An Attempt at Modelling of Cutting Forces in Oak Peripheral Milling

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An attempt was made to evaluate the non-linear, multi-variable dependency between the main (tangential) force, F_C, and machining parameters and properties of pedunculate oak (*Quercus robur*) during straight edge, peripheral milling. The tangential force, F_C, was found to be influenced by the feed rate per tooth, f_Z, cutting depth, c_D, rake angle γ_F, Brinell hardness, H, bending strength, R_B, and modulus of longitudinal elasticity, E. Several interactions between machining parameters and properties of wood were confirmed in the developed relationship $F_C = f(f_Z, c_D, \gamma_F, H, R_B, E)$.

Keywords: Oak; Properties of wood; Machining parameters; Peripheral milling; Main cutting force

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INTRODUCTION

It seems that contemporary theory of wood machining was developed on the basis of metal processing theory (Afanasev 1961; Beršadskij 1967; Glebov 2007; Manžos 1974; Amalitskij and Lûbčenko 1977). An attempt to base wood cutting theory on the mechanical properties of wood for three main and intermediate cutting directions (modes) has already been done (Deševoy 1939). Factors that impact the mechanics of wood cutting (and chip formation) can be divided into three groups (Eyma *et al.* 2004; Naylor and Hackney 2013): wood species, wood properties, and the cutting process itself. In the work of Franz (1958) aimed at main and normal cutting force analysis (with the use of a dynamometer equipped with an electrical resistance strain-gage bridge and strain-analyzing instrument), several mechanical properties (tensile strength perpendicular to grain, $R_{T\perp}$, modulus of elasticity in tensile parallel to grain, E, modulus of rupture in bending, R_B, modulus of elasticity in bending, E_B, crushing strength parallel to grain, $R_{C\parallel}$, and shear strength parallel to grain, $R_{S\parallel}$, and cleavage, R_{CL}) were evaluated for moisture contents of 3%, 11%, and 30%, and for wood specimens of yellow birch (*Betula alleghaniensis* Britt.), sugar pine (*Pinus lambertiana* Dougl.), and white ash (*Fraxinus americana* L.).

The papers of various researchers (Kivimaa 1950; Franz 1958; McKenzie 1960; Koch 1964; Woodson and Koch 1970; Axelsson *et al.* 1993; McKenzie *et al.* 1999) have discussed the impacts of the stated factors on cutting forces and chip formation, durability of tools, precision of processing, and quality of the processed surface. The primary objective of these studies was to better understand the interaction between a tool and processed item for the purpose of more efficient management of cutting processes. Energy and tool saving methods are also important. However, in this case, they are in the background.

In the literature dealing with mechanical wood processing, various methods for anticipating/calculating cutting forces can be found. In many approaches, these are coefficient methods, where authors start from a referent unit cutting resistance for a certain wood species, most frequently pine wood, which is calculated under strictly defined and controlled (standard) conditions (Afanasev 1961; Beršadskij 1967; Manžos 1974; Amalitskij and Lûbčenko 1977; Orlicz 1982; Goglia 1994; Glebov 2007). Specific cutting resistances for certain materials and certain cutting conditions (for all possible wood grains orientation angles or cutting modes) are obtained by multiplying referent unit cutting resistances and appropriate correction coefficients, which can be found in adequate tables. Orlicz (1982) pointed out that differences between forces calculated using the coefficient method and forces measured in experiments reached as high as 40%. In certain cases, this difference was more than 100% (Mandić *et al.* 2014). There are several possible reasons for the discrepancies, but one of the primary reasons is the fact that physical and mechanical properties of wood are not sufficiently and properly included in the stated models. It is known from the literature that physical and mechanical properties of wood depends on the environmental conditions in which the tree grew up, as well as on the position of the longitudinal and transverse cross-section of the trunk (Krzysik 1975).

The work of Eyma *et al.* (2004) would be an interesting, novel approach of employing several mechanical properties of wood of 13 species to cutting forces analysis. However, instead of main (tangential) cutting force, F_C, the resultant cutting force, $F_R=(F_C^2+F_N^2)^{0.5}$ was taken into account in the cited work, rendering all of the determined cutting forces useless, at least for the present purposes.

Better results in anticipating cutting forces can be achieved by establishing a model that would include both physical and mechanical properties of wood in addition to the cutting regime. These models are usually established based on extensive experiments involving various wood species and at various cutting conditions. The disadvantage is the fact that tests are most frequently conducted on specially designed equipment and/or at small cutting speeds, thus not providing a sufficient level of generality. The role of the applied measuring equipment in observed discrepancies should also be taken into consideration (Feomentin 2007).

Wood machining in practice rarely is conducted in pure general cases (modes): perpendicular (⊥), parallel (∥), or transversal (∦). These cutting cases were also examined in the past (Time 1870; Deševoy 1939). In the work of Kivimaa (1950), these cases were first defined as A, B, and C, respectively, and further used in many studies. A more general approach to applying the three wood grain orientation angles towards the cutting edge (φ_K), towards the vector of cutting speed velocity (φ_V), and towards the cutting plane (φ_S) can be found in work of Orlicz (1982). This approach allows for the analysis of all intermediate cutting cases as well as defining the general case.

Another model, published in the work of Porankiewicz *et al*. (2011) was made and verified on research results from the work of Axelsson *et al.* (1993). The model includes physical properties of the samples, tool characteristics, and characteristics of the processing regime. The paper gives statistical equations for tangential F_C and normal F_N cutting forces in the function of the wood grain orientation angles (from perpendicular to parallel), edge radius round up ρ, rake angle γ_F, mean thickness of a cutting layer (chip thickness), a_P, cutting speed, v_C, wood density, D, and moisture content, m_C (8% to 133%) and temperature of wood, T, developed from an incomplete experimental matrix.

Another approach, Eqs. 1 through 3, evaluating the relationship $F_C= f(\gamma_F, \rho, a_P, D, m_C)$, not taking into account wood grain orientation angles and cutting speed (v_C), was published by Ettelt and Gittel (2004), Tröger and Scholz (2005), and Scholz *et al*. (2009). These solutions were based on the work of Kivimaa (1950) and refer to defined cutting modes. In the authors' opinion, is difficult to find a justification for the use of the following equations:

$$k_{C0.5} = F_C \cdot a_P^{-1/2} \quad (1)$$

$$k_{C0.5} = f_t \cdot w_C \cdot a_P^{1/2} \quad (2)$$

and

$$f_t = f(\Omega, \Phi) \quad (3)$$

where k_C is the specific cutting resistance ($N \cdot mm^{-1/2}$), f_t is a linear function, taking into account cutting modes, Ω is the angle between wood grains and cutting edge, and Φ is the angle between the wood grains and the cutting velocity vector.

A linear model, $F_C = f(\gamma_F, \rho, a_P, D, m_C, \varphi)$, was recently published for rotary cutting of two African wood species for three basic cutting cases (modes) (⊥- A, ∥ - B and ∦ - C), employing one wood grain orientation angle φ (Cristóvão *et al*. 2012). Literature surveys show that the dependence of the main cutting force on all cutting parameters, in a wide range of variation, is non-linear, so the choice of linear model in the cited work does not appear to be appropriate. Furthermore, the use of only one wood grain orientation angle φ to define three basic cutting cases (modes) in one linear model, appears to be a fundamental error. Three orientation angles are necessary for precise definition of cutting cases: φ_E - between wood grain and the cutting edge, *CE;* φ_V - between wood grain and the vector of the cutting velocity, v_C, and φ_C - between the wood grain and the cutting plane (Orlicz 1982; Porankiewicz 2014).

The present study is an attempt to evaluate the relationship between the main (tangential) cutting force, F_C, by peripheral milling of oak wood and machining parameters: - cutting depth, c_D, and feed per edge, f_Z, rake angle γ_F, as well as physical properties of wood: - wood density, D, moisture content, m_C, and some mechanical properties: - bending strength, R_B, modulus of elasticity, E, and hardness, H. The other parameters: - cutting edge round up radius, ρ, cutting speed, v_C, - diameter of the cutter, D_C, cutter width, W_C, number of cutting edges, z, were kept constant. The present study will also make it possible to verify, to a certain degree, the relation $K=f(c_D, f_Z)$, between relatively cutting resistance K and the cutting depth c_D and the feed rate per edge f_Z, published in work of Orlicz (1984) for pine wood and applied for oak wood after the use of coefficient of wood species c_R=1.55 (1.5 - 1.6).

EXPERIMENTAL

The testing materials included radially cut samples for the determination of physical and mechanical properties of pedunculate oak (*Quercus robur*).

Physical and mechanical properties were tested in accordance with various standards, *i.e*., standards for density (SRPS D.A1.044, 1979), for bending strength (SRPS D.A1.046, 1979), for Brinell hardness, perpendicular to the wood grains, in radial direction (EN 1534, 2011) and for modulus of elasticity (SRPS D.A1.046).

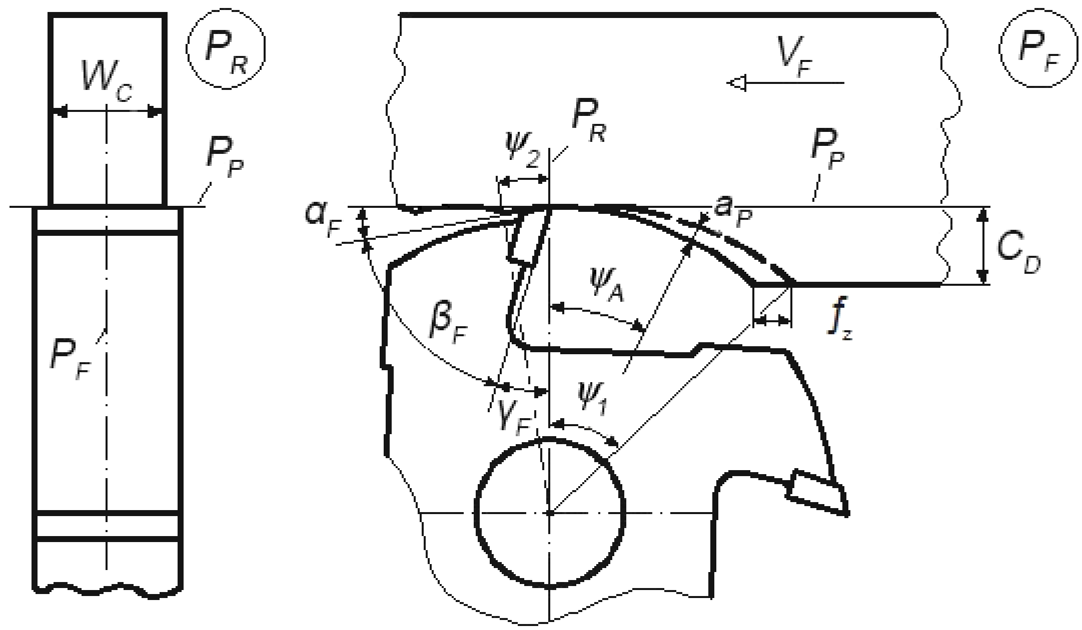

Fig. 1. Cutting situation by longitudinal milling with use of the cutter with hole; a_P - average thickness of cutting layer; ψ_A - average meeting angle; ψ_1 - maximum meeting angle; P_F - working plane; P_R - main plane; P_P - back plane

Testing during peripheral milling of wood was done on a table-mounted milling machine, MiniMax, equipped in an 3 kW three phase asynchronous electrical motor. Accessory motion was achieved using an Maggi Engineering feeding machine Vario Feed, with speeds ranging between 3 and 24 m·min^{-1} and an three phase asynchronous electrical motor nominal power of 0.45 kW. The milling case was up-milling, opened, and peripheral. Three cutters of diameter D_C = 125 mm, width C_W= 40 mm, equipped with four soldered plates, made of cemented carbide H302 , manufactured by Freud, Italy, were used. The edges had a radius of the round up ρ = 4 μm. The roughness of rake and clearance surfaces, after regrinding with grinding wheels, D76, D46 and D7, for respectively rough, finishing final finishing, were respectively of $R_{a\alpha}$ = 0.15 μm and $R_{a\gamma}$ = 0.18 μm. The cutters had a clearance angle of α_F = 15°, and three different rake angles, γ_F, namely: 16°, 20°, and 25°. The orientation angle of wood grains towards the cutting edge was φ_K= 90°. The average wood grain orientation angle toward the cutting speed velocity vector was φ_V <0.1241; 0.1908 rad>, and that toward the cutting plane was φ_S <0.1241; 0.1908 rad>.

Testing was done with a constant RPM of the working spindle, *i.e.*, n = 5,860 RPM; at a constant cutting speed, v_C= 38.35 m·s^{-1}; at three different feeding speeds, v_F= 4, 8, and 16 m·min^{-1}; and at cutting depths of wood c_D= 2, 3, and 4.5 mm. The rotational speed of the tool spindle n was measured under load, with the use of a digital, non-contact tachometer, type PCE-DT 65 manufactured by PCE Instruments.

Cutting power was measured indirectly using the power input of the machine driving the electric motor. The measuring device SRD1, connected with a computer, was used for measuring, with sampling frequency of 1kHz. The device uses the Power Expert program package developed in cooperation with the Centre for Wood Processing Machines and Tools and Unolux Company from Belgrade. Figure 2 presents a typical record of cutting power and the manner of its processing.

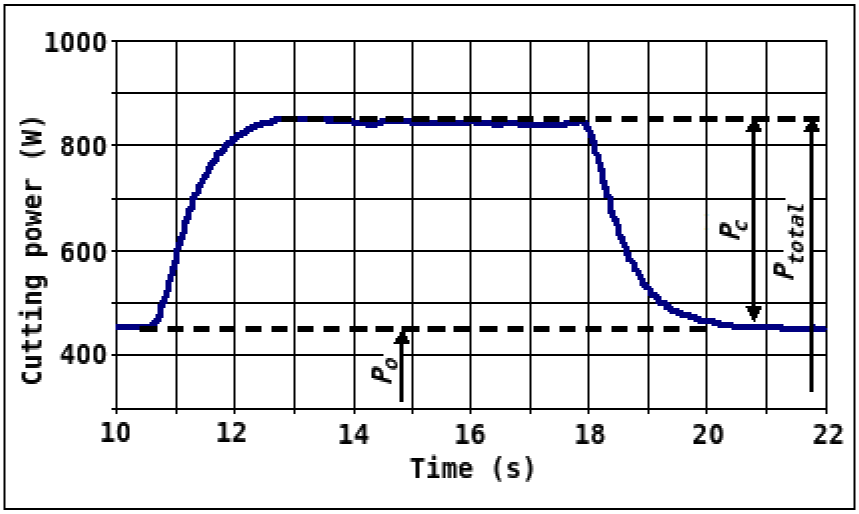

Fig. 2. Procedure for processing the results of cutting power measurements during peripheral milling

It can be seen in Fig. 2 that the shape of the record was a rounded trapezium. In the graph, values of total power, P_{total}, power when idle, P_O, and power required for cutting, P_C, are presented, with the power required for cutting being the difference between the first two values:

$$P_C = P_{total} - P_O \quad \text{(W)} \tag{4}$$

Average values of the measured powers, based on a higher number cuts on the same test sample, were used for calculating mean values of the main cutting resistance by using the following Eq. 5,

$$F_{mean} = P_{cmean} \cdot v_C^{-1} \text{ (N)} \tag{5}$$

where F_{mean} is the mean value of main cutting force for one revolution of the tool; P_{cmean} is the average cutting power (W), and v_C is the cutting speed (m·s^{-1}).

The calculated value of the mean force, F_{mean}, is the mean force for one rotation of the tool, which means that it also includes idle time between the cutting rotations of the four blades (Fig. 3).

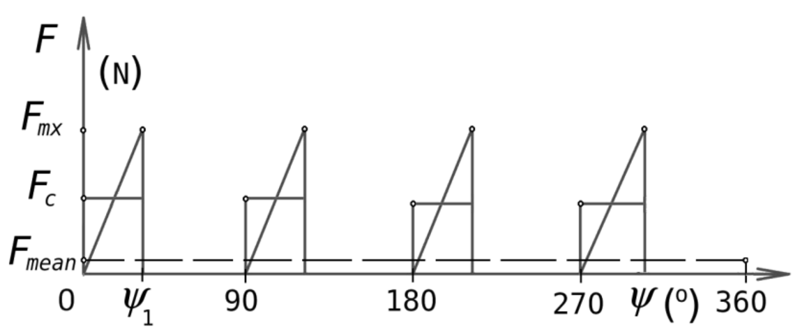

Fig. 3. Tangential cutting forces during one revolution of the cutter; ψ - cutting edge rotation angle; ψ_1 - maximum edge meeting angle

For obtaining the average main cutting force per one edge, F_C, it is necessary to correct the value of mean force, F_{mean}, as follows,

$$F_C = F_{mean} \cdot O_{gl} \cdot \Sigma\, l_{r1}^{-1} = F_{mean} \cdot 2 \cdot \pi \cdot (i \cdot y_M)^{-1} \text{ (N)} \quad (6)$$

$$y_M = y_1 + y_2 \quad \text{(rad)} \quad (7)$$

where O_{gl} is the circumference of the cutter (m); l_{r1} is the length of cutting of one edge (m); i is the number of cutting edges; R_C is the cutting radius (m); and y_M is the rotation angle of the cutting edges in the object (Fig. 1).

For each of the 25 specimens, at least four measurements of physical and mechanical properties were conducted. The measured density, D, values were within the range of 0.613 g·cm^{-3} to 0.790 g·cm^{-3}, and the calculated mean value was 0.737 g·cm^{-3}. In accordance with the differences in the density, oak hardness, bending strengths, and modulus of elasticity varied. Because the moisture content, m_C, of samples varied (from 6.67% to 7.83%, with an average of 7.2%), the values of the wood density computationally were normalized to a humidity of 8%.

As already mentioned, testing was conducted for various combinations of cutting process parameters. In Table 1, the peripheral milling process parameters are given.

Table 1. Peripheral Milling Process Parameters of Oak

Feed speed v_F	Feed per tooth f_Z	Cutting depth c_D	Rake angle γ_F	
(m·min^{-1})	(mm)	(mm)	(º)	(rad)
4	0.171	2	16	0.279
8	0.341	3	20	0.349
12	0.683	4.5	25	0.436

Table 2 shows the derived values of a mean thickness of cutting layer (chip thickness), a_p, and angle between the cutting velocity vector v_C and wood grains, φ_V.

Table 2. Derived Peripheral Milling Process Parameters of Oak

Feed per tooth f_Z (mm)	Cutting Depth c_D (mm)			Cutting Depth c_D (mm)		
	2	3	4.5	2	3	4.5
	The mean thickness of cutting layer, a_P (mm)			Angle between the cutting velocity vector v_C and wood grains φ_V (rad)		
0.171	0.022	0.026	0.032	0.032	0.026	0.032
0.341	0.043	0.053	0.063	0.065	0.053	0.065
0.683	0.086	0.106	0.13	0.13	0.106	0.13

The measurements were repeated at least eight times for each combination of processing parameters. Based on the calculated average forces, the relationship $F_C = f(D_8, m_C, H, R_B, E, f_Z, \gamma_F, c_D)$ was estimated in preliminary calculations for linear, polynomial, and power types of functions, with and without interactions. The model should fit the experimental matrix by the lowest summation of residuals square S_K, by the lowest standard deviation of residuals S_R, and by the highest correlation coefficient of predicted, and observed values R. The experimental matrix can be fitted with more simple models, but this will result in decreasing approximation quality, which means that the S_K and S_R values will increase and R will decrease. It must to be pointed out that any empirical formula can be valid only for ranges of independent variables chosen within the experimental matrix, especially for incomplete experimental matrices and complicated mathematical formula with interactions.

In this case, all predicted values of the dependent variable will have higher expected error. Many years of experience by the authors suggests that efforts to fit such data with overly simple models can be expected to hurt the quality of the approximation, especially in the case of variables with small importance, making such a model nonsensical. The proper influence of low importance variables can be extracted only from an experimental matrix when using a more complicating model. The most adequate formulas appeared to be the non-linear multi variable equations with interactions, Eqs. 8 through 11.

$$F_C^P = A+B+C \tag{8}$$

where

$$A = e_1 \cdot D_8^{e2} \cdot m_C^{e3} \cdot H^{e4} \cdot R_B^{e5} \cdot E^{e6} \cdot f_Z^{e7} \cdot \gamma_F^{e8} \cdot c_D^{e9} \tag{9}$$

$$B = e_{10} \cdot f_Z \cdot c_D + e_{11} \cdot f_Z \cdot \gamma_F + e_{12} \cdot R_B \cdot f_Z + e_{13} \cdot R_B \cdot \gamma_F \tag{10}$$

$$C = e_{14} \cdot D_8 \cdot E + e_{15} \cdot D_8 \cdot f_Z + e_{16} \cdot m_C \cdot R_B + e_{17} \tag{11}$$

Estimators were evaluated from an incomplete experimental matrix having 25 data points. During the evaluation process of chosen mathematical models, elimination of unimportant or low important estimators was carried out by use of coefficient of relative importance CRI. The CRI is defined by Eq. 12 and with the assumption $C_{RI} > 0.1$:

$$C_{RI} = (S_k + S_{RI}) \cdot S_k^{-1} \cdot 100 \qquad (\%) \tag{12}$$

In Eq. 12, the new terms are S_{K0k}, the summation of square of residuals for estimator $a_k = 0$. In this equation, a_k is the estimator of the number k in the statistical model evaluated.

Figure 4 shows a flow chart of the optimization program used.

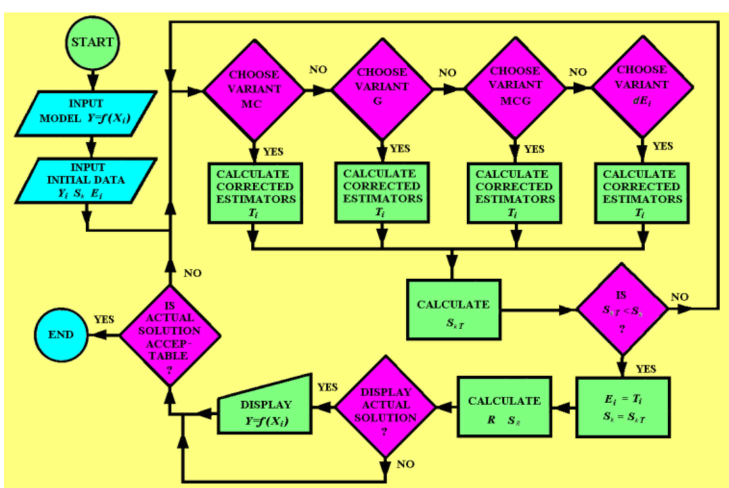

Fig. 4. Flow chart of the optimization program; variants: MC- Monte Carlo, G- gradient, MCG- combined MC and G, S_K- summation of square of residuals, R- correlation coefficient, S_R- standard deviation of residuals

Summation of the residuals squared, S_K, standard deviation of residuals, S_R, correlation coefficient, R, and the square of correlation coefficient between predicted and observed values, R^2, were used for characterization of approximation quality. The calculation was performed at the Poznań Networking & Supercomputing Centre (PCSS) on an SGI Altix 3700 machine using an optimization program based on a least squares method combined with gradient and Monte Carlo methods (Fig. 4).

RESULTS AND DISCUSSION

The final shape of approximated dependence, Eqs. 8 through 11, using the optimization program shown in Fig. 4, was obtained after $2.84 \cdot 10^{10}$ iterations. The following estimators for were evaluated: e_1= 6.89566; e_2= 0.071757; e_3= 0.44357; e_4= 0.01213; e_5= 0.4689; e_6= 0.2595; e_7= 0.056059, e_{28}= -0.22267; e_9= 0.013037; e_{10}= 3.072638; e_{11}= -1.7767; e_{12}=-3.40655, e_{13}=0.12584, e_{14}= -0.000040724; e_{15}= 0.41851; e_{16}= -0.86718; e_{17}= -552.38077.

A rounding of estimator values to the 5th decimal place produced an acceptable deterioration of the fit of less than 0.1%. Reducing the number of rounded decimal digits to 4, 3, and 2 would cause unacceptable deterioration of the fit as much as 0.5%, 5668%, 21045%, respectively.

The coefficients of relative importance, C_{RI}, for the estimators have the following values: C_{RI1}=78233531.9, C_{RI2}=11185239.7, C_{RI3}=26717460.2, C_{RI4}=157352.7, C_{RI5}=62689340.4, C_{RI6}= 64980315.8, C_{RI7}=362660, C_{RI8}=69246221.4, C_{RI9}=17637.1, C_{RI10}=729.8, C_{RI11}=10171.1, C_{RI12}= 1276115.8, C_{RI13}=3506721.7, C_{RI14}=4328076.4, C_{RI15}=697231.3, C_{RI16}=21189557.2, C_{RI17}=10851518.3.

For each of the 25 combinations of input data, the predicted tangential cutting forces, F_c^p, were calculated using Eqs. 8 through 11. These results are shown in Fig. 5. The approximation quality of the fit, also seen in Fig. 5, can be characterized by the quantifiers S_K= 61.86, R= 1.00, R^2= 0.99, and S_R= 1.76.

Equations 8 through 11, after substituting estimators from e_1 to e_{17} and substituting average values of the D_8 and the m_C, take the following forms,

$$F_C^p = A+B+C \quad (13)$$

where

$$A= 26.68307 \cdot H^{0.01213} \cdot R_B^{0.4689} \cdot E^{0.2595} \cdot f_Z^{0.056059} \cdot \gamma_F^{-0.22267} \cdot c_D^{0.013037} \quad (14)$$

$$B= 3.072638 \cdot f_Z \cdot c_D - 1.7767 \cdot f_Z \cdot \gamma_F - 3.40655 \cdot R_b \cdot f_Z + 0.12584 \cdot R_B \cdot \gamma_F \quad (15)$$

$$C=-0.030543 \cdot E + 313.8825 \cdot f_Z - 6.27838 \cdot R_B + 552.38077 \quad (16)$$

Figure 5 shows that the maximum deviation from Eqs. 13 through 16 was as high as S_R=4.79 N. For minimum and maximum values of the F_C, the points lay closer to the regression straight line. These equations provide a strong link between the observed and predicted cutting forces and can be used to analyze the impact of specific inputs on the predicted cutting forces.

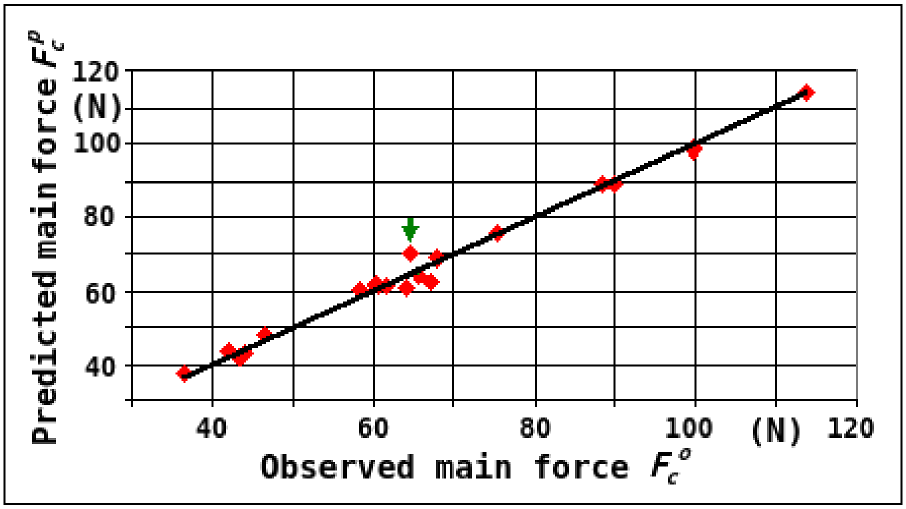

Fig. 5. Plot of observed main force F_C^O against predicted F_C^P values, according to Eqs. 13 through 16

Figure 6 shows a plot of the relationships among the main cutting force, F_c^P, the feed per tooth, f_z, and the rake angle, γ_F. Figure 6 shows that F_C^P strongly, non-linearly, in a parabolic decreasing manner depends upon the f_Z. An increase in the f_Z increased the F_C^P, more for lower γ_F. The average change of the F_C^P with an increase of feed per tooth, f_Z, by 0.1 mm is between 7 and 10 N, depending on the size of the rake angle γ_F. An increase in γ_F decreases the F_C^P for the largest feed rate per tooth, f_Z. As the f_Z fell below ~0.4 mm, a minimum started to appear. For the lowest f_Z, the influence of the γ_F on the F_C^P was small, with a minimum at γ_F ~21°.

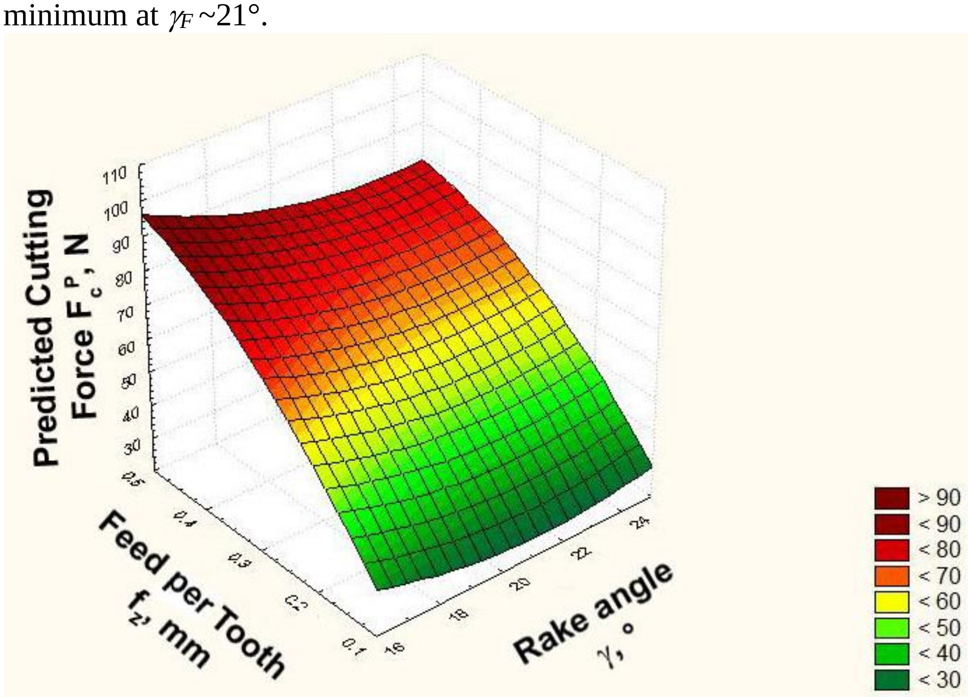

Fig. 6. Plot of the relationships among the predicted main force F_C^P (N), γ_F (°), and f_Z (mm), according to Eqs. 13 through 16; D_8 = 0.750 g·cm^{-3}; m_C = 7.24 %; H = 44.16 N·mm^{-2}; R_B =122.47 N·mm^{-2}; E = 11355.13 N·mm^{-2}; c_D = 3.25 mm

Figure 7 shows a plot of the relationships among the predicted main force, F_C^P, bending strength, R_B (N·mm^{-2}), and wood hardness, H (N·mm^{-2}). It can be seen that with an increase in H and R_B, the tangential cutting force also grows in a nonlinear, parabolic, decreasing manner. This is consistent with the theory that states that with higher values of bending strength, R_B, and/or hardness of wood, H, the resistance to cutting wood grows. This means that the cutting force, F_C, will also be greater. From Fig. 7, it can be seen that the influence of bending strength, R_B, on the F_C^P for oak was larger than that of the Brinell hardness, H. When bending strength, R_B, increases to 10 N·mm^{-2}, the average change of the cutting force will be as high as 5.3 N. In the case of an increase in hardness, H, by 10 N·mm^{-2} the cutting force grows by 4.2 N.

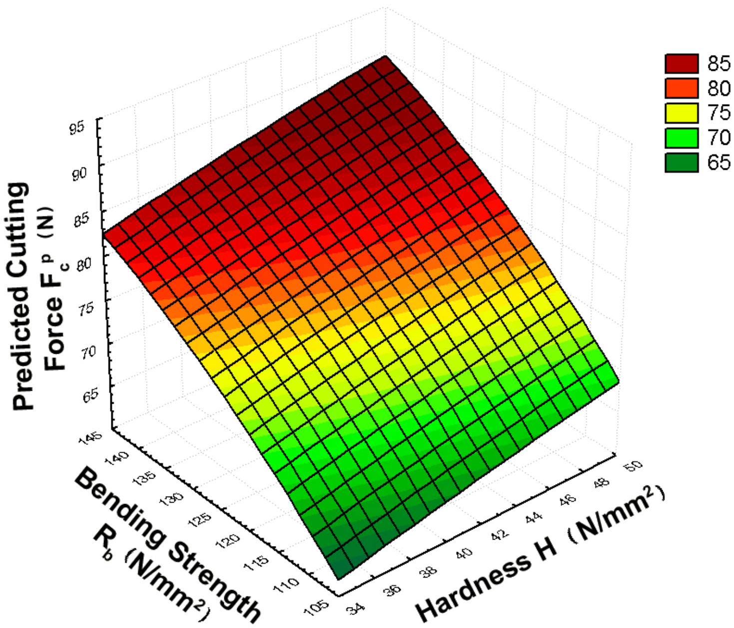

Fig. 7. Plot of the relationships among the predicted main force, F_C^P, H (N·mm^{-2}), and R_B (N·mm^{-2}), according to Eqs. 13 through 16; D_B =0.75 g·cm^{-3}; m_C =7.24 %; E =11355.13 N·mm^{-2}; γ_F=19.91°; f_Z =0.427 mm; c_D = 3.25 mm

Figure 8 shows the relationships among the observed main force, F_C^P, cutting depth c_D (mm), and oak wood modulus of elasticity E (N·mm^{-2}). The figure shows a strong, non-linear influence of E and the c_D on the main cutting force, F_C^P. The relationship $F_C^P = f(c_D)$ combines the influence of increasing thickness of a cutting layer (chip thickness), a_p, and to a lesser degree, an influence of increasing grain orientation angle φ_V. An increase of both independent variables increases the F_C^P. In the reference range, the overall increase was about 20 N. If the depth of cut increase is 1 mm, the average increase in force, F_C^P, will be between 7.2 and 7.9 N (lower values are valid for the lower modulus of elasticity E). The influence of the modulus of elasticity E is as follows. When there is an increase in E of 1,000 N·mm^{-2}, the cutting force, F_C^P, increases by approximately 4 and 16 N in the analyzed range. The analysis of results confirmed that the examined dependency, $F_C^P=f(H, R_B, E, f_Z, \gamma_F, c_D)$, is nonlinear in a range of variation of independent variables.

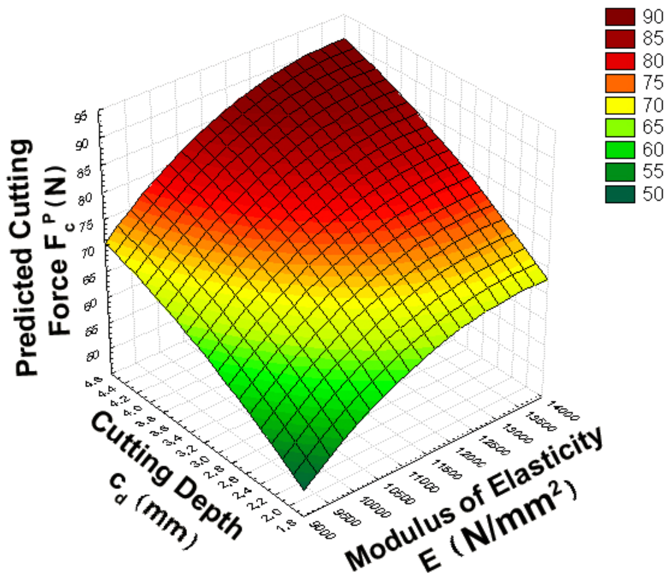

Fig. 8. Plot of the relationships among the predicted main force, F_C^P (N), c_D (mm), and E (N·mm^{-2}), according to Eqs. 13 through 16; D_8= 0.75 g·cm^{-3}; m_C = 7.24 %; H=46.14 N·mm^{-2}; R_B= 122.47 N·mm^{-2}; γ_F=19.91°

According to the evaluated formula of relation $F_C=f(D_8, m_C, H, R_B, E, f_Z, \gamma_F, c_D)$ the predicted main cutting force, F_C^P, decreases with an increase of the wood density D_8 in whole range of variation. Similarly, an increase in the moisture content, m_C, decreases the F_C^P, as calculated from this equation, which contradicts the information from the literature (Kivimaa 1950; Franz 1958). For these reasons, the variables D_8 and m_C were excluded from formula (11) through (16), with assumption D_8= 0.75 g·cm^{-3} and m_C = 7.24%, and the influence of D_8 and m_C was not discussed in the work.

The main cutting force, F_C (average in one cutting cycle), calculated from formula (11) - (16), evaluated for following average parameters: f_Z= 0.427 mm, c_D= 3.25 mm, γ_F= 19.91^0, D= 0.75 g·cm^{-3}, H= 44.16 MPa, R_B= 122.47 MPa, and E= 11355 MPa, was of value F_C=77.7N. The F_C calculated from formulas published in the works Orlicz (1984), Amalitskij and Lûbčenko (1977), Beršadskij (1967) and Orlicz (1982) (f_Z, c_D) were respectively: higher 78%, higher 60%, lower 17%, and lower 15%. It has to be mentioned that average values of: D, H, R_B and E taken from work of Wagenfür and Scheiber (1974) were respectively: 8% lower, 21 %lower, 28% higher, and 11% higher. It can be seen from this comparison that the values of the F_C obtained in the present study lay between values evaluated with use of formulas published in the literature.

CONCLUSIONS

The analysis of results of the calculations performed makes it possible to state the following:

1. An increase in the feed rate per tooth, f_Z, increased the main cutting force, F_C^P, in a parabolic, decreasing manner.
2. For a small rake angle γ_F of 16°, the established relationship $F_C^P=f(f_Z)$ was strongest.

3. A decrease in the rake angle, γ_F, for the largest examined feed rate per tooth, f_Z, of 0.7 mm increased the main cutting force, F_C, in a parabolic, increasing manner. A further decrease in feed rate per tooth, up to f_Z = 0.1 mm, resulted in a decrease in the relationship $F_C^P = f(f_Z)$, with a minimum of about 21°.
4. An increase of the bending strength, R_B, increased the main cutting force, F_C^P, in a parabolic, decreasing manner.
5. An increase in the Brinell hardness, H, increased the main cutting force, F_C^P, less than the bending strength R_B.
6. An increase in the modulus of elasticity, E, increased the main cutting force, F_C^P, in a parabolic, decreasing manner.
7. An increase in the cutting depth, c_D, increased the main cutting force, F_C^P, in a parabolic, decreasing manner.

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