Modeling the Cupping of Lumber

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Wood shrinks anisotropically as it loses hygroscopic moisture. While longitudinal shrinkage (parallel to the grain) is nearly negligible in normal wood, transverse shrinkage (across the grain) is significant and characterized as tangential and radial shrinkage. The application of average tangential shrinkage values to a rectangular cross section results in errors, especially for boards cut from near the center of the log. In addition, using a Cartesian coordinate system to calculate shrinkage cannot provide an estimate of cup. Calculating shrinkage and cup deformation using a previously developed model, this Excel model can provide a more realistic image of the final cross section and a more accurate estimate of shrinkage. The model is dependent on wood species, initial and final moisture contents, and location of the board within the log. This paper describes and illustrates uses of the model.

Keywords: Lumber; Radial; Tangential; Shrinkage; Cup; Crown; Deformation; Differential shrinkage; Polar geometry; Dry kiln; Drying defect; Modeling; Excel

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INTRODUCTION

The utilization of sawn lumber generally requires that it be dried from its natural, high-moisture content condition. Drying will improve subsequent manufacturing processes, such as gluing, machining, and finishing, and helps prevent stain and decay, reduces the weight, increases the wood's strength and stiffness, and pre-shrinks the wood for the dry atmospheric conditions typically found in service conditions. Wood shrinks by different amounts depending on the grain orientation, species, and by the decrease in moisture content (MC). Shrinkage along the grain in normal wood is small enough to be ignored, while radial shrinkage (the direction from pith to bark) is about half that of tangential shrinkage (the direction parallel to the growth rings and perpendicular to the radial direction). This anisotropic shrinkage of wood often results in unwanted deformation of lumber, both during manufacture and while in service.

One type of deformation is cup, which is defined as "a distortion of a board in which there is deviation from flatness across the width of the board" (Simpson 1991). It is the result of a much greater tangential shrinkage than radial shrinkage. Larger tangential shrinkage than radial shrinkage causes flat-sawn boards (boards in which the annual rings are approximately tangent to the wide face) to cup toward the bark during drying, unless they are restrained. Flat-sawn lumber cut near the pith will tend to cup more than a similar board cut near the bark because the curvature of the growth rings is greater near the pith. For the same reason, flat-sawn lumber from small-diameter trees will be more cup-prone than lumber from larger trees.

Restraint to minimize deformation during kiln drying is commonly achieved by the weight of the lumber stacked above, and some operations place additional top weights to minimize warp in the upper layers of drying lumber. Using good lumber stacking practices is the best way to minimize cup during the kiln drying process. Dry kiln schedules can also influence the development of cup by impacting the wood viscoelastic properties and having mechano-sorptive effects. Lumber with excessive cup may not completely achieve a smooth surface during planing, or the pressure of the planer rollers may split the severely cupped board. Additionally, cup can be reduced by not over drying the lumber during the kiln process.

The USDA Wood Handbook (Forest Products Laboratory 2010) publishes the average total percent of radial and tangential shrinkage for moisture losses from the fiber saturation point (fiber saturation point, or FSP, occurs at a moisture content of approximately 30%) to 0% moisture content for most domestic and some foreign wood species. These values are often used to calculate an estimate of the expected dimensional changes as wood moisture decreases or increases using the following equation (Hoadley 2000),

$$\Delta D = D\ S\ (\frac{\Delta MC}{FSP}) \tag{1}$$

where ΔD is the change in dimension, D is the initial dimension, S is the total fractional shrinkage from FSP to 0%, ΔMC is the change in moisture content (below FSP), and FSP is the fiber saturation point (average value of 30% MC)

This calculation assumes that the radial and tangential shrinkages occur parallel to the lumber surfaces. As a result, it has a built-in error in calculating the change in dimension, as it ignores the curvilinear nature of the growth rings. This was not a bad assumption when our trees were very large, but today's logs tend to be much smaller, and the arc of the growth rings and hence the angles imparted by shrinkage are much more significant. An additional deficiency is that this model of shrinkage is not able to predict cup.

Leavengood (2001) developed an Excel spreadsheet that estimates the dimensional change in wood with moisture loss (or gain) using a related approach, but does not provide an indication of cup magnitude. Ormarsson *et al.* (1998, 1999, 2000) have developed a complex model that uses finite element simulations and considers mechano-sorptive and visco-elastic behaviors of drying wood that predict dimensional changes and warp, but this model is beyond the scope of the current, more pragmatic approach to estimating cup.

A model that estimates the amount of cup that develops in drying lumber was initially developed by Booker (1992, 2003) and further modified and verified by Xiang *et al.* (2012). The objective of the current work was as follows: 1) develop a user-friendly version of the Booker-Xiang model; 2) demonstrate the application of the model; and 3) provide a graphic representation of the model results. This was done using an Excel spreadsheet to calculate and graph the cross section distortion that occurs during moisture loss (or gain) because of the anisotropic shrinkage (or swelling) that potentially results in cup formation. This visualization will help manufacturers, lumber users, and consumers to better understand the possible development and magnitude of cup and dimension change. As this approach does not consider visco-elastic and mechano-sorptive effects that would tend to reduce cup, the results represent a worst case scenario (as if the wood were dried without external restraint).

Use and Assumptions of the Booker-Xiang Shrinkage Model for Cupping

To use this Excel spreadsheet model, the user needs to input seven pieces of information: 1) initial board width; 2) initial board thickness; 3) x-y Cartesian coordinates of the board center (board centroid); 4) initial moisture content; 5) final moisture content; 6) species of wood; and 7) diameter of log. Based on this input, the model determines the perimeter points of the lumber and produces a visual graphic of the lumber cross-sectional profile and placement within the log using an x-y coordinate grid that assumes the origin is the log pith (Fig. 1).

The model uses the original size and position in the log (which determines ring orientation) to model the shrinkage. In modeling the lumber shrinkage, the following assumptions are made: 1) growth rings are circles that all have the same center, which coincides with the pith of the log; 2) radial shrinkage occurs toward the pith along a line from the pith to the outer circle, where the latter coincides with the outside of the log; and 3) tangential shrinkage is perpendicular to radial shrinkage and occurs toward the centroid of the lumber cross-section.

EXPERIMENTAL

Materials

Species and their total (green to oven-dry) radial and tangential shrinkage values were obtained from the USDA Wood Handbook for the domestic and foreign wood species listed there. These were imported into the spreadsheet model, and the appropriate shrinkage values are used based on the species selected by the user.

Methods

To account for the curvature of the growth rings and the imperfect estimations that result when tangential and radial shrinkage values are applied to rectangular dimension lumber, Booker *et al.* (1992, 2003) developed a theory to describe shrinkage and cupping using a polar coordinate system. Modifications of that approach by Xiang *et al.* (2012) improved the estimate of shrinkage and deformation. The current study uses the improved technique developed and described by Xiang *et al.* to adjust points along the board's perimeter to their new location after shrinkage.

Before perimeter points are adjusted, however, the amount of fractional radial and tangential shrinkage is calculated, based on the total amount of shrinkage possible for the species and the moisture content change that has occurred. Equations 1 and 2 found in Table 1 describe this calculation.

In the first step of determining the board's new shape, the board is assumed to shrink isotropically in both x and y directions by an amount equal to the fractional radial shrinkage (Table 1, Eqs. 3 and 4, respectively). Next, the x and y coordinates of the board's perimeter are converted to the polar coordinates R and θ using Eqs. 5 and 6, respectively. For the coordinates to be correctly displayed, θ may be adjusted, depending on which quadrant the original point is located (Eq. 7).

Next, tangential shrinkage along the growth rings is taken into account. Whereas the previous radial shrinkage moves all points toward the pith, tangential shrinkage is toward the line drawn through the board centroid and the origin.

Table 1. Summary of the Equations used in the Excel Spreadsheet Model for Shrinkage and a Description of their Functions

Equation Number	Function	Equation	Excel Function
1	Determine the amount of fractional radial shrinkage, S_{rp}, that will occur based on MC change and total radial shrinkage, S_r	$Srp = Sr\frac{(\frac{\Delta MC}{fsp})}{100}$	S_{rp} = (S_r*((M_i-M_f)/30))/100 where: M_i = Initial MC M_f = Final MC
2	Determine the amount of fractional tangential shrinkage, S_{tp}, that will occur based on MC change and total tangential shrinkage, S_t	$Stp = St\frac{(\frac{\Delta MC}{fsp})}{100}$	S_{tp} = (S_t*((M_i-M_f)/30))/100 where: M_i = Initial MC M_f = Final MC
3	Implement isometric shrinkage on the *x* coordinate	$x' = x(1 – S_{rp})$	x' = x*(1-S_{rp})
4	Implement isometric shrinkage on the *y* coordinate	$y' = y(1 – S_{rp})$	y' = y*(1-S_{rp})
5	Transform to the polar coordinate *R*, the radial distance of the point from the origin	$R = \sqrt{x'^2 + y'^2}$	R = SQRT(x'^2 + y'^2)
6	Transform to the polar coordinate θ, the angle in degrees the point makes with the *x*-axis	$\theta = tan^{-1}\frac{y'}{x'}$	θ = DEGREES(ATAN(y'/x')
7	Modify θ depending on in which quadrant the point is located	If point is in quadrant 1, $\theta_p = \theta$; If point is in quadrant 2 or 3, $\theta_p = \theta + 180$; If point is in quadrant 4, $\theta_p = \theta + 360$;	θ_{adj}=IF(quadrant=1,θ_p=θ, IF(quadrant=2,θ_p=θ+180, IF(quadrant=3,θ_p=θ+180, IF(quadrant=4,θ_p=θ+360, "undefined"))))
8	Adjust the point angle, θ_p, for tangential shrinkage, moving the point toward the centroid, calculating a new point angle, $\theta_{p'}$	If $\theta_p > \theta_c$: $\theta_{p'}=\theta_c+(\theta_p-\theta_c)\frac{(1-Stp)}{(1-Srp)}$ If $\theta_c > \theta_p$: $\theta_{p'}=\theta_c-(\theta_c-\theta_p)\frac{(1-Stp)}{(1-Srp)}$	$\theta_{p'}$=IF(θ_p>θ_c, θ_c+(θ_p-θ_c)*(1-S_{tp})/ (1-S_{rp}), θ_c-(θ_c-θ_p))*(1-S_{tp})/ (1-S_{rp}))
9	Transform the polar coordinates (R, $\theta_{p'}$) to the new x''.	$x'' = R (\cos \theta_{p'})$	x'' = R*COS(RADIANS($\theta_{p'}$))
10	Transform the polar coordinates (R, $\theta_{p'}$) to the new y''.	$y'' = R (\sin \theta_{p'})$	y'' = R*SIN(RADIANS($\theta_{p'}$))

Consider the board shown in Fig. 1 with the centroid, Point C, and the line OC connecting the origin of the graph to the centroid. When the board is shrinking, tangential movement will occur along the growth rings toward OC by a value equal to the fractional tangential shrinkage, excluding the radial shrinkage previously applied isometrically. For example, consider Point A during shrinkage: the angle θ_A between the x-axis and the line OA will increase, moving closer (physically and numerically) to the angle of the centroid, θ_C, according to Eq. 8. On the other hand, for Point B, the angle θ_B between the x-axis and line OB will decrease. [Note that the model can also be used to calculate and illustrate the new shape resulting from wood swelling in addition to shrinkage; in that case, the perimeter points would move away (*i.e.*, expand) from the line OC.]

The final step is to convert the polar coordinates given by Eqs. 5 and 8 back to their new *x*” and *y*” coordinates using Eqs. 9 and 10. The new coordinates, showing the now distorted shape of the cupped (or crowned) lumber, are plotted along with the original shape.

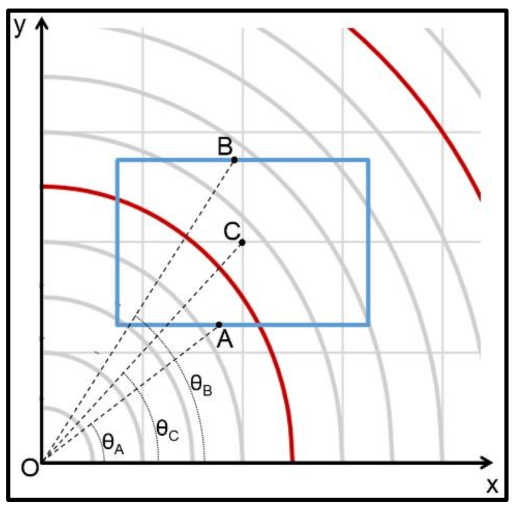

Fig. 1. Cross-section of board showing points on perimeter relative to the centroid of the board

RESULTS AND DISCUSSION

Table 2 presents the previously mentioned input variables, summarizes the model output results, and illustrates the output for a “bastard” sawn southern red oak (*Quercus falcata*) board (lumber that has annual rings that are 30 to 60 degrees to its wide face) that was originally 1” x 10” when sawn green. The first two lines of output are the published percent total tangential (11.3) and radial shrinkage (4.7) from the green to the oven-dry condition. The next two rows of output give the average board width (9.34) and thickness (0.94) (in the same units as the original board) after shrinkage has occurred, followed by the average dimension change in width (-0.66) and thickness (-0.06). The output includes the actual board percent dimensional change for both width (-6.64) and thickness (-5.57) and the total average percent shrinkage exhibited, for both the board width (8.66) and thickness (7.27). The former pair reflect the actual percent shrinkage based on the initial and final moisture content input, while the latter pair represent the total shrinkage if dried to the oven-dry condition. This output pair can be compared to the published values for the total radial and tangential shrinkage for the modeled species, and are shown to differ because of the curvature of the growth rings and natural variability. Thus, they are in near agreement with either flatsawn or quartersawn boards (annual rings are approximately perpendicular to the wide face of the board) that are distant from the pith, as shown for the flatsawn board (Table 2), whose exhibited total average percent width shrinkage was 11.06 and exhibited total average percent thickness shrinkage was 5.00.

In addition, the magnitude of cup (or crown) of the board's top was calculated, and the minimum distance between the top and bottom of the board was given as the maximum thickness possible for the board at the final moisture content. From Table 2, for the flatsawn board, these values were 0.082 and 0.876 inches, respectively.

Table 2. Example of Model Input and Output Variables for Two Southern Red Oak (*Quercus falcata*) Boards

Input Variables	Bastard Sawn Board	Flatsawn Board
Initial Board Width	10	10
Initial Board Thickness	1	1
X and Y Coordinates of Board Center	6, 7	0, 14
Initial Moisture Content (%)	30	30
Final Moisture Content (%)	7	7
Species of Wood	Oak, S. Red	Oak, S. Red
Diameter of Log	30	30
Output Variables		
% Total Tangential Shrinkage for Species	11.3	11.3
% Total Radial Shrinkage for Species	4.7	4.7
Final Board Avg. Width	9.34	9.15
Final Board Avg. Thickness	0.94	0.96
Average Width Dimension Change	-0.66	-0.85
Average Thickness Dimension Change	-0.06	-0.04
Actual Board Width % Dimension Change	-6.64	-8.48
Actual Board Thickness % Dimension Change	-5.57	-3.84
% Average Width Shrinkage Value Exhibited	8.66	11.06
% Average Thickness Shrinkage Value Exhibited	7.27	5.00
Board is Cupped: Valley to Peak Distance	0.197	0.082
Max. Planed Board Thickness (both sides)	0.731	0.876

Because the model assumes that shrinkage occurs linearly with the loss of moisture below the FSP, in similar fashion, the model predicts that the resulting amount of cup as a function of decreasing moisture content is also linear.

The model can be used to compare the magnitude of cup that develops depending on where it is sawn from the log. Figure 2 shows four flatsawn overcup oak (*Quercus lyrata*) boards modeled as having their centroids at distances of 0.6, 4.5, 9, and 13.5-inches from the pith center of the log. The predicted amount of cup increases more than threefold as the board center moves from near the bark inward toward the pith.

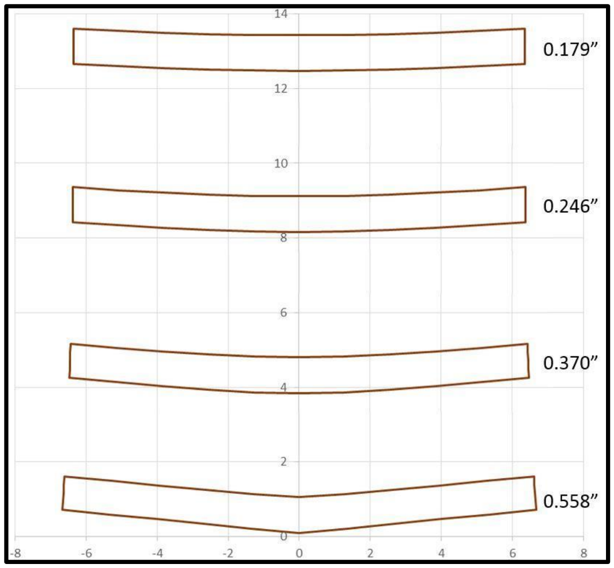

Fig. 2. The development of cup as a flatsawn overcup oak 1" x 14" board is positioned closer to the pith

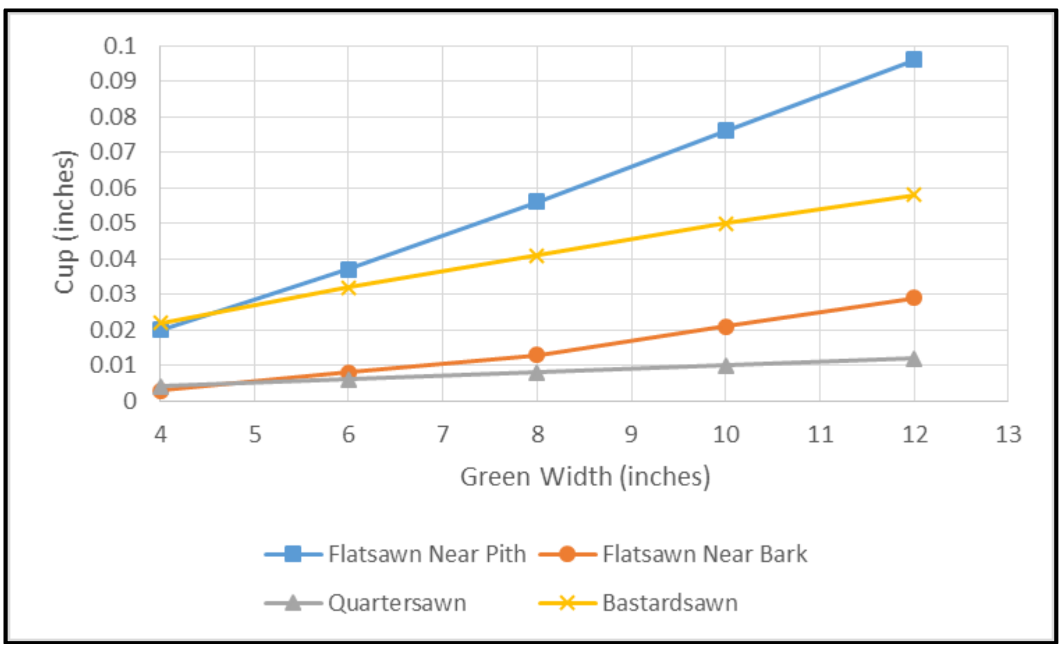

Fig. 3. Magnitude of cup in loblolly pine dried to 15% moisture content depending on the location of dimension lumber of various widths

The relative amount of cup that will be developed depending on product size and location can be predicted by the model. For example, consider the amount of cup that might occur in loblolly pine (*Pinus taeda*) dimension lumber dried to 15% moisture content. The amount of cup *versus* the green width of dimension lumber was modeled for 1) flatsawn lumber near the pith; 2) flatsawn lumber near the bark; 3) quartersawn lumber; and 4) for the bastard sawn board that lies between the quartersawn and flatsawn boards. Figure 3 illustrates that cup increases with increasing lumber width, that quartersawn lumber exhibits minimal cup, and that flatsawn lumber from near the pith was much more prone to cup than flatsawn lumber sawn from near the bark. This has implications in terms of being able to produce a dimension product that cleans up in the planer: severe cup may prevent the lumber from being planed on both sides. Boards that are cupped excessively might also split in the planer.

Different species of wood can also be modeled to predict the potential magnitude of cup that may develop in pieces that are located the same distance from the pith. As mentioned previously, many process factors will influence the development of cup; the model only predicts the potential of cup based on the magnitude and difference between the tangential and radial shrinkages, board location, and moisture content change. Species having a larger cup potential will require that good stacking practices are employed. Table 3 illustrates the range of cup that might occur in several species of 1" x 8" flatsawn boards centered 14" distance from the pith.

The effects of swelling from the adsorption of moisture can also be estimated by the model. A good example of this would be a flatsawn hickory floor plank originally ¾" x 6" wide at 7% MC that was flat when originally installed. If the home, and consequently the floor, are exposed to high humidity for an extended period of time, the plank might increase to 14% MC. Assuming that the top of the installed flatsawn board was the side nearest the bark, the result of moisture gain and the accompanying swelling is the development of crown (the top surface is convex).

Table 3. Comparison of Predicted Cup Magnitude for 1" x 8" Flatsawn Boards of Selected Species Dried to 8% Moisture Content and Located 14" from the Pith

Species	Cup (inches)
Alder, red (*Alnus rubra*)	0.023
Cedar, incense (*Calocedrus decurrens*)	0.015
Hickory, pignut (*Carya glabra*)	0.033
Maple, sugar (*Acer saccharum*)	0.040
Oak, southern red (*Quercus falcate*)	0.051
Pine, loblolly (*Pinus taeda*)	0.020
Poplar, yellow (*Liriodendron tulipifera*)	0.028
Walnut, black (*Juglans nigra*)	0.018
Teak (*Tectona grandis*)	0.026

Model Limitations

There are several limitations to this shrinkage and cup estimation model, which are listed here:

- The published average tangential and radial shrinkage values have been used in this model. Shrinkage actually varies between trees of the same species, within trees, and even within growth rings. Model results are therefore likely to differ from actual samples.
- The model approximates growth rings as being circular. Actual growth rings generally deviate from being perfect circles.

- Juvenile wood or abnormal (compression and tension) wood is not considered in the model. Also, sloping grain or cross grain is not taken into account.
- A uniform moisture content is assumed throughout the wood, both in the initial and final states.
- Modeling of cup assumes that the wood is free of any restraint and that the wood is completely an elastic material. Hence, the model does not consider internal restraints generated by drying stresses, nor does it consider external restraints such as top loading weight typically present in stacked lumber, nor restraints provided by glue, nails, or tongue and groove (T&G) installations. The calculated value of cup represents the potential maximum amount of cup if these forces are absent.
- Although the model produces error warnings if the user inputs an initial moisture content greater than 30%, or a negative final moisture content, the program will erroneously calculate a shrinkage result.
- While effort has been made at error prevention and/or generating error warnings, the author acknowledges that errors in the software are still possible.

CONCLUSIONS

1. Estimating wood shrinkage using the assumption of linearity for tangential shrinkage incorporates a "built-in-error" in its estimate and cannot predict or calculate cup. The current work has developed a user-friendly, graphic model of lumber shrinkage and swelling based on the modified Booker-Xiang's shrinkage model for cupping. It employs a polar coordinate system and thus produces a more accurate picture of shrinkage and shows the development of cup.
2. The model has the ability to estimate and graphically illustrate shrinkage and cup as dependent on board location within the log, change in moisture content, change in product size, and differences between species.
3. In addition to shrinkage, the model can estimate swelling and the deformation that results when wood gains moisture.

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