

Design Value of the Compressive Strength for Bamboo Fiber-Reinforced Composite Based on a Reliability Analysis

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The objective of this study was to determine the design value for the compressive strength (CS) for bamboo fiber-reinforced composite (BFRC) based on a reliability analysis. A total of 180 specimens were subjected to static compressive testing. The CS of the BFRC was significantly higher than that of raw bamboo and other bamboo-based composites, and its cumulative probability distribution accorded with a normal distribution. Furthermore, a calculation program of the reliability index (β) was developed by adopting the first-order second-moment method. Results of the reliability analysis indicate that β increases nonlinearly not only with the enhancement of the partial coefficient but also with the live-to-dead load ratio in all the simulation load cases. The simulation load case of the maximum and minimum β is $G+L_0$ and $G+L_s$, respectively. To meet the target reliability requirement, it is suggested that the design value of CS for BFRC be 43.247 MPa and the partial coefficient be 2.0.

Keywords: Design value; Compressive strength (CS); Bamboo fiber-reinforced composite (BFRC); Reliability index (β); Partial coefficient

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INTRODUCTION

In the 21st century, with the rapid development of renewable raw materials, which are environmentally friendly and sustainable (Cai *et al.* 2007), more and more people are interested in using natural plant fibers as reinforcements in polymer composites to make engineering materials (Kalia *et al.* 2009). Since 1990, bamboo has been widely used to fabricate bamboo-based composites, such as bamboo plywood, laminated bamboo lumber, and bamboo fiber-reinforced polymer composites, which have widespread applications in the building and civil engineering field, especially in China and India (Lee and Liu 2003; Sulaiman *et al.* 2006; Mahdavi *et al.* 2012).

A novel process, as previously reported (Yu *et al.* 2014), produces an innovative unit only by mechanical treatment without chemical treatment and without removing the inner or outer layer of the bamboo. This process has been further developed to produce a new type of bamboo fiber-reinforced composite (BFRC) using oriented bamboo fiber materials. This new type of BFRC with excellent mechanical performance is very popular in several applications including furniture, construction formwork, wind-power blades, and container flooring; however, there are few applications for this type of BFRC in building structures due to the lack of design value of mechanical performances (Yu and

Yu 2013; Yu *et al.* 2013). Although a few studies have been conducted on the effect of the manufacturing process on the mechanical performance of BFRC (Yu *et al.* 2006; Zhu *et al.* 2011), very little work has been focused on the determination of design values of mechanical properties with a reliability analysis. Thus, it is unsafe to use the new type of BFRC material in building structures without establishment of a suitable design value for mechanical performances.

The objective of this study was to determine the design value of compressive strength (CS) for BFRC based on a reliability analysis (Fig. 1). In practical engineering design, the obtained design value will significantly contribute to assessing the load bearing capacity of structural members that are made from BFRC.

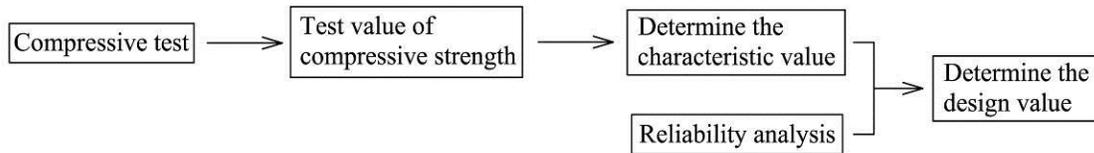


Fig. 1. The flowchart of the steps

MATERIALS AND METHODS

Materials

Ten pieces of commercial BFRC plates supplied by Sichuan Hongya Bamboo Co., Ltd with dimensions of 20 mm (thickness) × 1250 mm (width) × 2500 mm (length) were used in this study (Fig. 2). The raw material in the bamboo fiber-reinforced composite was Ci bamboo (*Neosino calamus affinis*) with an age of 3 to 4 years and cut from the Hongya Forest Reserve, Sichuan province, in the southwestern part of China. The matrix material used in this study was based on a commercially available low molecular weight phenol formaldehyde resin supplied by Beijing Dynea Chemical Industry Co., Ltd.

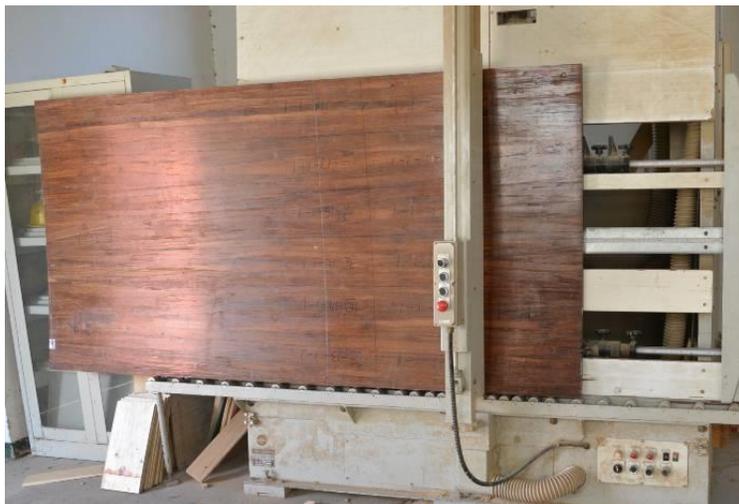


Fig. 2. The bamboo fiber-reinforced composite plate

Sample Preparation

Each plate was cut in half in the longitudinal direction. One half was used for static compressive testing, and the other half was used for future mechanical testing of its connection. In consideration of the effect of the raw bamboo material itself and the manufacturing process on the quality of the BFRC plates (Yu *et al.* 2006; Zhu *et al.* 2011), the specimens cut from the plates needed to be random and uniform; therefore, each plate was divided into 18 subzones, as shown in Fig. 3. A specimen from each subzone was used for compressive testing, for a total of 180 specimens. The specimens had dimensions of 20 mm (thickness) \times 20 mm (width) \times 30 mm (length), an average mass density of 1.231 g/cm³ with a standard deviation of 0.091 g/cm³, and an average moisture content of 1.932%.

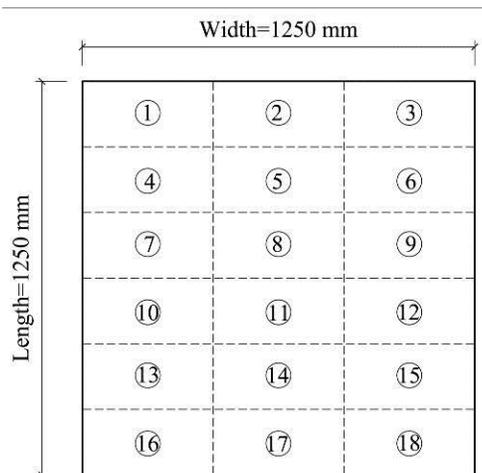


Fig. 3. Divided subzones of the bamboo fiber-reinforced composite plate

Compressive Testing

According to Chinese national standard GB1935 (2009), static compressive tests (Fig. 4) were conducted by an INSTRON 5582 machine (100 kN of load limit; Instron Corporation, the United States). The specimens were loaded along the length direction at a rate of 1 mm/min and continued until failure. The length direction was parallel to the bamboo fiber direction. The compressive strength (CS) of the bamboo fiber-reinforced composite (BFRC) was calculated using Eq. 1,



Fig. 4. Test set-up of compressive strength

$$f = F_{\max} / bt \quad (1)$$

where F_{\max} is the maximum compressive force applied to the specimens during the test (N), b is the width of the specimens (mm), and t is the thickness of the specimens (mm).

RESULTS AND DISCUSSION

Compressive Strength of BFRC

Bamboo fiber-reinforced composite (BFRC) is an anisotropic material. The CS of raw bamboo and various bamboo-based composites, such as BFRC, commercial bamboo scrimber, and laminated bamboo lumber, are shown in Table 1.

Table 1. Compressive Strength of Raw Bamboo and Bamboo-based Composites

Product	Density		CS	
	Avg (g/cm ³)	COV (%)	Avg (MPa)	COV (%)
BFRC	1.23	7.39	104.82	10.65
Bamboo ^a	0.68	11.93	66.24	5.74
BS ^a	1.10	3.11	77.9	10.81
LBL ^a	0.66	5.55	52.65	5.56

^aBamboo, BS, and LBL are raw bamboo, commercially bamboo scrimber, and laminated bamboo lumber, respectively (taken from Lu *et al.* 2005). The CS of all materials is obtained by static compressive test parallel to the bamboo fiber direction.

The CS of BFRC was found to be 0.6 times, 0.3 times, and 1.0 times higher than that of raw bamboo, BS, and LBL, respectively. The CS of a polymer-matrix composite depends on the interaction between the reinforcement (representative of the resins) and the matrix (representative of the bamboo fibers) (Wang *et al.* 2011). The interaction between bamboo fibers and resins is strongly dependent on the contact surface, in other words, the fineness of bamboo fiber bundles. Using a new manufacturing process of BFRC, the phenolic resin is distributed more uniformly with the increasing fineness of fiber bundles (Yu *et al.* 2014).

Probability Distribution

A histogram of the compressive strength results is presented in Fig. 5. The probability distribution of CS is assumed to be a normal distribution $N(\mu, \sigma^2)$, and the cumulative distribution function of CS is as follows,

$$F_0(x) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^x e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt \quad (2)$$

where x is the random variable of CS obtained by the static compressive test. The unknown numbers μ and σ are obtained by the maximum likelihood method used in the following analysis (Table 2).

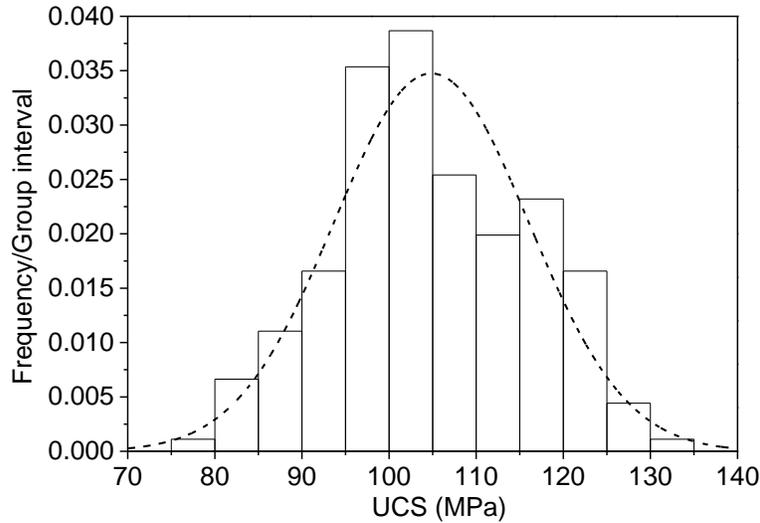


Fig. 5. Normal fit of compressive strength for BFRC

Table 2. Calculation Table of $\hat{\mu}$, $\hat{\sigma}$, and χ^2 of BFRC

Interval (x_i)	\bar{x}_i	Observed numbers (n_i)	Interval (u_i)	Probability (\hat{p}_i)	Predicted numbers $n\hat{p}_i$
[75, 80]	77.5	1	[-2.647, -2.203]	0.0097	1.7514
[80, 85]	82.5	6	[-2.203, -1.759]	0.0254	4.5814
[85, 90]	87.5	10	[-1.759, -1.316]	0.0549	9.8738
[90, 95]	92.5	15	[-1.316, -0.872]	0.0974	17.5339
[95, 100]	97.5	32	[-0.872, -0.429]	0.1425	25.6566
[100, 105]	102.5	35	[-0.429, 0.015]	0.1719	30.9351
[105, 110]	107.5	23	[0.015, 0.458]	0.1708	30.7358
[110, 115]	112.5	18	[0.458, 0.902]	0.1398	25.1638
[115, 120]	117.5	20	[0.902, 1.346]	0.0943	16.9762
[120, 125]	122.5	15	[1.346, 1.789]	0.0524	9.4369
[125, 130]	127.5	4	[1.789, 2.233]	0.0240	4.3224
[130, 135]	132.5	1	[2.233, 2.676]	0.0091	1.6312

$n = \sum n_i = 180$

The estimated values of the unknowns μ and σ were calculated as follows:

$$\hat{\mu} = \bar{x} \approx \frac{1}{180} \sum_{i=1}^{12} n_i \bar{x}_i = 104.833 \text{ MPa} \tag{3}$$

$$\hat{\sigma} = s \approx \sqrt{\frac{1}{180} \sum_{i=1}^{12} n_i (\bar{x}_i - \bar{x})^2} = 11.272 \text{ MPa} \tag{4}$$

By substitution of Eq. 3 and Eq. 4 into Eq. 2, $F_0(x)$ can be rewritten as:

$$F_0(x) = \frac{1}{11.272 \times \sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{(t-104.833)^2}{2 \times 11.272^2}} dt \quad (5)$$

Furthermore, $F_0(x)$ can be transformed into a standardized normal distribution $\Phi(u)$ when $u=(x-104.833)/11.272$. As a standardized normal distribution, the cumulative distribution probability and the predicted numbers of random variable on interval $[x_i, x_{i-1}]$ are \hat{p}_i ($\hat{p}_i = F_0(a_i) - F_0(a_{i-1})$), and $n\hat{p}_i$, respectively (Table 2).

$F(x)$ is the cumulative distribution function of CS obtained by the static compressive test. Whether or not the $F(x)$ is equal to the assumed $F_0(x)$ at the level of 0.05, it is analyzed by χ^2 testing (Mao *et al.* 2006). The formula for χ^2 is as follows:

$$\chi^2 = \sum_{i=1}^{12} \frac{(n_i - n\hat{p}_i)^2}{n\hat{p}_i} = 11.305 \quad (6)$$

For a probability level of 0.05 and 12 data points, the critical χ^2 -value is 16.919 (Mao *et al.* 2006). A χ^2 -value of 16.919 or less indicates a good fit. It is evident that the assumed $F_0(x)$ provides great fit to the $F(x)$ obtained by the static compressive test ($\chi^2 = 11.305 < 16.919$); therefore, the normal distribution was judged to be a good model for the actual distribution of CS in the case of BFRC.

Characteristic Value and Design Value

According to the Chinese national standards (GB 50068 2001; GB 50153 2003), the characteristic value of CS for BFRC could be determined by the 5% probability distribution. As a normal distribution, the characteristic value (f_k) and design value (f_d) are as follows:

$$f_k = \mu_f (1 - 1.645\delta_f) \quad (7)$$

$$f_d = f_k / \gamma_R \quad (8)$$

where μ_f is the mean value of compressive strength; δ_f is the coefficient of variance (COV) (Table 1); and γ_R is the partial coefficient of compressive resistance. The characteristic value of CS is 86.494 MPa, and the γ_R is determined according to the following reliability analysis.

Reliability Analysis

In the reliability analysis of bamboo structure design, the compressive resistance (R) of BFRC is calculated using Eq. 9. The mean value and coefficient of variance (COV) of R is as follows (Wang 2002; Pan and Zhu 2009),

$$R = k_1 k_2 k_3 f \quad (9)$$

$$\mu_R = \mu_{k1} \mu_{k2} \mu_{k3} \mu_f \quad (10)$$

$$\delta_R = \sqrt{\delta_{k_1}^2 + \delta_{k_2}^2 + \delta_{k_3}^2 + \delta_f^2} \quad (11)$$

where k_1 , k_2 , and k_3 are adjusting factors for the equation precision, geometric character, and the effect of long-term load, respectively. The parameter f is the experimental value of CS for BFRC, and its mean and COV are shown in Table 1. Furthermore, the random variables k_1 , k_2 , and k_3 are assumed to have a normal distribution, and the mean value and standard deviation are presented in Table 3; therefore, according to statistical theory, the R of BFRC is also distributed normally, and the calculated mean value and COV are 72.476 MPa and 17.841%, respectively.

Table 3. Statistical Parameters of Adjusting Factors (GB 50005 2003)

Parameters	k_1	k_2	k_3
Mean value	1.00	0.96	0.72
SD	0.050	0.058	0.086
COV (%)	5	6	12

The loads applied to the bamboo structure are divided into two groups: the dead loads (G) and the live loads (L). The former includes the self-weight of structural members and other materials while the latter includes the office occupancy load (L_O), residential occupancy load (L_R), wind load (L_W), and snow load (L_S). According to Chinese National Standard GB 50009 (GB 50009 2012), the data of dead loads are fitted to the normal distribution, while all the data of live load are fitted to the extreme type-I. The statistical parameters of the loads are shown in Table 4.

Table 4. Statistical Parameters of the Loads (GB 50009 2012)

Statistical parameters	Load types				
	G	L_O	L_R	L_W	L_S
Mean/nominal	1.060	0.524	0.644	1.000	1.040
COV (%)	7.0	28.8	23.3	19.0	22.0
Distribution types	Normal	Extreme-I	Extreme-I	Extreme-I	Extreme-I

The combinations of the two loads, including $G+L_O$, $G+L_R$, $G+L_W$, and $G+L_S$, were then used in the reliability analysis. The limit state design equation for compressive resistance can be expressed as:

$$a_D D_K + a_L L_K = K_S (f_k / \gamma_f) \quad (12)$$

where a_D and a_L are the dead load factor (1.2) and live load factor (1.4), respectively; D_K and L_K are the nominal dead load and nominal live load, respectively; and K_S is an adjusting factor for the service life and keeps to 1.0 for 50 years (GB 50009 2012).

The failure function, developed to assess compressive resistance and the effect of loads for first-order second-moment reliability analysis, is as follows (Zhuang 2004; Li 2011; Kimiaiefar *et al.* 2013):

$$Z = R - (D + L) \quad (13)$$

where R , D , and L are random variables representing the compressive resistance of BFRC, dead load (G), and live load (L_O , L_R , L_W , or L_S), respectively. The random variable R was assumed to be a normal distribution according to the above analysis.

By substitution of Eq. 12 into Eq. 13, the failure function can be rewritten as:

$$Z = R - \frac{k_s f_k / \gamma_R}{a_D + \rho a_L} (g + \rho l) \quad (14)$$

where ρ , g , and l are the equivalents of L_K/D_K , D/D_K , and L/L_K , respectively.

To determine the design value of CS for BFRC, first-order second-moment reliability analyses were performed for all data cells and simulation load cases, including $G+L_O$, $G+L_R$, $G+L_W$, and $G+L_S$. A calculation program for reliability index (β) was developed by use of Matlab 7 software. The results of reliability analysis indicate that the β increases nonlinearly with the enhancement of the live-to-dead load ratio (ρ) in all the simulation load cases. Similar results have been obtained in previous studies on the effect of ρ on the β (Folz and Foschi 1989; Zhuang 2004; Li 2011). For example, the relationship between β and ρ of BFRC, under dead load (G) plus live office load (L_O), is shown in Fig. 6.

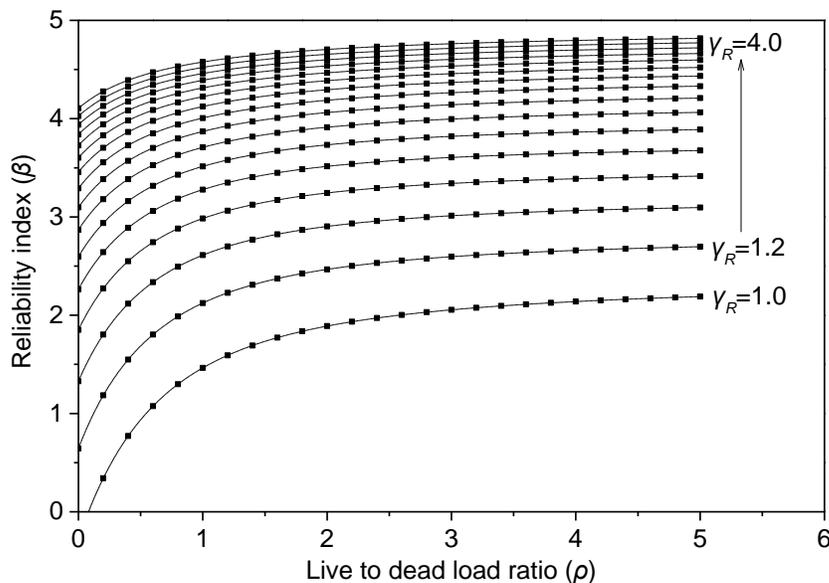


Fig. 6. Relationship between reliability index (β) and live-to-dead load ratio (ρ) under $G+L_O$

The reliability level, which needs to meet the target reliability level ($\beta_0=3.2$) in the determination of design value of CS (GB 50068 2001), is acquired by taking the average of the reliability index under the combinations of two loads, where $\rho=0.25, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75, 2.0, 2.25, 2.5, 2.75$, and 3.0 (Fig. 7).

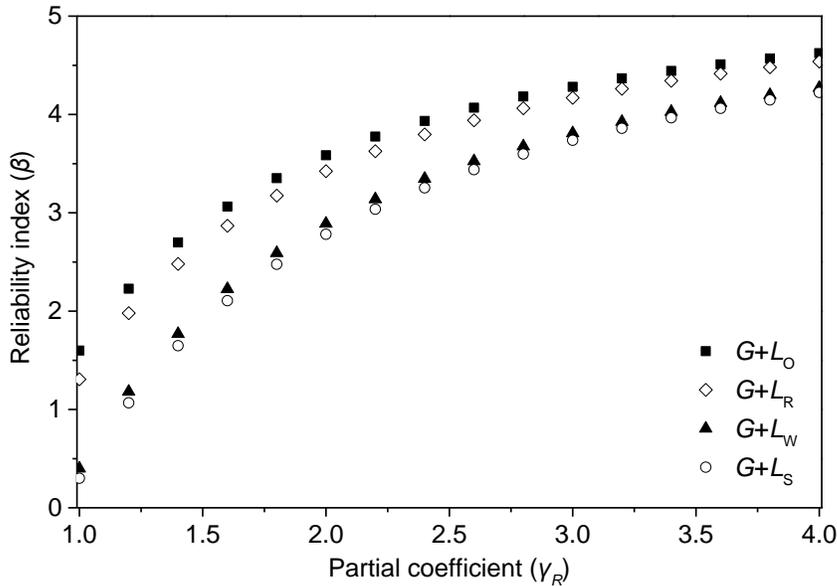


Fig. 7. Relationship between reliability index (β) and partial coefficient (γ_R) under different load combinations

Figure 7 shows that the reliability index (β) increases nonlinearly with the enhancement of partial coefficient (γ_R) in all the simulation load cases, including $G+L_O$, $G+L_R$, $G+L_W$, and $G+L_S$. For the same ratio of live-to-dead load (ρ), the simulation load case of the maximum and minimum β are $G+L_O$ and $G+L_S$, respectively. The results obtained in this work are similar to those of Zhuang (2004) and Li (2011). The partial coefficient (γ_R) of CS for BFRC, obtained by taking the average of all simulation cases, is shown in Fig. 8.

To assess the reliability index (β) of design value of CS, the cubic, logarithmic, and inverse models were used to describe the relationship between β and partial coefficient (γ_R) of compressive resistance for BFRC (Fig. 8).

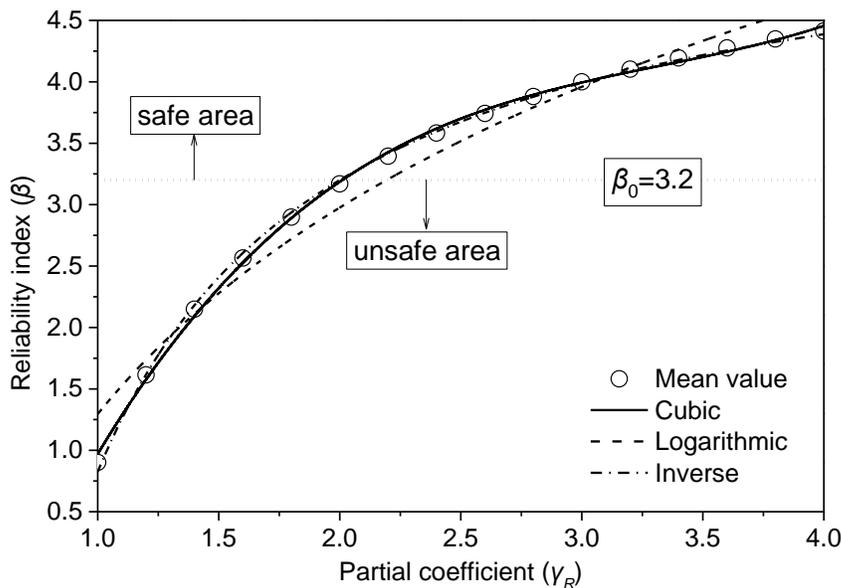


Fig. 8. Prediction of the reliability index (β) by partial coefficient (γ_R)

Fitting the results to various models indicated that the cubic, logarithmic, and inverse models could precisely predict the relationship between β and γ_R . The inverse mode fits the data slightly better than the other models. With a statistic r^2 of 0.999 and standard error of 0.031 (Table 5), the inverse model is selected for calculation of the reliability index.

Table 5. Results Fitted to Various Models

Models	Calculation formula	r^2	Std. Error	$\gamma_R (\beta=\beta_0)$
Logarithmic	$y=1.2951+2.4252\ln x$	0.972	0.181	2.197
Inverse	$y=5.5741-(4.7479/x)$	0.999	0.031	2.000
Cubic	$y=-3.7188+6.2784x-1.7670x^2+0.1771x^3$	0.999	0.040	2.011

According to the above reliability analysis and the requirement for the minimum reliability index ($\beta > \beta_0 = 3.2$) (GB 50068 2001), the partial coefficient that satisfies the condition $\gamma_R \geq 4.7479/(5.5741-\beta_0)$ is safe for engineering design for BFRC. To meet the target reliability level ($\beta_0 = 3.2$), it is suggested that the partial coefficient be 2.0, and the design value of CS calculated by Eq. 8 be 43.247 MPa.

CONCLUSIONS

The aim of this work was to determine the design value of CS for BFRC based on a reliability analysis. The results will significantly contribute to assessing the load bearing capacity of structural members in practical engineering design. Based on the results of this research, the conclusions are as follows:

1. The mean value of CS for BFRC was 104.822 MPa, which is significantly higher than that of raw bamboo and other bamboo-based composites.
2. The cumulative probability distribution of CS for BFRC accords with the normal distribution according to the χ^2 test.
3. For all simulation load cases, the reliability index (β) increased nonlinearly not only with the enhancement of the partial coefficient but also with the live-to-dead load ratio. For the same ratio of live-to-dead load, the simulation load case of the maximum and minimum β was $G+L_0$ and $G+L_s$, respectively.
4. To meet the target reliability level ($\beta_0 = 3.2$), it is suggested that the design value of CS for BFRC be 43.247 MPa and the partial coefficient be 2.0.

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