# RELIABILITY ANALYSIS OF WOOD-PLASTIC PLANKS BASED ON PREDICTED MODULUS OF RUPTURE

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The safety of wood-plastic planks based on predicted modulus of rupture (MOR) is presented in this paper. Three different nondestructive testing (NDT) methods were used as checking tools for dynamic modulus of elasticity (MOE) of wood-plastic planks. The MOR was determined by a three-point bending test. The regression relationship between various dynamic MOE and MOR was evaluated to predict MOR of other identical wood-plastic planks. Furthermore, improved first-order second-moment (FOSM) method was used to analyze reliabilities based on measured and predicted MOR, and evaluate safety of them in service. Results indicated that reliabilities of other identical wood-plastic planks based on predicted and measured MOR were almost the same. The greatest difference between them was 0.01%; therefore, their reliability could be analyzed by predicted MOR.

*Keywords: Wood-plastic plank; Modulus of elasticity; Modulus of rupture; Reliability analysis*

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## INTRODUCTION

Wood plastic composites (WPCs) have great application value for their out-standing advantages in environmental protection, recycling, and economic factors (Yeh *et al.* 2009; Thompson *et al.* 2010). WPCs have dual characteristics of wood and plastic, and their performance index can be even better than either wood or plastic by itself. In comparison to wood, WPCs are isotropic, resistant to weather, dimensionally stable, hard, wear-resistant, and many other features. In comparison to plastics, WPCs are better suited to wood processing methods, and their surfaces are easier to decorate, by such means as printing, paint spraying, and coating (Ouyang 2009). Similarly, WPCs have high economic and social benefits (Gao *et al*. 2009).

Non-destructive testing (NDT) methods have been used for sorting and grading wood products over the last few decades (Hu 2004; Hu *et al.* 2005a,b). If the elasticity and strength of wood-plastic planks can be estimated nondestructively with high accuracy by applying a small deformation or vibration, the confidence of them for structural uses will be increased. Many studies have been conducted to investigate the dynamic properties of different wood products and to predict the MOR of wood products by dynamic results. However, relatively little research on reliability analysis of such predictions have been reported (Hu and Afzal 2006). It is becoming increasingly important to analyze the reliability of MOR prediction by dynamic MOE (Cheng and Hu 2011).

Wood-plastic planks are widely used in wood-plastic pallets. Thus, it is necessary to study the reliability of wood-plastic planks used in the surfaces of pallets. Reliability research mainly has focused on metals, ceramics, and wood products (Zhang *et al.* 2007; Zhou *et al.* 2004; Cheng and Hu 2011). The reliability analysis methods employed for wood products have been the first-order second-moment (FOSM) and the improved FOSM method. The calculation accuracy of the improved FOSM method is higher than that of the FOSM method, and random loads withstood by wood-plastic planks are subject to normal distribution, to meet the requirements of the improved FOSM method. Thus, the improved FOSM method is used to analyze reliability of wood-plastic planks.

The objectives of this study was to establish prediction models of dynamic MOE and MOR of wood-plastic planks, to predict MOR of other identical wood-plastic planks with dynamic MOE, and to analyze the reliabilities based on measured and predicted MOR. Through the above analysis and comparison, appropriate NDT methods would be found to predict MOR of other identical wood-plastic planks and evaluate the safety performance of them in service, eventually achieving the purpose of saving resources. Dynamic MOE of wood-plastic planks were measured by longitudinal transmission (Hearmon 1961), longitudinal vibration (Chimoshienko 1956) and flexural vibration test (Hearmon 1961; Timoshenko 1921); the MOR values were measured by a three-point bending test (ASTM 2003).

## EXPERIMENTAL

### Materials

All test material used in the study was bought from Sanlite Company in Zouping, Shandong, China. The percentages of wood flour and polyethylene (PE) were 55% and 45%, respectively, and the density of wood-plastic structural planks was 1300 kg/m$^3$. The material was cut into specimens in the Key Laboratory of Bio-based Material Science and Technology of Ministry of Chinese Education; the dimensions of specimens were 516 mm × 105 mm × 26 mm.

All specimens were placed in a thermostatic chamber, in which the temperature was 20±2˚C and the relative humidity was 65±5%, until the weight of all wood-plastic structural planks remained constant, for the purpose of homogenization of moisture by volume before the experiments.

### Nondestructive Testing Methods

The dynamic MOE ($E_V$) was carried out using the longitudinal transmission method. In this test, the sound transmission time propagating through a specimen was measured with a fast Fourier transform (FFT) analyzer. The sound velocity and dynamic MOE were calculated based on Eqs. 1 and 2 (Hearmon 1961),

$$V = l/T \tag{1}$$

$$E_V = \rho V^2 \tag{2}$$

where $V$ is sound velocity; $l$ is the length of the specimen; $T$ is the transmission time; $E_V$ is dynamic MOE; and $\rho$ is the density of the specimen.

The dynamic MOE ($E_P$) was obtained by using the longitudinal vibration method. In this test, a specimen was held lightly by the fingers at the center while it was tapped by a small hammer at an end. The tap tone was detected with a microphone at the other end, and the resonance frequencies of tap tone were identified by a FFT analyzer. The dynamic MOE $E_P$ was calculated using Eq. 3 (Chimosiento 1956),

$$E_p = \rho\left(\frac{2lf_n}{n}\right)^2 \quad n = 1.2.3...... \tag{3}$$

where $E_P$ is the dynamic MOE; $\rho$ is the density of the specimen; $l$ is the length of the specimen; and $f_n$ is the frequency of the sound wave.

The dynamic MOE ($E_f$) was evaluated by using the flexural vibration method. Two strings supported a specimen, the supporting positions of the strings were 0.224 L (length of specimen) from both ends, and this position corresponded to the nodal points for the fundamental mode of this vibration system. A high-sensitivity microphone was connected to an amplifier, and a FFT analyzer detected the vibrating frequency. The dynamic MOE was obtained from the Timoshenko-Goens-Hearmon (TGH) flexural vibration method including the influence of shear and rotatory inertia.

## Static Testing Methods

There were a lot of random variables impacting on the performance of the specimen. Some of these variables could be measured, while some could not, and they could be speculated only via experimental analysis (Xue and Hu 2009; Chen and Soares 2007; Yu 2011).

The MOR of specimens were measured by a three-point bending test with a 100 kN capacity universal test machine and an applied loading speed of 10 mm/min to reach the maximum load. The MOR in bending $f_r$ was calculated based on Eq. 4(ASTM 2003),

$$f_r = aF_{\max}/(2W) \tag{4}$$

where $f_r$ is the static modulus of rupture (MOR); $a$ is distance between a loading position and the nearest support in a bending test; $F_{max}$ is maximum load; and $W$ is the section modulus.

## Procedure

There were two groups of identical specimens: Group 1 and Group 2. There were 20 specimens in each group.

In Group 1, the dynamic MOE of specimens were measured by longitudinal transmission, longitudinal vibration, and flexural vibration tests. The three-points bending test, in accordance with the ASTM D790-03, measured the MOR of specimens of Group 1. The prediction models of MOR based on various MOE were obtained from relativity analysis, respectively.

In Group 2, the dynamic and static properties of specimens were also measured by the above dynamic and static test methods, and predicted MOR was calculated based on various dynamic MOE of specimens of Group 2 according to the previously mentioned prediction models. Furthermore, reliabilities based on measured and predicted MOR were analyzed with the improved FOSM method.

## RESULTS AND DISCUSSION

### Results for Mechanical Test of Group 1

The mean values of the dynamic MOE and MOR are shown in Table 1. According to Table 1, the symbols $E_V$, $E_p$, and $E_f$ stand for dynamic MOE obtained by longitudinal vibration, longitudinal transmission, and flexural vibration tests, respectively. The $E_V$ was the greatest in all dynamic MOE, and $E_f$ was in the middle of $E_V$ and $E_p$. The variation coefficient of dynamic MOE was almost the same.

**Table 1.** Mechanical Test Results for Wood-plastics Planks of Group 1

| Parameter | $E_V$ | $E_p$ | $E_f$ | MOR |
|---|---|---|---|---|
| Mean value | 6.318 GPa | 4.572 GPa | 4.576 GPa | 19.199 MPa |
| Standard deviation | 0.091 GPa | 0.067 GPa | 0.069 GPa | 0.201 MPa |
| Variation coefficient (%) | 1.43 | 1.47 | 1.51 | 1.05 |

### Regression Formula for Dynamic MOE and MOR of Group 1

The regression curve between $E_V$ and MOR is shown in Fig. 1. From the regression analysis between $E_V$ and MOR, the following linear regression formula was obtained: MOR = 2.0611$E_V$ + 6.1809, $R_V$ = 0.9197 >$R_{18,0.02}$ = 0.516 ($R_V$ is the correlation coefficient, $R_{18,\,0.02}$ is a standard correlation coefficient selected according to the number of specimens and accuracy class); therefore there was a strong correlation between $E_V$ and MOR.

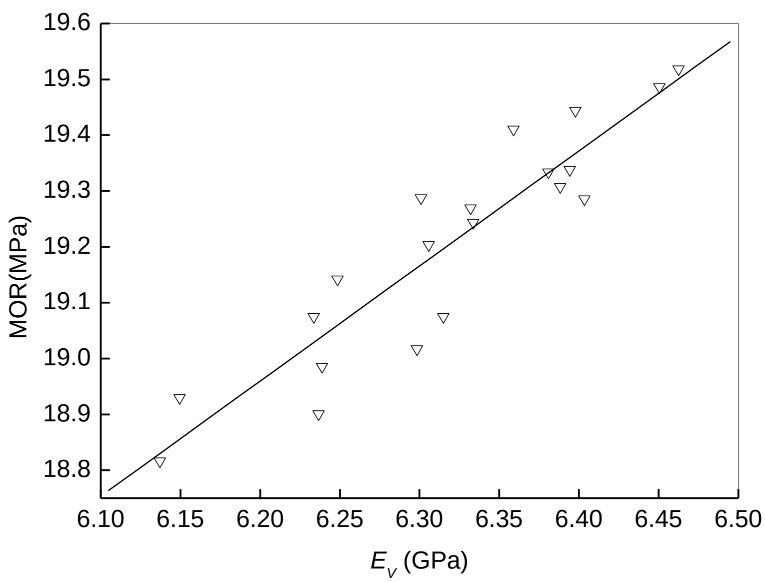


**Fig. 1.** Regression curve between $E_V$ and MOR of wood-plastic planks of Group 1

The regression curve between $E_P$ and MOR is shown in Fig. 2. From the regression analysis between $E_P$ and MOR, the following linear regression formula was obtained: MOR=3.225$E_P$+4.4574, $R_P$=0.9282> $R_{18,\ 0.02}$ = 0.516 ($R_P$ is the correlation coefficient, $R_{18,\ 0.02}$ is a standard correlation coefficient selected according to the number of specimens and accuracy class); therefore the correlation between $E_P$ and MOR was strong.

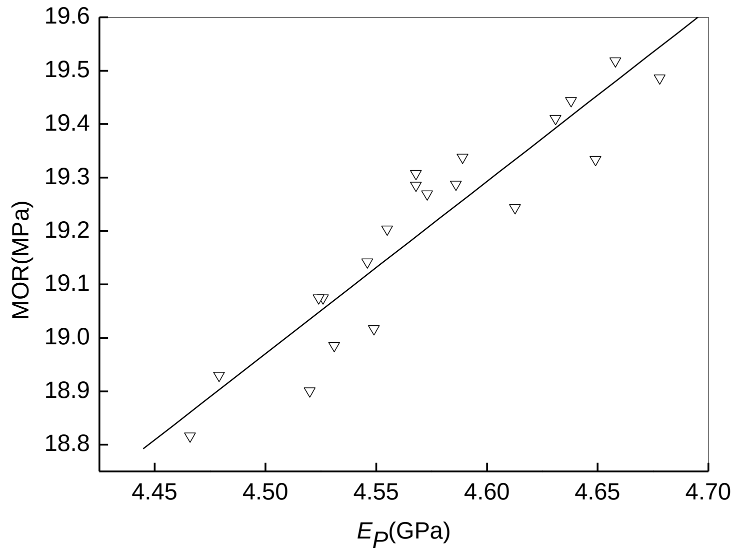


**Fig. 2.** Regression curve between $E_P$ and MOR of wood-plastic planks of Group 1

The regression curve between $E_f$ and MOR is shown in Fig. 3. From the regression analysis between $E_f$ and MOR, the following linear regression formula was obtained: MOR = 0.9771$E_f$ +14.717, $R_f$ = 0.9120 >$R_{18,\ 0.02}$ = 0.516 ($R_f$ is the correlation coefficient and $R_{18,\ 0.02}$ is a standard correlation coefficient selected according to the number of specimens and accuracy class); therefore $E_f$ and MOR had a strong correlation.

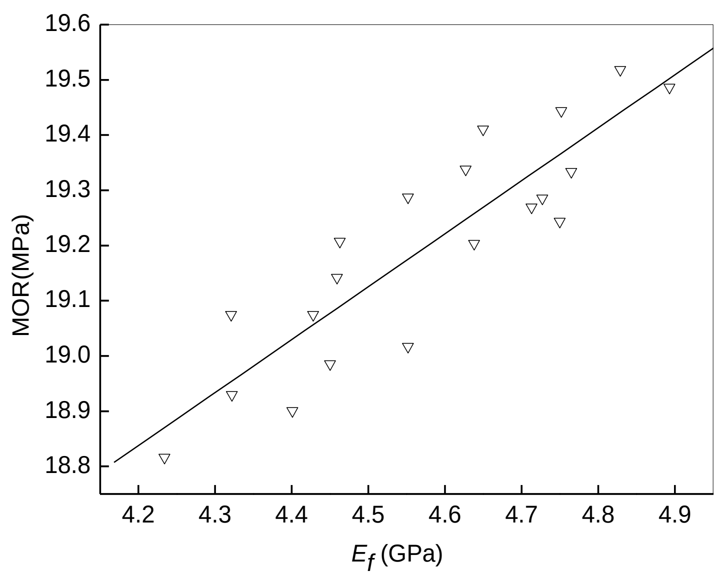


**Fig. 3.** Regression curve between $E_f$ and MOR of wood-plastic planks of Group 1

## Prediction of MOR for Wood-plastic Planks of Group 2

The MOR of specimens of Group 2 was predicted based on the prediction models (the regression formulae) between dynamic MOE and MOR of Group 1, respectively. The results of measured and predicted MOR are shown in Table 2. $MOR_2$ is the measured MOR of specimens of Group 2; $MOR_V$ is the predicted MOR based on $E_V$; $MOR_P$ is the predicted MOR based on $E_P$; and $MOR_f$ is the predicted MOR based on $E_f$. Results show that the mean values of predicted MOR were slightly greater than that of $MOR_2$, and the mean value of $MOR_V$ was the greatest of all. The standard deviation of predicted MOR was slightly less than that of $MOR_2$. The maximum difference between them was 0.022 MPa, so the mean values of predicted MOR had high accuracy.

**Table 2.** Predicted and Measured MOR of Wood-plastic Planks of Group 2

| Parameter | $MOR_2$ | $MOR_v$ | $MOR_p$ | $MOR_f$ |
|---|---|---|---|---|
| Mean value (MPa) | 19.198 | 19.203 | 19.202 | 19.199 |
| Standard deviation (MPa) | 0.200 | 0.186 | 0.188 | 0.178 |
| Variation coefficient (%) | 1.05 | 0.97 | 0.98 | 0.93 |

## Reliability Analysis of Wood-plastic Planks of Group 2

The limit state function of wood-plastic planks, $g(k, f, Q)$, is described by Eq. 5. The mean value of random load ($Q$) is 10 MPa, and the standard deviation of $Q$ is 2 MPa. The mean value of adjusting factor ($k$) is 0.8, and the standard deviation of $k$ is 0.1. The results of $k$, $f$, and $Q$ are subject to normal distribution; thus, the improved FOSM method can be used to analyze reliability of wood-plastic planks of Group 2. Without considering variability of other random variables and inaccuracy of formula, Eq. 10 could estimate the reliability index of the third iteration. Meanwhile, the reliability index and reliability could be obtained by iteration with Eqs. 5 through 9 (Liu and Wang 2005),

$$g(k, f, Q) = kf - Q = 0 \tag{5}$$

$$\alpha_i = \frac{\sigma_{X_i}\left(\frac{\partial g}{\partial X_i}\right)_{X^*}}{\sqrt{\sum_{i=1}^{3}\left(\frac{\partial g}{\partial X_i}\sigma_{X_i}\right)_{X^*}^2}} \tag{6}$$

$$X_i^* = \mu_{X_i} - \alpha_i \beta \sigma_{X_i} \tag{7}$$

$$\beta = \frac{\sum_{i=1}^{3}\left(\mu_{X_i} - X_i^*\right)\left(\frac{\partial g}{\partial X_i}\right)_{X^*}}{\sum_{i=1}^{3}\left[\alpha_i \sigma_{X_i}\left(\frac{\partial g}{\partial X_i}\right)_{X^*}\right]} \tag{8}$$

$$P_r = \phi(\beta) \tag{9}$$

$$\beta_{n+1} = \beta_n - g_n \frac{\Delta\beta}{\Delta g} \tag{10}$$

In these equations, $\alpha_i$ is a sensitivity factor of random variable; $x_i$ is the random variable, when $i$ is 1,2,3, $x_i$ stands for $k$, $f$, and $Q$, respectively; $f$ is resistance stress, and its mean values and standard deviation are equal to those based on measured and predicted MOR of wood-plastic planks of Group 2, respectively; $k$ is an adjusting factor selected according to creep and fatigue properties of wood-plastic planks; $Q$ is random load; $\mu_{X_i}$ is the mean value of $x_i$; $\sigma_{X_i}$ is the standard deviation of $x_i$; $x_i^*$ is figure point of $k$, $f$ and $Q$; $\beta$ is the reliability index, and $P_r$ is the reliability.

Iterative process of reliability index ($\beta$) based on measured MOR is shown in Table 3. The initial value of $\beta$ was 2.5, and the second value of $\beta$ was 4. Eq. 11 calculated the third value of $\beta$, the iterative process was ended after three steps, and when the limit state function was equal to zero, $\beta$ was 3.68. Other iterative processes were omitted. The results of reliability index and reliability based on measured and predicted MOR are shown in Table 4. As shown, the reliability index based on $MOR_v$ was the largest of all. The reliability index based on $MOR_p$ was less than that based on $MOR_2$, but the reliability indexes based on $MOR_f$ and $MOR_2$ were the same. The reliability index and reliability based on $MOR_2$ were in between those based on $MOR_v$ and $MOR_p$. The maximum difference of reliabilities based on predicted and measured values was 0.01%; therefore, the values of reliability based on predicted MOR had high accuracy. The reported test data applies only to wood-plastic planks used in this study.

**Table 3.** Iterative Process of Reliability Index Based on Measured $MOR_2$

| Parameter | First iteration | Second iteration | Third iteration |
|---|---|---|---|
| $\alpha_f$ | 0.0904 | 0.0894 | 0.1025 |
| $\alpha_k$ | 0.0452 | 0.9637 | 0.9575 |
| $\alpha_Q$ | -0.2510 | -0.1266 | 0.2649 |
| $f^*$ (Mpa) | 19.1528 | 17.7671 | 19.1225 |
| $k^*$ | 0.8887 | 0.9506 | 0.5457 |
| $Q^*$ (Mpa) | 10.3137 | 10.2532 | 10.4985 |
| $g(f,k,Q)$ (Mpa) | 6.6075 | 6.6368 | -0.0238 |
| Reliability index $\beta$ | 2.5 | 4 | 3.68 |

**Table 4.** Reliability Index and Reliability Based on Measured and Predicted MOR

| Limit State Function | $g(k, f_2, Q) = 0$ | $g(k, f_v, Q) = 0$ | $g(k, fp, Q) = 0$ | $g(k, f_f, Q) = 0$ |
|---|---|---|---|---|
| Final value (MPa) | -0.0238 | -0.0219 | 0.0186 | -0.0242 |
| Reliability index | 3.68 | 3.70 | 3.67 | 3.68 |
| Reliability/% | 99.9882 | 99.9892 | 99.9873 | 99.9882 |

## CONCLUSION

1. There was a strong correlativity between dynamic MOE ($E_V$, $E_P$, and $E_f$) and modulus of rupture (MOR) of wood-plastic planks of Group 1, respectively.
2. The reliabilities of other identical wood-plastic planks based on predicted and measured MOR were almost the same, and the reliability of them could be analyzed by predicted MOR.
3. The reliabilities of other identical wood-plastic planks based on predicted and measured MOR were greater than 99.9873%; these wood-plastic planks meet the safety requirements.
4. Three nondestructive testing (NDT) methods were suitable to predict MOR of other identical wood-plastic planks and evaluate the safety of them in service.

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