Stiffness Coefficients of Mortise and Tenon Joints used on Wooden Window Profiles

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Samples of corner joints of wooden rectangular windows, with widths of 78 and 92 mm, were used to determine the stiffness of tenon and mortise joints. Two series of samples were loaded statically in the angular plane of compression and tension, so that the bending moment could be derived. The objective of the experiment was to determine the existing correlations between the stiffness in maximum strength and the stiffness in the elastic area for both types of tests. After strength tests were carried out, the annual ring width of the samples was measured to determine whether this factor affects the stiffness of the joints. The results showed that there was a relatively strong correlation between the stiffness in the elastic area and the maximum load. A two-factor analysis of variance confirmed that the type of load did not affect the stiffness of the joint, but the type of joint (width) does significantly affect the stiffness. Therefore, the width of annual rings was positively correlated with the stiffness of the joints.

Keywords: Corner joint; Stiffness; Mortise and tenon; Deformation; Load

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INTRODUCTION

When assessing the strength of wooden structures, the mechanical properties of either the entire structure or just a particular joint are tested. The strength properties of individual structural joints are tested under various conditions (Atar *et al.* 2009; Oktaee *et al.* 2014; İmirzi *et al.* 2015). When the tests are carried out, a maximum force of compression or tension is applied to the joint until total failure of the joint is observed for the purpose of subsequent assessment (extension or shortening, load force, change of angle, bending moment, and stiffness of joint). Knowledge of the mechanical behaviour of the individual joints is important for assigning a proper application and dimensioning in the construction sector. The second case investigates the behaviour of the joints in the entire construction. In this case, a load that the wooden structure is actually exposed to during normal use is simulated (*e.g.* weight of a person sitting on furniture) (Eckelman and Haviarova 2006; Tas *et al.* 2014).

One of the most frequently used joints in wooden structures is the mortise and tenon joint. There are many articles dealing with the properties of the tenon and mortise in relation to the size of the tenon (Wilczyński and Warmbier 2003), the shape of the tenon (Tankut and Tankut 2005), the glue-line thickness of the bonded joints (Ratnasingam and Ioras 2013), the wood species (Kasal *et al.* 2013), the type of adhesive, and wood moisture content (Tankut 2007), as well as temperature and relative air humidity (Jivkov *et al.* 2008). As previously shown, the strength of the joints may be affected by various factors, *i.e.*, the internal conditions (type of joint, joint geometry, joint material, and/or type of adhesive), or external conditions (type of loading forces and abiotic factors). These factors are usually resolved in use for furniture purposes.

A little investigation into the mechanical properties of mortise and tenon in the application of wooden windows has been conducted. In Hrovatin *et al.* (2013), the mortise and tenon was compared with different corner joints (dowel and wooden ring) for tensile testing, but only load force was observed. Moreover, in Joščák and Kollár (2007), the bending moment, deformation, and stiffness were observed for tensile and compression testing. The profiles of windows 68 mm were used in both research projects. More recently, Pantaleo *et al.* (2014) have carried out a profile of 68 mm for tensile testing. Nowadays, the requirements for the size of windows are increasing, and there is increasing concern about the energy sufficiency of windows. For these reasons there is greater use of profile 92 mm. The objective of this experiment was to ascertain the stiffness of the mortise and tenon joints, thereby discovering the points of weakness in the structure of different window frames. Generally, these joints are critical for structural integrity, since they weaken the profile in the cross-sectional dimension. The aim of this paper is to examine the extent to which the stiffness of the maximum load correlates with the stiffness in the elastic area. The stiffness in the region is influential in the overall assessment of structural strength in a joint.

EXPERIMENTAL

Materials and Methods

The testing samples were the corner joints of casements, made from glued-laminated timber. All of the slats were made of spruce wood, in accordance with the EN 14080 testing standard (2013). The glued prisms were compiled from length-adjusted slats using finger joints. Series of completed corner joints were supplied from window manufacturers (Janošík Okna-Dveře Ltd., and Davelo Ltd., Czech Republic) in their standard profiles and dimensions, as specified in Fig. 1.

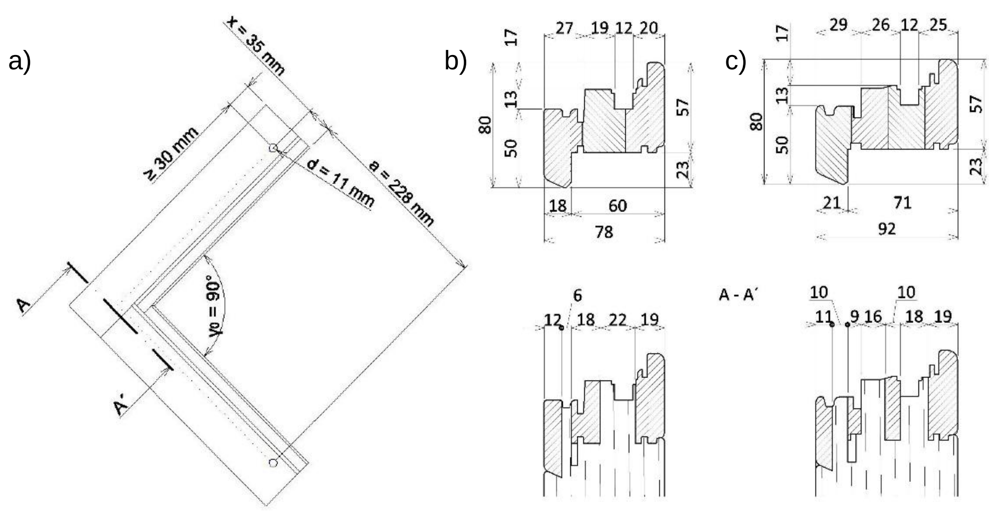

Fig. 1. a) Basic measurements of the testing samples, b) Janošík Okna-Dveře Ltd., Czech Republic, and c) Davelo Ltd., Czech Republic

The first series of tests was run on samples with a profile width of 78 mm (IV 78). Samples from IV 78 were provided from Janošík Okna-Dveře Ltd. A window profile was made from a three-layer prism, with a slat thickness of 28 mm. The second series of testing samples were made from a four-layer bonded prism, with a width of 92 mm (IV 92) and a slat thickness of 24 mm. These samples were obtained from Davelo Ltd. For the strength tests, 50 samples were supplied from each profile. For each type of stress test, 25 corner joints were available from each series. The tenon and mortise was used as a corner joint, glued with poly(vinyl acetate) adhesive (class D4, EN 204 (2001)) in manufactures standard.

The corner joints were prepared in the laboratory of Faculty of Forestry and Wood Sciences at Czech University of Life Sciences Prague. At the start of the test, the equilibrium moisture content of the samples was stabilized in a conditioning chamber (BMT Medical Technology Ltd., Czech Republic) at the environmental conditions of 20 °C (± 2 °C) and 65% (± 5%) relative humidity. After conditioning, the samples were immediately used for strength testing. The corner joints were loaded in the angular plane of compression and tension on the material testing machine, TIRA 50 kN. The testing machine was originally a UTS 50 unit; however it had been rebuilt as a TIRA 50 (TIRA system GmbH, Germany). Universal testing fixture was designed for fastening samples with different widths. The samples were secured in the fixture with drilled hole and steel pin construction (circular diameter, D = 10 mm), which was secured via a cotter pin (Fig. 2).

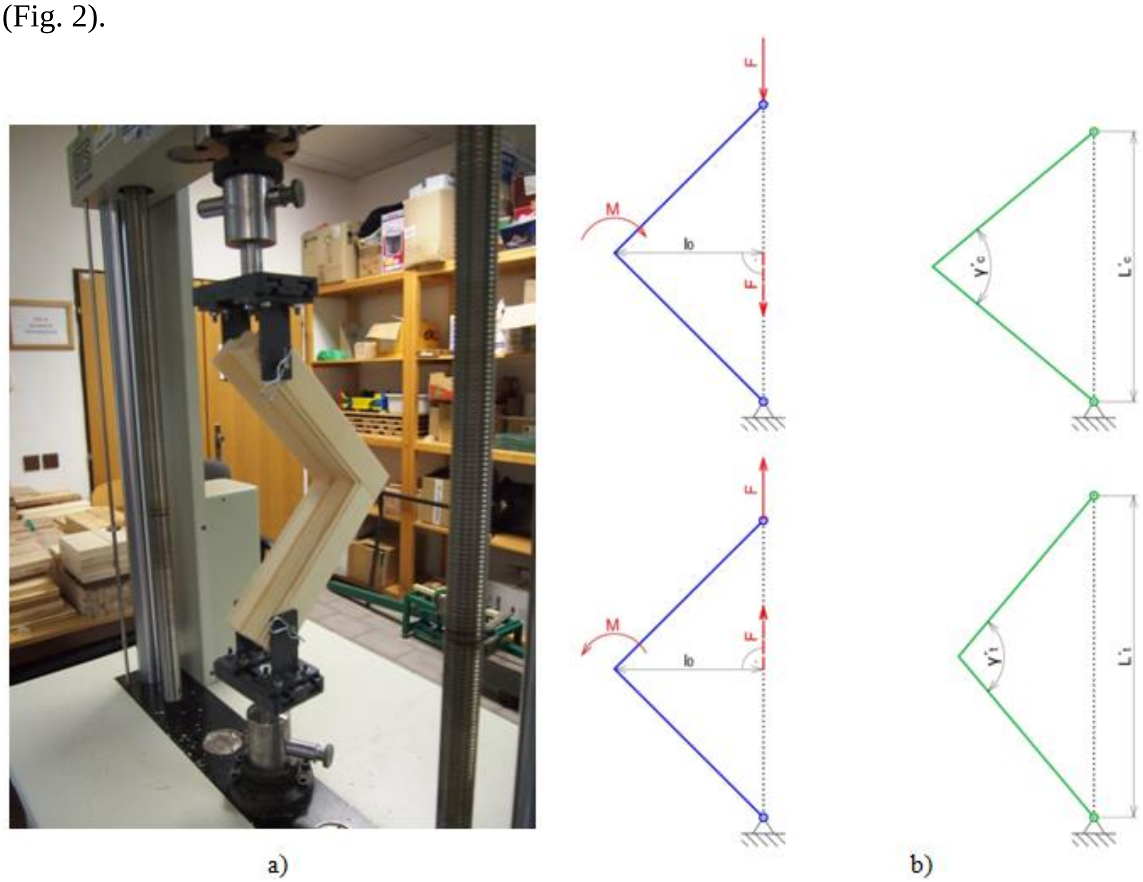

Fig. 2. a) The testing procedure and b) schematic of the bending test for compression and tensile testing

The load was applied with a crosshead speed of 5 mm/min to run the entire test in 1 to 2 min. The tests were automatically ended when the loading force was decreased by 15%. During testing, the force (F) and extension were recorded, representing the shortening at the maximum load and the elastic region, *i.e.*, at 10% and 40% (Fig. 3).

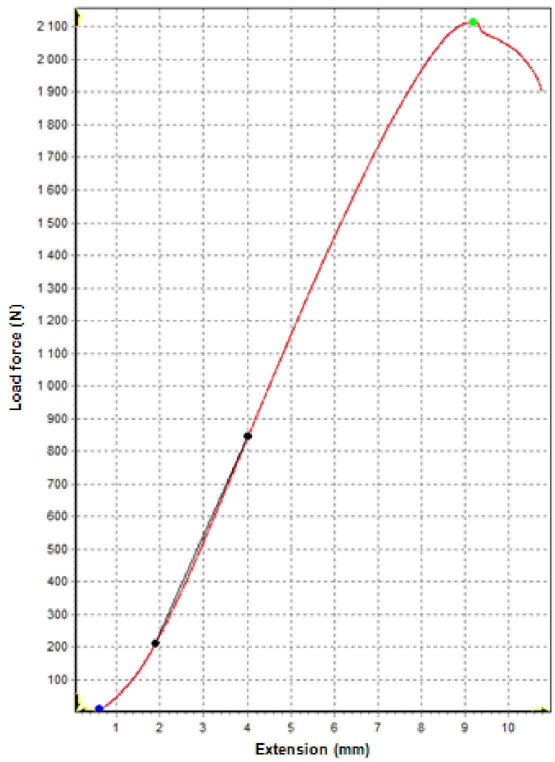

Fig. 3. Working diagram of tensile strength testing (IV 92, No. 7)

The curve on the graph in Fig. 3 shows the dependence on the load force and the extension. The loading force (F) acting on the arm (l_0) deduced the bending moments in the area of joint connections. The bending moments were calculated for maximum load (M_{max}) and for bending moment ($\Delta M_{elast.}$), which was calculated for statuses in the elastic region as a difference between M_{40} and M_{10}. These limits were stated for both testing methods because the curves of corner joint loadings exhibited linear behavior in this range as is reported in Fig. 3. During the calculations, perfectly rigid members were considered, and the effects of the bending and deformation at the location of the tenon were neglected. The creep of the joints in the fixture at the beginning of the test was reduced in the software configuration of the testing program, and the actual amount that the joint extended or contracted was recorded.

The bending moments for both window profiles and types of load were calculated according to the following Eq. 1 (Joščák and Kollár 2007; Jivkov *et al.* 2008):

$$M = F\ (N) \times\ l_0\ (m) \qquad (\mathrm{N} \times \mathrm{m}) \qquad (1)$$

The size of the arm force was a constant value (l_0 = 186 mm) for all of the samples. This size was derived from the product of the distance (a + x) and cosine of half of the original angle (γ_0 = 90°). The result of the acting force (F) changed the original distance between the pins of the fixture (L'), which resulted in a deviation in the internal angle of the joints (γ'). This was calculated using the following Eq. 2 (Podlena *et al.* 2015):

$$\gamma' = 2 \arcsin \frac{L'\ (\text{mm})}{2\ (a\ (\text{mm}) + x\ (\text{mm}))} \qquad (rad) \qquad (2)$$

The overall resulting stiffness was calculated for the elastic area (c_{elast}) and maximum stiffness of joints (c_{max}), as a proportion of the change in the corresponding moments ($\Delta M_{elast.}$, M_{max}) and the change in the relevant angles ($\Delta\gamma_{elast.}$, γ_{max}), according to Eqs. 3 and 4 (Warmbier and Wilczyński 2000):

$$c_{elast.} = \frac{\Delta M_{elast.}\ (N \times m)}{\Delta\gamma_{elast.}\ (rad)} \qquad (\text{N} \times \text{m/rad}) \qquad (3)$$

$$c_{max} = \frac{M_{max}\ (N \times m)}{\gamma_{max}\ (rad)} \qquad (\text{N} \times \text{m/rad}) \qquad (4)$$

After the tests were carried out, the average width of the annual rings, near the corner joints, was measured on the samples (Fig. 4.). Cross sections of the samples were scanned and evaluated using image analysis software (NIS Elements AR, Laboratory Imaging, Czech Republic). The average width of the annual rings was measured for each sample in pixels and converted to millimeters. In particular, the images were scanned in the resolution 600 DPI. This means that, the one pixel is equal to 0.0423 mm in the real object. The impact of this factor was evaluated together with the results of the measurement.

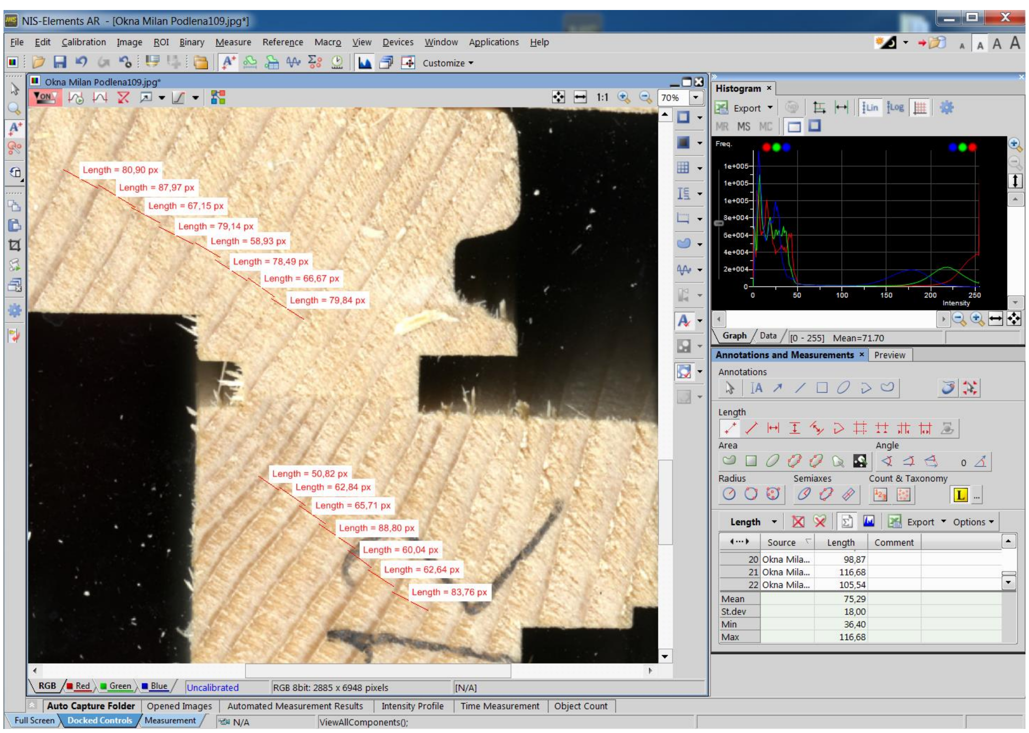

Fig. 4. Width measurement of the annual rings by software NIS-Elements AR (IV 78, No. 12)

RESULTS AND DISCUSSION

The results of the load tests are shown in Figs. 5 and 6 for tensile and compression tests of profiles IV 78 and IV 92, respectively. Figure 5 shows the results of the tensile tests, with a dependency between stiffness in the elastic area ($c_{t,\ elast}$) and stiffness at the maximum load ($c_{t,\ max}$).

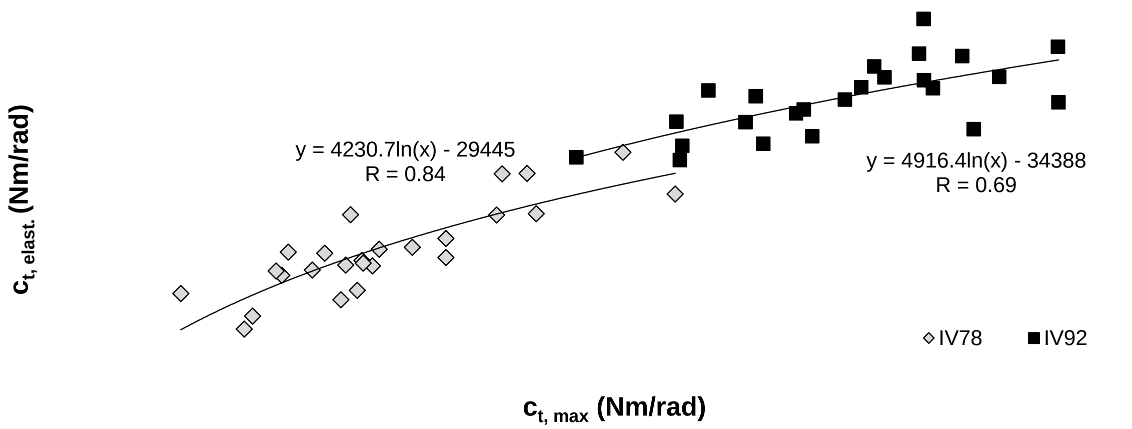

Fig. 5. Stiffness results for the tensile test

Figure 6 shows the results of compression tests, with a dependency between stiffness in the elastic area ($c_{c,\ elast.}$) and stiffness at the maximum load ($c_{c,\ max}$).

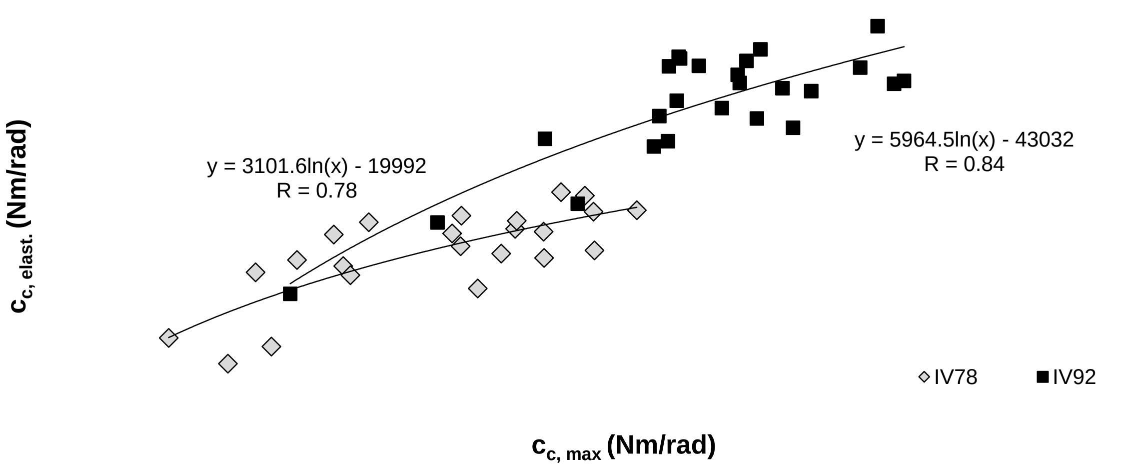

Fig. 6. Stiffness results for the compression test

Measured values were introduced from all of the tests into Eqs. 1 to 4, and from these calculations, the values for stiffness were acquired (Table 1). The statistics for the material parameters, *i.e.*, the width of annual rings for the relevant type of load and dimension of test profile, is specified in Table 2.

Table 1. Stiffness Values According to the Types of Testing Procedures and Size of Profiles

Stiffness	Tensile testing				Compression testing			
	IV 78		IV 92		IV 78		IV 92	
	Elastic area	max	Elastic area	max	Elastic area	max	Elastic area	max
Minimum (Nm/rad)	3536	2421	8053	5686	3413	2303	5034	3290
Maximum (Nm/rad)	8262	6501	11824	9666	7396	6105	11250	8276
Median (Nm/rad)	5450	3916	9712	7970	6143	4680	9809	6797
Mean (Nm/rad)	5667	4130	9680	7882	5989	4485	9377	6658
Standard deviation (Nm/rad)	1185	989	977	1055	1065	1090	1400	1124
Coefficient of variation (%)	21	24	10	13	18	24	15	17

Table 2. Width of Annual Rings According to the Type of Testing Procedures and Size of Profiles

Width of annual rings	Tensile testing		Compression testing	
	IV 78	IV 92	IV 78	IV 92
Minimum (mm)	2.1	2.1	1.9	2.2
Maximum (mm)	4.5	4.5	4.4	4.4
Median (mm)	3.1	3.1	3.2	3.2
Mean (mm)	3.2	3.2	3.1	3.1
Standard deviation (mm)	0.6	0.6	0.7	0.7
Coefficient of variation (%)	18	18	23	22

As shown in Table 1, the resulting stiffness values from comparing both the compression and tensile testing methods were very similar to each other. The differences between the average value of maximum stiffness in the compression and tensile tests for profiles IV 78 and IV 98 were 355 Nm/rad and 1224 Nm/rad, respectively. These values can be compared with the results specified by Joščák and Kollár (2007) for profile 68 mm (IV 68). Their results showed the difference 5220 Nm/rad for maximum stiffness between compression and tensile testing. Considerable differences were also found for changes in internal angles, which indirectly and proportionally affect the size of joint stiffness. As shown in Fig. 1, the depth of the joints was constant (80 mm) for used profiles, but there was a difference in used width of profiles, which is reflected in the uses of double (IV 68, IV 78) or triple mortise and tenon (IV 92). If one compares these joints, the dimension of used joints will play the most significant role. The size of bonded area was larger for IV 92 profile, which means that stiffness for triple mortise and tenon should be larger. The adhesives and their appropriate application method also had a significant influence on the strength of produced window. The stiffness of joints was shown to be affected by the width of annuals rings (Table 2) according to the same range of coefficient of variation of joint stiffness (Table 1).

The present results were unable to demonstrate which type of load on the joints exhibited the greatest effect on stiffness, as the standard deviation of joints stiffness was determined by small differences in the average values of maximum stiffness. In order to make the evaluation more relevant, the acquired results were evaluated (software STATISTICA 12) according to an analysis of variance (ANOVA) to determine the effects of the load, joint, and load and joint interaction on elastic stiffness and maximum angle change (Tables 3 and 4).

Table 3. The Effect of Load, Joint, and Load*Joint on the Elastic Stiffness*

Factor	Sum of Squares	DF	Variance	Fisher's F-test	*P*-value
Intercept	5.711585E+09	1	5.711585E+09	4160.732	< 0.01
Load	2.264197E+03	1	2.264197E+03	0.002	0.97
Joint	3.317402E+08	1	3.317402E+08	241.664	< 0.01
Load*joint	2.362323E+06	1	2.362323E+06	1.721	0.19
Error	1.276644E+08	99	1.372735E+06		

*Significance Level of 99%

Table 4. The Effect of Load, Joint, and Load*Joint on the Maximum Angle Change*

Factor	Sum of Squares	DF	Variance	Fisher's F-test	*P*-value
Intercept	637.4798	1	637.4798	3207.622	< 0.01
Load	0.5840	1	0.5840	2.938	0.09
Joint	2.7410	1	2.7410	13.792	< 0.01
Load*joint	3.7959	1	3.7959	19.100	< 0.01
Error	19.6752	99	0.1987		

*Significance level of 99%

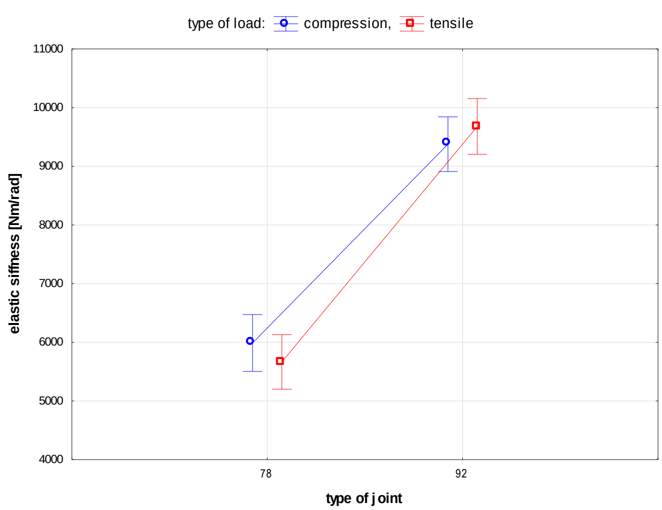

Fig. 7. Graphic depiction of the impact of the load and joint on the elastic stiffness

The type of joint (width) significantly (P < 0.01) affected the elastic stiffness of the joint; however, there was no difference detected in the type of load (P = 0.97) or the combination of load and joint (P = 0.19) on the elastic stiffness (Fig. 7). Similar results were demonstrated for the effect of the load, joint, and the load and joint interaction on the maximum angle change. In this case, the interaction effect of the type of load and the joint type (width) were statistically significant (P < 0.01).

The thresholds that were set in the elastic area reduced the probability of errors that might have occurred during this measurement. When a sample was affixed to the apparatus, a certain amount of clearance in the tenon joint was also created. A minimum of 10% threshold was attributed to the 'start-up' at the beginning of the tests, and was therefore eliminated. The upper limit of 40% was used as a standard, which is commonly used for ascertaining static modulus of wood elasticity. In this study, the focus remained on maximum stiffness and the yield strength of joints. However, the strength of the joints may vary substantially between the same type of joints and materials. Therefore, the testing was unable to detect whether a joint failed in the glued area, or in some other area. Further research is required in this area to expand the elastic stiffness database and determine the mechanical properties of other types of joints with respect to relative humidity.

CONCLUSIONS

1. For the tensile test of profile IV 78, an average of 5667 and 5989 Nm/rad were obtained for the elastic area in the tensile and the compression tests, respectively. For profile IV 92, the stiffness test resulted in an average of 9680 and 9377 Nm/rad for the elastic area in the tensile and the compression tests, respectively.
2. The correlation between elastic stiffness and stiffness at the maximum load were similar, ranging from 0.69 to 0.84 for profile IV 92 and profile IV 78.
3. A two-factor analysis of variance demonstrated a significant effect for the type of joint (joint width) on the elastic stiffness, but failed to demonstrate a statistical influence of the type of load or the combination of load and joint.
4. The variance for the average width of the annual rings, as one of the material factors that affects stiffness, ranged from 18% to 23%. The coefficient of variation for joint stiffness was approximately within the same range of values, *i.e.*, from 10% to 24%. Thus, the variability of joint stiffness is clearly influenced by the structure of the material.
5. The loading of samples within the elastic area proved to be a more relevant method. The range of the elastic area is clearly defined by the upper and lower limits. Within this region, the trajectory of forces for all types of joints will always be the same, without a permanent breach of the test samples. The results are more accurate in comparison to loading up to the ultimate strength.

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