RELIABILITY ANALYSIS OF TIMBER STRUCTURE DESIGN OF POPLAR LUMBER WITH NONDESTRUCTIVE TESTING METHODS

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The safety of timber structure design based on the predicted Modulus of Rupture (MOR) of poplar lumber with nondestructive methods is presented in this paper. Dynamic Modulus of Elasticity (MOE) of poplar lumber was measured with three different nondestructive methods, and static MOE and MOR were obtained by a static bending test. The regression relationship between various MOE and MOR was evaluated to predict MOR with various MOE. Then timber construction design was conducted on poplar lumber based on measured and predicted MOR. Furthermore, reliability of timber structure design was analyzed with advanced first-order second-moment method. Results indicated that mean values of predicted MOR were slightly greater than those of measured MOR, but Coefficient of Variation (COV) of them were less than those of measured MOR. The reliability index of timber structure design based on predicted MOR, varying from 2.404 to 2.574, was less than that on measured MOR as 2.831.

Keywords: Fast-growing timber; Reliability analysis; Nondestructive testing; Performance prediction; Timber structure design

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INTRODUCTION

Poplar is one of the most important fast-growing species in China, and the proportion of poplar in wood products market has been increasing. With a great deal of fast-growing poplar planted in China, it becomes increasingly important to investigate the mechanical properties of fast-growing poplar and extend its range of application. In addition, technological advances in nondestructive testing has allowed for lumber to be graded on the basis of its mechanical properties, rather than visual grading alone (Hernandez et al. 1996; Green et al. 1994).

Non Destructive Evaluation (NDE) techniques have been used for sorting and grading wood products over the last few decades (Nzokou et al. 2006). If the elasticity and strength of wood for structural uses can be estimated nondestructively with high accuracy by applying a small deformation or vibration to a wood sample, the confidence of wood for structural uses will increase; moreover, various woods can be more appropriately utilized according to their elasticity and strength. Therefore, it is useful for the forest industry that a wood's elasticity and strength be easily estimated with high accuracy by non-destructive testing (Yang et al. 2002). Many studies have been conducted to investigate the dynamic properties of different wood products and predict the MOR of wood products by dynamic results, but relatively little research on reliability analysis of this prediction have been reported (Hu and Afzal 2006; Lin et al. 2006; Sun and Arima 1999). As the recent reliability-based design of timber structure has been put into practice, it becomes increasingly important to analyze the reliability of MOR prediction by dynamic MOE, which has practical significance on design and assembly of timber construction. Especially, after Standardization Administration of China published the latest Code for design of timber structures, there was an instant need to conduct the reliability assessment of timber structure design based on the nondestructive results.

The objectives of the study were to predict MOR of poplar lumber with dynamic MOE, analyze the reliability of timber construction design based on measured MOR, and predicted MOR of poplar lumber, and compare the reliabilities of them. Through the above analysis and comparison, we could investigate the safety of nondestructive methods when being used in actual mechanics performance measurement of structural lumber. Dynamic MOE of poplar lumber were measured by longitudinal vibration, flexural vibration (Hu 2004), and longitudinal transmission test (Hu et al. 2005a,b).

EXPERIMENTAL

Materials

The experimental material used in this study was poplar (*Populus tomentosa* Carr.) lumber. A special emphasis was put on the selection of the wood material. Accordingly, non-deficient, proper, knotless, and normally grown (without zone line, reaction wood, decay, insect and fungal damages) wood samples were selected. Besides, the samples were taken as far from the pith as possible to reduce the influence of the curvature of the annual rings (Serrano 2000), as shown in Fig. 1. The timber was conditioned in a room environment for a year, and the final average moisture content was 8.3%, which was measured by weighing and was close to the local equilibrium moisture content of wood.

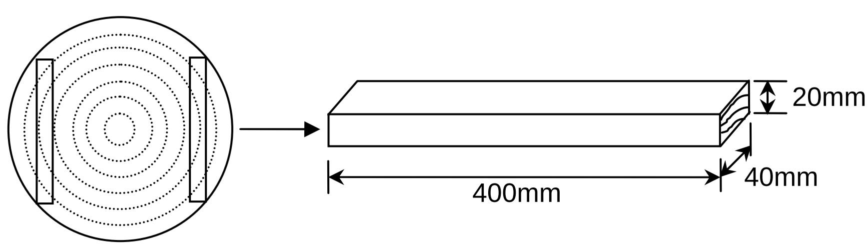


Fig. 1 Experimental materials

Nondestructive Testing Methods

Longitudinal vibration, flexural vibration, and longitudinal transmission tests were conducted on poplar samples in a room maintained at 20°C and 65% relative humidity.

The experiment was first carried out with the longitudinal transmission method as shown in Fig. 2. In the test, the sound transmission time propagating through the specimen was measured with a fast Fourier transform (FFT) analyzer. The sound velocity and dynamic MOE were calculated based on Eqs. 1 and 2 (Hu 2004),

$$V=L_s/T \tag{1}$$

$$E_v=\rho V^2 \tag{2}$$

where V is sound velocity; L_s is length of the specimen; T is transmission time; E_v is dynamic MOE; and ρ is density of the specimen.

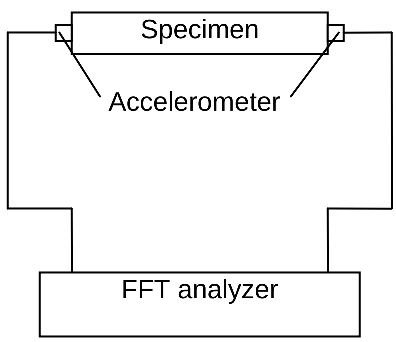


Fig. 2 Longitudinal transmission test

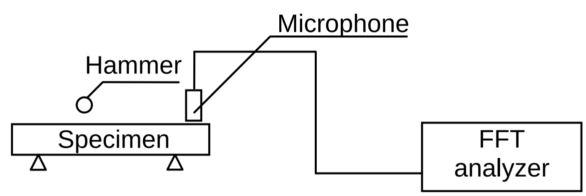


Fig. 3 Flexural vibration test

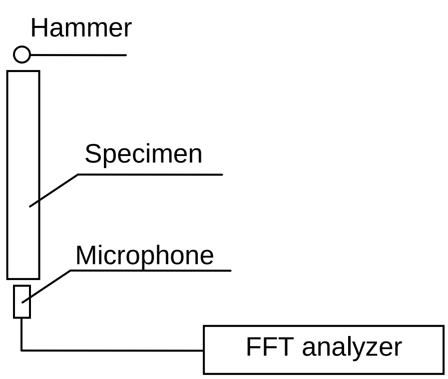


Fig. 4 Longitudinal vibration test

The experiment was then carried out with the flexural vibration method as shown in Fig. 3. In the test, specimens in the freely vibrating free-free beam test were supported by two planks. The supporting positions of the planks were 0.224L_s (length of specimen) from both ends. This position corresponds to the nodal points for the fundamental mode of this vibration system. The vibrating frequency was detected by a high-sensitivity microphone connected to a FFT analyzer. The resonant frequencies of the 1st, 2nd, 3rd, and 4th modes were obtained by giving a blow to an edge of the beam and recording the results with the FFT analyzer. The dynamic MOE was obtained from Timoshenko-

Goens-Hearmon (TGH) flexural vibration method including the influence of shear and rotatory inertia (Hu 2004).

The experiment was last carried out with the longitudinal vibration method, as shown in Fig. 4. In the test, the specimen was held lightly by the fingers at the center of the specimen while they were tapped by a small hammer at the end of the specimen. The tap tone was detected by a microphone at the other end of the beam. The resonance frequencies of the tap tone were identified by a FFT analyzer. The dynamic MOE of free-free longitudinal vibration E_p was calculated by Eq. 3 (Hu 2004).

$$E_p = \rho\left(\frac{2L_s f_n}{n}\right)^2,\ n = 1,\ 2,\ 3...... \tag{3}$$

where E_p is dynamic MOE of specimen; L_s is length of the specimen; f_n is resonance frequency.

Static Testing Methods

A static bending test was conducted on poplar samples in a room maintained at 20°C and 65% relative humidity.

Static MOE and MOR of lumber in bending were measured according to the EN 408 standard by using a 100 kN capacity universal test machine and applying a loading speed of 10 mm/min to reach the maximum load within 300±120 s. The static MOE in bending E_b and MOR in bending f_b was calculated based on Eqs. 4 and 5,

$$E_b = 1242(P_1 - P_2)/[b(y_1 - y_2)] \tag{4}$$

$$f_b = aF_{max}/(2W) \tag{5}$$

where E_b is static MOE; P_1-P_2 is an increment of load on the regression line; y_1-y_2 is the increment of deformation corresponding to P_1-P_2, b is the width of specimen; f_b is static MOR; a is distance between a loading position and the nearest support in a bending test; F_{max} is maximum load; and W is section modulus.

Procedure

There were two groups of poplar lumber (400×40×20mm), Group 1 and Group 2, with 48 samples in each group conducted in this experiment.

In Group 1, dynamic MOE of samples were measured by longitudinal transmission, longitudinal vibration, and flexural vibration test. Then the static MOE and MOR of them were measured by static bending test. The prediction model of MOR based on various MOE was established through regression analysis on various MOE and MOR.

In Group 2, mechanics properties of samples were also measured by the above methods. And predicted MOR of them was calculated based on various MOE according to the above prediction model. In accordance with the current National Standards of China, timber structure design was conducted based on predicted and measured MOR of samples, respectively. Furthermore, reliabilities of timber construction design based on measured and predicted MOR were analyzed with advanced first-order second-moment method in respective.

RESULTS AND DISCUSSION

Results of Mechanical Tests and Prediction of MOR

Results of dynamic and static tests of poplar lumber of Group 1 are shown in Table 1. The symbols E_v, E_f, E_p, and E_b stand for MOE obtained by longitudinal transmission, flexural vibration, longitudinal vibration, and static bending tests, respectively. The table indicated that MOE of the three dynamic tests were higher than those of static test, and the COV of dynamic MOE was larger than static MOE.

Table 1. Mechanical Test Results of Poplar Lumber of Group 1

Parameter	E_v	E_f	E_p	E_b	MOR
Mean Value	10.845GPa	10.859GPa	11.128GPa	9.973GPa	78.147MPa
Standard Deviation	1.561GPa	1.112GPa	1.128GPa	0.864GPa	7.163MPa
COV	14.397%	10.244%	10.137%	8.666%	9.166%

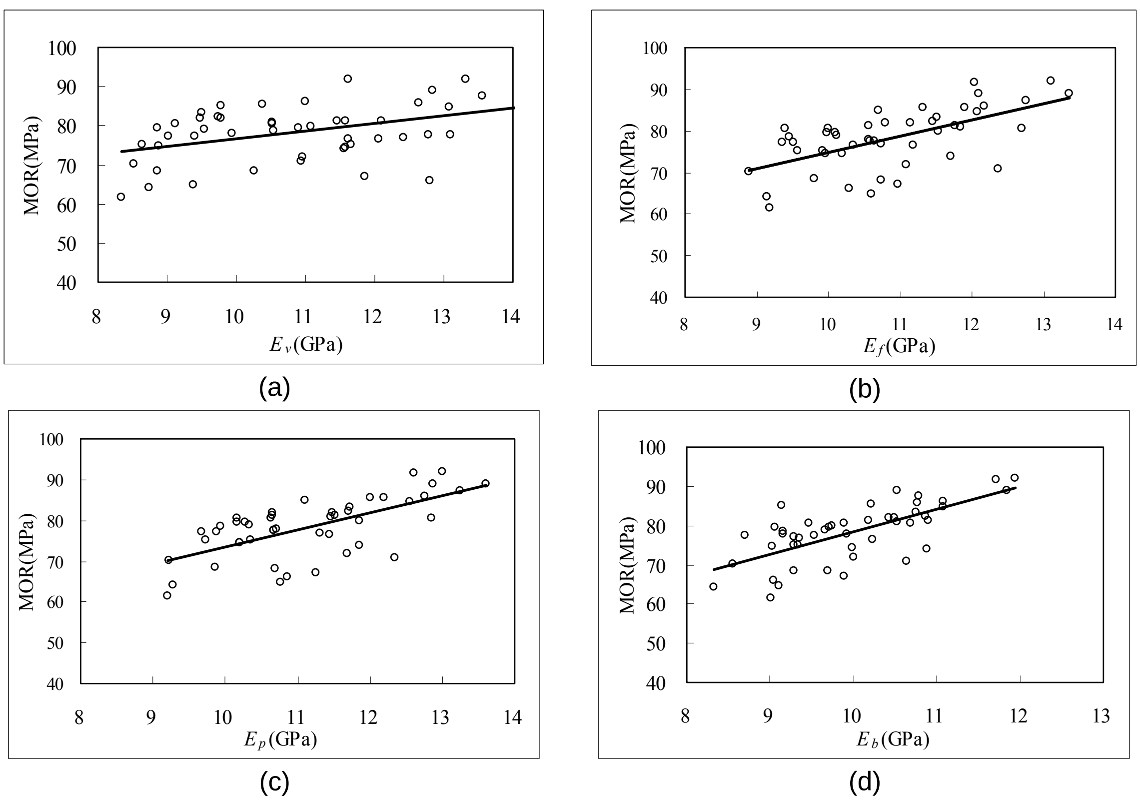


Fig. 5. Linear regression correlation between various MOE and MOR of Poplar Lumber, Group 1

To obtain the relationship between various MOE and MOR, regression analysis was conducted on mechanical test result of poplar lumber of Group 1. The regression correlation between various MOE and MOR are shown in Fig. 5, and the linear regression formula and regression coefficient are presented in Table 2. Results indicated that all the regression coefficients were much greater than $R_{46,0.01}$ = 0.369, and the regression coefficients between E_f, E_p, E_b and MOR were greater than $R_{46,0.001}$ = 0.461 in particular. Consequently, there was a strong linear correlation between various MOE and MOR.

Table 2. Linear Regression Formula and Regression Coefficient between Various MOE and MOR of Poplar Lumber of Group 1

Parameter	Linear Regression Formula	R
E_v	MOR = 1.960E_v + 56.887	0.427
E_f	MOR = 3.931 E_f + 35.457	0.611
E_p	MOR = 4.160 E_p+ 31.870	0.655
E_b	MOR = 5.738 E_b + 20.939	0.692

MOR of poplar lumber of Group 2 was predicted based on the above prediction model based on the regression relationship between various MOE and MOR, and results of measured and predicted MOR are presented in Table 3. MOR_2 is the measured MOR of poplar lumber of Group 2. MOR_v, MOR_f, MOR_p and MOR_b stand for MOR predictions for the corresponding samples according to the value and formula of E_v, E_f, E_p and E_b, respectively. Results show that mean values of predicted MOR were slightly greater than those of measured MOR, but the COV values of predicted MOR were less than those of measured MOR. Consequently, load-carrying capacity of poplar lumber was overestimated when MOR of poplar lumber was predicted based on the linear correlation between various MOE and MOR.

Table 3. Predicted and Measured MOR of Poplar Lumber of Group 2

Statistical Parameter	MOR_2	MOR_v	MOR_f	MOR_p	MOR_b
Mean Value(MPa)	73.757	80.090	77.651	77.478	76.313
Standard Deviation(MPa)	8.529	3.643	4.888	5.291	4.822
COV/%	11.564	4.548	6.295	6.829	6.318

Timber Structure Design

Timber structure design was conducted according to the National Standards of China: GB 50005-2403 Code for design of timber structures; GB 50068-2001 Unified standard for reliability design of building structure. The characteristic value of bending strength of poplar lumber was calculated by Eqs. 6 and 7 (Mo 2006). The poplar lumber was assumed to used in the condition as follows: all of poplar lumber were used in hinged truss as a beam; the constant load on the beam followed normal distribution, and COV of load was 0.07 (Mo 2006); design service life of this structure was 50 years; and the adjusting factor for structural significance was 1.1,

$$f_k = \mu_f(1 - u_a\delta_f) \tag{6}$$

$$f_m = f_k\gamma_1\gamma_2\gamma_3/(\gamma_f\gamma_0) \tag{7}$$

where f_k is standard value of poplar lumber strength; μ_f is mean value of poplar lumber strength, which equals the above-mentioned mean value of MOR of poplar lumber (MOR_2, MOR_v, MOR_f, MOR_p and MOR_b); δ_f is the COV of poplar lumber strength, which equals to the above-mentioned COV of MOR of poplar lumber (MOR_2, MOR_v, MOR_f, MOR_p and MOR_b); u_a is factor determined according to distribution style of poplar lumber strength, which equals 1.645; f_m is characteristic value of bending strength of poplar lumber, γ_f is subentry factor of poplar lumber strength, which equals 4.2; γ_0 is an adjusting factor for structural significance, which equals to 1.1; γ_1 is an adjusting factor for service condition, which equals 0.8; γ_2 is an adjusting factor for service life, which equals to 1.0; and γ_3 is an adjusting factor for obtaining the characteristic value from standard value, which equals to 1.06.

The checking computation of load carrying capacity of poplar lumber beam was conducted according to Eq. 8 (Mo 2006), when the beam was bearing the pure bending load. Obviously, the equation indicated that σ_m was the largest stress that the poplar lumber could bear when it equaled to f_m. The largest characteristic values of bending stress of poplar lumber are shown in Table 4. The symbols σ_m, $\sigma_{m,v}$, $\sigma_{m,f}$, $\sigma_{m,p}$, and $\sigma_{m,b}$ stand for the largest characteristic values of bending stress of poplar lumber based on the mean value and standard deviation of MOR_2, MOR_v, MOR_f, MOR_p, and MOR_b, respectively,

$$\sigma_m = \frac{M}{W_n} \leq f_m \tag{8}$$

where σ_m is characteristic value of bending stress of poplar lumber, M is characteristic value of bending moment of poplar lumber, W_n is cross section moment of poplar lumber and f_m is characteristic value of bending strength of poplar lumber.

Table 4. Largest Characteristic Values of Bending Stress of Poplar Lumber

Statistical Parameter	σ_m	$\sigma_{m,v}$	$\sigma_{m,f}$	$\sigma_{m,p}$	$\sigma_{m,b}$
Mean Value(MPa)	10.963	13.601	12.777	12.623	12.551
Standard Deviation(MPa)	0.767	0.952	0.894	0.884	0.879
COV	0.070	0.070	0.070	0.070	0.070

Reliability Analysis

In reliability analysis of timber structure design, the resistance stress of poplar lumber was calculated by Eqs. 9, 10, and 11 (Wang 2002). The statistical parameters of adjusting factors are shown in Table 5.

$$f_r = Kf \tag{9}$$

$$K = K_p K_{q1} K_{q2} K_{q3} K_{q4} \tag{10}$$

$$\delta_K = \sqrt{\delta_p^2 + \delta_{q1}^2 + \delta_{q2}^2 + \delta_{q3}^2 + \delta_{q4}^2} \tag{11}$$

In these equations, f_r is resistance stress of poplar lumber; K is an adjusting factor for changing f into f_r; f is bending strength of clear wood, and its mean value and standard deviation is equal to those of above-mentioned MOR of poplar lumber (MOR_2); K_p is an adjusting factor for equation precision; K_{q1} is an adjusting factor for natural defect of wood; K_{q2} is an adjusting factor for drying defects in wood; K_{q3} is an adjusting factor for effect of long-term load on wood; and K_{q4} is an adjusting factor for dimensions of wood.

Table 5. Statistical Parameters of Adjusting Factors

Statistical Parameter	K_p	K_{q1}	K_{q2}	K_{q3}	K_{q4}
Mean Value	1.000	0.750	0.850	0.720	0.890
Standard Deviation	0.050	0.120	0.034	0.086	0.053
COV	0.050	0.160	0.040	0.120	0.060

The limit state function of poplar lumber beam, g (f, K, σ_m), was described by Eq. 12. The results of f, K, and σ_m were assumed to be subject to normal distribution; thus, advanced first-order second-moment method can be used to calculate the reliability index. Without considering variability of other random variables and inaccuracy of the formula, reliability index and reliability of the poplar lumber beam can be calculated by Eqs. 13, 14, and 15 (Wu 2005). The reliability index was obtained through several iterative calculations of Eqs. 13 and 14. The initial values of f^*, K^* and $\sigma_m{}^*$ were mean values of f, K, and σ_m, and the iterative calculation was continued until $g(f^*, K^*, \sigma_m{}^*) = 0$.

$$g(f, K, \sigma_m) = fK - \sigma_m = 0 \tag{12}$$

$$\alpha_i = \frac{\sigma_{X_i}\left(\frac{\partial g}{\partial X_i}\right)_{X^*}}{\sqrt{\sum_{i=1}^{3}\left(\sigma_{X_i}\frac{\partial g}{\partial X_i}\right)_{X^*}}}, i=1,2,3 \tag{13}$$

$$X_i^* = m_{X_i} - \alpha_i \beta \sigma_{X_i}, i=1,2,3 \tag{14}$$

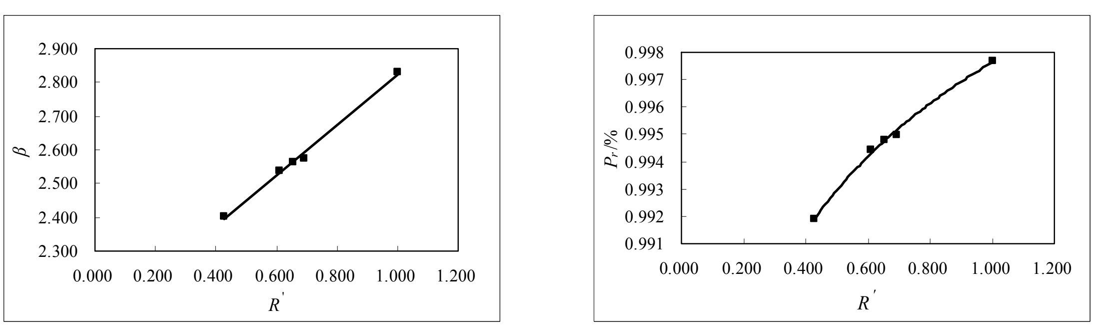
$$P_r = \Phi(\beta) \tag{15}$$

In the equations above, α_i is the sensitivity factor of random variable; X_i is random variable, when i is 1, 2, 3, X_i stand for f, K and σ_m, respectively; m_{Xi} is the mean value of random variable; σ_{Xi} is the standard deviation of random variable; and X^* is figure point.

The results for the reliability index and reliability of the poplar lumber beam are shown in Table 6. The largest were obtained from measured MOR of bending test as 2.831. And the reliability index of poplar lumber beam based on predicted MOR with longitudinal transmission test results were the least of all, as 2.404, which was 15% lower than the reliability index of poplar lumber beam based on measured MOR. The reliability indices of poplar lumber beam based on other predicted MOR were about 10% lower than that on measured MOR, and larger than 2.500. This showed that timber structure design based on predicted MOR with MOE of static bending, flexural vibration and longitudinal vibration test were nearly under the same safety level. Meanwhile, it was less safe to carry out timber structure design based on predicted MOR with longitudinal transmission test, comparing with the prediction with other three tests.

Comparing the regression coefficient R' (Table 2) between different MOE and MOR with the values of reliability index β and reliability P_r, it could be found that the values of β and P_r increased with the increase of R', and there could be regression correlations between R' and β, R' and P_r. From the regression analysis between R' and β, R' and P_r, the following regression formulas were obtained: $\beta = 0.746R' + 2.077$, $R = 0.997 > R_{3,0.001}=0.991$ for β and R'; $P_r = 0.0068\text{Ln}(R') + 0.998$, $R = 0.999 > R_{3,0.001}=0.991$ for P_r and R'. Thus, there were strong regression correlations between R' and β, P_r for various tests, as shown in Fig. 6. Besides, results also showed that it was more reliable to carry on timber structure design based on predicted MOR with various test results when R' was larger than regression coefficient of 0.1-percentile ($R_{\text{n}, 0.001}$). And the predicted MOR should be adjusted to satisfy the requirement of reliability, before the timber structure design was conducted.

The present study analyzed only differences between the reliability of timber structure design based on predicted MOR and that on measured MOR. Also, the size and quality of the timber was different from commercial timber. Further study will focus on how to adjust the predicted MOR with nondestructive methods to ensure that the safety of timber structure design based on predicted MOR is same as that on measured MOR. And the effect of the size and quality of the timber on the application safety of the nondestructive testing would also be investigated in later research.



(a) (b)

Fig. 6. Regression correlation of R' with reliability index and reliability of the poplar lumber beam

Table 6. Results of Reliability Index and Reliability of the Poplar Lumber Beam

Limit State Function	$g(f,K,\sigma_m)=0$	$g(f,K,\sigma_{m,v})=0$	$g(f,K,\sigma_{m,f})=0$	$g(f,K,\sigma_{m,p})=0$	$g(f, K,\sigma_{m,b})=0$
Reliability Index	2.831	2.404	2.538	2.562	2.574
Reliability/%	99.768	99.189	99.443	99.480	99.497

CONCLUSIONS

1. There were strong linear correlations between different MOE and MOR of poplar lumber. Load-carrying capacity of poplar lumber was overestimated in the prediction of MOR based on these linear correlations.
2. Reliability of timber structure design based on predicted MOR poplar lumber was less than that on measured MOR.
3. The results showed that it was less safe to carry out timber structure design based on predicted MOR with a longitudinal transmission method, comparing with the prediction with other three methods.
4. It was suggested that timber structure design could be carried out with nondestructive methods when regression coefficient between various MOE and MOR was larger than that of 0.1-percentile. And the predicted MOR should be adjusted to satisfy the requirement of reliability, before the timber structure design was conducted.

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