Simulating Stresses Associated with the Bending of Wood Using a Finite Element Method

Milan Gaff, Miroslav Gašparík,* Vlastimil Borůvka, and Marián Babiak

This article examines the stress-strain curves of various thicknesses of soft and hard wood when bent during three-point loading. The finite element method was used to simulate the course of stresses that occurred during the bending of these materials. Reference curves obtained by bending real specimens offered a basis for simulation. The results showed that with increasing material thickness, deflection values decreased and the proportionality limit increased; eventually, the bendability coefficient value decreased and the loading force necessary for bending increased. Moreover, it was apparent when bending hard materials that higher loading forces were necessary for different materials of the same thickness. It is possible to determine the stress-strain curves without having to perform experiments (except for indispensable reference ones) under real conditions.

Keywords: Layered wood material; Finite-element method; Stress simulation; Bending; Stress-strain curve; Bendability coefficient

*Contact information: Department of Wood Processing, Czech University of Life Sciences in Prague, Kamýcká 1176, Praha 6 - Suchdol, 16521 Czech Republic; *Corresponding author: gathiss@gmail.com*

INTRODUCTION

Great attention is paid to the creation of new materials that have specific properties for a particular purpose. The properties of newly created structures can be modified by combining materials with distinct properties. The properties can be modified with respect to the composition of individual layers, layer thicknesses, and the use of various wood-based and non-wood materials.

For such procedures, first of all, the properties of the individual materials that compose the layered structure must be known, as well as their behavior under specific stress modes. Further, the behavior of these materials in combination under such stresses needs to be determined. New methods are being developed with new materials. In order to know the materials' behavior under cyclic loading, non-destructive methods of modelling and simulation are beginning to be used. They provide a realistic image of the materials' behavior under specific loading modes (Gaff and Gašparík 2013). These methods of modelling and simulation also allow deeper scientific knowledge, since they provide integral information about the material.

A material's stratification is very important in industrial practice, both in civil engineering and the wood processing industry (Glos *et al.* 2004; Frese and Blaß 2006). In the wood processing industry, material stratification provides greater homogenity, thereby reducing the portion of critical areas causing damage to the material under stress. This process also allows the utilization of lower-rated wood species in the middle layers, with more valued wood species, or those with better mechanical properties, in the outer layers. First of all, we are able to modify the properties of the individual layers of the structure according to the required purpose.

Usually, tasks involving continuous specimens require partial differential equations to be solved for marginal or, eventually, initial conditions (Farlow 1993). Often, both areas and specimens have complicated three-dimensional shapes, with local discontinuities and complicated marginal conditions. Non-stationary, non-linear, and unstable tasks have to be solved for frequently. These issues do not provide an exact solution for the differential equations for most practical engineering tasks, and therefore, an application able to simulate the process and calculate the required results must be used. Today, the finite-element method (FEM) is among the most universal, most efficient, and most used numeric methods for the solution of engineering or field tasks. This engineering work is impossible without the use of computer-aided technology (CAT). Within this context, the FEM for stress analysis coordinates with design tools (computer aided design, or CAD) and the simulation of mechanical systems (Benča 2006).

The main goal was to determine the impacts of various compositions of the layered materials and of the thickness of the individual layers on the stress-strain course during bending. The simulation of the course of stresses was accomplished by means of FEM while modelling bending stress.

EXPERIMENTAL

This study included two basic tests of hard and soft materials, where the thicknesses were 2, 4, 6, 8, 10, 12, 14, 16, 18, and 20 mm (Fig. 1).

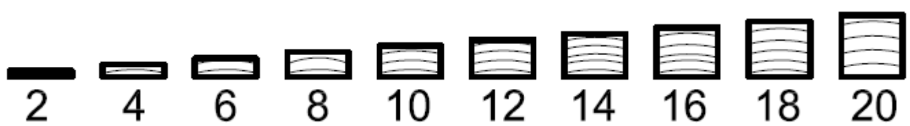

Fig. 1. Graphic illustration of the model material creation with the thickness increase by 2 mm

Materials

Regarding the input properties of the materials, the parameters of European beech (*Fagus sylvatica* L.), were used for hard materials and quaking aspen (*Populus tremula* L.) for the soft material. The physical and mechanical properties of these species were suited to the requirements for the modelling of both hard and soft materials (Požgaj *et al.* 1993). To prepare the specimens, a two-step selection was carried out. The first step was the selection of lumber from the University Forest Enterprise of the Technical University in Zvolen. Specimens with the dimensions of 20 × 20 × 410 mm (Fig. 2) were formed in the second step. Thirty specimens of each wood species were used for the measurements (a statistically significant number).

Fig. 2. Bending strength specimen

The specimens were conditioned to 12% moisture in a conditioning chamber (APT Line II, Binder, Germany), at the following conditions: moisture content (ϕ) = 65± 3% and temperature (t) = 20± 2 °C. Moisture content verification was carried out in accordance with ISO 13061-1 (2014).

Methods

Bending

The specimens were bent using the three-point bending principle according to ISO 13061-3 (2014). Bending was carried out in universal testing machine FPZ 100/1 (HECKERT, Germany), in accordance with ISO 13061-3 (2014), which contained a special jig for flexural tests and data logger ALMEMO 2690-8 (AHLBORN, Germany) for recording the loading forces and maximum deflection. Specimens were placed on supporting pins (l= 18 × h= 360 mm) so that the loading force acted in a radial direction with respect to the length of the specimen (Fig. 3), and a load was applied until it broke.

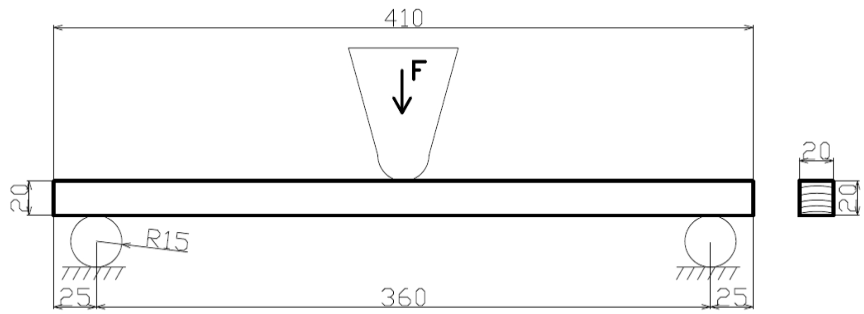

Fig. 3. Schematic representation of the bending principle

Measurements

The values for maximum loading force and moduli of elasticity were directly downloaded from the data logger onto a computer, and the bending strength (*i.e.*, modulus of rupture), and shear strength were calculated. The maximum deflection, measured at the midpoint of the specimen (mid-span deflection), was accurate to 0.01 mm. Tensile and compression values were obtained from preliminary tests according to ISO 13061-6 (2014) and ISO 3787 (1976).

The dimensions of the specimens, used for calculating the moisture content and density, were measured with a digital caliper 500/150/20 (Mitutoyo; Japan) to a precision of 0.1 mm.

Calculations and evaluation

The ultimate shearing strength parallel to grain was calculated according to ISO 3347 (1976) and Eq. 1,

$$\tau_s = \frac{F_{\max}}{bh} \quad (1)$$

where τ_t is the ultimate shearing strength of wood (MPa), F_{max} is the maximum (breaking) force (N), b is the width of the specimen (mm), and h is the height (thickness) of the specimen (mm).

The bending strength (MOR) was calculated according to ISO 13061-3 (2014) and Eq. 2,

$$\sigma_b = \frac{3F_{\max} l_0}{2bh^2} \tag{2}$$

where σ_b is the (ultimate) bending strength of wood (MPa), F_{max} is the maximum (breaking) force (N), l_0 is the distance between supporting pins (mm), b is the width of the specimen (mm), and h is the height (thickness) of the specimen (mm).

To determine the relation between the bending strength and deflection at three-point bending, *i.e.*, without the shear stresses, Eq. 3 was applied to figure the linear area,

$$\Delta y = \frac{\Delta F l_0}{4bh^3 E} \tag{3}$$

where Δy is the deflection increase of the wood (mm), ΔF is the loading force increase (N), l_0 is the distance between supporting pins (mm), b is the width of the specimen (mm), h is the thickness of the specimen (mm), and E is the modulus of elasticity (MPa)

Externally, the bending deformation results in the specimen's deflection. The maximum deflection was measured. As the wood compression strength alongside the fibers was much less than the tensile strength, deformation occurred in the compression area due to the fibers' densification. This cannot be seen under normal conditions. Reaching the maximum bearable value of densification causes a rupture in the tensile area. Additionally, shear stresses arise while bending. These stresses are trying to shift the individual layers of wood against each other in the direction of the support distance. The relationship between longitudinal and transversal stresses is defined by the law of associated shear stresses, which says that the shear stresses in two mutually perpendicular planes of an elementary specimen are equal and directed either toward or from the intersection of their planes (Požgaj *et al.* 1993).

A comparative characteristic for the final evaluation is the bendability coefficient, which was calculated according to Kurjatko *et al*. (2010) and Eq. 4,

$$k_{bend} = \frac{h}{R_{\min.}} \tag{4}$$

where k_{bend} is the bendability coefficient (-), h is the height (thickness) of the specimen (mm), and R_{min} is the minimum bending radius (mm).

The minimum bending radius was calculated according to Kurjatko *et al*. (2010) and Eq. 5,

$$R_{\min} = \frac{l_0^2}{8y_{\max.}} + \frac{y_{\max}}{2} \tag{5}$$

where R_{min} is the minimum bending radius (mm), y_{max} is the maximum deflection of wood (mm), and l_0 is the distance between supporting pins (mm).

Density was calculated according to ISO 13061-2 (2014) and Eq. 6,

$$\rho = \frac{m}{hbl} = \frac{m}{V} \quad (6)$$

where ρ is the density of the specimen (kg/m^3); m is the mass (weight) of the specimen (kg); h, b, and l (the height, width and length) are dimensions of the specimen (m); and V is the volume of the specimen (m^3). The specimens for density measurement were cut off from the middle part of the specimens for bending. The dimensions of these specimens were in compliance with standard ISO 13061-2 (2014).

The moisture content was determined during testing. These calculations were carried out according to ISO 13061-1 (2014) and Eq. 7,

$$w = \frac{m_w - m_0}{m_0} * 100 \quad (7)$$

where w is the moisture content of the specimens (%), m_w is the mass (weight) of the specimen at certain moisture w (kg), and m_0 is the mass (weight) of the oven-dry test specimen (kg).

An oven-dry state was achieved according to the ISO 13061-1 (2014) standard. Specimens were weighed and then dried at a temperature of 103 ± 2 °C. Specimens reached constant moisture content when the weight change between two weighings at intervals of 6 h did not exceed 0.5% of the mass of the specimen. After drying, the specimens were cooled in a desiccator and subsequently rapidly weighed to insure the moisture content did not increase more than 0.1% under the influence of air humidity. Weighing was carried out with an accuracy of 0.5%.

Simulation

For simulation of real material behavior it was necessary to define both the density and mechanical properties of the materials in the SolidWorks® (Dassault Systèmes S.A., France) application. The material was modeled on the orthotropic model. Moduli of elasticity in bending, Poisson's ratio in a transverse plane, tensile, compression strengths and bending strengths, as well as shear strengths in direction parallel to the grain were calculated and subsequently recalculated for 12% moisture content (Table 1).

A stress-strain average curve was plotted using the average measured values. This curve was subsequently used to define the stress-strain model curve in the SolidWorks® application (Vláčilová et al. 2007). This way, the basic stress-strain curve was created for the hard and soft wood species specimens.

The wood bending simulation was divided into the following steps:

1. Plotting of the individual 3-D models in SolidWorks 2009 application.
2. Definition and calculation of material properties.
3. Creation of a network of finite elements and definition of the loading force.
4. Simulation of stresses and deflection in the non-linear area using the CosmosWorks 2009 application, using the non-linear visco-elastic model.
5. Results evaluation.

Table 1. Properties of Materials

Properties of wood	Beech	Aspen
Density (kg/m^3)	650	460
Elasticity Modulus (MPa)	12,600	10,700
Poisson's Ratio TR	0.394	0.496
Tensile Strength Parallel to Grain (MPa)	135.0	108.0
Bending Strength Parallel to Grain (MPa)	123.0	84.2
Compression Strength Parallel to Grain (MPa)	56.3	47.0
Shear Strength Parallel to Grain (MPa)	14.5	7.7

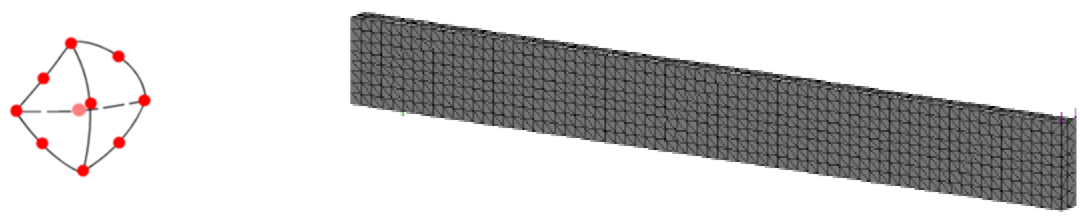

Fig. 4. A parabolic tetraedric element and illustration of the network of tetraedric elements in the model

When dealing with design issues, each node had three degrees of freedom representing the shifts in three orthogonal directions. For the problem's formulation, SolidWorks used the directions X, Y, and Z of the global Cartesian system of coordinates. For our analyses, the network of volume tetraedric elements was used (Fig. 4). Reference curves obtained after the specimens were bent on the tensile testing machine were the base for the simulation. Figure 5 shows the real curve obtained by the mean of measurements and its comparison with the curve obtained by the mean of the simulation. The figure proves that the simulated course of the curve is identical to that of the real specimen, thus confirming the correctness of the model simulation.

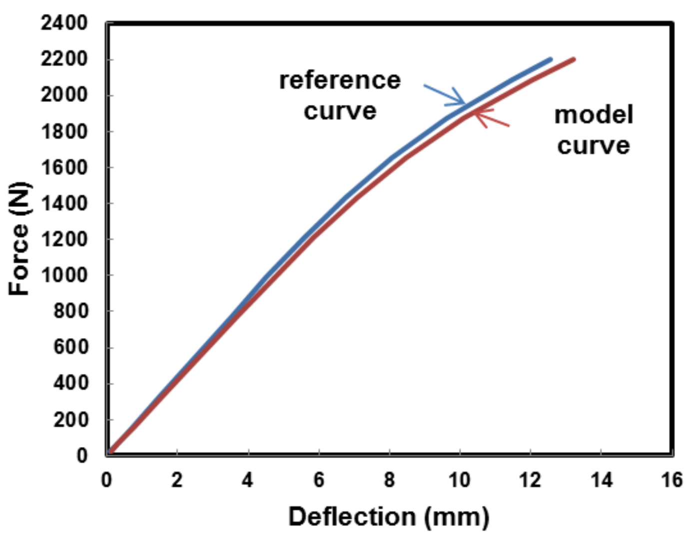

Fig. 5. Comparison of reference and model curves for hard material

RESULTS AND DISCUSSION

Figure 6 shows the courses of force and deflection as a function of hard material thickness. Deformations of inner structures could be observed at the material thicknesses of 2, 4, and 6 mm. In the remaining cases, *i.e*., the specimens measuring 8 to 20 mm, the course of the force-deflection curves was identical.

A decrease in deflection values and increase in the proportionality limit could be observed with increasing material thickness. The use of the material is based on the proportionality limit, whereas the non-linear area is essential for bending during the manufacturing process.

Wood constructions are a different case since the defining strength parameters are inside the linear area, *i.e*., up to the proportionality limit.

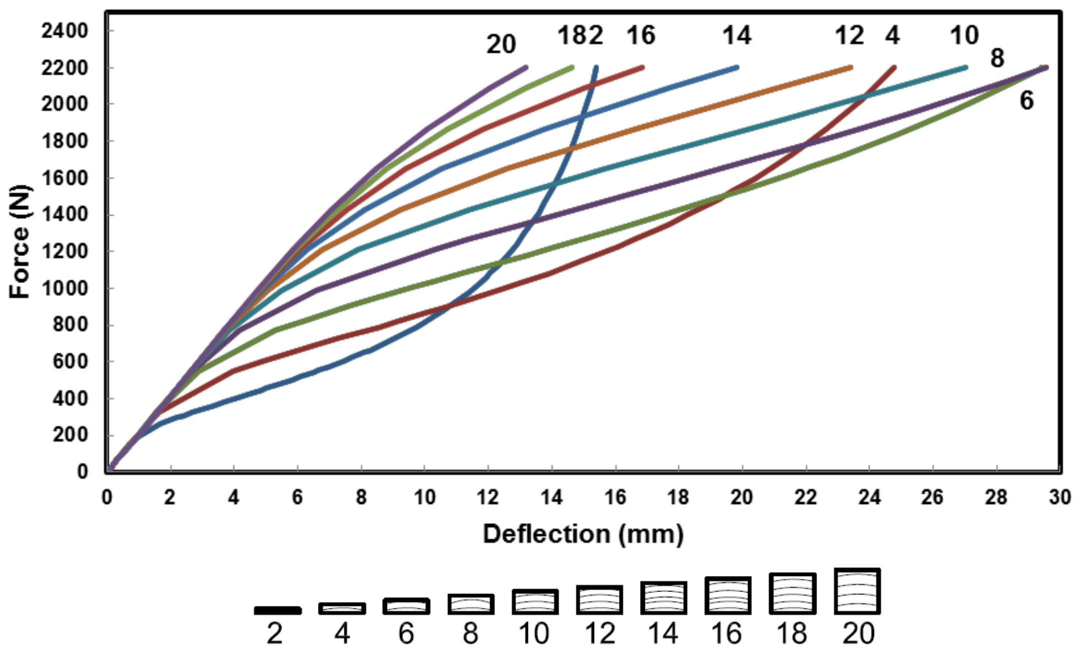

Fig. 6. Graphical representation of force and deflection of the individual thicknesses of the hard material.

Figure 7 shows the courses of force and deflection as a function of thickness in soft material bending. As can be seen from the resulting curves, the force necessary for bending decreased with material thickness. This was also confirmed by the decreasing values of the bendability coefficient (Fig. 8). A sudden transition from linear to non-linear areas can be seen on the curves 2 to 6. For the curves 2 and 4, the calculation was interrupted, and after repeated simulation, the same result was obtained as the material broke.

As can be seen from the results shown in Fig. 8, the material bendability increased with its decreasing thickness. The bendability coefficient (given as the ratio of the bent material thickness vs. minimum bend radius) value decreased and the force necessary to bend increased with the increasing thickness.

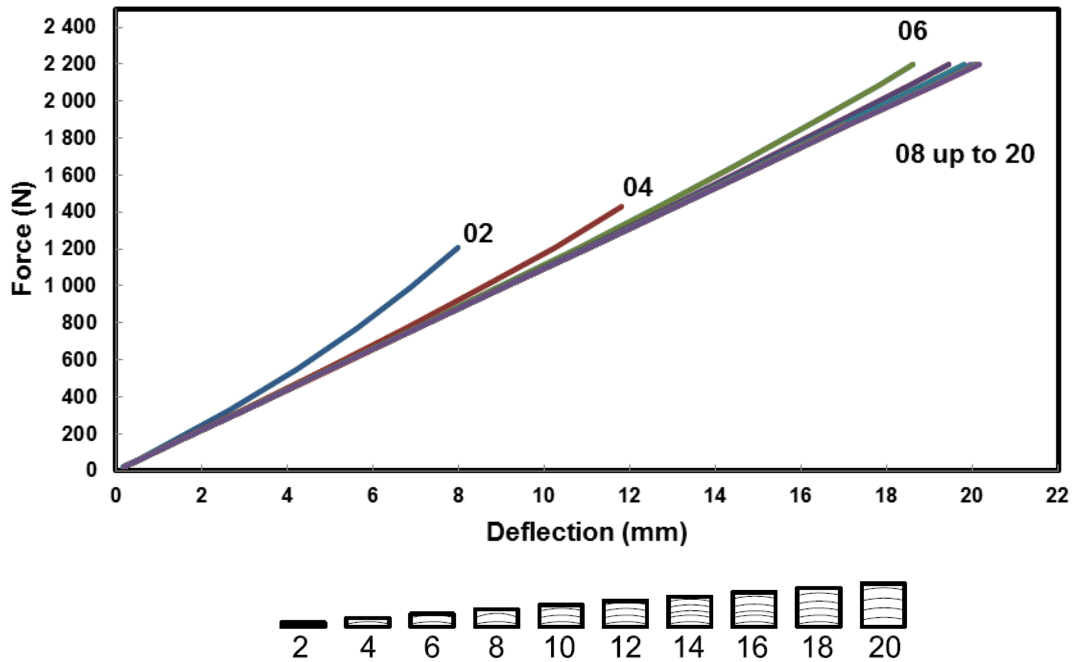

Fig. 7. Graphical representation of force and deflection of the individual thicknesses of the soft material.

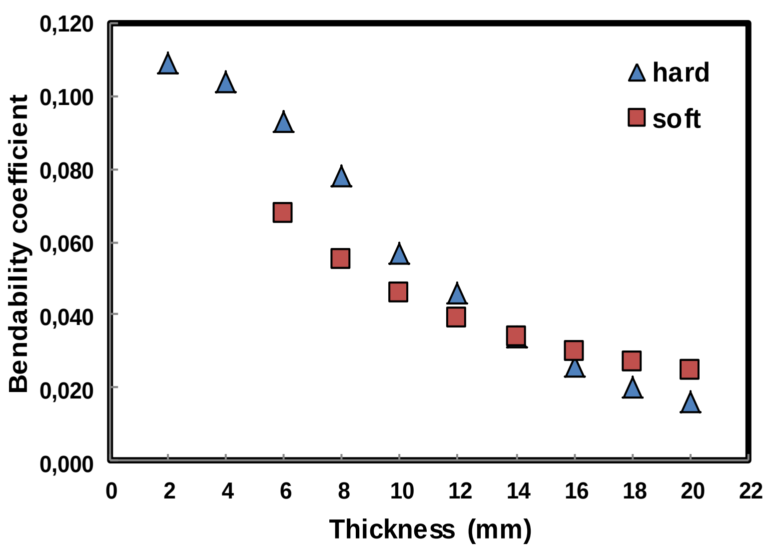

Fig. 8. Comparison of coefficients of bendability for each thickness of hard and soft materials

When comparing the results shown in Fig. 6 and Fig. 7, it is apparent that greater force was necessary to bend hard materials of the same thickness. The calculation was interrupted for soft materials 2 and 4, and therefore these were not included in Fig. 8.

From Table 2 it can be seen that the thickness of 6 mm was the most appropriate in terms of the bendability soft material, as the bendability coefficient was 0.068.

Similar results for the bendability coefficients were also achieved by other relevant authors, such as Kurjatko *et al.* (2010). The minimum comparable factor values for soft aspen wood exceeding a certain minimum thickness were emphasized in comparison with hard beech wood, as shown in Fig. 9. The possible combinations of various thicknesses of these two wood species, of their number and of their order in the composition, provides obviously more suitable and therefore more effcient utilization in practice.

Table 2. Bendability Coefficients of Hard and Soft Materials for All Thicknesses

Material	**Bendability coefficient** (-)									
	Thicknesses (mm)									
	2	4	6	8	10	12	14	16	18	20
Hard	0.109	0.104	0.093	0.078	0.057	0.046	0.034	0.026	0.020	0.016
Soft	-	-	0.068	0.055	0.046	0.039	0.034	0.030	0.027	0.025

Another simulation outcomes was a graphic illustration of the stress distribution under monitored loading (Fig. 9). The figure shows the specimens loaded by three-point bending. Since the SolidWorks® application makes possible symmetric calculations, only a half of the specimen under the load is shown in the figure.

The stress values and distributions in the individual areas of the specimen as well as the tensile strength on the bottom (outer) side of the specimen and the compression strength on its top (inner) side can be seen in Fig. 9. Obviously, there are also shear stresses (in the tangential plane), which tried to shift in the direction of the support distance. The maximum shear stress is in the middle neutral layer, particularly under the supports. Therefore, it is suitable to keep the slenderness ratio *l/h* within the range from 14 to 20; this is also due to the optimum values of the proper bending strength of the material.

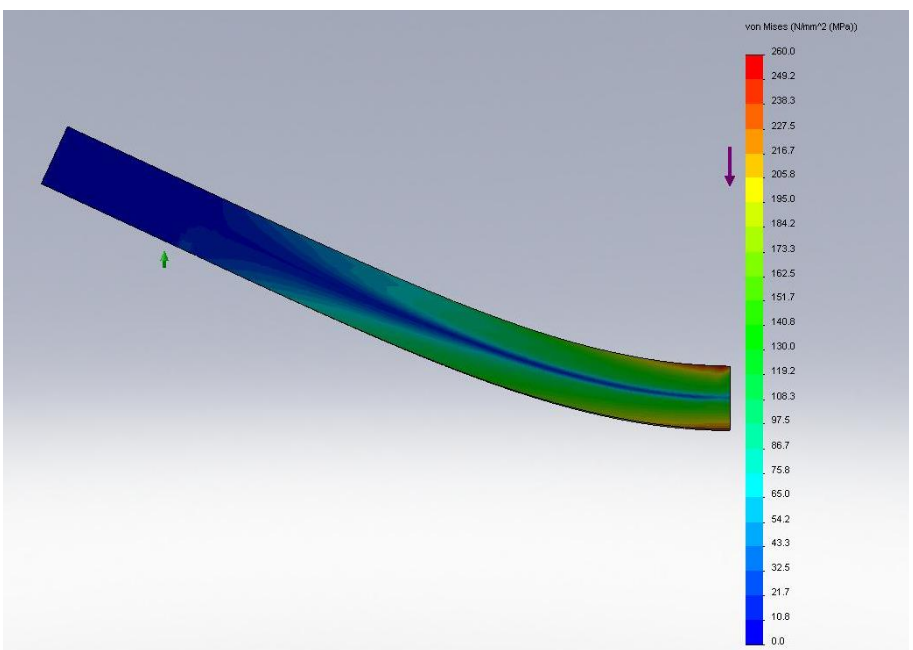

Fig. 9. Stress distribution in the specimen

Improvement of bendability can be provided by optimal layering of ideal thickness of hard and soft materials (no account is taken account the effect of factors such as humidity and temperature, as well as the use of a steel strap on the tensile zone). This is because during bending, the neutral layer shifts to the area with higher rigidity. During bending is an effort to shift the neutral axis to the tensile zone, *i.e*. increase in volume of pressure zone in the cross-section of wood. The reason is the low (insufficient) deformation under tension loading along the fibers compared with the pressure in the identical direction.

CONCLUSIONS

1. With increasing material thickness, the deflection values decreased and the proportionality limit increased; eventually, the bendability coefficient value decreased and the force necessary for the bending increased.
2. As expected, the specimens with material thicknesses of 2 and 4 mm were broken, and as far as the bendability was concerned, 6 mm was the most suitable thickness.
3. At the same time, materials use in practice are chosen by their proportionality limit, while for the bending process in fabrication the non-linear area is critical; the opposite case applies for wooden structures, where the strength features within the area must not exceed the proportionality limit.
4. Another important fact is that with sufficiently reliable material parameters, it is possible to model the material's behavior before its creation within the composition or layered structure.

ACKNOWLEDGMENTS

The authors are grateful for the support of the Grant Agency of the Faculty of Forestry and Wood Science, project No. 20123144, "Evaluation of the properties of materials used in wooden structures."

REFERENCES CITED

Benča, Š. (2006). "Výpočtové postupy MKP pri riešení lineárnych úloh mechaniky [FEM calculation procedures for solving linear mechanics problems]," Slovak Technical University, Bratislava, 150 pp.

Farlow, S. J. (1993). *Partial Differential Equations for Scientists and Engineers*, Dover Publications, New York, USA, 414 pp.

Frese, M., and Blaß, H. J. (2006). "Characteristic bending strength of beech glulam," *Materials and Structures* 40(1), 3-13. DOI: 10.1617/s11527-006-9117-9

Gaff, M., and Gašparík, M. (2013). "Shrinkage and stability of thermo-mechanically modified aspen wood," *BioResources* 8(1), 1136-1146. DOI: 10.15376/biores.8.1.1136-1146

Glos, P., Denzler, J. K., and Linsenmann, P. (2004). "Strength and stiffness behaviour of beech laminations for high strength glulam," in: Proceedings *Meeting 37 CIB Working Commission W18-Timber Structures*, paper CIB-W18/37-6-3.

ISO 13061-1 (2014). "Physical and mechanical properties of wood -- Test methods for small clear wood specimens -- Part 1: Determination of moisture content for physical and mechanical tests," International Organization for Standardization, Geneva, Switzerland.

ISO 13061-2 (2014). "Physical and mechanical properties of wood -- Test methods for small clear wood specimens -- Part 2: Determination of density for physical and mechanical tests," International Organization for Standardization, Geneva, Switzerland.

ISO 13061-3 (2014). "Physical and mechanical properties of wood -- Test methods for small clear wood specimens -- Part 3: Determination of ultimate strength in static bending," International Organization for Standardization, Geneva, Switzerland.

ISO 13061-6 (2014). "Physical and mechanical properties of wood -- Test methods for small clear wood specimens -- Part 6: Determination of ultimate tensile stress parallel to grain," International Organization for Standardization, Geneva, Switzerland.

ISO 3347 (1976). "Wood-Determination of ultimate shearing stress parallel to grain," International Organization for Standardization, Geneva, Switzerland.

ISO 3787 (1976). "Wood-Test methods -- Determination of ultimate stress in compression parallel to grain," International Organization for Standardization, Geneva, Switzerland.

Kurjatko, S., Čunderlík, I., Dananajová, J., Dibdiaková, J., Dudas, J., Gáborík, J., Gaff, M., Hrčka, R., Hudec, J., Kačík, F., Kúdela, J., Lagaňa, R., Laurová, M., Mamoň, M., Mišíková, O., Račko, V., Rohanová, A., and Solár, R. (2010). "Parametre kvality dreva určujúce jeho finálne použitie [Wood quality parameters determining its end use]," Ivan Makovíny, Štefan Šteller (eds), Technical University in Zvolen, 352 pp.

Požgaj, A., Chovanec, D., Kurjatko, S., and Babiak, M. (1993). *Štruktúra a Vlastnosti Dreva* [*Structure and Properties of Wood*], Príroda a. s., Bratislava, 486 pp.

Vláčilová, H., Vilímková, M., Hencl, L. (2007). "Základy práce v CAD systému SolidWorks [The basics of working of CAD system SolidWorks]," Computer Press a.s. Brno, Czech Republic, 319 pp.

Article submitted: November 4, 2014; Peer review completed: January 19, 2015; Revised version received and accepted: January 15, 2015; Published: February 4, 2015.
DOI: 10.15376/biores.10.2.2009-2019