Investigation of Elastic Properties of Paper Honeycomb Panels with Rectangular Cells

Jerzy Smardzewski,* Adam Gajęcki, and Marlena Wojnowska

Multilayer panels that have paper cores with hexagonal cells continue to have a limited application in furniture production. In contrast, there are no sandwich honeycomb panels with cores containing rectangular cells employed in this industry. Such cores should distinguish themselves by strong orthotropic advantages, in particular, for designing shelves and partitions of cabinet furniture. Hence, the objective of this study was to determine the effect of core rectangular paper cells on the mechanical properties of three-layer furniture panels. The authors decided to ascertain relative density and elasticity constants of the designed cells. The results of empirical experiments of cell elasticity moduli were compared with the results of the analytical calculations. The impact of sample width on their mechanical properties was determined. It was demonstrated that cores with hexagonal cells in furniture panels could be replaced by cores with rectangular cells.

Keywords: Honeycomb panels; Rectangular cells; Stiffness; Strength; Wood composite

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INTRODUCTION

Wood is a renewable, ecological raw material employed to manufacture high quality furniture and everyday products. Its versatile utilization in numerous branches of the wood industry exerts considerable influence on the intensive exploitation of wood resources. The above-mentioned factors clearly show that there are reasons to replace traditional panel materials, such as plywood (PW), particleboard (PB), oriented strand board (OSB), medium-density fiberboard (MDF), and high-density fiberboard (HDF), with lightweight sandwich honeycomb panels. These panels are characterized by relatively high strength and stiffness (Khan 2006; Schwingshackl *et al.* 2006; Jen and Chang 2008; Smardzewski 2013). According to Negro *et al.* (2011), the density of light honeycomb panels should not exceed 500 kg/m^3.

The use of honeycomb panels with paper cores manufactured from hexagonal cells is quite widespread. However, during the manufacturing process these cells acquire irregular shapes of non-regular hexagons (Xu *et al.* 2008). In a study conducted by Smardzewski and Prekrat (2012) it was demonstrated that the core of a honeycomb panel made of irregular hexagonal cells placed between two HDF panels equalizes quite well the stresses that develop in the facings. The above researchers observed that the stiffness and strength of the honeycomb panels were affected significantly by the paper grammage as well as the cell shapes and dimensions.

Honeycomb structures find widespread application in the motor, airplane, and military industries (Schmueser and Wickliffe 1987). In the furniture industry, due to economic reasons, honeycomb panels with thicknesses exceeding 25 mm (Barboutis and

Vassiliou 2005; Smardzewski 2015; Smardzewski and Jasińska 2016) are preferred. Furthermore, physico-chemical properties of honeycomb panels with hexagonal cells manufactured from light metals are commonly known (Paik *et al.* 1999; Schwingshackl *et al.* 2006; Said and Tan 2008).

To increase the stiffness of wood-based honeycomb panels, the type and thickness of their facings (Meraghni *et al.* 1999; Sam-Brew *et al.* 2011; Chen and Yan 2012) were changed, the paper used to manufacture them was impregnated, and the dimensions as well as the shapes of the core cells were changed (Majewski and Smardzewski 2012). In addition, recommendations were made regarding factors that should be taken into account during the production process of paper cores of honeycomb panels intended for the furniture industry (Sam-Brew *et al.* 2011). For the core with hexagonal cells, these suggestions included: cell dimension, filling height, filling density, as well as cell orientation with respect to the panel sheet. In addition, it was confirmed many times that the honeycomb panel stiffness depends on the stiffness of the external facings. On the basis of a four-point bending, it was demonstrated that to reduce deflection of a honeycomb panel with a paper core and wood base facings, core cells should be as small as possible, whereas the core height should be as large as possible (Sam-Brew *et al.* 2011). It has been shown that honeycomb panels have higher values of shear modulus and higher stiffness when planes of common core cell walls are oriented parallel to the longer side of the panel (Bitzer 1997). The enlargement of the inclination angle of the cell walls increases the panel density and, by doing so, it significantly enhances its strength and stiffness (Majewski and Smardzewski 2013).

Hexagonal, regular core cells ensure panel isotropy, whereas the elongated cells affect their orthotropy (Côté *et al.* 2004; Smardzewski and Prekrat 2012). The honeycomb panel core and facing isotropy exert a positive influence on the processes of their cutting by minimizing the amount of waste during the production process. Simultaneously, isotropy assures the uniform bending stiffness in mutually perpendicular directions. In contrast, orthotropy interferes with panel cutting efficiency, although it does has an advantageous influence on improved stiffness and strength of rectangular panels along one preferred direction. This is an exceptionally useful property when designing shelves and horizontal partitions in cabinet furniture. Rectangular cells constitute a special case of core polygonal cells. Their shape and arrangement in the honeycomb panel core can have a crucial impact on improved multilayer panel stiffness. Based on the available literature, it has not yet been analyzed to what extent elongated, rectangular paper core cells affect the mechanical properties of furniture honeycomb panels and the orthotropic strength of such panels.

The aim of this study was to determine the effect of the orientation of the rectangular cells of the paper core on the mechanical properties of three-layer furniture panels. The cognitive objective of the experiments was also to ascertain relative density and elasticity constants of the designed cells. The authors decided to compare the results of the empirical experiments of cell elasticity moduli with the results of analytical calculations. The practical goal of the investigation was to show the possibilities of substituting cores with hexagonal cells used in furniture panels with cores with rectangular cells.

EXPERIMENTAL

Elastic Properties and Relative Density of a Polygonal Cell

Elastic properties of a hexagonal cell for directions 1 and 2 of orthotrophy, depending on the wall inclination angle, are described by the general formulas (1 through 5) (Masters and Evans 1996; Smardzewski 2013; Wojnowska *et al.* 2017),

$$E_{\mathrm{H1}} = \frac{t(h + l\sin\varphi)}{l^2\cos\varphi\left(\frac{l^2\cos^2\varphi}{E_{\mathrm{MD}}t^2} + \frac{\cos^2\varphi}{G_{\mathrm{MDCD}}} + \frac{(2h + l\sin^2\varphi)}{lE_{\mathrm{MD}}}\right)} \tag{1}$$

$$E_{\mathrm{H2}} = \frac{t}{(h + l\sin\varphi)\left(\frac{l^2\sin^2\varphi}{E_{\mathrm{MD}}t^2\cos\varphi} + \frac{\sin^2\varphi}{G_{\mathrm{MDCD}}\cos\varphi} + \frac{\cos\varphi}{E_{\mathrm{MD}}}\right)} \tag{2}$$

$$\upsilon_{\mathrm{H12}} = -\sin\varphi\left(\frac{h}{l} + \sin\varphi\right)\left(\frac{-\frac{l^2}{E_{\mathrm{MD}}t^2} - \frac{1}{G_{\mathrm{MDCD}}} + \frac{1}{E_{\mathrm{MD}}}}{\frac{l^2\cos^2\varphi}{E_{\mathrm{MD}}t^2} + \frac{\cos^2\varphi}{G_{\mathrm{MDCD}}} + \frac{(2h + l\sin^2\varphi)}{lE_{\mathrm{MD}}}}\right) \tag{3}$$

$$\upsilon_{\mathrm{H21}} = \frac{\sin\varphi\cos\varphi\left(\frac{l^2}{E_{\mathrm{MD}}t^2} + \frac{1}{G_{\mathrm{MDCD}}} - \frac{1}{E_{\mathrm{MD}}}\right)}{\left(\frac{h}{l} + \sin\varphi\right)\left(\frac{l^2\sin^2\varphi}{E_{\mathrm{MD}}t^2\cos\varphi} + \frac{\sin^2\varphi}{G_{\mathrm{MDCD}}\cos\varphi} + \frac{\cos\varphi}{E_{\mathrm{MD}}}\right)} \tag{4}$$

$$G_{\mathrm{H12}} = \frac{1}{\left(\frac{lh^2(l+2h)\cos\varphi}{E_{\mathrm{MD}}t^3(h + l\sin\varphi)}\right) + \frac{1}{G_{\mathrm{MDCD}}t}\cos\varphi\left(\frac{h(h+2l)}{(h + l\sin\varphi)}\right) + \frac{l\sin\varphi\,(l\cos^2\varphi + (h + l\sin\varphi))}{E_{\mathrm{MD}}t}\left(\frac{\cos\varphi}{(h + l\sin\varphi)} + \frac{\sin\varphi}{\cos\varphi}\right)} \tag{5}$$

where E_{H1} (MPa) and E_{H2} (MPa) are the linear elasticity modulus (MPa) of the cellular panel core in directions 1 and 2; G_{H12} (MPa) is the shear elasticity modulus of the core in plane 12; ν_{H12} and ν_{H21} are the Poisson's ratios of the core in plane 12 and 21, respectively; E_{MD} (MPa) is the linear elasticity modulus of paper; G_{MDCD} (MPa) is the shear elasticity modulus of paper; L_x (mm) and S_y (mm) are the length and width of cell l; h (mm) is the length of the sides; t (mm) is the thickness of the cell wall; and φ (°) is the inclination angle of the cell walls (Fig. 1). The adopted direction of orthotropy 2 was parallel to the plane of the common wall of the neighbouring cells.

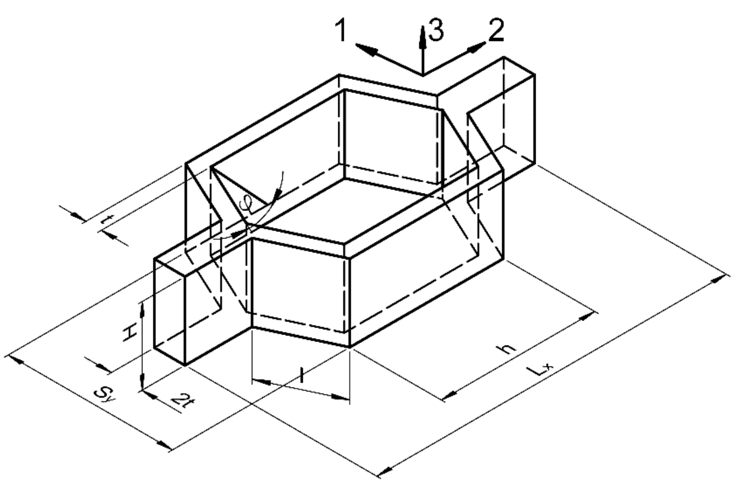

Fig. 1. Hexagonal cell

For the formation of the rectangular cell, angle φ = 0°. In these special conditions, the elastic properties of the rectangular cell can be expressed by the following equations,

$$E_{\mathrm{R1}} = \frac{th}{l^2\left(\frac{l^2}{E_{\mathrm{MD}}t^2} + \frac{1}{G_{\mathrm{MDCD}}} + \frac{2h}{lE_{\mathrm{MD}}}\right)} \quad (6)$$

$$E_{\mathrm{R2}} = \frac{tE_{\mathrm{MD}}}{h} \quad (7)$$

$$\upsilon_{\mathrm{R12}} = 0 \quad (8)$$

$$\upsilon_{\mathrm{R21}} = 0 \quad (9)$$

$$G_{\mathrm{R12}} = \frac{1}{\frac{lh^2\,(l+2h)}{E_{\mathrm{MD}}t^3h} + \frac{h(h+2l)}{G_{\mathrm{MDCD}}\mathrm{th}}} \quad (10)$$

where E_{R1} (MPa) and E_{R2} (MPa) are the linear elasticity modulus (MPa) of the panel core with rectangular cells in direction 1 and 2; G_{R12} (MPa) is the shear elasticity modulus of the core in plane 12; and ν_{R12} and ν_{R21} are the Poisson's ratios of the core in plane 12 and 21, respectively. For identical E_{MD} (MPa), G_{MDCD} (MPa) l (mm), h (mm), and t (mm) values, the values of elastic constants of the hexagonal and rectangular cells depend entirely on the inclination angle of the cell walls. Assuming a typical value of this angle for a hexagonal cell in furniture panels at $\varphi = 34°$, the relationships between the elastic constant values of the rectangular cells E_{R1} (MPa), E_{R2}, (MPa) and G_{R12} (MPa), and the respective values of the elastic constants of hexagonal cells E_{H1} (MPa), E_{H2} (MPa), and G_{H12} (MPa), amount to, respectively:

$$\frac{E_{\mathrm{R1}}}{E_{\mathrm{H1}}} = 0.065,\ \frac{E_{\mathrm{R2}}}{E_{\mathrm{H2}}} = 1572,\ \frac{G_{\mathrm{R12}}}{G_{\mathrm{H12}}} = 0.067 \quad (11)$$

The above numbers illustrate that rectangular cells are characterized by a very strong orthotrophy as well as by a very high value of linear elasticity modulus E_{R2} in the direction parallel to the surface of common cell walls. It should be stressed here that the change of cell geometry from hexagonal to rectangular exerts a strong influence on the change of its relative density, respectively ρ_{H} and ρ_{R}. Relative density is the ratio of the density of a substance to the density of a given reference material. Because the core was made of the same materials, the relative density can be expressed as the ratio of the surface area of the substance to the core surface:

$$\rho_{\mathrm{H}} = 1 - \frac{F_1+F_2+F_3}{F^*}, \quad (12)$$

where F_1, F_2, F_3 partial surfaces of the core substance, F^* surface of the core:

$$F^* = 4(l\cos(\varphi) + t)\left(h + l\sin(\varphi) - t\cot\left(\frac{\varphi+90^{\mathrm{o}}}{2}\right)\right), \quad (13)$$

$$F_1 = 2l\cos(\varphi)\left(h - 2t\cot\left(\frac{\varphi+90^{\mathrm{o}}}{2}\right) + l\sin(\varphi)\right), \quad (14)$$

$$F_2 = 2(l\cos(\varphi) - t)\left(h - 2t\cot\left(\frac{\varphi+90^{\mathrm{o}}}{2}\right)\right), \quad (15)$$

$$F_3 = 2l\sin(\varphi)\cos(\varphi)\left(l - t\cot\left(\frac{\varphi+90^{\circ}}{2}\right)\right), \tag{16}$$

and

$$\rho_R = 1 - \frac{(h-2t)(2l-t)}{2(l+t)(h-t)} \tag{17}$$

For dimensions identical as earlier: l, h, t, and the relationship between the relative density of the rectangular cell and hexagonal cell wall inclination angle $\varphi = 34°$ has the value of:

$$\frac{\rho_R}{\rho_H} = 2.1289 \tag{18}$$

Therefore, for the adopted assumptions, the relative density of the rectangular cells was more than two times higher when compared with the density of the hexagonal cells. Bearing this in mind, the authors decided to prepare two types of paper cores, namely cores with rectangular cells and with reference hexagonal cells, typical for the furniture industry. It was assumed that the model cores would be characterized by similar relative density as well as identical cell wall height and thickness. They would differ with respect to the length of cell walls.

Properties of Paper and Core Cells

Two kinds of cells, hexagonal and rectangular, were prepared for the experiments. Figure 2a presents the shape of a core section with hexagonal cells, while Table 1 collates the mean dimension values of this cell. The cores with rectangular cells (Fig. 2b) were obtained by stretching in an expander of appropriate paper structures from which reference cells were also obtained, as in Fig. 2a. Table 1 collates the mean dimensional values of single rectangular cells.

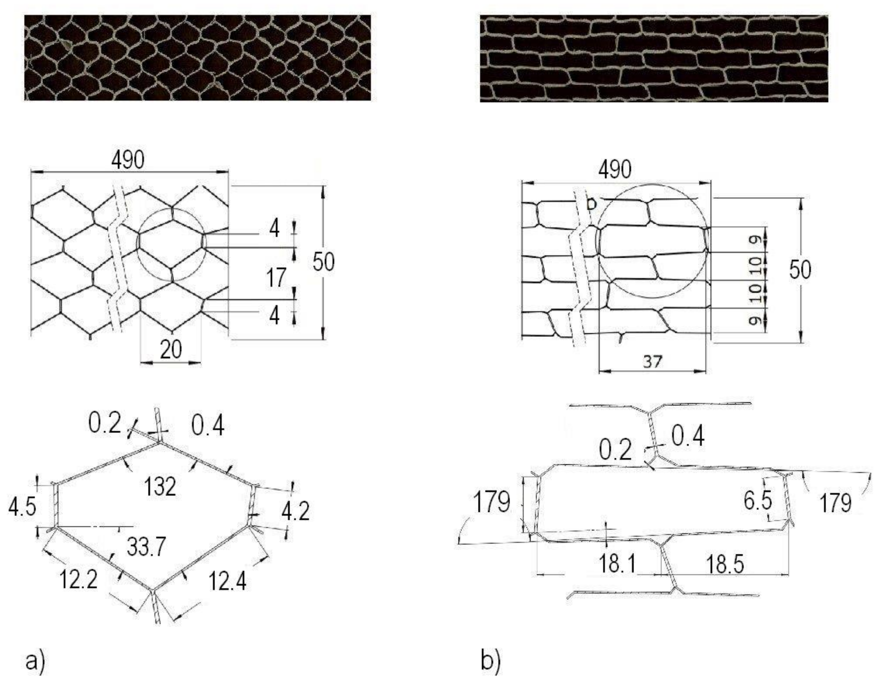

Fig. 2. Core section and dimensions of a) hexagonal and b) rectangular cell (dimensions in mm)

Table 1. Cell Dimensions Used in Experiments

Parameter	Unit	Shape of Cells	
		Hexagonal	Rectangular
h	mm	4	7
l		12	18
t		0.223	0.217
h_c		17	17
φ	°	34	0

h_C = Cell wall height (core height)

The discussed structures were made from waste paper supplied by Axxion Industries Polska, Ltd. (Zbąszynek, Poland). For the core with hexagonal and rectangular cells recycled paper was used. The paper samples were conditioned prior to the experiments for 12 h in air at 23 ºC ± 1 ºC and with an air relative humidity of 50% ± 2% in accordance with BS EN 20187 (1993). The examination of the paper's physicochemical properties was conducted in a facility in which the same climatic conditions were maintained as during sample conditioning. Paper grammage was determined according to BS EN ISO 536 (2012) and its thickness according to BS EN ISO 534 (2011). Measurements of the examined samples were: 200 mm × 250 mm. A total of 20 samples of each paper were examined, and the measurement results were given as a mean value from the twenty measurements. Paper linear elasticity moduli E_{MC} and E_{CD} for the machine direction (MD) and crosswise direction (CD) were determined in accordance with BS EN ISO 1924-2 (2008). Tensile tests were conducted with the assistance of a Zwick Z010 universal testing machine (Zwick GmbH & Co. KG, Ulm, Germany) for 20 mm/min velocity. The tests were performed on samples 15-mm-wide and 180 mm in length. Ten samples were prepared for each paper and direction. In the tensile test with the assistance of a mechanical extensimeter (Dantec Dynamics A/S, Skovlunde, Denmark), values of the appropriate Poisson's ratios ν_{MDCD} and ν_{CDMD} were also determined. The results of the performed paper examinations are presented in Table 2.

Table 2. Paper Physicochemical Properties

Shape of Cells	Thickness	Grammage	Linear Elasticity Modulus		Shear Elasticity Modulus	Poisson's Ratio	
			E_{MD}	E_{CD}	G_{MDCD}	$\nu_{MDCD,}$	ν_{CDMD}
	(mm)	(g/m^2)	(MPa)				
Hexagonal	0.223	151	4460	2070	915	0.39	0.81
Rectangular	0.217	149	4300	1940	894	0.36	0.86

Table 3. Cell Elastic Constants and Relative Density

Shape of Cells	Linear Elasticity Modulus		Shear Elasticity Modulus	Poisson's Ratio		Relative Density
	(MPa)					
Hexagonal	E_{H1} = 0.045	E_{H2} = 0.085	G_{H12} = 0.163	ν_{H12} = 0.726	ν_{H21} = 1.376	0.027
Rectangular	E_{R1} = 0.003	E_{R2} = 133.3	G_{R12} = 0.011	ν_{R12} = 0	ν_{R21} = 0	0.029

On the basis of the determined cell dimensions as well as the paper elastic properties, the cell elastic constants as well as cell relative densities were calculated employing Eqs. 1 through 10 and 12 through 17. The results are presented in Table 3.

At comparable relative density of the hexagonal (ρ_H = 0.027) and rectangular (ρ_R = 0.029) cells, the linear elasticity modulus for the direction 1 amounted to E_{H1} = 0.045 MPa and E_{R1} = 0.003 MPa, respectively. The highest linear elasticity modulus was recorded for a rectangular cell for direction 2. This value amounted to E_{R2} = 133.3 MPa, while for the hexagonal cell it was E_{H2} = 0.085 MPa. The shear elasticity modulus of the hexagonal cell was almost 15 times greater than that of the rectangular cell. The Poisson's ratio of the hexagonal cell ν_{H21} was nearly twice as high in comparison with ν_{H12}. In contrast, the Poisson's ratios of the rectangular cell, whose wall inclination angle equalled $\varphi = 0^o$, assumed the value of 0 for both directions.

Empirical Experiments of Sandwich Furniture Panels

Two variants of sheets measuring 22 mm × 600 mm × 560 mm were prepared for empirical experiments on sandwich furniture panels in industrial conditions. In the first variant, the cores with hexagonal cells were covered with 2.5-mm HDF facings (h_F = 2.5 mm; SD = 0.03 mm). The core height amounted to h_C = 17 mm, while the entire thickness of the honeycomb panel reached H = 22 mm (Fig. 3a). In the second variant, the cores with rectangular cells were also covered with HDF facings obtaining identical measurements of the core height and thickness of the honeycomb panel (Fig. 3b). Physico-mechanical properties of the applied HDF boards are presented in Table 4. The polyvinyl acetate glue (PVAc D3) was used to glue the facings. It was decided to use 40 g of adhesive per 1 m^2 of facing. In industrial conditions, the temperature and air relative humidity amounted to 20 ºC and 50%, respectively.

Table 4. Physico-mechanical Properties of HDF Board

	Moisture Content	Thickness h_F	Density	MOE	MOR
	(%)	(mm)	(kg/m³)	(MPa)	
Average	5.85	2.5	922	6531	71
Standard Deviation	0.09	0.03	25.06	440.8	5.6

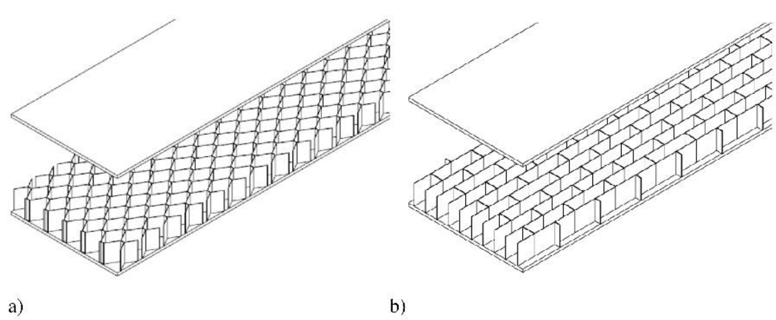

Fig. 3. Variants of panels prepared for tests with: a) hexagonal core and b) rectangular core

Following two weeks of conditioning in production conditions, appropriate samples were cut out from the formed furniture panel sheets intended for three-point bending tests. The first group of samples was characterized by measurements complying with the EN 310

(1993) standard, namely: H × 50 mm × 20 H + 50 mm. The second group of samples measuring H × 100 mm × 20 H + 50 mm was made to evaluate the effect of the sample width on the mechanical properties of furniture honeycomb panels. In the case of the crosswise arrangement of these cells, *i.e.*, along the width of the sample equaling 50 mm, only two samples occurred. Both dimensional sample groups were additionally divided into two types, considering the directions of cell orthotrophy 1 and 2. The first type comprised of samples whose surfaces of common core cell walls were oriented perpendicularly to the longer side of the sample (P). The second type included samples whose surfaces of common core cell walls were oriented parallel to the longer side of the sample (L). Ten samples were prepared for each variant, for a total of 80 samples. Three-point bending tests were conducted with the assistance of a Zwick Z010 universal testing machine (Zwick GmbH & Co. KG, Ulm, Germany) employing a loading velocity of 10 mm/min. The force was measured with 0.01 N accuracy, while the deflection was with the accuracy of 0.01 mm. The elasticity moduli (MOE) and bending strength (MOR) of the examined furniture honeycomb panels were determined on the basis of direct experimental results.

Modulus of Linear Elasticity of the Honeycomb Panel Core

The linear elasticity modulus of the honeycomb panel was calculated in accordance with previous studies (Smardzewski and Prekrat 2012; Smardzewski 2013), that calculated beam stiffness as a sum of the stiffness of the individual layers (Fig. 4), hence Eq. 19,

$$E = 2E_{\mathrm{F}}\left(\frac{h_F}{H}\right)^3 + 6E_{\mathrm{F}}\frac{h_{\mathrm{F}}(h_{\mathrm{C}}+h_{\mathrm{F}})^2}{H^3} + E_{\mathrm{C1}}\left(\frac{h_{\mathrm{C}}}{H}\right)^3 \tag{19}$$

where E (MPa) is the linear elasticity modulus of the sandwich beam, E_{F} (MPa) is the linear elasticity modulus of facing, E_{C1}/ E_{C2} (MPa) are the linear elasticity modulus of the core, respectively in 1 and 2 direction, b (mm) is the beam width, H (mm) is the beam thickness, h_{F} (mm) is the facing thickness, and h_{C} (mm) is the core thickness.

Converting this equation, appropriate core linear elasticity moduli were calculated from the formula:

$$E_{\mathrm{C1}} = \frac{E-2E_{\mathrm{F}}\left(\frac{h_{\mathrm{F}}}{H}\right)^3-6E_{\mathrm{F}}\frac{h_F(h_{\mathrm{C}}+h_{\mathrm{F}})^2}{H^3}}{\left(\frac{h_{\mathrm{C}}}{H}\right)^3} \tag{20}$$

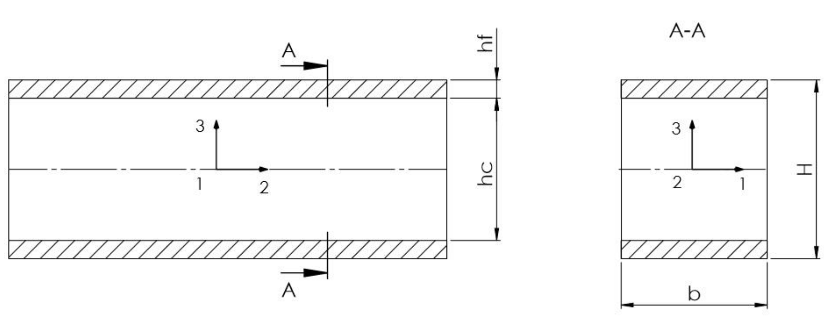

Fig. 4. Measurements of the sandwich beam

The obtained results of the empirical experiments were compared with the results of the analytical calculations collated in Table 3. Bearing in mind the types of the planned investigations, in Table 5 the authors presented the way of designation for the individual samples.

Table 5. Designations of Samples Used in Bending Tests

Type of Test	Shape of Cells	Width of Samples	Orientation of Cells	Code
Experimental	Hexagonal	50 mm	P	EH5P
			L	EH5L
		100 mm	P	EH1P
			L	EH1L
	Rectangular	50 mm	P	ER5P
			L	ER5L
		100 mm	P	ER1P
			L	ER1L
Analytical	Hexagonal	50 mm	P	AH5P
			L	AH5L
		100 mm	P	AH1P
			L	AH1L
	Rectangular	50 mm	P	AR5P
			L	AR5L
		100 mm	P	AR1P
			L	AR1L

Statistical analysis

The authors decided to verify the significance of the above presented interrelationships by subjecting them to the statistical analysis of the Statistica v.13.1 program (StatSoft Polska Sp. z o.o., Kraków, Poland). For this purpose, the t-Student test was employed for the variable-independent samples. The confidence interval of basic statistics equalled 0.95.

RESULTS AND DISCUSSION

Stiffness and Strength of Honeycomb Panels

Figure 5 presents the dependence of loading in the deflection function of the samples of 50 mm and 100 mm width. An impact of the width sample on its deflection was noticeable. This dependence is particularly noticeable in the case of samples with longitudinal (L) plane orientation of the common core cell walls. In this case, samples with 100 mm width were characterized by at least two times higher loading at the same value of deflection. For example, for the deflection of 4 mm, the force bending samples with a hexagonal core 50 mm wide equalled 68.40 N, while for 100-mm-wide samples it equalled 150.89 N. For the same direction but with the rectangular core, these values amounted to 66.25 N and 145.01 N, respectively. In the case of samples with a crosswise (P) plane orientation of the common core cell walls, the value of the bending force for samples with a 50-mm-wide hexagonal core equalled 64.13 N, whereas with the rectangular core equalled 14.57 N. For the 100-mm-wide samples, these forces amounted to 142.43 N and 65.22 N, respectively.

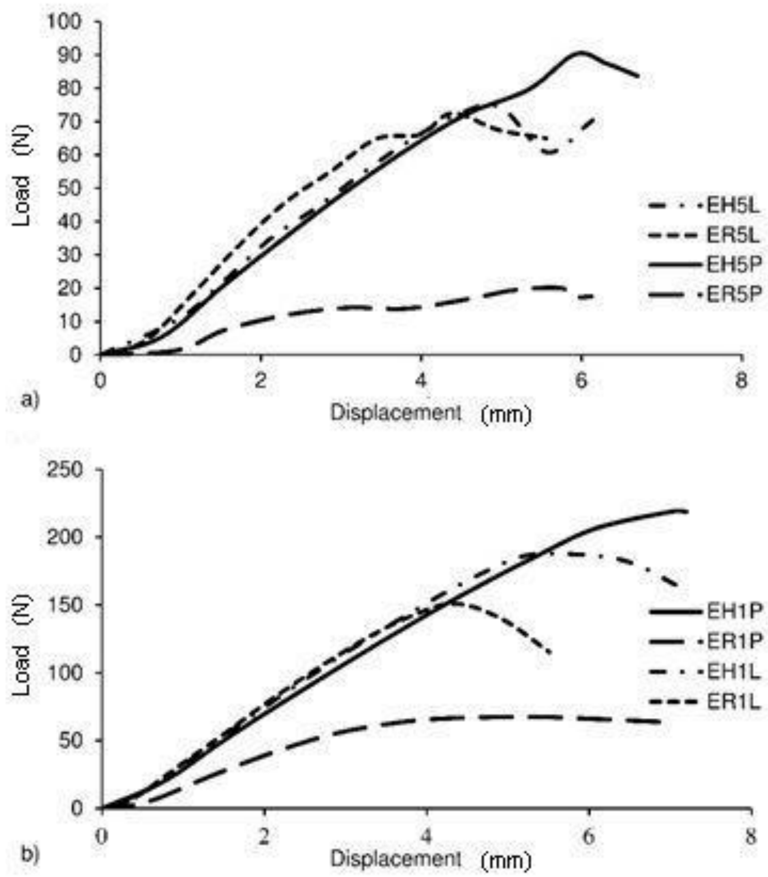

Fig. 5. Stiffness of samples: a) 50-mm-wide and b) 100-mm-wide

It follows that for the 4 mm deflection, the dependence of force and deflection was connected with the correlation in the form of the linear function $f(x) = Ax$, where the directional coefficients A are collated in Table 6. The R^2 coefficient for these correlations is equal from 0.96 to 0.98.

Table 6. Directional Coefficients A for the Linear Correlation Function

Sample Type	EH5L	ER5L	EH1L	ER1L	EH5P	ER5P	EH1P	ER1P
A	12.381	13.708	26.967	27.203	14.184	3.055	31.874	10.233

The character of the course of the load-deflection dependence exerted a direct influence on the elasticity modulus value of the bent material. The high values of the directional coefficient A indicate considerable stiffness of the examined honeycomb panel sample. The highest values of this coefficient were determined for the ER1L and EH1L panels with longitudinal arrangement of the rectangular and hexagonal cell walls for the 100-mm-wide samples and were 27.203 and 26.967, respectively. A similar regularity was observed for the 50-mm-wide ER5L and EH5L panels, the values were 13.708 and 12.381, respectively. In the case of the ER1P and EH1P panels with a crosswise arrangement of the rectangular and hexagonal cell walls, the hexagonal cells ensured higher values of the *A* coefficient. For the 100-mm-wide samples, these values equalled 10.233 and 31.874, respectively. The same regularity was found for the 50-mm-wide ER5P and EH5P panels, the values were 3.055 and 14.184, respectively. Therefore, on the basis of the performed experiments, it can be said that panels with cells oriented in the (L) direction (Fig. 6a) were characterized by the highest value of the linear elasticity modulus. For the 50-mm-wide sample with a rectangular cell core $MOE_{(L)}$ = 999 MPa, whereas for the samples of the same width with a hexagonal core, the $MOE_{(L)}$ = 814 MPa. In the case of the identical 100-mm-wide samples, the $MOE_{(L)}$ values amounted to 842 MPa and 852 MPa, respectively. For the (P) direction, the stiffness of the honeycomb panels was noticeably lower (Fig. 6b). For the 50-mm-wide sample with a rectangular cell core, the $MOE_{(L)}$ = 397 MPa, while for samples of the same width but with a hexagonal core $MOE_{(L)}$ = 760 MPa. In the case of the identical samples, but 100-mm-wide, the $MOE_{(L)}$ assumed values of 480 MPa and 788 MPa, respectively.

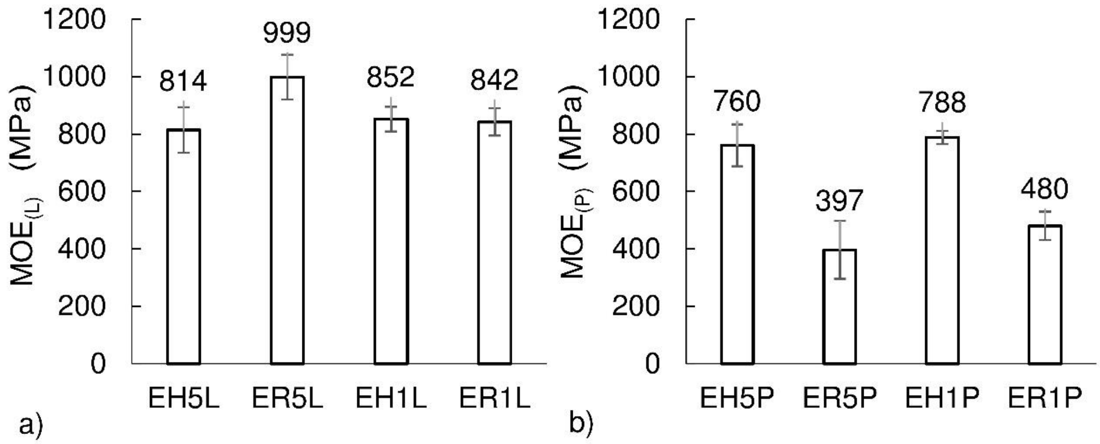

Fig. 6. Modulus of elasticity of the honeycomb panels for direction: a) (L) and b) (P)

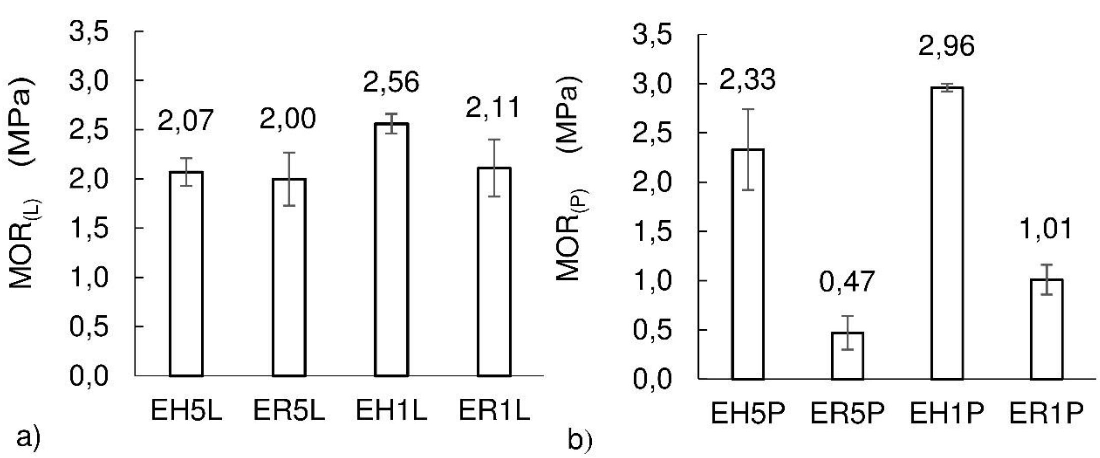

Fig. 7. Bending strength of the panels for direction a) (L) and b) (P)

Figure 7 presents the results of the MOR studies. For the (L) direction, the highest mean $MOR_{(L)}$ value was observed in the case of the 100-mm-wide sample with a hexagonal core (2.56 MPa). The sample of the same width but with a rectangular core was characterized by approximately 20% lower strength (2.11 MPa). The $MOR_{(L)}$ values of 50-mm-wide samples were more similar to each other and amounted to 2.07 MPa and 2.00 MPa, respectively, for the panels with a hexagonal and a rectangular core. In the case of the panels with (P) oriented cores, the bending strength was more diversified (Fig. 7b). The highest mean $MOR_{(L)}$ value was determined for the 100-mm-wide sample with a hexagonal core (2.96 MPa), while this value for the sample with a rectangular core was 1.01 MPa. A similar high variability of bending strength was observed for the 50-mm-wide samples. The $MOR_{(L)}$ value for the sample with a hexagonal core equalled 2.33 MPa, while the value for the rectangular core sample was 0.47 MPa.

The width of the sample and the length of the cells oriented along to width of samples influence on relationships between MOE and MOR. For samples 50 mm wide, the number of full rectangular cells, 36.6 mm long, filling the width of the sample is equal 1, and hexagonal cells (24.6 mm long), respectively 2. For 100 mm wide samples, the number of rectangular cells is equal 3, while hexagonal cells 4. For this reason the MOE is about two times higher for EH5P samples than ER5P. In case of MOR, the ratio between EH5P and ER5P is equal about 5/1.

It is evident from the performed experiments that rectangular cells increased the MOE value only for the (L) direction and for the 50-mm-wide samples. This increase, in relation to the MOE of the identical hexagonal samples, amounted to 23%. In contrast, the linear elasticity modulus of the 100-mm-wide sample with a rectangular core was 1.2% smaller in comparison with the MOE of the identical panel with a hexagonal core. For the panels with rectangular cell orientation for the (P) direction as well as the 50-mm- and 100-mm-wide samples, the MOE decreased 42% and 39%, respectively, in relation to the MOE of the panels with hexagonal cells. In addition, rectangular cells decreased the MOR values. For the 50-mm- and 100-mm-wide samples, the decline in the MOR values amounted to 3.4% and 18%, respectively, in comparison with the reference panels of the hexagonal cells. A dramatic deterioration of the panel bending strength was observed for the (P) direction. For the identical 50-mm- and 100-mm-wide samples, the reduction in the MOR value reached 80% and 34%, respectively. Furthermore, it was observed that the core cell orientation in relation to the longer sample side exerted a noticeable influence on the change of the MOE values. For the hexagonal 50-mm- and 100-mm-wide cells, a change in the arrangement direction of the common cell wall from (P) to (L) resulted in an increase of the MOE values 7.1% and 8%, respectively. Equally important was the orientation of the rectangular core cells. In this instance, for identical sample dimension as before, the values of elasticity moduli increased 152% and 75%, respectively. The core cell orientation also affected the panel bending strength. For the 50-mm- and 100-mm-wide samples with hexagonal cells, the change of direction from (P) to (L) decreased the MOR values 11% and 15%, respectively. In contrast, for the identical panels with rectangular cells, a considerable increase of 325% and 109%, respectively, in the MOR values was recorded. Sample width increase from 50 mm to 100 mm also increased the MOR values. For the (L) direction and hexagonal cells, this increase amounted to 19%, while for the rectangular cells it amounted to 5% and for the (P) direction it amounted to 21% and 53%, respectively.

Table 7. List of Significant Correlations for MOE Values

Combination 1 – Combination 2	t	p	p Variations
EH5L - ER5L	-5.2255	0.000128	0.421542
EH5L - ER5P	9.6357	0.000000	0.773386
EH5L - ER1P	9.5456	0.000000	0.189525
EH5P - EH1L	-2.9925	0.009695	0.136886
EH5P - ER5L	-6.9089	0.000007	0.471163
EH5P - ER5P	8.4917	0.000001	0.709479
EH5P - ER1L	-3.4301	0.004062	0.000158
EH5P - ER1P	8.1054	0.000001	0.218005
EH1L - EH1P	3.8765	0.001678	0.126798
EH1L - ER5L	-5.6144	0.000064	0.425598
EH1L - ER5P	12.8024	0.000000	0.067859
EH1L - ER1P	15.7065	0.000000	0.784855
EH1P - ER5L	-9.4591	0.000000	0.025410
EH1P - ER5P	11.7008	0.000000	0.001934
EH1P - ER1L	-7.2213	0.000004	0.185366
EH1P - ER1P	15.4324	0.000000	0.075920
EH1P - ER1P	5.2255	0.000128	0.421542
ER5L - ER5P	15.6710	0.000000	0.278927
ER5L - ER1L	6.6841	0.000010	0.001022
ER5L - ER1P	18.7114	0.000000	0.597556
ER5P - ER1L	-14.0712	0.000000	0.000059
ER5P - ER1P	-2.6636	0.018532	0.114248
ER1L - ER1P	20.4080	0.000000	0.003796

Table 8. List of Significant Correlations for MOR Values

Combination 1 – Combination 2	t	p	p Variations
EH5L - EH1L	-8.6624	0.000001	0.535961
EH5L - EH1P	-18.4345	0.000000	0.007178
EH5L - ER5P	19.6070	0.000000	0.428629
EH5L - ER1P	13.7104	0.000000	0.592939
EH5P - EH1P	-4.1179	0.001045	0.000002
EH5P - ER5P	10.4649	0.000000	0.027686
EH5P - ER1P	7.4495	0.000003	0.015550
EH1L - EH5L	8.6624	0.000001	0.535961
EH1L - EH1P	-9.8554	0.000000	0.029252
EH1L - ER5L	4.0254	0.001252	0.001820
EH1L - ER5P	28.0607	0.000000	0.165574
EH1L - ER1L	2.6839	0.017808	0.000064
EH1L - ER1P	22.4258	0.000000	0.254272
EH1P - ER5L	6.8205	0.000008	0.000004
EH1P - ER5P	37.4498	0.000000	0.001019
EH1P - ER1L	4.3328	0.000688	0.000000
EH1P - ER1P	32.2731	0.000000	0.001959
ER5L - ER5P	9.3122	0.000000	0.047619
ER5L - ER1P	6.0152	0.000032	0.027445
ER5P - ER1L	-5.7108	0.000054	0.002459
ER5P - ER1P	-6.2075	0.000023	0.794118
ER1L - ER1P	3.5782	0.003027	0.001285

Together with the increase of sample width, for the (L) direction, the elasticity modulus of the panel with the hexagonal core increased 4.7% and for the panel with the rectangular core it dropped 16%. For the (P) direction, the MOE values increased 3.7% and 18%, respectively, for appropriate samples.

Table 7 collates significant correlations between the MOE values, while Table 8 presents correlations between the MOR values for selected honeycomb panel combinations. The following designations were used: t – test result; p – confidence interval; and p variation – basic variability measure of the observed results.

It is evident from Tables 7 and 8 that the discussed differences in the MOE and MOR values were caused by a change in the width of the selected samples as well as the cell shape and orientation, which were statistically significant. The value of parameter t indicated the scale of significance between the individual combinations. A higher t value resulted in a higher correlation significance. A positive value of parameter t showed that as the value of MOE or MOR increased, together with the increase of the first combination, the value of the second decreased. In contrast, a negative value of parameter t meant that together with the decrease in the MOE or MOR value, a decrease of the first combination led to an increase in the second one.

Modulus of Linear Elasticity of the Honeycomb Panel Core

Experimentally determined values of the linear elasticity moduli of the honeycomb panels were compared with the results of the analytical calculations. Figure 8 presents a comparison of the values obtained employing equation 20 as well as the values of the linear elasticity moduli of the facings (Table 4) and core cells (Table 3). It is evident from this figure that the MOE calculated on the basis of Eq. 20 for individual types of honeycomb panels was characterized by almost identical values. Therefore, although in the case of the analytical model the influence of the MOE of the core cells on the honeycomb panel, the MOE was negligible and did not exceed 1.7%, the results of experimental studies showed that the cell type and the direction of its position in the core affected this stiffness. However, the obtained values were 3.5 to 8.8 times lower in comparison with the results of the theoretical calculations. Therefore, it was decided to calculate the values of the core elasticity moduli knowing the MOE experimental data for the panels and their facings and to compare the results with those presented in Table 3. Table 9 collates the calculated values of the core linear elasticity moduli of the honeycomb panels.

Table 9. Collation of Linear Elasticity Moduli of Honeycomb Panel Cores

Panel Type	MOE According to Eq. 19	MOE According to Eqs. 1 and 2; 6 and 7
	(MPa)	
EH5L	E_{C2} = -5859	E_{H2} = 0.085
EH5P	E_{C1} = -5976	E_{H1} = 0.045
ER5L	E_{C2} = -5458	E_{R2} = 133.3
ER5P	E_{C1} = -6776	E_{R1} = 0.003

On the basis of analytical calculations in accordance with Eq. 20, the authors obtained negative values of the linear elasticity moduli of the honeycomb panels. For the cores with rectangular cells, the linear elasticity modulus for direction 2 exhibited the value of E_{C2} = -5458 MPa, whereas for direction 1 the value E_{C1} = -6776 MPa was exhibited. In contrast, the values of corresponding moduli determined in accordance with Eqs. 6 and 7 amounted to: E_{R2} = 133.3 MPa and E_{R1} = 0.003 MPa, respectively.

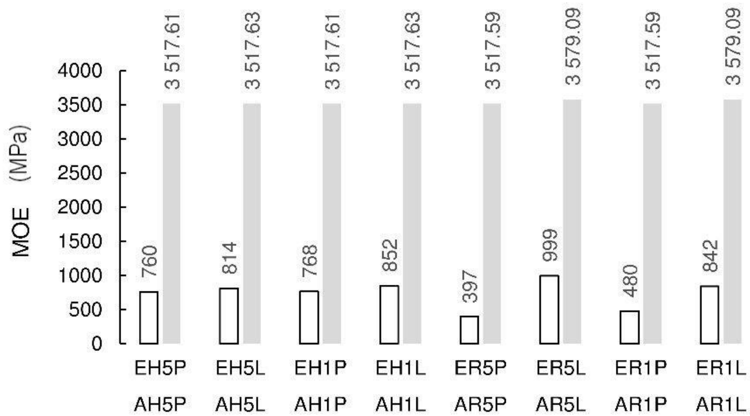

Fig. 8. Modulus of elasticity of honeycomb panels

A similar tendency was observed for the cores with hexagonal cells. The linear elasticity modulus for direction 2 assumed the value of E_{C2} = -5859 MPa, whereas direction 1 assumed the value of E_{C1} = -5976 MPa. According to Eqs. 1 and 2, these values corresponded to E_{H2} = 0.085 MPa and E_{H1} = 0.045 MPa, respectively. The results presented above clearly show that Eq. 19 failed to properly calculate the elastic properties of the cell core. In this case, the stiffness of the honeycomb panel depended, primarily, on the elastic properties of the facings as well as their distance from the panel center. In contrast, the effect of the core was negligibly small, in comparison with the facings, due to its infinitesimal value of the linear elasticity modulus.

Summarizing, Eq. 19 can be used only for homogenous systems of sandwich panel cores. In the case discussed in this study, it will be expedient to utilize numerical calculation.

CONCLUSIONS

1. It is unequivocally clear from this study that rectangular cells of the core increased the stiffness of furniture panels most advantageously in the (L) direction and, simultaneously, decreased this stiffness in the (P) direction. Therefore, a furniture panel with a rectangular cell core was characterized by strong orthotropy, which is valuable in furniture designing.
2. The performed analytical calculations ruled out the possibility of the application of Eq. 19 for the MOE calculations of a paper core in honeycomb panels. In this situation, it is necessary to elaborate on appropriate numerical models.
3. To evaluate the mechanical properties of honeycomb panels, especially those with slender core cells, the application of 100-mm-wide samples is recommended. Advantageous mechanical properties of honeycomb panels with rectangular cell cores point to the possibilities of their application in light furniture panels.
4. It can be expected that, similarly to increased stiffness and strength of the examined samples, the stiffness of the shelves and partitions of cabinet furniture manufactured from these materials will also increase.

ACKNOWLEDGMENTS

The present work was conducted as part of research project No. 2016/21/B/ ST8/01016 financed from the funds of the National Science Centre.

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Article submitted: October 21, 2018; Peer review completed: December 15, 2018; Revised version received: December 17, 2018; Accepted: December 29, 2018; Published: January 8, 2019.
DOI: 10.15376/biores.14.1.1435-1451