Reliability Analysis for the Bending Strength of Larch 2×4 Lumber

Yong Zhong [a, b] and Haiqing Ren [a, b,*]

A safety analysis for Larch (*Larix gmelinii*) 2×4 lumber, based on the requirements of Chinese national standards, is presented in this paper. Static, third-point bending tests were conducted using an MTS universal testing machine. The bending strength was adjusted to 15% moisture content (MOR_{15}) and was fitted with three different probability distribution models. The best fitting MOR_{15} model for Larch 2×4 lumber possessed lognormal distribution. Furthermore, a procedure for calculation of a reliability index *(β*) was developed using the first-order second-moment method. Results of reliability analysis indicated that the reliability index decreased nonlinearly as both the design value of bending strength increased and the reduction of live to dead load ratio *(ρ*). Finally, it was suggested that the design values for the bending strength of Larch 2×4 lumber were 18.4 MPa for grade SS, 11.6 MPa for grade No.1, 12.7 MPa for grade No.2, and 10.3 MPa for grade No.3, to meet the target reliability level.

Keywords: Larch; Lumber; Bending strength; Reliability index; Design value

*Contact information: a: Research Institute of Wood Industry, Chinese Academy of Forestry, Beijing, 100090, China; b: Research Institute of Forestry New Technology, Chinese Academy of Forestry, Beijing, 100090, China; *Corresponding author: renhq_wood@163.com*

INTRODUCTION

With the increasingly rapid development of light wood frame construction in China, more and more dimension lumber products are being imported from other countries. Specifications regarding the timber structure design are different for each country; moreover, the design value for the bending strength of dimension lumber with equal grade is also different, due to differing national economic progress and finance tolerance. There are two common methods to determine the design value of the bending strength. One is the analysis method of allowable properties, which has been adopted by the American and Canadian standards organizations, among others (ASTM D1990 2007); another is the reliability analysis method specified in China (GB50068 2001; GB50153 2008).

To ensure the safety of a timber structure design, the design value for the bending strength of dimension lumber is provided in the Chinese national standard GB 50005 (2003). For example, the design value of bending strength is 20 MPa for SS grade Southern Pine dimension lumber. However, for forest resources located at China, the design value for the MOR of dimension lumber (which bears no correlation to the grade of the dimension lumber) is only dependent upon the tree species (Pan and Zhu 2009; Zhu and Pan 2011). For instance, Larch (*Larix gmelinii*) belongs to the TC 17-A group, and its design value of the bending strength is 17 MPa, regardless of its grade.

Although China relies on imports to supply the resources required to make dimension lumber, the Chinese government, along with some research institutions, have begun to prepare for manufacturing dimension lumber using China's own forest resources. Mechanical properties for Larch dimension lumber were studied, using the full-size test method, by the Research Institute of Wood Industry, Chinese Academy of Forestry (Long and Lü 2007). The bending strength (MOR), the relationship between the MOR and modulus of elastic (MOE), and the characteristic values of MOR and MOE have been obtained (Zhao *et al*. 2009; Zhao *et al*. 2011; Lou *et al*. 2011; Zhong *et al*. 2012a,b). The design value for the bending strength of Larch, based on reliability analysis, is expected to be important for engineering applications; however, it has not been investigated.

The objectives of this study were to evaluate the effect of grade on MOR of Larch 2×4 lumber, as well as to analyze and compare the fitting degree between the normal, lognormal, and 2P-Weibull probability distribution models for MOR. The final goal of the study was to determine the design value of the bending strength, by conducting a reliability analysis, in accordance with the target reliability level in the Chinese national standards (GB50068 2001; GB50153 2008). Through the previous analysis, the safety design problem for Larch dimension lumber could potentially be solved.

EXPERIMENTAL

Materials

Larch (*Larix gmelinii*), collected from the Cuigang and Pangu forest farms of Heilongjiang Province, China, was selected for this study. The average tree age was 35 years, with the diameter of logs ranging from 160 to 340 mm. The material was cut into specimens of 2×4 lumber. According to the Chinese national standards (GB50005 2003), the single digit of dimensions for lumber was rounded to zero or five, and millimeter (mm) was used as the unit of dimensions; therefore, unlike other countries the actual dimensions of 2×4 lumber are 40×90 mm, in China. The length of dimension lumber was 4000 mm. Since there are no codes for visually grade rules about dimension lumber, the structural lumber appearance quality of the Larch lumber (such as knots, slope of grain, checks, and splits) was evaluated by the NLGA standard (NLGA 2014). Besides, the grade SS, No.1, No.2, and No.3 in NLGA standard (NLGA 2014) represented the grades Ic, IIc, IIIc, and IVc in Chinese national code (GB50005 2003), respectively. There were 429, 201, 285, and 165 pieces of lumber for grade SS, No.1, No.2, and No.3, respectively. All specimens were conditioned to equilibrium moisture content (EMC) in a standard room, which was maintained at 20 °C and 65% relative humidity (RH) before the bending test was conducted. The measured average EMC was 11.4% with a standard deviation of 1.26% (ASTM D4442 2007).

Static Testing Method

The edgewise bending test was carried out on an MTS universal testing machine, according to ASTM D198 (2009), which uses the third-point loading method (Fig. 1). The specimens were loaded at a rate of 5 mm/min and continued until failure. Furthermore, the span to depth ratio was 18 to 1 (1620 mm to 90 mm). The specimens were not placed in the test setup in a manner that maximizes strength, but rather loaded randomly, to better simulate actual use. The MOR of each specimen was calculated using Eq. 1, shown next (ASTM D198 2009).

$$\mathrm{MOR} = aF_{\max} / 2W \tag{1}$$

In the aforementioned equation, *a* represents the distance (540 mm) between a loading position and the nearest support, $F_{\max}$ is the maximum load, and *W* is the section modulus.

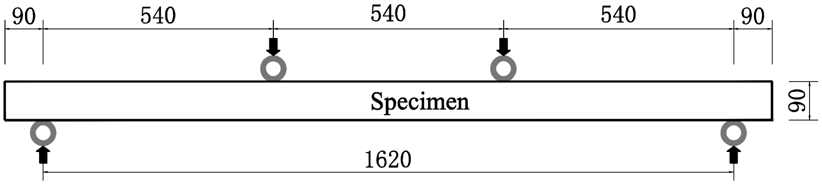


Fig. 1. Bending test for Larch 2×4 lumber (size: mm)

The bending strength for each specimen, adjusted to 15% MC (MOR_{15}), and in accordance with ASTM D1990 (2007), was calculated using Eq. 2,

$$\mathrm{MOR}_{15} = \begin{cases} \mathrm{MOR} & \mathrm{MOR} \le 16.66MPa \\ \mathrm{MOR} + (M_1 - 15) \times (\mathrm{MOR} - 16.66) / (40 - M_1) & \mathrm{MOR} > 16.66MPa \end{cases} \tag{2}$$

where M_1 is the moisture content of the specimen, expressed as a percentage.

Statistical Analysis

The statistical analysis, and the associated graphics, was performed using Origin 9. Each specimen was tested to determine the degree of fitting in terms of MOR_{15} using normal, lognormal, and 2P-Weibull distributions. The mean, median, standard deviation, and coefficient of variation (COV) were determined for the MOR_{15}. The design value of the bending strength was also calculated using the estimated lognormal distribution.

RESULTS AND DISCUSSION

Results of Bending Test

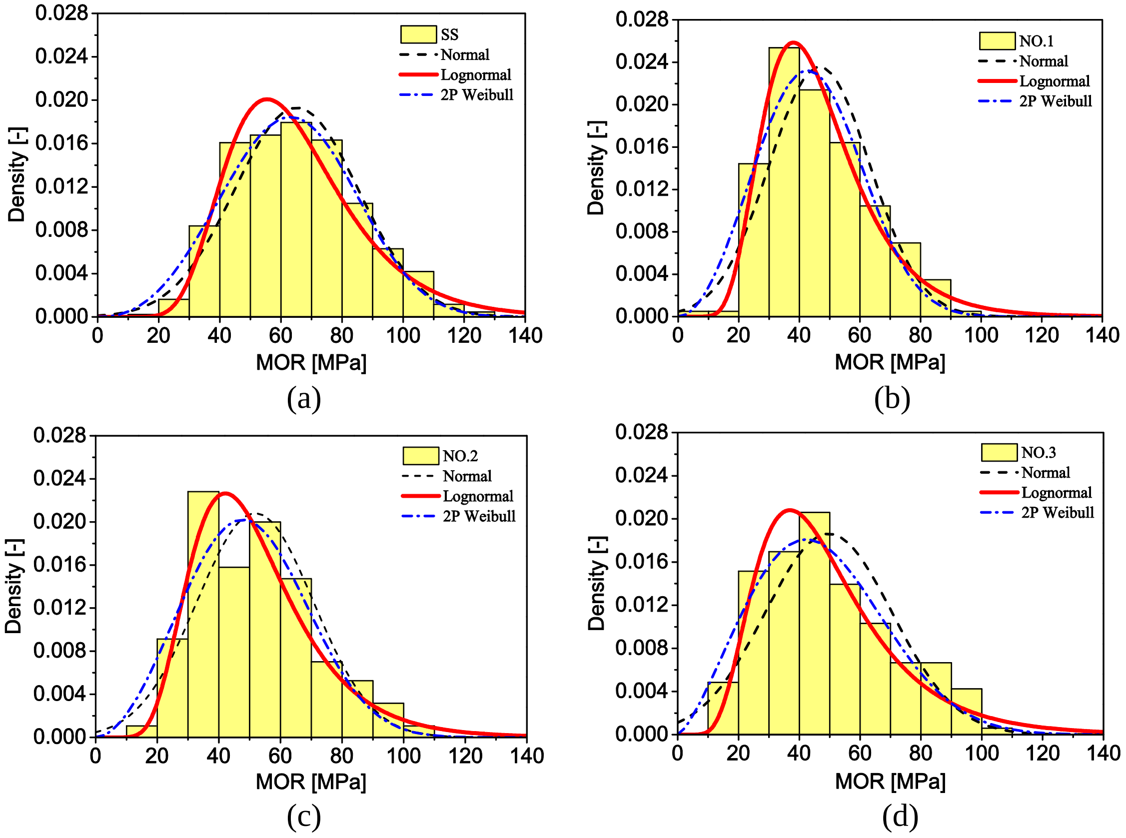
Results for the mean value of MOR_{15}, median value, standard deviation, and COV of different grades for Larch 2×4 lumber are shown in Table 1.

Table 1. Summary of Statistics for Bending Strength Adjusted to 15% Moisture Content (MOR_{15}) of Larch 2×4 Lumber

Statistical parameters	MOR_{15} (MPa)			
	SS	NO.1	NO.2	NO.3
Mean value	65.12	46.69	51.93	49.65
Median	63.94	44.21	50.77	46.41
Standard deviation	20.34	16.65	18.88	20.90
COV (%)	31.24	35.65	36.36	42.10

The mean value of MOR_{15} for SS grade, which was larger than that for other grades, was 65.117 MPa. Grade SS showed less variation in MOR_{15}, possessing a COV of 31.235%. Although grade No.1 had the lowest mean value of MOR_{15} (46.693 MPa), the COV (35.648%) of grade No.1 was lower than that of grades No. 2 and 3.

The normal, lognormal, and 2P-Weibull fitting MOR_{15} for Larch 2×4 lumber are shown in Fig. 2. Greater differences between the normal and lognormal distributions were found in MOR_{15}, where the lognormal distribution fitted the data more adequately for both grades. This is particularly evident in the left tail region for both grades, where the lognormal data followed the actual data much more accurately than the normal distribution, which is consistent with numerous findings (Zhao *et al*. 2009; Dahlen *et al.* 2012). The lognormal distribution fit the data slightly better than the 2P-Weibull distribution in MOR_{15} for both grades, with the exception being SS grade. The distribution finding is similar to that of the study Dahlen *et al.* (2012) conducted on Douglas-fir and southern pine No.2 2×4 lumber. Therefore, the lognormal distribution for MOR_{15} was selected for the reliability analysis of design value in this paper.



(a) (b)

(c) (d)

Fig. 2. Normal, lognormal, and 2P-Weibull best-fit modulus of rupture adjusted to 15% moisture content (MOR_{15}) for Larch 2×4 lumber

Reliability Analysis

Due to a lack of previous full size test data, wood member resistance is based on the material strength of small clear wood specimen test results, in accordance with the Chinese National Standard GB 50005 (GB 50005 2003). There is significant difference in the mechanical properties between small clear specimens and full sized members (Zhu and Pan 2011; Zhong *et al*. 2012a,b). Considering differences in the accuracy and reliability, some countries have new regulations to determine the design value for the bending strength of dimension lumber through full size tests instead of small specimen tests (AF&PA 1997; CSA 1994). In this paper, the resistance stress of Larch 2 x 4 lumber, based on full size test results, was calculated using Eqs. 3 and 4 (GB 50068 2001; Wang 2002). The statistical parameters of the adjusting factor are shown in Table 2.

$$R = K_P K_A K_L f_0 \tag{3}$$

$$\delta_R = \sqrt{\delta_P^2 + \delta_A^2 + \delta_L^2 + \delta_0^2} \tag{4}$$

In the equations above, R is the resistance stress of Larch 2×4 lumber; K_P, K_A, K_L, are adjusting factors for the equation precision, the geometric characters and the effect of long-term load on wood. Furthermore, f_0 is the bending strength of full size wood, and its mean and COV is equal to the above-mentioned MOR_{15} of Larch 2×4 lumber in Table 1. Additionally, δ_R is the COV of R of Larch 2×4 lumber and the mean value and standard deviation of R are shown in Table 3.

Table 2. Statistical Parameters of Adjusting Factors

Statistical parameters	K_P	K_A	K_L
Mean value	1.00	0.94	0.72
COV (%)	5	8	12

Table 3. Summary of Statistics for the Resistance Stress (R) of Larch 2×4 Lumber

Statistical parameters	R (MPa)			
	SS	NO.1	NO.2	NO.3
Mean	44.07	31.60	35.15	33.61
Standard deviation	15.32	12.26	13.86	15.05
COV (%)	34.8	38.8	39.4	44.8

The loads applied to the timber structure were divided into two groups: the dead load, represented as G, which includes the weights of structural members and other materials, and the live load, which includes the office occupancy load (L_O), residential occupancy load (L_R), wind load (W), and snow load (S). According to Chinese National Standard GB 50009 (2012), dead load data was fitted to the normal distribution, and all the live load data were fitted to the Extreme type I. The statistical parameters of the loads are shown in Table 4.

Table 4. Statistical Parameters of Loads

Statistical parameters	Load types				
	G	L_O	L_R	W	S
Mean/Nominal	1.060	0.524	0.644	1.000	1.040
COV (%)	7.0	28.8	23.3	19.0	22.0
Distribution types	Normal	Extreme-I	Extreme-I	Extreme-I	Extreme-I

Next, two load combinations, including $G+L_O$, $G+L_R$, $G+W$, and $G+S$, were used in the reliability analysis (Wang 2002; Zhuang 2004). The limit state design equation for the bending strength is expressed as,

$$(a_D D_K + a_Q Q_K) \times (6L^2 / 8bh^2) = fK_S \tag{5}$$

where a_D and a_Q are the dead load factor (1.2) and live load factor (1.4), respectively; D_K and Q_K are the nominal dead load and nominal live loads, respectively; L is the span of the beam; b is the width of the beam; h is the height of the beam; f is the design value of the bending strength; and K_S is an adjusting factor for the service life (1.0) (GB 50005 2003; GB50009 2012).

The failure function, developed to relate MOR_{15} and the effect of loads for first-order second-moment reliability analysis, is as follows (Zhuang 2004; Li 2011),

$$Z = R - (D+Q) \times (6L^2 / 8bh^2) \tag{6}$$

where R, D, and Q are random variables representing the resistance stress of *Larch* 2×4 lumber (Table 3), the dead load (G), and the live load, respectively (L_O, L_R, W, or S) as shown in Table 4. The random variable R for different grades was assumed to be lognormal, according to above analysis (Fig. 2).

The failure function can be rewritten as,

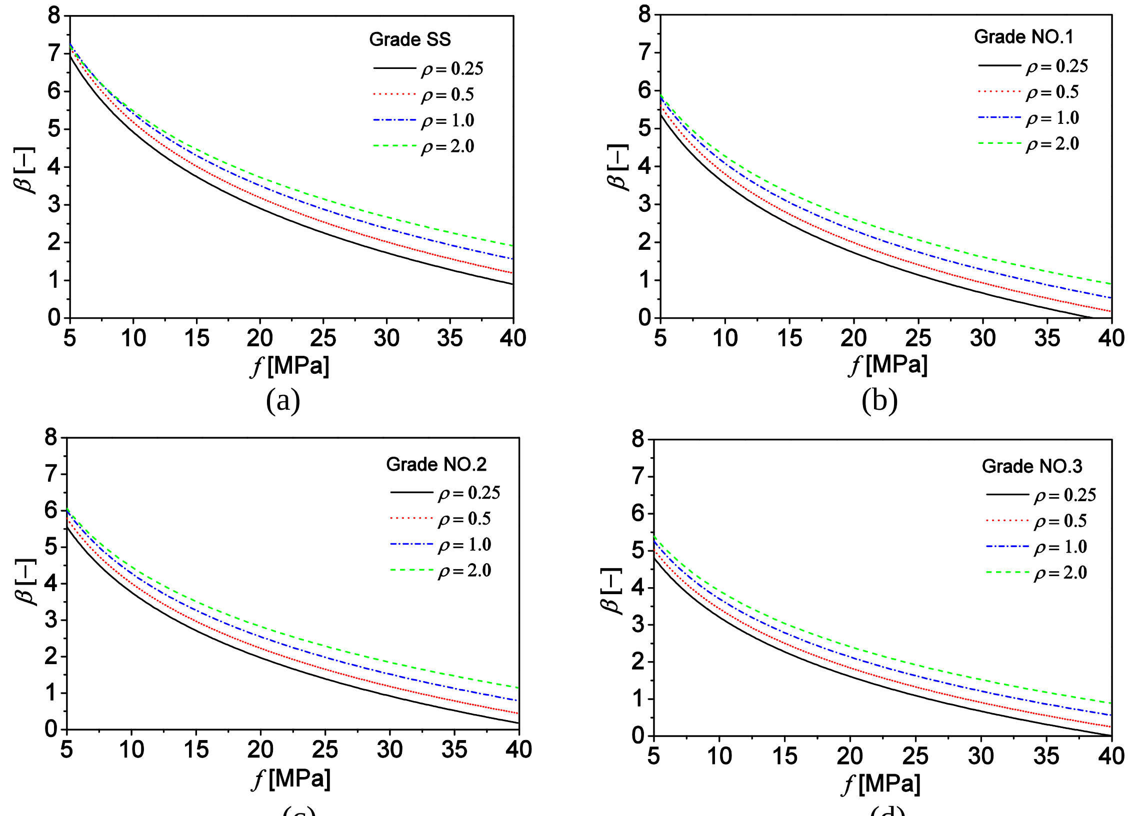
$$Z = R - \frac{fK_S}{a_D + a_Q \rho}(g + q\rho) \tag{7}$$

where ρ equals Q_K/D_K, g is equal to D/D_K, and q is the equivalent of Q/Q_K.

According to the survey of the timber structure in China (GB 50005 2003; GB50009 2012), the live to dead load ratio (ρ) was specified as 0.25, 0.5, 1.0, and 2.0, respectively. The calculation program for the reliability index (β) was developed using Matlab 7 software. First-order second-moment reliability analyses were performed for all the data cells and simulation cases, including $G+L_O$, $G+L_R$, $G+W$, and $G+S$ (Zhuang 2004; Li 2011; Yang 2013). For example, the relationship between the reliability index (β) and the design value of bending strength (f) of Larch 2 x 4 lumber, for both office live load (L_O) and dead load (G), is shown in Fig. 3.

The results of reliability analysis indicated that the reliability index (β) decreased nonlinearly with the increase of design value of the bending strength (f). Furthermore, the reliability index (β) decreased nonlinearly with the enhancement of the design value of bending strength (f), and increased as the live to dead load ratio (ρ) increased. Similar results have been obtained in previous studies on the effect of f and ρ on the reliability index (β) (Foschi *et al*. 1989; Zhuang 2004; Li 2011). The reliability level, which needed to meet the target reliability level (β_0=3.2) for f (GB50068 2001), was acquired by taking the average of the reliability indices under two load combinations.

According to the reliability analysis and the requirements for the minimum reliability index ($\beta>\beta_0$) (GB50068 2001), the average of the reliability index (β) for all load combinations were 3.215 for grade SS, 3.226 for grade No.1, 3.215 for grade No.2, and 3.213 for grade No. 3.



(a) (b) (c) (d)

Fig. 3. Reliability index (β) versus the design value of bending strength (f) of Larch 2×4 lumber for office live load (L_O) plus dead load (G)

Table 5. Reliability Index *B* of Larch 2×4 Lumber for Different Load Combinations

Load combinations	ρ	*B*			
		SS (18.4 MPa)	NO.1 (11.6 MPa)	NO.2 (12.7 MPa)	NO.3 (10.3 MPa)
G+L_O	0.25	3.150	3.156	3.144	3.141
	0.5	3.428	3.411	3.396	3.368
	1.0	3.738	3.704	3.686	3.637
	2.0	3.945	3.919	3.903	3.855
G+L_R	0.25	3.080	3.093	3.082	3.085
	0.5	3.307	3.300	3.286	3.269
	1.0	3.557	3.536	3.520	3.485
	2.0	3.734	3.715	3.700	3.662
G+S	0.25	2.855	2.890	2.882	2.907
	0.5	2.908	2.941	2.933	2.955
	1.0	2.930	2.969	2.962	2.988
	2.0	2.910	2.960	2.955	2.992
G+W	0.25	2.880	2.912	2.903	2.926
	0.5	2.958	2.984	2.975	2.992
	1.0	3.021	3.047	3.038	3.053
	2.0	3.044	3.077	3.069	3.089
Avg (all)	—	3.215	3.226	3.215	3.213

Meanwhile, the design value for the bending strength (*f*) was 18.4 MPa for grade SS, 11.6 MPa for grade No.1, 12.7 MPa for grade No.2, and 10.3 MPa for grade No.3 (Table 5). The design value of MOR_{15} of grade SS was 0.57 times, 0.45 times and 0.79 times higher than that of grade No. 1, grade No. 2 and grade No. 3, respectively. So, the results of reliability analysis indicated that the grade of the dimension lumber had a significant effect on the design value of the bending strength; therefore, it is unreasonable that the design value of the bending strength for Larch is set to 17 MPa for all grades in the Chinese national standard 50005 (GB 50005, 2003).

Additionally, researchers have reported that the simulation load case of the maximum and minimum reliability index (β) are the load combination $G+L_O$ and G+S, for the same live to dead load ratio (ρ), respectively (Zhuang 2004; Li 2011). For example, the value of the reliability index (β) for grade SS lumber was 3.150 under $G+L_O$, and 2.855 under G+S, when ρ is equal 0.25.

CONCLUSION

1. The best fitting model of MOR_{15} for Larch 2×4 lumber was the lognormal distribution.
2. The results of the reliability analysis showed that the reliability index (β) decreased nonlinearly as both the enhancement of the design value of bending strength (*f*) and the reduction of the live to dead load ratio (ρ).
3. To meet the target reliability level ($\beta_0 = 3.2$), it was suggested that the design value of bending strength (*f*) for Larch 2 x 4 lumber be 18.4 MPa for grade SS, 11.6 MPa for grade No.1, 12.7 MPa for grade No.2, and 10.3 MPa for grade No.3.

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