Failure Criteria for Shear Strength Evaluation of Bonded Joints According to Grain Slope under Tension Load

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Failure criteria from six theories were applied to estimate the shear strength of the adhesive line in terms of grain slope when under loaded tension stress. The shear stresses of the adhesive line as a function of the angle of the wood grain were determined by experimental tests. Specimens were obtained from 12 *Eucalyptus saligna* wood beams. They were prepared with varying angles of the grain (0˚, 15˚, 30˚, 45˚, 60˚, 75˚, and 90˚) in relation to load application, following the requirements of the Brazilian standard. From the results of the six failure criteria and experimental results, robust statistical analysis was carried out; it was thus possible to adapt the models to determine the shear strength of the adhesive line as a function of the angle of the wood grain. The six mathematical models evaluated do not show statistical significance ($p < 0.05$) in their original format. With modifications, the models showed statistical significance only with the formulations of DIN 1052 and Karlsen.

Keywords: Shear strength; Slope of wood grain; Bonded joints

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INTRODUCTION

The behavior of wood under compressive load when applied at a slope to the wood grain has been studied (Kollmann and Côté 1968; Wangaard 1968; Bodig and Jayne 1982; Dinwoodie 1975; Grekin and Surini 2008), with the ultimate effect depending on the type of wood. There have been few studies regarding the effect of grain slope on shear strength in bonded joints when compression stress is applied and, moreover, there are controversies about which theory of failure is the most appropriate (Burdurlu *et al.* 2006; Follrich *et al.* 2007; Carrasco and Mantilla 2013). For the influence of grain slope on shear strength of bonded joints when under tension stress, no published studies were found. Studies in regard to the behavior of adhesives for shearing under tension stress parallel and normal to the grain have been carried out by Blyberg *et al.* (2012), Follrich *et al*. (2010), Patel *et al.* (2013), Serrano (2004), Tannert *et al.* (2012), Raftery *et al.* (2009), and Vallée *et al.* (2011). These authors used test specimens of varied shapes and sizes. Justifications for adoption of the test specimens are diverse and include the equipment available for performing the tests. In the present study, the test specimens were adapted from the Brazilian standards NBR-7190 (1979) and from Hellmeister (1973).

Researchers have studied the effect of direction of application of stress in relation to wood grain on shear strength and used principles and theories applied to wood compression strength (Kollmann and Côté 1968; Szücs 1992; Liu and Ross 1998; Forest Product Laboratory 1999; Logsdon *et al.* 2010). Some established theoretical models generally considered orthotropic wood, while others established empirical models based on test results. The types of failures that may occur in the test specimens, due to the anisotropy of the material, are difficult and complicated to evaluate because of the interaction of diverse mechanisms such as defects of materials, conditions of load application, environmental conditions, and the mechanical properties of wood in different directions (Oh 2011).

Wood is a unique and variable material due to its natural defects and anisotropic characteristics. Thus, failure theories are quite complicated (Woodward and Minor 1988; Oh 2011). There are failure criteria for compressive, tensile, and shear stresses on wood (Liu 1984; Woodward and Minor 1988; Liu *et al.* 1999; Liu 2001). Carrasco and Mantilla (2013) presented six failures criteria to determine the shear strength for glued joints under compressive stress as a function of wood grain slope. (Hankinson, Karlsen, DIN-1052, Keylwerth, hyperbolic, and the Tsai-Hill). The authors concluded that, with the changes in the exponents of the equations, only three failure criteria were appropriate to estimate the shear stresses due to the inclination of the fibers. The authors also suggested that a study should be developed applying these criteria failure for glued joints under actions of tension, flexion, and torsion.

The purpose of this study is to apply these six failure criteria to determine the shear strength of glued joints under tension stress as a function of wood grain slope. Also the goal was to modify the exponents of the equations to determine the most appropriate failure criteria to estimate the shear stress of these bonded joints.

MATERIALS AND METHODS

Theoretical Analysis

The test specimens for tension stress consist of two wooden plates covering the splice (glued lap joint), as shown in Fig. 1a. The aim of this analysis is to determine the TS maximum load rupture. The load transfer in this type of bond occurs through shearing stress of the adhesive line. Since there is a discontinuity of the material in the test specimens, the rupture may occur through tension or compression perpendicular to grain. Thus, it is necessary to verify this rupture mode. In this simplified analysis it was considered the classical theory of materials. Figure 1b shows half of the connection. The N/2 force applied to the gravity center of the half splice will be transmitted to the joint-cover through the shear stresses in the glued area (axLc) (Fig. 1a). According to Leicester (1971) the distribution of the shear stresses along the adhesive line (detail A) in the elastic phase is not linear, being characterized by the stress concentration at the beginning and end of the adhesive line. Conversely, in the plastic phase these tensions are uniform along the adhesive line (Fig. 1b). Figure 1c shows an infinitesimal element. Making balance of forces and adopting a uniform distribution of shear stress, we have:

$$A(\rho - d\sigma) + 2\tau(x).a.dx = Adx \;\; \rightarrow \;\; d\sigma = \frac{2\cdot\tau(x)\cdot a\cdot dx}{A} \;\; \rightarrow \;\; \int d\sigma = \frac{a}{A}\int_0^{L_c} 2\tau(x)dx$$

$$\sigma = \frac{2a}{A}\int_0^{L/c}\tau(x)dx\,, \;\; A=a.b, \;\; \sigma = \frac{N}{a.b}\,, \;\; \text{considering } t(x)=f_v \;\; \text{(uniform)}$$

$$\sigma = \frac{2a}{a.b}\int_0^{L/c} f_v dx \;\; \rightarrow \;\; \frac{N}{a.b} = \frac{2a}{a.b} f_v.L_c \;\; \rightarrow \;\; N = 2a.L_c.f_v$$

$$N = 2.A_c.f_v \qquad (1)$$

Therefore, the rupture of the glued joint occurs when the shear stress, at all points of the adhesive line, reaches the plastification shear stress value of the adhesive or adhesive-wood interface (f_v).

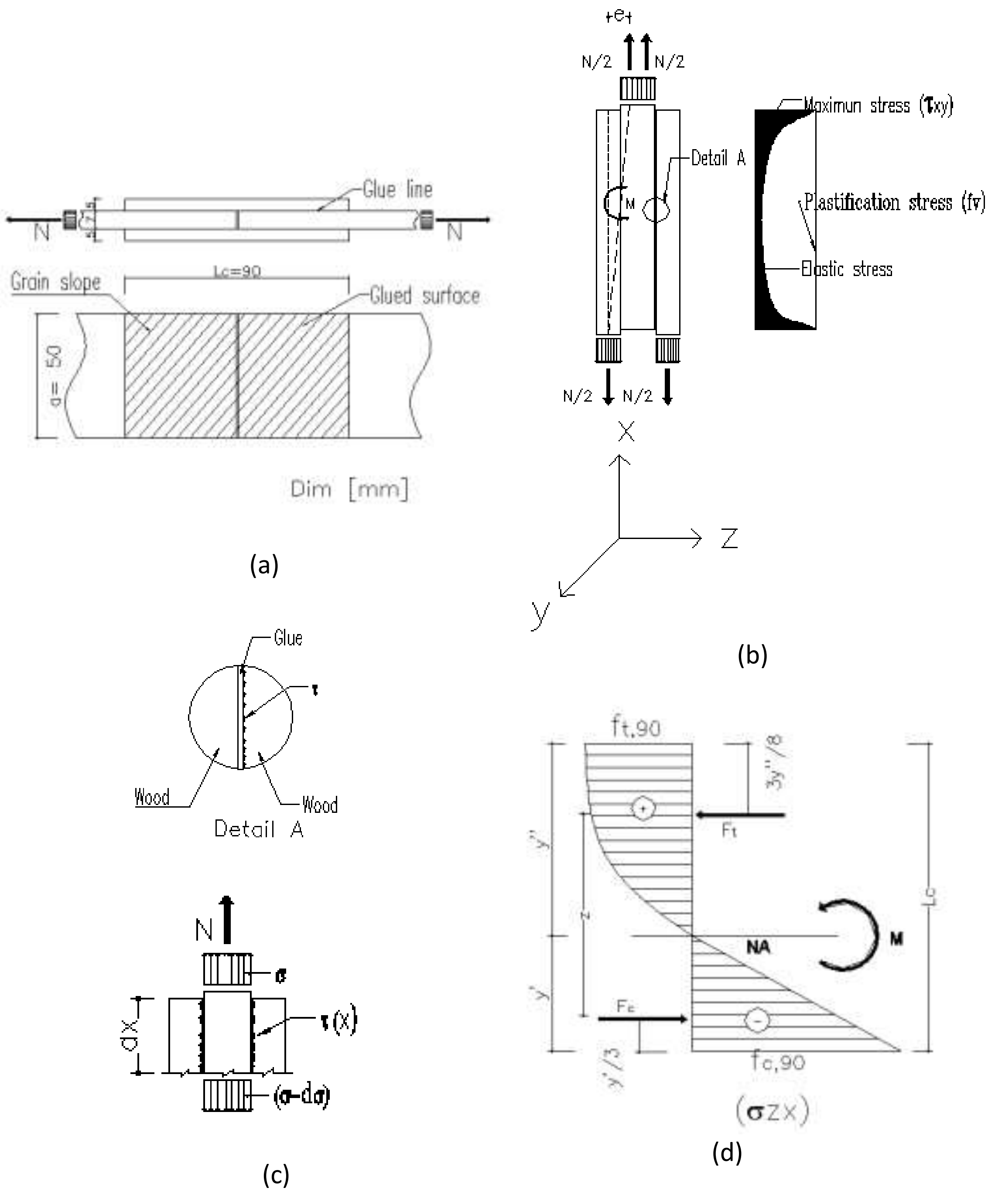

Fig. 1. (a) Test model for determination of shear stress in tension, **(**b) static balance, (c) infinitesimal element, and **(**d) distribution of the stresses perpendicular to the glue line

Another tension, seemingly unimportant, is the traction stress normal to the fibers. This tension appears due to the eccentricity of the applied load. Figure 1b represents the moment (M) and the torque (F) applied perpendicularly to the adhesive film plane (due to the eccentric load application). The stress distribution according to Carrasco (1984), experimentally determined, is shown in Fig. 1d. This stress distribution along the adhesive line has a triangular form in the compression zone and a parabolic form in the tensile zone. This occurs because the compression resistance perpendicular to the grain ($f_{c,90}$=13.00 MPa) is greater than the resistance traction of the fibers ($f_{t,90}$=8.30 MPa).

Making balance of forces, one obtains,

$$\sum F_h = 0 \quad \rightarrow \quad F_c = F_t \quad \rightarrow \quad \frac{f_{c,90}\cdot y'\cdot a}{2} = \frac{2\cdot f_{t,90}\cdot y''\cdot a}{3} \tag{2a}$$

$$\sum M_{L.N.} = 0 \quad \rightarrow \quad M = F_c\left(y' - \frac{y'}{3}\right) + F_t\left(y'' - \frac{3\cdot y''}{8}\right), \quad \text{and} \quad L_c = y' + y'' \tag{2b}$$

the location of the NA is $y' = \frac{L_c}{\left(\frac{3f_{c,90}}{4f_{t,90}}+1\right)}$ and $y'' = \frac{L_c}{\left(\frac{4f_{t,90}}{3f_{c,90}}+1\right)} \rightarrow z = \frac{2y'}{3} + \frac{5y''}{8}$ (2c)

$$M = N.e \quad \rightarrow \quad N = \frac{M}{e} \tag{3}$$

where $f_{c,90}$ is the wood resistance to compressive stress perpendicular to grain, $f_{t,90}$ is the wood resistance to tensile stress perpendicular to grain, M is the bending moment, NA is the neural axial, L_c is length of the glue line, y′= distance of the upper grain to the neutral line, y″ is the distance of the lower grain to the neutral line, *a* is the width of the piece, f_v is the wood resistance to shear stress, *e* is the eccentricity of the applied load, and A_c is the glued surface.

Applying Eqs. (1, 2c, and 3), the maximum loads at which the test specimens will rupture is obtained: rupture by normal tensile stress 27,400 N, rupture by normal compressive stress 25,250 N, and rupture by shearing of glue 19,200 N. This shows that the test specimens are recommended for determining shear strength of the adhesive.

Failure Theories for Wood

The six formulas presented below are derived from the application of diverse failure criteria used in obtaining compressive stress strength at an angle to grain.

According to Kollmann and Côté (1968), the empirical model proposed by the “Hankinson equation”, Eq. (4), with exponent n=2, may also be used to predict strength at an angle to the grain under tensile stress and shear stress. Karlsen *et al.* (1967), for their part, recommend the empirical model presented in Eq. (5), with exponent n=3, the “Karlsen equation”. The German standard DIN-1052 (2008) uses Eq. (6) with exponent n=1, the “DIN 1052 equation”. According to Kollmann and Côté (1968), Keylwerth also developed an empirical expression with exponent n=2, but for variation of the elasticity module according to grain slope, which for the strength values, assumes the form of Eq. (7), the “Keylwerth equation”.

Woodward and Minor (1988) and Liu (2001), present the Tsai-Hill theory as being an extension of the Von Mises criteria for isotropic materials. In the case of tests with loading in one direction, there is the form of Eq. (8), with exponent n=2, the “Tsai-Hill equation”. Moreover, Woodward (1986) affirms that the Tsai-Hill theory and the Hankinson formula grossly overestimate the strength for small wood grain slopes and propose a “hyperbolic equation” that has the form of Eq. (9), with the value of n=0.01.

In Eqs. 4 to 9, the indexes were modified to indicate that they may be used to predict shear strength of the adhesive line under tension stress at an angle to wood grain,

$$f_{vat,\theta} = \frac{f_{vat,0} \times f_{vat,90}}{f_{vat,0} \times \sin(\theta)^n + f_{vat,90} \times \cos(\theta)^n} \quad (4)$$

$$f_{vat,\theta} = \frac{f_{vat,0}}{1 + \left(\frac{f_{vat,0}}{f_{vat,90}} + 1\right) \times \sin(\theta)^n} \quad (5)$$

$$f_{vat,\theta} = f_{vat,0} - \left(f_{vat,0} - f_{vat,90}\right) \times \sin(\theta)^n \quad (6)$$

$$f_{vat,\theta} = \frac{f_{vat,0}}{\left(\cos(\theta)^n - \frac{f_{vat,0}}{f_{vat,90}} \times \sin(\theta)^n\right) \times \cos(2\theta) + \frac{f_{vat,0}}{f_{vat,45}} \times \sin(2\theta)^n} \quad (7)$$

$$\frac{1}{{f_{vat,\theta}}^2} = \frac{\cos(\theta)^{2n}}{{f_{vat,0}}^2} - \frac{\cos(\theta)^n \sin(\theta)^n}{{f_{vat,0}}^2} + \frac{\sin(\theta)^{2n}}{{f_{vat,90}}^2} \quad (8)$$

$$f_{vat,\theta} = \frac{f_{vat0} \times f_{vat,90}}{f_{vat,0} \sinh(n\theta) + f_{vat,90} \cosh(n\theta)} \quad \text{or} \quad f_{vat,\theta} = \frac{2 f_{vat,0} \times f_{vat,90}}{e^{n\theta}(f_{vat,0} + f_{vat,90}) + e^{-n\theta}(f_{vat,90} - f_{vat,0})} \quad (9)$$

where $f_{vat,\theta}$ is the shear strength of the adhesive line under tension stress at an angle to the grain, $f_{vat,0}$ is the shear strength of the adhesive line under tension stress parallel to the grain, $f_{vat,90}$ is the shear strength of the adhesive line under tension stress perpendicular to the grain, $f_{vat,45}$ is the shear strength of the adhesive line under tension stress at 45° in relation to the grain, and θ is the angle between the grain and load application.

There is controversy among authors regarding the value of the exponent (n) of the trigonometric terms in (4 to 9). Some authors indicate a different coefficient according to the type of stress application, while others indicate a different coefficient for different wood moisture content.

Material

To obtain a representative sample, wood beams were randomly selected in small lots from sawmills at varying times.

The wood used for the test specimens, *Eucalyptus saligna* (*E. saligna* Sm., Myrtaceae), had a maximum thickness of 55 mm and was found in varied widths. The wood moisture ranged from 11.44% to 13.95% and apparent density ranged from 690 kg/m^3 to 860 kg/m^3.

According to Logsdon (1995), the variation of shear strength parallel to the grain over a piece of Amescla (*Trattinikia burserifolia*) is so small that it can be ignored. Thus, similar test specimens may be obtained from a single beam.

The choice of adhesives was based on the principle that wood, a highly polar material, will have a good affinity with a polar adhesive or an adhesive with intermediate polarity (Carrasco 1984; Plaster *et al.* 2012). According to Carrasco (1984), the most recommended adhesives, for structural applications, are polyvinyl acetate, phenol formaldehyde, resorcinol-formaldehyde, urea formaldehyde, melamine, and melamine-urea-formaldehyde. For these tests, the polar resorcinol-resin based adhesive was used. This adhesive is composed of a liquid resin used with a powder catalyst, the proportions of which were recommended by the manufacturer (Hexion Chemical Industry and Trade). Both the resin and the catalyst were weighed on an electronic balance to a precision of the hundredth gram, and then mixed until completely uniform. The temperature during preparation and at the time of application ranged from 20 ˚C to 30 ˚C.

The test specimens were idealized according to the test specimens under tension stress proposed by Hellmeister (1973) and the Brazilian standard NBR-7190 (1979). The side plates of the test specimens were prepared by varying the slope of the wood grain in relation to the applied load (Fig. 2).

The test specimens were made from twelve beams chosen at random and the beam surfaces were carefully prepared (through a knife-planing machine). The adhesive was applied through brushing, and the pressure recommended by the manufacturer (1.5 MPa) was quickly applied. This pressure remained for the minimum period of eight hours. A total of ninety-six test specimens were prepared: 84 test specimens for shear strength tests of the adhesive line, with the wooden side plates glued with the grain at an angle in relation to load application, n=12 per angle and n=12 test specimens for moisture and density. This follows the recommendations of the Brazilian Standard NBR-7190 (1979). The dimensions of the test specimens for shear strength tests were 550.0 mm x 50.0 mm x 20.0 mm, and a glued area of 50.0 mm x 90.0 mm (Fig. 2). According to Carrasco (1984), with the pressure applied in the manufacture of test specimens and considering that the wood has medium density, the thickness of the glued area will vary minimally. Also, there will be no influence on bond strength since is used the same wooden beam for the preparation of test specimens (in the same series).

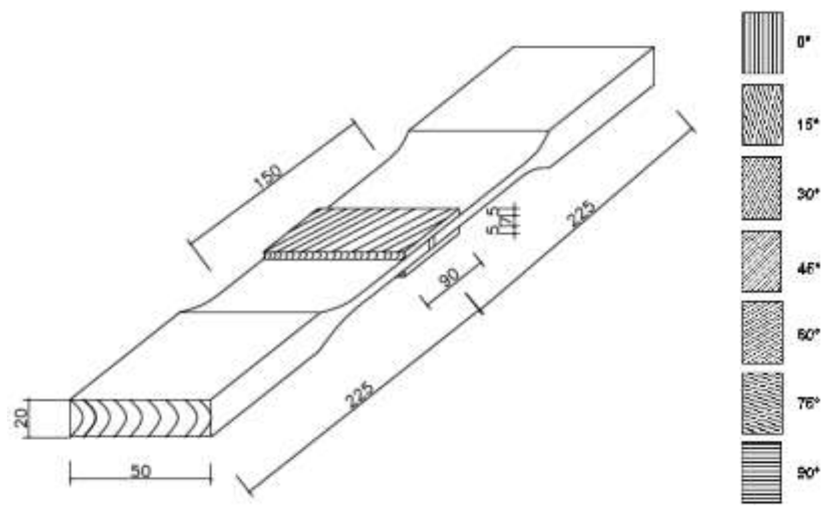

Fig. 2. Test specimens with varied grain angles in relation to the load applied (mm)

Shear strength tests of the adhesive line under tension stress were performed on a universal machine with 300 kN capacity (EMIC DL-300). The load applied was monotonic with a speed of 2.5 MPa per minute until reaching rupture of the test specimens. Abrupt and instantaneous rupture of the test specimens were observed, characterizing a fragile rupture. After termination of each shear strength test, moisture and density tests were performed for obtaining values at the time of testing to thus allow correction of the result to the reference moisture level of 12%. For that purpose, Eq. (10) was used, as recommended by NBR-7190 (1979), valid for range, 8% to 20% of moisture content.

$$f_{vat,\theta,12} = f_{vat,\theta,U\%}\left(1+\frac{2\times(U\%-12)}{100}\right) \quad (10)$$

where $f_{vat,\theta.12}$= shear strength of the adhesive line to tension stress at angle θ, in relation to wood grain, at reference moisture content of 12%; $f_{vat,\theta,U\%}$= shear strength of the adhesive line to tension stress at angle θ, in relation to wood grain at U% moisture content at the time of testing.

For statistical analysis of the test results, we used the Minitab 16 program, especially for null hypothesis testing and one-way ANOVA.

RESULTS AND DISCUSSION

The results of the shear stresses at rupture of the adhesive under tension stress are presented in Table 1. The data has been corrected to the reference moisture level of 12%.

Figure 3 is a graph of shear stress of the adhesive under tension stress relative to the grain slope of the experimental values (including the confidence interval) and the six equations with exponent as suggested by the authors. Note that the curves of all six equations are outside of the confidence interval of the experimental results for at least one of the wood grain slopes. This indicates that the “n” exponents of the equations must be different from those suggested by the authors.

Table 1. Shear Stresses at Rupture of the Adhesive under Tension Stress, Corrected for Standard Moisture of 12% (MPa)

Beam	0°	15°	30°	45°	60°	75°	90°
1 - bonded wood	10.19 15.65	9.27 12.87	8.96 12.25	3.09 13.28	2.99 10.09	1.96 9.06	1.75 8.44
2 - bonded wood	7.97 17.14	7.07 15.44	7.37 14.35	1.59 11.36	1.00 12.95	0.80 10.16	0.40 9.76
3 - bonded wood	9.60 18.68	9.60 16.95	8.68 14.19	5.92 11.84	3.27 11.64	2.25 11.74	0.82 12.56
4 - bonded wood	12.00 17.85	10.03 16.12	7.57 14.75	5.80 12.68	2.85 13.08	1.87 13.47	1.28 11.31
5 - bonded wood	11.11 14.29	10.16 9.53	9.21 9.84	6.56 12.28	3.60 11.43	2.75 7.94	1.91 7.94
6 - bonded wood	11.60 14.08	10.02 14.19	8.22 11.60	6.76 10.47	2.93 10.14	1.46 9.01	0.45 8.22
7 - bonded wood	13.84 12.64	11.68 12.04	11.92 10.95	5.06 9.63	2.65 8.43	1.56 7.34	1.08 6.50
8 - bonded wood	13.29 14.47	11.84 11.97	11.84 10.00	3.82 9.60	2.24 9.74	1.45 6.71	1.18 6.97
9 - bonded wood	12.62 15.87	12.49 15.35	10.01 13.40	7.67 15.35	4.03 9.23	1.17 10.93	1.04 9.49
10 - bonded wood	14.42 17.78	13.72 16.10	13.16 12.32	7.70 13.44	4.48 11.90	1.82 11.06	0.84 11.48
11 - bonded wood	10.26 15.19	9.31 12.98	8.51 11.97	4.78 11.08	2.66 9.76	1.56 8.85	1.01 8.30
12 - bonded wood	10.18 14.05	9.23 12.14	8.42 11.33	4.68 10.83	2.61 9.72	1.50 8.78	0.90 8.22

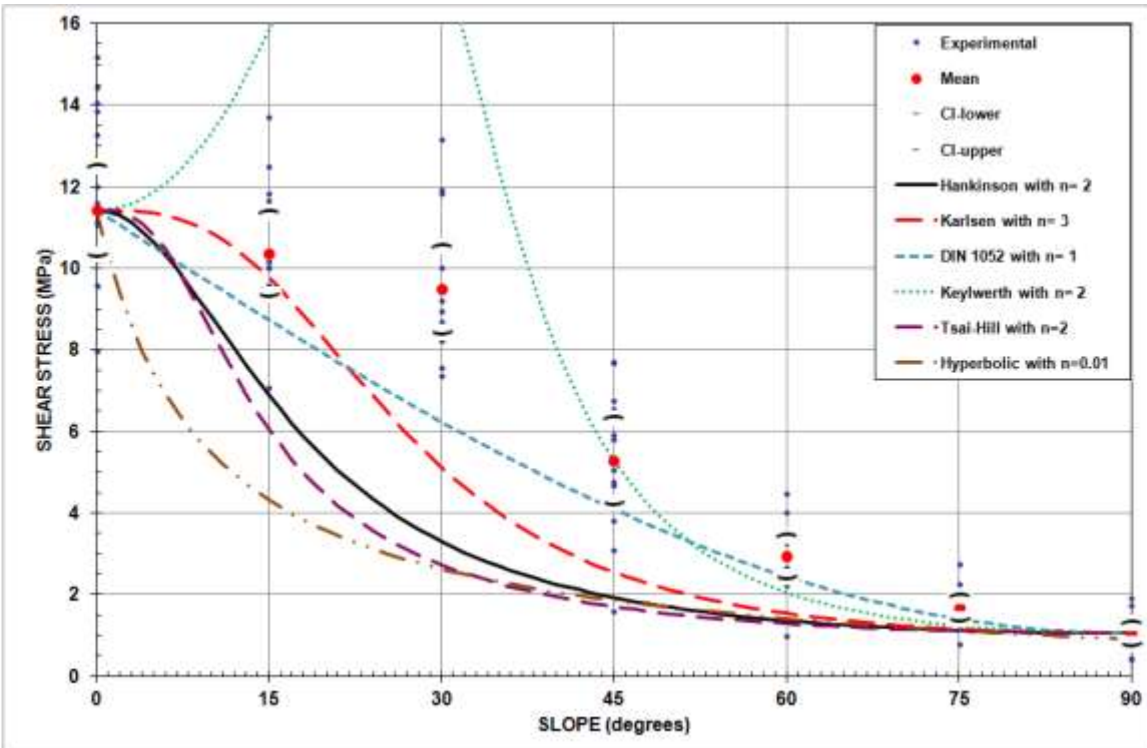

Fig. 3. Shear stress of the adhesive under tension stress relative to grain slope

Assuming the experimental results have a normal distribution, as shown by the Anderson-Darling test (p-value > 0.05) and through statistical analysis of the null hypothesis for comparison between pairs of individuals and groups of individuals, the experimental values were compared with the equivalent values calculated by (4 to 9) using variation of the exponents (n) of the trigonometric functions. Thus, the value of the n exponent which has the best fit for each equation was obtained. Table 2 presents the report of results of the null difference testing, obtained through application of the Minitab 16 program and its analysis. The shaded lines indicate the results of the null hypothesis testing carried out with application of "n" exponents suggested by the authors for compressive stress and shear stress of wood.

Upon analyzing Table 2, it may be observed that the Hankinson expression (Eq. 4) has statistical validity as long as the "n" exponent is in the interval $3.97 \leq n \leq 4.78$. The exponent that provides the best fit is $n = 4.346$. The traditionally used value of $n = 2$ does not have statistical validity and has high rejection. The values found by Logsdon *et al.* (2010) for Perobamica wood (*Aspidosmerma populifolium*), when the effect of the direction of the wood grain on wood shear strength is analyzed, also do not have statistical validity ($n = 2.05$ and interval of $1.785 \leq n \leq 2.868$).

For the Karlsen expression (Eq. 5) to have statistical validity, the "n" exponent must be within the interval of $5.60 \leq n \leq 7.63$. The exponent that provides the best fit is $n = 6.485$. The traditionally used value, recommended by Karlsen *et al.* (1967), of $n = 3$ has statistical validity; the value found by Logsdon *et al.* (2010) is different, $n = 2.13$, does not have statistical validity, and shows high rejection.

Table 2. Results of Null Difference Testing between the Experimental Values and the Equations from 4 to 9, with Variation in the "n" Exponent (Significance Level of 95%)

Null difference between the experimental results and the equations						tcr = 1.992
Eq.	n	Mean diff.	T	p	Confidence interval	Comments
Hankinson	2.00	2.326	8.55	0.000	1.785 ; 2.868	Null hypothesis rejected (significant)
	2.05	2.277	8.50	0.000	1.745 ; 2.810	Null hypothesis rejected (significant)
	3.97	0.380	1.99	0.050	-0.000 ; 0.760	Null hypothesis accepted (lower limit)
	4.346	0.000	0.00	1.000	-0.402 ; 0.402	Null hypothesis accepted – best curve fit
	4.78	-0.461	-1.98	0.051	-2.925 ; 0.003	Null hypothesis accepted (upper limit)
Karlsen	2.13	2.218	8.45	0.000	1.696 ; 2.740	Null hypothesis rejected (significant)
	3.00	1.548	7.25	0.000	1.548 ; 1.973	Null hypothesis rejected (significant)
	5.60	0.284	1.97	0.052	-0.002 ; 0.569	Null hypothesis accepted (lower limit)
	6.485	0.000	0.00	0.999	-0.284 ; 0.284	Null hypothesis accepted – best curve fit
	7.63	-0.301	-1.98	0.051	-0.604 ; 0.001	Null hypothesis accepted (upper limit)
DIN-1052	1.00	1.082	6.78	0.000	0.765 ; 1.399	Null hypothesis rejected (significant)
	1.21	0.765	5.41	0.000	0.483 ; 1.046	Null hypothesis rejected (significant)
	1.64	0.242	1.93	0.057	-0.007 ; 0.492	Null hypothesis accepted (lower limit)
	1.887	-0.000	0.00	0.999	-0.249 ; 0.249	Null hypothesis accepted – best curve fit
	2.19	-0.254	-1.96	0.053	-0.512 ; 0.004	Null hypothesis accepted (upper limit)
Keylwerth	-----	-----	----	----	-----	Discontinuity and inconsistency
	2.00	-1.80	-0.89	0.378	-5.84 ; 2.24	Null hypothesis accepted (I.C. very large)
	2.04	-0.78	-0.45	0.657	-4.27 ; 2.71	Null hypothesis accepted (I.C. very large)
	3.60	-0.262	-1.16	0.250	-0.712 ; 0.188	Null hypothesis accepted – best curve fit
	4.79	0.439	-1.99	0.050	-0.879 ; 0.000	Discontinuity and inconsistency
Tsai-Hill	2.00	1.817	7.27	0.000	1.320 ; 2.315	Null hypothesis rejected (significant)
	3.31	0.429	1.98	0.051	-0.002; 0.859	Null hypothesis accepted (lower limit)
	3.722	0.001	0.00	0.996	-0.454 ; 0.456	Null hypothesis accepted – best curve fit
	4.205	-0.543	-1.98	0.051	-1.089 ; 0.002	Null hypothesis accepted (upper limit)
Hyper- bolic	0.00239	0.478	1.97	0.052	-0.004 ; 0.960	Null hypothesis accepted (lower limit)
	0.00184	0.000	-0.00	0.999	-0.485 ; 0.484	Null hypothesis accepted – best curve fit
	0.0014	-0.501	-1.97	0.051	-1.001 ; 0.001	Null hypothesis accepted (upper limit)
	0.01	2.784	8.64	0.000	2.143 ; 3.425	Null hypothesis rejected (significant)

The expression of DIN-1052 (Eq. 6) has statistical validity as long as the "n" exponent is in the interval from $1.64 \leq n \leq 2.19$. The exponent that allows the best fit is n = 1.887. The value used by this standard, n = 1, is outside of the interval, which indicates that it does not have statistical validity. The value found by Logsdon *et al.* (2010), n = 1.21, does not have statistical validity and shows high rejection.

For the Keylwerth expression (Eq. 7), it was not possible to find the lower limit of the interval of statistical validity for the exponent of the expression. It seems that in the expression there is a singularity that allows finding another value of the exponent that provides a highly significant fit of the model. The interval is "open value" $\leq n \leq 4.79$ and the exponent that provides the best fit is n = 3.60. The traditionally used value, n = 2, recommended by Kollmann and Côté (1968), has statistical validity. The value found by (Logsdon *et al.* 2010) is also different, n = 2.04, with statistical validity, but with high average difference and very broad confidence interval (CI).

Furthermore, analyzing Table 2, it may be observed that the expression provided by Tsai-Hill theory (Eq. 8) has statistical validity as long as the "n" exponent is in the interval $3.31 \leq n \leq 4.205$. The exponent that provides the best fit is n = 3.722. The value indicated by Woodward and Minor (1988) and Sei-Chang Oh (2011) of n = 2 does not have statistical validity and shows very high rejection. The Hyperbolic formula (Eq. 9), for its part, has statistical validity as long as the "n" exponent is in the interval $17.4\text{x}10^{-4} \leq n \leq 28.3\ \text{x}10^{-4}$. The exponent that provides the best fit is $n = 22.24\ \text{x}10^{-4}$. The value indicated by Woodward and Minor (1988) and Sei-Chang Oh (2011) of $n = 100\ \text{x}10^{-4}$ does not have statistical validity and shows high rejection.

For a more robust evaluation in the choice of the best model that represents the experimental results, a one-way ANOVA was performed. In this regard, the Dunnett, Tukey and Fisher methods were used. Only the equations of DIN and Karlsen have high significance.

Finally, to visualize the variation of the shear stresses as a function of grain slope of all the equations within the confidence interval of the experimental shear stresses of the adhesive line under tension stress (significance level of 95%), a graphic representation of the shear stresses of these equations was created using the "n" exponent, which resulted in greater statistical significance (Fig. 4).

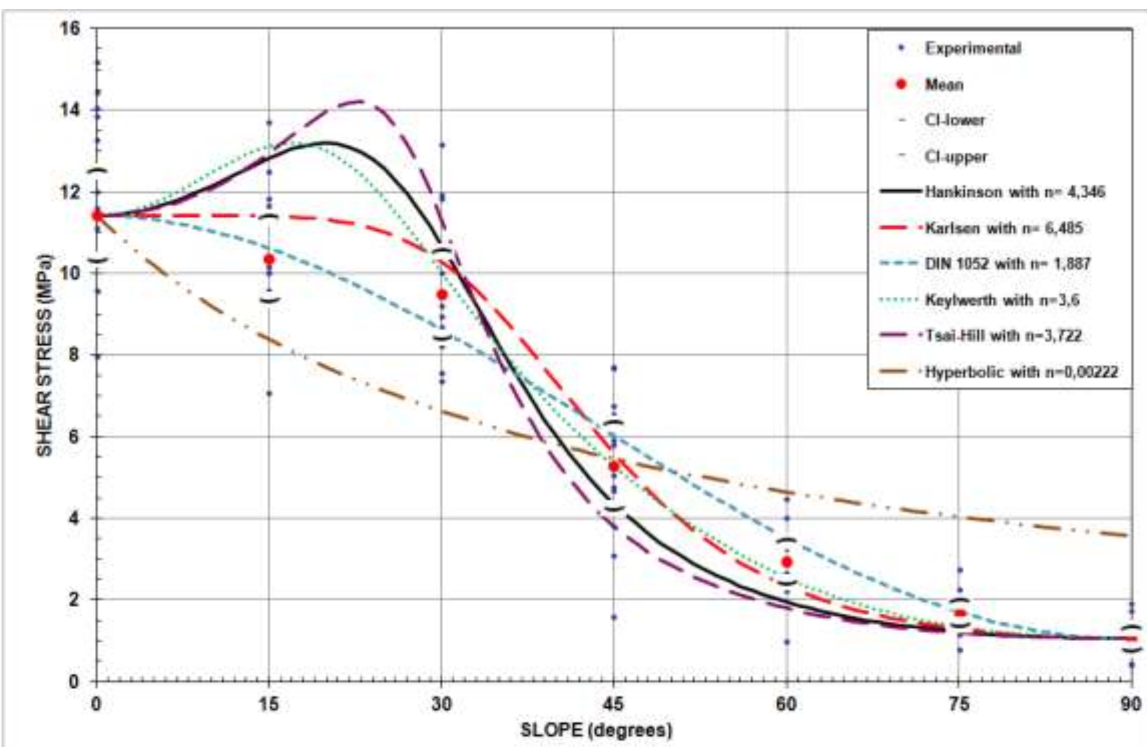

Fig. 4. Stress of the adhesive in tension stress relative to grain slope. Equations with "n" exponent that exhibited greatest statistical significance

From analysis of the graphs of Fig. 4 and taking the statistical analysis into consideration, it may be affirmed that, in order of significance, the equations that best represent the experimental values are: DIN 1052 (Eq. 11) and Karlsen (Eq. 12).

$$f_{vat,\theta} = f_{vat,0} - \left(f_{vat,0} - f_{vat,90}\right) \times \sin(\theta)^{1,887} \quad (11)$$

$$f_{vat,\theta} = \frac{f_{vat,0}}{1+\left(\frac{f_{vat,0}}{f_{vat,90}}+1\right)\times\sin(\theta)^{6,485}} \quad (12)$$

The research results will allow determination of the behavior of glued structural bonds in wood structures as a function of the slope of the wood fibers. Nevertheless, a limitation of the use of this type of bond is that tensile stresses perpendicular to the glued surface arise. These stresses must be absorbed by another type of connection mechanism, such as nails or screws.

CONCLUSIONS

1. None of the six mathematical models evaluated show statistical significance in their original format.
2. With the modifications made in the models, only the formulations of DIN 1052 and Karlsen showed significance.
3. Because of the ease of application, the models most recommended for estimating the shear strength values of the adhesive line under tension stress as a function of grain slope are the DIN 1052 and Karlsen equations, (11) and (13), respectively.

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