE Composites

Mechanical Properties		ROM	IROM	Hirsch	New modified ROM model
Flexural Properties	σ_F (MPa)	3.308	3.098	3.181	0.515
	E_F (GPa)	0.063	0.081	0.071	0.037
Tensile Properties	σ_T (MPa)	0.286	0.271	0.265	0.202
	E_T (GPa)	0.306	0.278	0.289	0.034
Impact Strength	σ_I (KJ/m^2)	13.306	9.713	10.343	8.050
* The value of α is 1 in ROM model, and in Hirsch model the value of α is 0.4 according to previous experience (Kalaprasad *et al.* 1997)					

Table 5. SSE Value of ROM, IROM, Hirsch, and New Modified ROM Model for F/W/PP Composites

Mechanical Properties		ROM	IROM	Hirsch	New modified ROM model
Flexural Properties	σ_F (MPa)	1.519	1.523	1.521	1.492
	E_F (GPa)	0.730	0.930	0.847	0.081
Tensile Properties	σ_T (MPa)	3.727	3.887	3.823	0.026
	E_T (GPa)	2.629	2.660	2.647	0.415
Impact Strength	σ_I (KJ/m^2)	5.350	5.347	5.349	3.897
* The value of α is 1 in ROM model, and in Hirsch model the value of α is 0.4 according to previous experience (Kalaprasad *et al.* 1997)					

To verify the new model, polypropylene (PP) composites reinforced with flax fiber /wood fiber were tested. The mechanical properties (flexural strength and modulus, tensile strength and modulus, and impact strength) of F/W/PP composites were calculated using ROM, IROM, Hirsch, and the new modified ROM models. For our new modified model, the values used for λ and ε are listed in Table 3. In comparison with the actual measured values, the new model gave the least SSE. This indicated that the new modified ROM model represented the best adaptation to describe F/W/PP composites (Table 5). It is assumed that the mechanical properties of any thermoplastic-based composite reinforced with flax fiber and wood fiber in various proportion could be calculated with our new modified models. The parameters λ and ε can take the value listed in Table 3.

Future research may be suggested to address concerns about statistical justification for use of the new model presented here (Eq. 5). On the one hand, it is unsurprising that a model having a larger number of fitting parameters has been able to achieve reduced SSE values (Table 4). On the other hand, if a data set is sufficiently large and well designed, as well has having sufficient precision of replication, then it should be possible to justify a model having a greater number of terms. Future researchers are cautioned that the new model presented here still needs further verification before it can be applied to new situations.

CONCLUSIONS

1. Adding flax fiber to a wood fiber/HDPE composite influenced the mechanical properties of the resulting flax fiber/wood fiber/HDPE composite. This influence was most significant on impact strength than on the other properties. Volume fraction of flax fiber almost did not affect flexural strength.
2. As the volume fraction of flax fiber increased, the tensile strength, tensile modulus, and impact strength of the composite generally increased. However, the flexural modulus decreased. The flax fiber decreased the stiffness, while it increased the tensility and toughness of composites.
3. Both our new modified model and other traditional models (ROM, IROM, and Hirsch) are good at describing the flexural modulus and the tensile strength of F /W/HDPE composites. The new modified ROM model that we have developed is more appropriate for describing the flexural strength, the tensile modulus, and impact strength of F/W/HDPE composites. This is due to the introduction of two coefficients, λ and ε. The coefficient λ represents the contribution proportion of flax fiber to the corresponding flax fiber/HDPE composite, and it is independent of the volume fraction of flax fiber. The physical meaning of ε is the fluctuations around the average mechanical properties value of flax fiber reinforced composite and wood fiber reinforced composite ($\left(\frac{\sigma_{fc}+\sigma_{wc}}{2}\right)$ or $\left(\frac{E_{fc}+E_{wc}}{2}\right)$).
4. Based on the SSE, it can be concluded that the new modified ROM model is more appropriate for predicting the mechanical properties (especially the tensile modulus) of F/W/HDPE composites and F/W/PP composites.

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