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DYNAMIC CONSOLIDATION OF PAPER DURING CALENDERING

Dynamic compressibility of paper

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Synopsis—Two dynamic compressibility testers are described and the accuracy of one of them is examined by means of an energy balance and found to be within 5 per cent. The relationship of dynamic and static compressibility is discussed, the effects of moisture and temperature briefly examined and the results of testing a number of papers on the dynamic tester are presented. The changes in compressibility through the calender stack of two papers are given as examples and the dwell time/pressure relationship for caliper reduction is given for three different papers. Printability is examined as a function of compressibility and the relative compression of press packing and various papers is examined. A graphical method for determining the specific pressure distribution in a calender nip is given and the implications of the results are discussed. Two extreme cases of materials in a calender nip, one completely elastic, the other completely plastic are briefly discussed and a rheological model for the calender action is presented in an appendix.

Introduction

In the manufacture of most types of paper and board, calendering on the machine is a part of the process. In spite of the general use of machine calenders, a lack of understanding of the calendering process exists and there are possibilities for the making of substantial improvements. One of the major alterations to the paper induced by calendering is the change in compressibility that occurs. Testing of the paper compressibility at various stages of calendering and under controlled conditions offers a direct method

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of gaining a better understanding of this part of the papermaking operation. Fig. 1 illustrates diagrammatically how the paper behaves under compression when hit with a flattened hammer.

The present contribution discusses the effect of such tests on paper and the deductions that can be drawn from them. The apparent compressibility of an elastic material under a piston, confined by the walls of a cylinder, is less in most cases than the compressibility of the same material under the same piston with the cylinder removed. This is because most materials have to expand in a lateral direction when compressed in a longitudinal direction: the pressure of the cylinder prevents lateral expansion and the pressure of



Fig. 1—Definition sketch for paper compressibility

the cylinder walls is transmitted to and felt by the piston. This property of lateral expansion is due to the inability of the constituent parts of the material to fit together easily while under pressure. The ratio of the material's lateral strain (expansion divided by width) to its longitudinal strain (compression divided by length) is its Poisson's ratio. This is always between 0 and 0.5, as exemplified by styrofoam 0 and by water or rubber 0.5.

Because a sheet of paper is as much as 50 per cent air, the constituent parts may fit together fairly easily and without lateral expansion when under pressure. Although adjacent fibres may move laterally with respect to one another, this lateral motion will be on random azimuths at all points in the sheet. The total lateral motion will therefore be negligible for a reasonably square sheet. It may be concluded that, for compression of paper, in which the biggest controlling factor is the amount of void space in the paper, Poisson's ratio is zero. (It is not zero for extremely dense papers whose density approaches that of pure cellulose nor for tensile strains across the sheet in either direction.)

It may therefore be considered that the bulk modulus of paper in

compression is equal to the Young's modulus of paper in compression. This means that testing under a 1 ft square piston would indicate the same modulus for the paper as with a 1 in square piston, even with a still smaller piston, provided the area under test is much larger than the typical void area. This conclusion is very important in the theory of paper compression, because it implies that the paper modulus of the sheet compressed between rollers is equal to the paper modulus given by compression between flat plates.* Because of the small expansion in the nip, a flat plate tester may be considered a suitable substitute for an experimental calender.

In an investigation⁽¹⁾ of the operation of high-speed machine calender stacks, Howe & Lambert showed that the effect is two-dimensional: the surface finish and caliper in the machine-direction are continuously modified and the finish and caliper across the machine are altered at the same time by localised nip pressures that occur across the face of the stack. These authors investigated the change in paper quality after each nip in the stack by measuring smoothness, total wire mark, proof press printability, softness and porosity. The mechanism of calendering was indicated as being one of rolling friction combining the compressional and shear stresses due to roll weight and rotation over the viscoelastic paper web. The effect of the stack on paper quality was found to be in both the machine- and cross-directions; the change of printability through successive nips of the stack and the change in smoothness were shown to be linear, whereas the other quality parameters such as caliper, wire mark and softness were mainly affected by the upper two nips of the stack, with the subsequent nips having very little effect.

Wahlström *et al.*⁽²⁾ investigated the mechanism of calender barring and its possible elimination. In order to calculate the dynamic properties of a calender stack operating with paper, the rheological properties of the paper had to be measured. They built an apparatus to do this on a single sheet at pressures varying 0.2–5 kgf/mm² and at times greater than a minimum value of 5.10^{-4} sec, which is approximately equivalent to the time of a compression cycle in a calender at speeds of 600 m/min.⁺ They stated that the compression of a paper sheet was known to vary with the rate of compression, the substance,

* This assumption may be criticised, because there is about 1 per cent non-recoverable extension in the machine-direction through the stack and the immediate extension may be greater

† A table of comparison of the initial rates of loading and dwell time used by the various authors and experienced on a typical nip of a machine calender is given below—

Autnor		
Brecht ⁽³⁾		
Wahlström ⁽²⁾		
Oxford (4, 5)		
23 in roll at 1 200 ft/min		
12 in roll at 1 200 ft/min		

Dwell time 0.002, 0.1, 2.5 sec 0.0005 sec and up 0.0002 sec and up 0.00079 sec 0.00054 sec paper quality, moisture content of the paper and the state of compression of the paper when the compression cycle started. By means of the paper compression curves that were obtained, they pointed out that it is possible to calculate the nip force in a calender nip, if the distance between the rolls and the state of compression of the paper before the nip were known. Using such measurements, these workers calculated the dynamic behaviour of a calender stack by the use of a digital computer.

Brecht & Schadler⁽³⁾ have provided a valuable report on the dynamic compressibility of paper. The rate of loading in their experiments was 0.001, 0.1 and 2.5 sec per test. The first loading rate gave slightly less compression than the slower loading rates and this was attributed to the creeping action that takes place when loads are applied for an appreciable time. The absolute relaxation or regain was found to be about equal for the loading rates employed, but the faster loading rates gave a larger relative relaxation. This relative regain was considered a direct measure of the elastic deformation. The paper tested proved to be harder and more elastic when the dynamic pressure applied was of shortest duration, but the time to reach maximum pressure appeared to have a surprisingly small influence on the elasticity. When the paper was test loaded several times in succession, no densification was noted after the first pressure application. Besides, when paper was pressed in its initial wet state, the compressibility of the finished paper was reduced, yet the relative regain became slightly higher. The experiments showed also that it seemed to be a combination of compressibility and smoothness that controlled printability.

The present group of workers has shown earlier⁽⁴⁾ that the compressibility of a non-groundwood coating base paper was progressively reduced by the machine calender, the coating operation and the supercalender. The paper properties were found to change more rapidly through the first nips and more slowly through the following nips for the heavier stacks, whose performance was examined, than for the lighter stacks. Caliper was found to exhibit the greatest change in the first three nips, whereas smoothness continued to develop at an approximately uniform rate. Printability and gloss were found to develop at the same rate as smoothness. It was concluded by these authors that uniform compressibility across the machine width could be maintained only by means of meticulous attention to the sheet on the wire, the pressing, the drying and, perhaps most important of all, the operation of the calender stack; if the moisture is uneven going into the stack, then even with uniform basis weight the compressibility will be irregular. The regions of greater paper compressibility across the machine are generally the drier parts of the sheet that have not been compacted so much by the calender. By plotting the linear compression at any one pressure against web position, cross-direction

compressibility profiles were produced and these indicated that the paper showed less uniformity at higher pressures, leading to the conclusion that paper should be tested for linear compression at printing pressure, if compressibility is to be used as a parameter relating to printability.

Consideration of the literature summarised above confirms that, even though various phases and results of the calendering process have been studied with valuable results, the operation as a whole is incompletely understood. A theory to explain and correlate the various results is lacking: the work on compressibility has so far been restricted to relating compression to various paper properties with no theory about how the calender affects compression.

The purpose of a long-range study of the machine calendering process may be stated to be to improve and to extend the present knowledge of calendering, to discover what happens between roll nips and, ultimately, to refine the calendering process.

The most exacting way of studying the calendering process would be to instrument a set of rolls completely, but this leads to involved problems, as it is found that many uncontrolled variables exist. The most practical approach is to design a tester that will nearly duplicate a calender nip, yet will eliminate all but a few variables. The method decided upon was to use two flat plattens, with one driven against the other by air pressure. The major disadvantage of this type of tester is the elimination of any shearing action such as is experienced in the calender nip. It is difficult to effect temperature changes and all testing in the work described was carried out at room temperature, except where otherwise specified. Work using the dynamic tester based on the principles outlined has been reported.⁽⁵⁾

Many types of paper have been tested on the dynamic compressibility tester, most of which were also tested in parallel on a static tester. At the same time, various other properties of the paper were measured to enable comparisons to be made at different stages of calendering. Some new relationships have been constructed to give a more complete understanding of the results of compressibility testing and a theoretical model relating tester results to the machine calender is presented.

The present paper is divided into four sections. In the first, the dynamic compressibility tester and the mode of analysis of typical results are described. In the second section, the experimental results are presented in two groups, the first of which deals with tests on various selected materials and the second with the testing of papers at various stages of calendering. The third portion consists of a discussion of the experimental results with relation to calendering and, in the fourth part, the results are evaluated and the appropriate conclusions drawn.

The compressibility tests

Two types of compressibility tester were developed and used—the first essentially an air-driven hammer, the second a free-falling weight. The greater amount of the data reported in the present paper was produced with the air hammer; the second tester was brought into use to compare compression data gained by using single sheets of paper with that obtained with multisheet packs.

Air hammer

The tester consists of a 3.6 lb hammer hitting on to a circular cup of 2 in^2 area, fitted with strain gauges so that the developed force can be measured. The hammer mechanism (shown schematically in Fig. 2) is a commercial single-stroke pneumatic hammer. Air at 70 lb/in² is reduced in pressure by an automatic control valve, then vented to the atmosphere. To fire the



Fig. 2—Diagrammatic view of the air hammer tester 38—c.p.w. I

hammer, this vent is closed, but the hammer is restrained from moving by a pneumatic mechanical timer; this piston is not released until enough air has seeped through a needle-controlled hole to push back a release catch. This makes it possible to apply a reproducible pressure to the piston each time the hammer is fired. At about two thirds of full piston stroke, the piston passes a part in the cylinder that releases the air pressure on the piston. A spring is attached to the hammer to return it to its original position. The strain gauge cup was backed by a massive instrument base to minimise movements of the cup and to reduce the shock wave as much as possible.

The light beam shown in Fig. 2 is approximately at rightangles to the motion of the hammer. It is cut off by a knife edge adjusted so that when the hammer is just touching the paper the light is just touching the mirror. As the hammer face sinks into the paper, more mirror is exposed and more light is reflected into the phototube. The electron beam intensity is controlled by a 38 000 cps signal generator switch, so that a dotted line is seen on the oscilloscope where the signal is displayed. The amount of compression is presented on the Y axis and the pressure felt by the strain gauge is shown on the X axis of a dual beam oscilloscope. By this means, a hysteresis loop is produced, which is photographed for further analysis; a typical trace is shown in Fig. 3. The average slope of the compression part of the stress/strain curve gives the paper modulus. The area of the loop, obtained when the hammer rebounds, gives the energy lost to the paper during the compression cycle.

In simulating all machine calender nips with a single hammer apparatus, considerable control over the tester is needed. The only control on the hammer is the striking velocity, although additional control is possible by varying the number of sheets between the hammer and the platten. More paper causes a softer landing, thus reducing the peak stress and increasing the dwell time or time of contact between hammer and paper. This is almost enough roughly to simulate nip conditions (specified load and rate of compression), except that nip conditions vary over a wide range. Because it was impossible to simulate all nips, it was decided to test all papers in exactly the same way, then to extend the results to different nip conditions.

Each of the many samples was hit by the hammer at 110 in/sec and 20 sheets were used in each sample tested. The peak compression was generally not controlled and depended on the paper characteristics. The classical mechanics terms are defined below to give clarity to the ensuing discussion. The hammer has an initial impact energy of $\frac{1}{2}mV^2$ and an initial momentum of mV; its final energy and momentum as it leaves the paper depend on the velocity. The difference between the impact and final energies is the energy lost to the paper and platten during the compression cycle as work, heat and

noise. That part lost to the paper is the hysteresis energy. The stress is the force on the strain cup divided by the cup area. Strain is the ratio of the amount of compression to initial paper thickness; thus, the strain of the 20 sheet pack and the strain of a single sheet in the pack are the same. Strain rate is a rheological term in common use; it is the rate of compression divided



Fig. 3—Typical oscilloscope trace from the dynamic compressibility test

by the thickness. Thus, for 20 sheets approximately 0.003 in thick and the initial compression rate of 110 in/sec, the strain rate is 1 840 in per inch per second. This is a very high strain rate compared with that experienced in most rheological studies in other fields.

The stress/strain hysteresis loop gives much information: one should not talk about the modulus of paper, because it is not constant, since it depends on paper history, on pressure level and probably on the strain rate. For perfectly elastic materials, the modulus is constant and is the ratio of stress to strain (the slope of the stress/strain curve). Because, in compression only, the curves produced by the compression tester have an almost constant slope for a given sample, in the analysis of the results, the average slope has been incorrectly called modulus for convenience. It must be appreciated that this modulus has nothing to do with the other part of the test, the expansion part. Finally, instead of working with the modulus, it has been found more convenient to work with its reciprocal, properly called compliance with units of in^2/lb . Instead of being a measure of rigidity, therefore, compliance is a measure of deformability.

Free-fall hammer

The free-fall hammer consists of a rectangular block weighing 4.05 lb that can be dropped from a predetermined height. The falling hammer is guided by two parallel vertical guide rods on to a strain gauged cup, which operates in the same manner as that of the air hammer. The light beam passes through an open shaft in the hammer to the photo tube and is cut off by a knife edge on the hammer (Fig. 4). The instrument was designed to measure the compressibility of a single sheet, thus the sensitivity of the light system was much greater than that for the air hammer. With the high sensitivity of compression measurement, the amount that the stainless steel cup compressed became a significant factor and a correction had to be applied: except for this, the results were analysed in a similar manner to those from the air hammer.

Application of the dynamic compressibility tests

The testers were used with the objectives of extending the theory of calendering and of studying the various properties of paper with regard to compression. As a part of the complete programme designed to understand the instrument, materials of relatively known compression characteristics were tested, the relationships between static and dynamic compressibilities were examined, energy balances were made around the tester and the relationship between the compression of single sheets and multi-sheet packs was examined. The main part of the experimental work was devoted to examining paper samples of a wide range and at various stages of calendering.

Examination of the operation of the dynamic compressibility tester

Testing rubber samples—Fig. 5 shows the results of hitting rubber squares of size $\frac{1}{50}$ in $\times \frac{1}{8}$ in $\times \frac{1}{8}$ in. The rubber appears inelastic, as a perfectly elastic test would have resulted in the two sides of the loop coinciding. The results indicate a compliance of about 2.6×10^{-4} in²/lb, which is the same as a modulus of about 4 000 lb/in². This value may be compared with an accepted modulus of 12 kg/cm² (170 lb/in²) for unrestrained compression in which the

rubber is allowed to flow laterally—there is no change in the volume of the rubber, only the shape. For a rubber sheet 11 mm thick, the modulus is 10 000 lb/in²; as the sample tested was about a quarter of this thickness, the 4 000 lb/in² value appears in a comparable range. If a block of rubber is



Fig. 4-Diagrammatic view of the free-fall hammer

compressed by more than 10 per cent, the force/compression relationship becomes non-linear, the incremental stiffness increasing the strain. A comparable effect may occur with paper and this complicates the interpretation of the results. To find the compression of a solid rubber block, a $\frac{1}{2}$ in thick neoprene rubber sample was tested on the free-fall hammer: the result is shown in Fig. 6 with about 0.125 in or 25 per cent compression. The modulus for this sample is 6 400 lb/in² or a compliance of 1.55×10^{-4} in²/lb. The figure illustrates that the rubber sample returns to nearly its initial caliper during the time of the test.

Aluminium foil and block tests—The aluminium foil was crumpled into $\frac{1}{4}$ in balls and hit by the hammer. As can be seen in Fig. 7, the foil compressed very easily and required little stress. As the foil began to compact, the stress



cubes in compression



built up to about 1 000 lb/in² at peak compression. It had been expected that the foil would compress in a completely plastic manner and would not regain its caliper after impact. This result is observed in the very flat return portion of the curve.

Solid aluminium was tested in the form of in $\frac{3}{32}$ in $\times \frac{3}{16}$ in $\times \frac{3}{16}$ in sheet aluminium cuttings, polished flat before the test. The loop shown in Fig. 8 shows considerable plasticity of the squares. The apparent modulus of the aluminium in compression is 3×10^5 lb/in² compared with a handbook value of 100×10^5 lb/in². It might be expected that in compressing such hard material the strain gauge cup might compress enough to contribute significantly to the measured compression. This is not the case and the cup compression is shown in Fig. 8 as relatively small. *Energy balances*—The initial or striking energy of the hammer is $\frac{1}{2}mV^2$. This section of the paper accounts for the distribution of this energy during the compression cycle. One part of the initial energy is used to raise the hammer equal to $\frac{1}{2}mV_1^2$, where V_1 is the upward velocity. The remaining energy is mostly lost to the paper and is represented by the hysteresis loop area. Other energy losses occur to noise and to shock waves in the base.





Fig. 8-Stress/strain curve for solid aluminium

The results of energy balances are shown in Table 1; the unaccounted for energy is less than 7 per cent of the initial energy in every case and is usually less than 5 per cent. The greater discrepancies were for the less compressible coated and supercalendered paper. The initial and final velocities of the hammer were calculated from the timing marks on films of the compression cycle taken with the Fastex camera. The initial velocity was also measured electronically with good agreement between the two values. The mass of the hammer was checked and the hysteresis loop was measured by planimeter.



Fig. 9—Typical comparison of stress/strain plot in compression for static and dynamic tests

Comparison of static and dynamic compressibilities—Many samples have been tested on a flat plate tester and with the air hammer. A typical comparison of the two results is shown in Fig. 9. The samples were first tested on the air hammer and the corresponding samples were then compressed statically to the same peak pressure. The static test usually produced a higher compression than the dynamic test for equal peak pressure. The static hysteresis loops are usually smaller indicating less energy loss to the paper,

Paper used	Initial energy per sheet, ft lb	Hysteresis loop energy, ft lb	Return hammer energy, ft lb	Hysteresis loop plus return energies	Unaccounted for energy, ft lb	Unaccounted for energy as percentage of initial energy
Machine-	0.218	0 184	0.0266	0.210	0.008	2 67
Machine-	0.210	0.104	0.0200	0.210	0.008	5.07
finished Coated and	0.216	0.198	0.0289	0.226	-0.0109	- 5.05
super- calendered Coated and	0.218	0.169	0.0612	0.231	-0.0125	- 5.75
calendered	0.218	0.172	0.0612	0.233	-0.0150	- 6.9
groundwood Containing	0.218	0.185	0.0364	0.221	-0.0031	-1.56
groundwood	0.216	0.177	0.0301	0.207	- 0.0084	3.87

TABLE	1-ENERGY	BALANCE	DURING	COMPRESSION	CYCLE

as would be expected, since sufficient time is available for recovery to take place.

Further information about the comparison of static and dynamic compressibilities is given elsewhere,⁽⁵⁾ in which it was concluded that the dynamic compressibility is usually about 0.7 of the static compressibility and that hardwood kraft fibre in the furnish raises and filler lowers the dynamic compressibility relative to the static value.



Fig. 10—Peak pressure used for different numbers of sheets tested

Comparison of single sheet and multi-sheet compression—The free-fall hammer was used to find the effect of compressing more than one sheet compared with a single sheet. The experiments commenced by testing a 20 sheet pack and were continued by progressively reducing the number of sheets tested one at a time until one sheet was tested; finally, the cup deflection was examined by experiments without paper. The peak pressure was held constant for these tests as far as possible, as indicated in Fig. 10. The peak compression per test was plotted as a function of the number of sheets used (Fig. 11), which indicates that compression data for more than one sheet may be considered typical of a single sheet, as the line passes through the point for no sheets in use. Fig. 12 shows the peak compression per sheet as a function of the number of sheets used; the fact that there is a slight wave in the line is due to the peak pressures not being identical, as may be seen from Fig. 10.

The exact shape of the stress/strain curve changes as the number of sheets tested is reduced. Fig. 13 compares the curves for 1 and 20 sheets.



Fig. 11-Total compression as a function of the number of sheets tested



Fig. 12-Compression per sheet as a function of the number of sheets tested



Fig. 13—Stress/strain curves in compression for 1 sheet and 20 sheets

Test results from different papers

The work on dynamic compressibility has been carried out over a period of three years, during which much experimental data was obtained. In the following section, selected data are presented to illustrate the scope of the work done and to suggest the variety of results that can be obtained.

Compliance as a function of void space—The compressibility and compliance of paper should depend upon the fraction of the paper volume represented by the void space in the paper and upon the stiffness of the network of interwoven fibres. From a knowledge of the proportions and densities of the paper constituents and of the caliper and substance of the paper, the approximate void space can be calculated for any sheet. The compliance is estimated from the oscilloscope photographs as the average slope of the initial compression part of the curve. The relationship between the fraction of voids present and compliance is shown in Fig. 14 as a common line through the points obtained from testing paper from successive nips of ten different calender stacks on grades ranging from paper contents of 70 per cent groundwood through 70 per cent hardwood kraft to 70 per cent softwood kraft fibre. The slope of the common line indicates that, as the amount of voids decreases, the fibre network becomes progressively less compliable. From this general tendency and with a knowledge of the void fraction of a given sheet, the compliance will be predictable when more data are available.

	Compression per sheet × 10 ⁻³ in at 1 000 lb/in ²	0.25 0.1725 0.1725 0.1725 0.1725 0.306 0.305 0.355 0.375 0.375 0.375 0.375 0.375 0.375 0.375 0.375 0.375 0.375 0.375 0.375 0.375 0.237 0.220 0.455 0.203 0.203 0.203 0.203 0.225 0.225 0.225 0.220 0.237 0.225 0.202 0.237
ІПТҮ	Print rating	$\begin{array}{c} -\frac{60}{55} \\ -\frac{15}{70} \\ -10}{15} \\ -100 \\ -100 \\ -100 \\ -100 \\ -50 \\ -50 \\ -50 \\ -50 \\ -50 \\ -50 \\ -50 \\ -50 \\ -90 \\ $
R COMPRESSIE	Gardner gloss	45.6/45.6 6.2/6.4 5.9/555.0 4.6/5.9 25.8/25.5 60.6/60.8 9.5/10.1 6.0/6.1 7.7/9.1 8.7/9.5 28.6/24.9 7.9/8.7 43.0/43.6 71.5/68.1 8.6/9.3 57.5/63.7 63.4/64.3 63.4/64.3
RS TESTED FO	Ingersoll gloss	84/85.3 26.9/24.9 89.6/87.3 56.5/57.1 66.1/64.8 89.4/89.4 30.2/30.2 30.2/30.2 30.2/30.2 30.2/30.2 30.2/30.2 30.2/30.2 30.2/30.2 30.2/30.2 30.2/30.2 30.2/30.2 30.2/30.2 30.2/30.2 30.2/30.2 88.5/88.5 88.5/88.8 88.2/87.6 88.2/87.6 88.9/88.8
VARIOUS PAPE	Sheffield smoothness	31/27 221/29 21/29 29/32 31/32 29/32 33/32 29/36 53/69 33/34 72/79 29/26 77/9 29/26 11/11 50/56 12/13
PROPERTIES OF V	Bekk smoothness	680/790 12/10 902(440 6/7 395/255 495/393 2 840/2 280 43/35 66/57 119/91 490/630 64/57 80/740 1 905/1 685 1 170/1 320 910/945 910/945 1 405/1 425
TABLE 2—1	Initial caliper, × 10 ⁻³ in	3.202 3.240 3.240 3.240 3.258 3.73 3.65 3.07 3.23 3.07 3.23 3.07 3.23 3.07 3.23 3.07 3.23 3.07 3.23 3.33 3.07 3.23 3.33 3.02 5.28 3.23 3.02 5.28 3.23 3.07 3.05 5.28 3.07 5.28 3.07 5.28 3.07 5.28 3.07 5.28 3.07 5.28 3.07 5.28 5.28 3.07 5.28 5.28 5.28 5.28 5.29 5.28 5.28 5.29 5.28 5.29 5.28 5.29 5.29 5.29 5.20 5.20 5.20 5.20 5.20 5.20 5.20 5.20
	Grade of paper	Machine-coated, supercalendered, 60 g/m ² Machine-finished Machine-finished Off-machine coated Off-machine coated off-machine coated off-machine coated Newsprint After stack Before stack Machine-finished Machi

592

Dynamic consolidation during calendering

Effect of caliper, bulk and previous peak pressure on compliance—An interesting relationship was found to exist between initial caliper and compliance (Fig. 15). In this example, the initial caliper was that for the same furnish after various nips in a calender stack; the curve shows that the compliance is reduced commensurately with the initial caliper decrease, but the change becomes nearly linear as the caliper is further reduced. Fig. 16 illustrates the interrelationship between bulk and compliance; the curve shows a very weak relationship, indicating an approximately linear increase with bulk. In Fig. 17, the previous peak pressure is plotted against com-



Fig. 14—Compliance as a function of sheet void space

pliance: as the previous peak pressure increases, the compliance decreases and tends towards a constant value; this indicates that a paper's ability to compress after it is compressed to a certain peak pressure will not change with additional pressure.

Relationship among compressibility, printability, smoothness and gloss—In the investigation of the relationship between compressibility and the parameters associated with printability, 19 samples of various grades of paper were tested for each property (Table 2). The relationships between compressibility and printability for the grades examined are shown in Fig. 18 for



Fig. 15-Compliance as a function of caliper



Fig. 16—Compliance as a function of bulk



Fig. 17—Compliance as a function of previous peak pressure



Fig. 18-Print rating against compression



Fig. 20-Print rating against Sheffield smoothness

different smoothness ranges: the upper line is drawn through the points for the papers which had a Sheffield smoothness value of 12 and below, the second line is for smoothness values of 29–31 and the lowest line for smoothness values of 50 and above. In other work,⁽⁵⁾ a relationship was found between compressibility and printability, because the papers then tested were rougher and more compressible. In the present examples, only in the rougher paper could any relationship between compressibility and printability be found. Fig. 19 and 20 illustrate that printability at lower smoothness



Fig. 21-Gloss against compression

is critically dependent upon smoothness; at high smoothness, it is the actual structure of the paper surface that controls the printability. In the investigation, a tenuous relationship was found also between gloss and compression (Fig. 21).

Compression as affected by the number of calender nips—Fig. 22 shows a typical example, for newsprint, of the results of testing samples taken after successive nips of the calender stack. The samples taken before the stack had a large compression at low peak pressure; as the paper progressed through the calender stack, there was a decrease of the peak compression and an increase of the peak pressure reached in the testing. Fig. 23 and 24 for papers containing groundwood and containing no groundwood, respectively, show the manner in which the paper caliper changes as the sheet progresses through the stacks, which were of approximately the same size and weight.

39—с.р.w. і



Fig. 22—Compressibility of paper after successive calender nips—newsprint



Fig. 23—Paper history through the calender stack groundwood-free paper

In these figures, the depth of the bar represents the estimated compression of the paper in the nip and the height of the step shows the resultant caliper change. The figures show up the much greater compression experienced by the groundwood paper; the later nips develop greater caliper change with groundwood-free papers than the earlier nips, which is the reverse of the caliper development characteristic of groundwood grades.

A special example of the change in compression with pressure as paper passes through successive nips is the case of glassine body paper passing through a glassine supercalender. Glassine has in its final state a density very close to that of pure cellulose (nearly 1.5). The results of tests carried out on glassine samples taken from successive nips of the supercalender are given



in Fig. 25. There is very little void space, even in the machine-finished sheet and the paper compression for approximately the same thickness sheet is one fifth that of newsprint: the total change in compressibility is thus small as the paper goes through the supercalender and most of it occurs in the first nip.

Effect of centrifugal cleaning of stock—It had been noted from printing observations that centrifugal cleaners affected the properties of groundwood papers. When the dynamic compressibility tester became available, samples of newsprint made with and without centrifugal cleaning were tested; the paper made with such cleaners exhibited greater compressibility, indicating that the cleaners were removing small dense material used elsewhere in the machine room, so allowing a lighter, bulkier and more compressible sheet to form.



Fig. 25—The compression of glassine body paper sampled from successive nips of the supercalender



Fig. 26—Compression of newsprint made with and without centrifugal cleaners

Dynamic consolidation during calendering

Compressibility through a machine with a breaker stack—Samples of newsprint were tested from a machine with a breaker stack at the following points —before the breaker stack, after the breaker stack, before the first calender stack and before and after the second calender stack. The results (Fig. 27) indicate that the breaker stack had a surprisingly small effect on compression. The major change occurred at the first stack, with the second stack having a minor effect.

The breaker stack is known to be a major factor in the reduction of the tendency of the paper to bar in the calender stack and its action is believed



breaker stack

to be due to a change in the paper compressibility characteristics. A considerable difference in paper compressibility before and after the breaker stack was therefore anticipated. The magnitude of the change actually obtained suggests that, at 25–30 per cent moisture content, the effect of a linear nip pressure approximately equivalent to the first two or three rolls in the calender stack is lessened by the amount of water present.

Effect of moisture on compression—The effect of moisture content on the compression of a hardwood kraft and softwood kraft mixture is shown in Fig. 28. The results indicate a linear increase in compression with increasing percentage moisture content; the increase was slightly greater with dynamic than with static testing. When the small moisture range is considered, the change in compressibility is surprisingly large. A related series of tests was run to determine the effect of temperature upon compressibility: it is seen



Fig. 28a—Static compression of groundwood as a function of moisture content



Fig. 28b—Dynamic compression of groundwood as a function of moisture content

from Fig. 29 that temperature has no effect on compression over the fairly wide range of moistures. The effect of steam rolls in the calender stack must therefore be attributed to increasing the plasticity rather than the compressibility of the paper.

Relative compressibilities of paper and of packing material used in printing— All paper is printed backed with some kind of packing, depending upon the type of paper and printing process. Samples of press packing were tested in combination with paper for compressibility in five steps ranging from 100 per cent paper to 100 per cent packing. As the tested materials were not of the same initial caliper, the compression was expressed as a percentage of the initial caliper and plotted against the percentage of paper in the pack tested



Fig. 29--Dynamic compression against moisture content at two temperatures

(Fig. 30). For coated paper—but not supercalendered—the percentage compression decreased as the amount of packing was increased; for supercalendered papers, however, the percentage compression was found to increase with increasing percentage of packing. Thus, the packing is more compressible than coated and supercalendered printing papers, but less compressible than coated paper not supercalendered.

With a press packing more compressible than the paper, it appears unlikely that compressibility would be an important factor in printability, but all the force in letterpress printing is applied through the surface of the paper in irregular small areas. Thus, the paper must be deformed and it is the absolute rather than the relative compression therefore that must be considered in the printing process.



Fig. 30-Compression of paper and printing packing



Fig. 31—Compression of newsprint packing without paper

Fig. 32—Compression of newsprint packing plus a single sheet of rotogravure newsprint



Fig. 33— Compression of newsprint packing plus a single sheet of low caliper newsprint



Fig. 34—Compression of newsprint packing plus a single sheet of high caliper newsprint



Fig. 35-Time against pressure at various final caliper percentages

A highly compressible press packing used for newsprint presses was tested on the dynamic compressibility tester without paper, then with single sheets of three samples of newsprint—rotogravure, low caliper standard newsprint and high caliper standard newsprint. The results are shown in Fig. 31 for the







packing, in Fig. 32 for the packing and one sheet of rotogravure newsprint, in Fig. 33 for packing and one sheet of low caliper newsprint and in Fig. 34 for packing and one sheet of high caliper newsprint. Because of the high compressibility of the packing, the effect of the single paper sheet was small.

The total compression of the system was slightly greater with paper present; the compression increased with the various samples in the order rotogravure, low caliper and high caliper newsprint. The increase in compression for the system using high caliper newsprint indicated that the paper compressed to nearly one half its initial caliper; although this is not significant in the total compression, it is significant for the individual paper samples.

Effect of time and pressure on the final percentage caliper change—Three samples of paper were subjected to intensive testing to determine the effect of peak pressure and dwell time on final caliper change. Samples were impacted under as many conditions of peak pressure and dwell time as possible by deliberate variation of the hammer velocity and the number of sheets. In addition, a sample of each grade was tested statically under a variety of pressures from the shortest possible time (about 3 sec) to longer times (about 2 min). In both the static and the dynamic tests, the caliper change, the peak pressure and the test duration were tabulated. The dynamic tests do not give the ideal results, because a sustained pressure of 2 000 lb/in² for 1 msec would produce a different effect from an impact lasting 1 msec, during which the pressure peaks at 2 000 lb/in², but which averages only about 60 per cent of the peak pressure.

The results presented in Fig. 35, 36 and 37 are semi-log plots of duration time and peak pressure: the lines are lines of constant caliper change. In Fig. 35, the relationship shown indicates that a 10 per cent caliper change in the paper could be obtained by either a 5 000 lb/in^2 stress for 0.28 msec or by a 1 000 lb/in^2 stress for 3 sec. The bulky sheet tested for the results shown in Fig. 35 had a wider range than the harder papers, the results for which are given in Fig. 36 and 37. These figures show that caliper change depends on some function of pressure and time of pressure duration.

Discussion of the experimental results

THE tests that have been carried out on the dynamic compressibility tester indicate that the instruments are operating in a reliable manner. The tests on rubber, aluminum foil and solid aluminium gave results of the type expected and the calculations for the energy balances accounted for all the energy, except for a small percentage believed to be lost in a shock wave created on impact. In comparing static and dynamic compressibility tests, a strong correlation is found. The static loops contain a smaller area because of the larger recovery time involved: the static compression was greater than the dynamic compression because of the larger time for the paper properties to decay. The most interesting tests in this section were those involving single sheets. The results show that compression of a multi-sheet pack of paper gives the same results per sheet as compression of a single sheet. The shape of the single sheet hysteresis loops are different from the 20 sheet loops with the peak compression occurring later in the compression cycle. This difference is now under investigation.

The experimental work has included a wide range of comparisons between various paper properties. The results have shown that compressibility depends on many factors. The compressibility has a strong relationship with the percentage voids as the compliance (ability of the paper to compress) was found to decrease linearly with decreasing amount of voids. The compliance was related also to initial caliper, bulk and to the previous peak pressure to which the paper had been subjected.

In comparing compressibility and printability, the effect was found to depend upon the smoothness range in which the samples could be classified.

The chilled iron calender rolls are so hard that their deformation is very small compared with the deformation in the paper. Thus, the change in thickness of the paper in the nip is very nearly—

$$Z = [X_p^2 - X^2] \left[\frac{1}{D_1} + \frac{1}{D_2} \right] \qquad . \qquad . \qquad (1)$$

where X_p is the distance from the leading edge of the nip to mid-nip,

X is the distance from where Z is measured to mid-nip,

 D_1 and D_2 are the diameters of the rolls.

This equation states that the paper compression at any point in the nip depends on the roll diameters and the nip width, but nip width depends in turn on nip load and on how the load is distributed through the nip. Because the paper is not elastic, the load is less beyond mid-nip than on the entering side of the nip. Thus, how the paper is compressed must be known in order to determine the pressure distribution through the nip.

Fig. 38-42 illustrate the results of a graphical trial and error method to evade the confusing interdependency first reported.⁽⁵⁾ In each of the figures, an air hammer compressibility result is plotted in quadrant I as inches of compression per sheet Z against pressure. In quadrant II, equation (1) is plotted as Z against $(X_p - X)$ or distance into the nip. Quadrant IV is the result of combining quadrants I and II; pressure is found as a function of distance through the nip, thus the area of the curve is the nip load in lb/linear in. This value must correspond to that of the nip load for the nip that is being simulated and the dynamic compressibility curve must be adjusted by varying the number of sheets and impacting force until agreement is reached. Fig. 38-41 were drawn for a nip with 23 in rolls: Fig. 38 is the plot for a 104 lb raw stock, which is very compressible, giving a curve that shows a wide nip width with low peak pressure. Fig. 39 is for a much harder paper, which gave a high peak pressure and a narrow nip. In Fig. 40, the results are



g. 38—Four quadrant graph giving hip pressu distribution—Grade A



given for newsprint that had been previously compressed to $4\ 000\ \text{lb/in}^2$; the compression and nip width lie between the two previous results. Fig. 41 has been drawn for a 40 lb offset printing sheet: a similar result to Fig. 40. Fig. 42 is given to illustrate the effect of changing the roll diameters; the curves show for this example that, as the roll diameter decreases with a consequent reduction in lb/in, the nip width or pressure duration is reduced, but the specific nip pressure remains the same.



Fig. 40—Four quadrant graph giving nip pressure distribution—Grade C

It should be noted that the peak pressure comes before the geometric middle of the nip because of the visco-elasticity of the paper. The nip is only about 50 per cent wider than the half nip width X_p . The load borne on the outgoing side of the nip is only about 20 per cent of the load borne by the entering side of the nip. It is this difference in load before and after the geometric roll centre that makes the upper rolls difficult to drive. This is the so-called rolling friction, even though the only friction present is inside the sheet of paper.

The compression relaxation curves of papers under the impact of the dynamic compressibility tester are very similar to those obtained for rubber squares, suggesting that the paper (like the rubber squares) is composed of elastic units with considerable free space between them. The curves obtained with paper are different from those obtained with solid but compressible aluminium or with the compressible aluminium foil balls, since these have relatively little or no recovery.



distribution—Grade D

The fact that a paper of low void space such as glassine has the same dynamic compressibility characteristics as the normal papers, though with higher peak pressures and lower compressibilities, suggests that the results with the dynamic compressibility tester are to a large extent due to the compression of the fibres.

After supercalendering glassine, the paper is inelastic compared with the machine-finished paper, presumably because the voids give a spongy characteristic to the paper, which gives more recovery. The glassine thus behaves somewhat like the balls of rolled aluminium foil.

In the results presented earlier, the compressibility was investigated as a function of void space; a full-scale investigation of the effect of different fibres remains to be carried out. The work so far completed shows^(4,5) that groundwood is the most compressible fibre. The grinding method affects the compressibility, a slow fine groundwood being less compressible than a coarser free stock. It was shown that papers containing groundwood change compressibility and other properties faster in the earlier nips of a calender



Fig. 42—Four quadrant graph giving nip pressure distribution changes for changes of roll diameter on the same paper

stack, whereas groundwood-free papers develop compressibility changes and other properties later in the stack. In the previous work, attention was drawn also to the fact that hardwood kraft fibre in the furnish raises and filler lowers the dynamic compressibility relative to the static value.

In the full-scale investigation of the effect of fibre type on dynamic compressibility that will be undertaken, the cross-sections of the paper at different stages in calendering will show the degree of the individual fibre flattening, thus the relationship of paper compressibility to fibre compressibility. It may be predicted, too, that compressibility studies on individual fibres will be undertaken and that the change in bonded area on calendering will also be investigated.

The principal objective of the machine calender and the supercalender is to improve surface properties by producing a smoother paper. For printing papers, the smoothness is most important in the obtaining of good printability.

In calendering, we are concerned with the development of smoothness without impairment of other properties and, in the investigation of calendering, one of the objectives is to find how to obtain the desired smoothness by the most appropriate method. With a full understanding of what happens in calendering, it will be possible to design more efficient equipment, to secure better operation of calenders and to gain better smoothness levels from a given raw material than is at present possible. An example of the manner in which failure to understand the calendering process was given in another paper,⁽⁶⁾ where it was demonstrated that passing the coated paper through an excessive number of supercalender nips caused smoothness to decrease. The printability was thus actually made worse in the last few nips of the supercalender.

For papers of low compression at 1 000 lb/in², it has been found that a large gloss change may be associated with a small change in compression. This is considered to be due to the fact that the differences in compressibility characteristics, which show as compression differences at 1 000 lb/in², give different nip widths and thus different specific pressures.

For the papers so far tested, the relationship of caliper change with pressure and with duration of pressure application differ according to the characteristics of the furnish. The pressure time integral is equally as important as the peak nip pressure, but at present the equipment for studying this is lacking It is not meant to convey that pressure and time are necessarily of equal importance; doubling the pressure results in a larger caliper change than doubling the dwell time under otherwise constant conditions. The relative importance of pressure and time of pressure on caliper change is not yet well enough known to put it in mathematical form. It must also be taken into account that the shearing stress in the nip could enable compression to take place more easily, since the movement of one fibre relative to another in the machine-direction may tend to roll the fibre into a void, thereby compacting the sheet.

Acquisition of families of curves such as Fig. 35–37, specific to different papers at different stages of calendering, together with the availability of the void space/compressibility relationship and the graphical method of computing nip pressures from dynamic compressibility results and calender roll $40-c_{P,W,I}$

sizes, will enable the prediction of the effect of the calender stack on caliper change for a given machine speed.

Since it appears that for most papers a definite relationship can be established between caliper and other paper properties such as smoothness and printability, it may finally be possible to predict these properties also with a reasonable degree of accuracy. Though a very considerable amount of work will be necessary before this objective can be achieved, it appears that the methods outlined in this paper will lead to such a result.

Because the effect of smaller calender rolls, giving lower line pressure, can be balanced by a narrower nip to give the same peak pressure, it is possible to draw certain conclusions. To achieve maximum compression (low bulk and high densities), if the time dependence curve for caliper reduction shows that dwell time of the order of the time that the paper is in the nip is important, then large rolls will be more effective. If, however, the dwell time/caliper relationship curve shows that dwell time does not greatly affect caliper, then small rolls with additional loading will be just as effective.

It is also known⁽⁵⁾ that increased smoothness is best obtained by high specific pressures—thus the combination of minimum caliper and compressibility reduction, at a given specific pressure—and optimum smoothness appears likely to be obtained by a stack construction with small rolls and the required additional loading—the opposite of that now being furnished for most new machines.

The results on the breaker stack were interesting, but testing of paper made on the same machine with and without the breaker stack is needed. The effect of moisture on compressibility was further documented, though surprisingly the temperature had little effect.

Discussion of the calendering process

In the appendix, a one-dimensional rheological model is presented that takes into account several of the rheological properties of paper when subjected to a simple one-dimensional stress/strain—a rate of strain sequence. This is subject to severe limitations and it is likely that a model based on a material with simpler definable properties should be first examined as it would behave when passed between rollers.

The simplest material to be considered would be a purely elastic material. The problem can be simplified by assuming that any strain in the rollers is small compared with the strains on the elastic sheet. Whatever strains or stresses are in the material as it emerges from the rollers, they are exactly the same as those when it entered the rollers. Thus, the deduction would be made that no work is done by the rollers. This would be correct if (a) the rolls were perfectly smooth so that no tangential stress acts anywhere on their

surface; but, in that case, the material would not be forced through when the rollers were caused to rotate; (b) alternatively, the friction is so great that the material sticks to the roller as though by an adhesive as soon as it has any contact. In this case, no work would be done if the work of compressing the material were recovered. When the material expands at the exit of the nip, the stresses would be symmetrical about the line of nearest approach of the rollers.

In general, the coefficient of friction would be finite and work would be done by rolls, even though the elasticity of the material were perfect. A corollary is then that the distribution of stress and strain in the material would not be symmetrical, if the coefficient of friction is neither zero nor sufficiently great to cause the material to have no slip.

The passage of a plastic-rigid material between rollers is discussed by Hill.⁽⁷⁾ Hill uses an expression for the distorted radius of curvature of the rolls, which he regards as constant in the nip. This is a different concept from that of the appendix.

The most important point brought out by Hill is the existence of a neutral point in the length of the nip where the friction reverses sign. When only one roller is driven,* the two rolls of a nip cannot be expected to rotate at exactly the same peripheral speed, so that the neutral point where friction reverses sign will not be the same on both sides and, in the part between these points the strip will be subjected to a shearing stress, but to no variation in the longitudinal stress. The neutral point at the top surface is nearer the point of entry than that on the lower surface, so that there is a tendency to reduce the length of the upper surface compared with the lower, thus to move the neutral axis towards the lower (driven) roll. This should cause the strip to curve towards the upper roller and this has been found to be the case with lead foil.

Paper is clearly more nearly describable by the plastic than by the elastic model, thus the difference in speed through the calender stack is an important measurement in the definition of the calendering process. At the same time, it is necessary to measure the increase in length of the paper.

In reading the appendix and in considering the applicability of the results of the flat plate tester, it has been assumed (as earlier stated) that the calender is changing the caliper, but is not altering the mass per unit area. Were the mass per unit area altered, then the method of using two parallel plates rather than the rolling cylinders of the calender—carries with it some implicit assumptions.

Given a test in the tester described in the text such that the pressure P was caused to vary with time t in a number of ways and the thickness between the

* In Hill's work, both rolls are assumed to be driven

plate measured as a function of t: assuming that the tester completely duplicates the nip, it has the disadvantage of not being able to explain any effect that calendering may have in making mass per unit area more uniform, because it assumes that there is no expansion in the plane of the paper. This is not a trivial point as one can see at once by considering what happens when lateral expansion is prevented in uni-directional compression of an *elastic* material of Young's modulus E and Poisson's ratio σ . Here, lateral expansion is not allowed and the strain e is—

$$e = \frac{P}{E} \left(1 - \frac{2\sigma^2}{1 - \sigma} \right)$$
 or $P = Ee \left| \left(\frac{1 - \sigma - 2\sigma^2}{1 - \sigma} \right) \right|$

and, if you allow free expansion, P = Ee. The ratio of these is $(1-\sigma)/(1-\sigma-2\sigma^2)$. One can see that, as σ goes from 0 to $\frac{1}{2}$, this factor goes from 1 to ∞ ($\sigma = \frac{1}{2}$, of course, corresponds to incompressible material). If one compresses a sheet between plates that allow perfect slipping, one obtains P = Ee; whereas, with even a small degree of friction, the pressure everywhere except near the edge of the sheet is $Ee[(1-\sigma)/(1-\sigma-2\sigma^2)]$.

Conclusions

1. The general reliability of the dynamic compressibility testers developed has been shown to give results accurate to within 5 per cent. The less compressible the paper, the less reliable are the results.

2. For equal peak pressures, the static compressibility is greater than the dynamic compressibility.

3. Compression per sheet is the same for the same peak pressure, regardless of the number of sheets used. For a single sheet, the peak compression is delayed beyond the point of peak pressure more than for multi-sheet tests.

4. For relatively rough and compressible papers, there is an important relationship between compressibility and printability; for the very smooth supercalendered coated papers of relatively low compressibility, however, the differences that exist in compressibility are of minor importance compared with the smoothness differences. This is at least partly attributable to the greater compression of the press packing material in a paper super-imposed on press packing.

5. Papers with a very low void space such as glassine give similar compressibility curves to normal papers.

6. From the limited results available, the temperature of compression over the range studied has no effect on compressibility.

7. Compressibility increased linearly with moisture content from 2 per cent to 17 per cent.

8. Caliper change is a function of pressure and of duration of pressure application increasing with dwell time and pressure.

9. Smaller calender rolls with a lower line pressure give a narrower nip, thus a shorter pressure duration than larger rolls, but the maximum specific pressure may be the same in both cases for a given sheet entering the nip.

10. It appears that better calendering—that is, controlled caliper reduction (a minimum for a given loading)—with optimised smoothness will be best obtained with a stack built of small diameter rolls with external loading.

11. By further work, it may be considered likely that the action of a stack on a given paper will be predictable by using the experimental methods discussed in this paper.

12. Paper is more nearly plastic than elastic and the discussion given suggests that the difference in speed through the stack is an important calendering parameter.

Appendix

This appendix presents a theoretical analysis of a calender stack in the form of equations and ideas to be related to experimental results and applied to practical operation. For such an analysis to be of use, the mechanics must be in a form that it can eventually explain the basic changes in paper—

- (a) Caliper and associated bulk-based properties.
- (b) Smoothness and associated surface-based properties.
- (c) Formation and other arrangement-based properties.

The response of paper to pressure determines how the loading per linear inch shall be distributed across the nip in the machine-direction or the pressure in lb/in^2 , thus indirectly relating the latter loading to the response. The rolls impose a geometrical restraint on the nip mechanics and the line pressure imposes a limit to the integral of vertical pressure across the nip. The paper itself basically imposes the pressure (lb/in^2) distribution and, for this reason, how the paper behaves dynamically under a very brief geometrical deformation is important.

A model to simulate the response of paper to a stress or strain is made by combining a number of springs, dashpots and perhaps friction slides in various combinations. The idea of using springs and dashpots is not new and many papers have been published showing how simple two- or three-element models can, in certain cases, partially predict the paper flow. Much of the work has been on tensile strength rather than on compression. In general, the utility of such theory has not been great enough to offset the complexities of the mathematics. To complete any theory, it is suggested that the third elementary body—the friction slide—has to be used, which could explain permanent set in paper as easily as a dashpot. Besides, all work has been with either series or parallel combination of elements, whereas paper mechanics is basically a network or diagonal system. A diagonal combination of elements might easily explain yield phenomena. For example, a spring at an angle θ to the stress direction might have its constant K replaced by $K \cos x$, where x is the amount of stretch. Such use of non-constant coefficients should be applied to other elements as well. Nearly all work so far has been with constant coefficients such as Kx for a spring and K' dx/dt for a dashpot; as a result, it is often pointed out that a larger number of simple combinations of elements ought to be used. The entire combination would have as its coefficients a full spectrum of constant coefficients with different constants from each simple combination. If instead a spectrum coefficient or at least a variable coefficient were assumed for each element, one simple combination of elements might then suffice. The objective of the present study is the explanation of the total behaviour of the paper.

Element coefficients would be determined using an analogue computer or, in some cases, by trial and error in a longhand mathematical analysis. Some coefficients that can be separated might be measured by the proper experiment that resembles the calendering process as closely as possible. The following is a rheological model of paper in compression. The four paper characteristics to be described are—

- (a) Reversible springiness of the sheet.
- (b) Non-reversible creep of the sheet under prolonged stresses.
- (c) Reversible creep of the sheet under prolonged stresses, and the recovery of the creep.
- (d) Non-reversible instant set under yield loads.



Fig. 43

The four phenomena are modelled by these elements-

- 1. Spring (a).
- 2. Dashpot (b).
- 3. Parallel combination of spring (c) and dashpot (d)— known classically as a Voigt element.
- 4. A friction slide (e)-non-reversible in character, one way only.

For given step impulses as indicated in Fig. 44, the total response or amount of compression might be indicated in the corresponding parts of Fig. 45, where the role of each element is indicated by the letter.







Fig. 45

General mathematics for the model-

Let P = the total instantaneous pressure (lb/in²) on the model, X = the total compression, t = time.

The various elements are referred to by their assigned letters. Thus-

$$t = t_a = t_b = t_c = t_d = t_d$$

$$\begin{cases}
P = P_a = P_b = P_e \\
P = P_e + P_d \\
\begin{cases}
X = X_a + X_b + X_{cd} + X_e \\
X_{ed} = X_c = X_d
\end{cases}$$

Consequently, if for each separate element, it can be written-

$$X_a = f_a(P_a, t_a...); \quad X_b = f_b(P_b, t_b...):$$
 etc.

then the solution is $X = f_a(P_a, t_a...) + f_b(P_b, t_b) + ... X_e$

or, complementarily, $P = f_a'(X_a, t_a, \ldots) = f_b'(X_b, t_b, \ldots) \ldots P_e$

To give an example solution of first approximation, these functions were selected to describe the elements—

$$X_a = K_4 P_a; \quad X_c = K_5 P_c; \quad X_e = K_6 (P - P_M) \text{ for } P > P_M$$

where P_M is the maximum pressure before the time t = 0.

Also $dX_b/dt = K_7 \cdot P/t$ and $dX_d/dt = K_8 \cdot P_d/t$

so that

$$X_{b} = K_{7} \int_{0}^{t} PK \, dt$$
 and $X_{d} = K_{8} \int_{0}^{t} P_{d}/t \, dt$.

At this point, it becomes necessary to assume a P = f(t) expression to describe the pressure function (lb/in²) through the nip. $P = h_1 t - h_2 t^2$ was assumed, where $h_1/h_2 = t$ when the paper leaves the nip and $t = h_1/2h_2$ at mid-nip. Thus, nip width w was $w = V(h_1/h_2)$, V being the roll speed.

Total load $W = V(h_1^3/6h_2^2)$ and total pressure $(lb/in^2) = 6WV/w^2[t - (V/w)t^2]$. It was shown, that under the assumption $X = 3W/w(K_7 + K_6/2)$ as $t \to \infty$, w was a function of caliper x and roll diameter.

The shape of the nip is determined mostly by the steel rolls, which impose a geometrical restraint on the paper (Fig. 46).



FIG. 40

 R_1 = radius of one roll; R_2 = radius of the other roll,

 h_{m_n} = thickness of paper on mid-nip,

 h_1 = change in thickness of the top half of the sheet as far as mid-nip,

 h_2 = same change for the bottom half of the sheet,

 $h_1' + h_2' =$ expansions in the halves of the sheet after the mid-nip, but only for so long as the contact lasts, distance X'.

$$X =$$
 distance from first contact to mid-nip.

If
$$\theta_1 = X/R_1$$
, then $h_1 = R_1 - R \cos \theta_1 = R_1(1 - \sqrt{R_1^2 - X^2}/R_1)$
 $h_1 = R_1 - \sqrt{R_1^2 - X^2}$
 $R_1 - h_1 = \sqrt{R_1^2 - X^2}$
 $X^2 = 2R_1h_1 - h_1^2$, but $h_1^2 \ll 2R_1h_1$

Dynamic consolidation during calendering

Therefore, $h_1 \approx X^2(1/2R_1) = X^2(1/D_1)$. Similarly, $h_2 \approx X^2(1/D_2)$. Paper thickness before the nip was $h_1 + h_{m_n} + h_2$. At some point in the nip X'', thickness is $h_1'' + h_{m_n} + h_2''$. If Z is the change in thickness of the sheet from the time it entered the mid-nip to point $X''(X'' \leq X')$, then—

 $Z = h_1 + h_2 - (h_1'' + h_2'')$ = $X^2(1/D_1 + 1/D_2) - X''^2(1/D_1 + 1/D_2)$ $Z = (X^2 - X''^2)(1/D_1 + 1/D_2)$

X" can be either side of the mid-nip. If X'' = 0, the amount of compression at mid-nip (that is, maximum strain) is $Z_{mn} = X^2(1/D_1 + 1/D_2)$.

The expression for Z is general for any calender nip, to the approximation that the rolls are not themselves deformed. Fig. 46 shows the paper going straight through the roll nip, but it can equally well be shown wrapping the top roll as it enters the nip and wrapping the bottom roll as it leaves the nip, with no loss of utility in the expression.

The rolls are not entirely undeformed as they hit the paper, although the radial acceleration of deformed material is small enough, so no pressure arising from the radial inertia of the steel is involved—but the nip does change shape. Sir G. I. Taylor suggested using the theory of roll deformation when steel sheet is striprolled, as set out in Hill.⁽⁷⁾ Hill presents a formula by Hitchcock (pages 190, 191⁽⁷⁾) for the *change in roll radius* as the roll surface passes through the nip. He first points out that the *nip* radius of the roll is very nearly constant through the nip, according to the theory of elastic contact between bodies, that is, the roll is cylindrical but of different radius R' instead of R. He says that this R' assumption and Hitchcock's formula cannot be bettered without great calculation.

Since steel strip-rolling presents a pressure distribution on each roll not too different probably from paper rolling, we examined Hitchcock's formula $1/R - 1/R' = P/CL^2$ (Hitchcock's formula modified by Hill),

where P = line pressure, lb/linear in, $C = \pi E/16(1-\gamma^2)$, $E = 30 \times 10^6$ for steel, γ = Poisson's ratio ≈ 0.275 for steel, L = nip arc length $\approx 2x$. We had $Z = (X^2 - X''^2)(1/D_1 + 1/D_2)$. Instead, $Z' = (X^2 - X''^2)(1/D_1' + 1/D_2')$.

The ratio Z'/Z tells how serious roll deformation is—

$$\frac{Z'}{Z} = \frac{1/2R_1' + 1/2R_2'}{1/2R_1 + 1/2R_2} = \frac{(1/2R_1 - P/2CL^2) + (1/2R_2 - P/2CL^2)}{1/2R_1 + 1/2R_2}$$

Where P/CL^2 is the same for each roll, then—

$$Z'/Z = 1 - \frac{P/CL^2}{(1/D_1 + 1/D_2)}$$

41-C.P.W. I

or

For the case (considered the extreme) where P = 300 lb/linear in



Fig. 47

Thus, consideration of roll deformation will decrease Z by up to 6 per cent anywhere through the nip, under the assumptions. It is reasonable to assume that nip width will be accordingly about 6 per cent larger, at most, than it would be with rigid rolls and that the pressure (lb/in^2) distribution through the nip will be somewhat flattened, since it has to total the same value as the line pressure (lb/linear in); the value in lb/in^2 may be (at most) about 6 per cent greater at the 'quarter points' of the nip and (at most) about 12 per cent smaller at the midpoint, owing to roll deformation. The effect is not serious, but is easy to include if experience shows it to be necessary.

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ADDENDUM

IN PRESENTING the paper, Mr Mardon pointed out that the direct approach to the investigation of the changes in paper structure during calendering has also been pursued and illustrated the changes by slides of paper sections at various stages of calendering.

Fig. 48 shows a speciality groundwood paper before the calender stack; very considerable void space characteristic of free-beaten papers in the uncalendered state can be seen. The fibres are undamaged from compression. The fact that the bulk of the compression occurs after one nip in the machine stack has been pointed out by the present authors in previous papers. Fig. 49 shows that the first nip works by compacting the sheet, but that the fibres remain uncompressed. Fig. 50 shows the paper as it appears after six nips and illustrates that the fibres are now flattened.

Fig. 51 shows standard newsprint before and after calendering and Fig. 52 illustrates similar cross-sections of newsprint made of chip groundwood with only 5 per cent of chemical woodpulp.



Fig. 48-Speciality groundwood paper before calender



Fig. 49—Speciality groundwood paper after first nip



Fig. 50—Speciality groundwood paper after calender



Fig. 52-Chip groundwood newsprint

After calender

Before calender

Transcription of Discussion

Discussion

Dr J. A. Van den Akker—It is very important, in our opinion, that laboratory devices intended to simulate the action of calenders or any other rolling devices should incorporate the element of shear. In unpublished work a number of years ago, Nolan and I attempted a laboratory study of the effect on the optical properties of paper of mechanical compaction of the damp web. In our initial effort, we set up a precise device for the mechanical compression of the web between plane-parallel steel bosses. On measuring the change in the Kubelka-Munk scattering coefficient for wide ranges in both compaction stress and moisture content (arrived at by desorption from a never-dried web), it was found that the largest changes were very much smaller than that found in the production machine operation, which involved a rolling action on the damp web. In seeking an explanation for this great disparity, I found an important experimental paper by the late Prof. Bridgman (Harvard University) 'Effects of high shearing stress combined with high hydrostatic pressure' (on various organic and inorganic materials).*

This important paper, which showed that the combination of intensive shear and compressive stresses can induce both mechanical and chemical changes at the molecular level, suggested to us that shear should be involved in our laboratory work and, on introducing this factor, we found that we could get on with our work satisfactorily. Although our study was not concerned with calendering, the role of shear in calendering suggested itself and this was later made the subject of an invited article.[†]

Mr M. I. MacLaurin—Mardon's measurements have been based essentially upon standard stress/time relationship. In a printing press or a calender stack, the more clearly defined relationship is that between strain and time. For example, in a flatbed letterpress machine, the compressive strain of the paper is related to a cosine² function of time. This is the sort of relationship he refers to in the fourth quadrant of his nip pressure distribution diagrams.

Some years ago, work in this field was done at Butler's Court, but not

^{*} Bridgman, P. W., Phys. Rev., 1935, 48 (10), 825-847

[†] Paper Ind., 1946, 28 (1), 57-59

completed owing to changing priorities. The apparatus that was being developed recorded the stress/time for any standard strain/time relationship. Fig. F illustrates the principle.

We used an L.V.D.T. strain pick-up, but Mardon's system appears to be an improvement on this. Do you think the use of this apparatus or other equipment based upon the strain/time reference would give rise to different conclusions from those stated in your paper?



Fig. F

Mr C. B. Ihrman—As a comment to Peel's contribution, I should like to mention a test run on the double-roll compactor with a set speed difference of about 6 per cent. We started with a rather low pressure and the paper had a moisture content of about 43 per cent. The paper came out of the nip neatly cut in strips across the web, which indicated a slip on the entering part of the nip. When the pressure was increased, the same paper was shrunk the normal way. It must be borne in mind, however, that on this run the web covered the steel roll on the in-going side of the nip at least 180° . The trials show that the locked region can be moved in the nip, so that the accelerating rubber on the in-going side of the nip will be found outside of the nip, where it will not destroy the running web.

Mr J. R. Parker—I should like to comment on the relationship between compressibility and printability. Firstly, I believe it is important to distinguish between the compressibility of the body of the sheet of paper and the compressibility of the paper surface, because only a very small proportion of the paper surface can be brought into contact with the printing press—only 10-

Discussion

20 per cent in the case of newsprint. The stress distribution in the paper surface may be quite different therefore from that within the paper and there may not be a high degree of correlation between the compression of the surface and the compressibility of the body of the paper.

Secondly, the smoothness referred to in the paper has been measured both by the Bekk and Sheffield instruments. Each of these exerts a pressure on the paper that is small compared with the pressure in the nip of a printing press. The pressure in the Sheffield instrument is only 1.46 kgf/cm² compared with a possible pressure of 10–40 kgf/cm² in the nip of the press. If print rating is to be predicted from the results given by such instruments, therefore, correction must be made for the varying compression of different paper surfaces. I have found that, when smoothness is measured under printing pressures by a suitably designed instrument, no such correction appears to be necessary. I suggest that the compressibility of letterpress papers is not really a fundamental printing property: it is merely a factor introduced to correct for the deficiencies of some smoothness testers.

One final point is in connection with Mardon's Fig. 19 and 20 (page 596). The authors comment that the effect of smoothness on print quality is critical for smooth papers. The shape of these curves, however, is largely a consequence of the method of measuring smoothness. The rate of laminar flow of air between parallel planes is proportional to the cube of the distance between the planes, so the air flow in a smoothness instrument should vary as the cube of mean depth of the surface defects. We have not been given full details of the method of assessing print rating, but if we assume it to be inversely proportional to the depth of the surface pits, the curves shown ought to be of the form— $y = x^3$ and $y = x^{-3}$. This largely explains the results that have been obtained.

Mr J. Mardon—Taking Peel's contribution first, our paper was dealing only with machine calenders, because of the additional difficulties that occur in supercalenders: the only mention of supercalenders was in considering the rather interesting case of glassine. I should point out we were fully aware of the details of rolling friction, with particular reference to Dritowski's work. As a result of discussions with Sir Geoffrey Taylor, the equivalent of Peel's picture is in words in the paper (about four paragraphs on supercalenders). In Peel's particular case, the paper does not come out straight as shown in the diagram, but will run up the slower roll as indicated again in the paper. Although, basically, I agree with him, we may possibly be getting into that region where angels fear to tread.

My only comment on the first part of Chapman's contribution is the fact that there is a variation of one order of magnitude more in his results than ours of the variation in the time/pressure duration relationships. I would not attempt to decide on that basis which results were perhaps more appropriate; it should be possible for him to make an energy balance in the way he did, which would give a very good indication of the accuracy of his instrument. We have given the accuracy of ours.

On the comment (to which I take a little exception) about the similarity of characteristics of the curves for different papers, there is a statement in the paper (page 611) that the voids give a spongy character to the paper, which gives more recovery and so we are aware that the voids play a very great part. As the earlier photographs showed, there is no question that the fibres compressed.

I would like to mention that the specific pressure in the calender stack varies according to the size of the stack approximately $1\ 000-2\ 500\ lbf/in^2$. The bigger stacks do not give the expected increase in specific pressure, because one is widening the nip; so the specific pressure does not increase at the rate at which the total line pressure is going up. It is important to appreciate that the pressure on the fibres is very much greater than the pressure in the nip.

I have no comment on what Van den Akker said, except that our reasons for believing that the flat plate tester gives useful results appear in the text.

I think our results were sufficient for us to be reasonably certain that these very hard, smooth papers were in fact so incompressible after being in the supercalender stack that the differences in compressibility (although quite large compared with the total compressibility) were in fact so small that the compressibility plays a very small part in these particular papers. What plays a more important part in the printability of these papers is the actual surface structure of the base sheet that protrudes through the coating; the screening of the fibres is the vital thing, provided the coating weight is sufficient.

MacLaurin's apparatus looks ingenious, but the only way one can really answer his question is to do the experiment.

Ihrman's comment concerned the passing of the paper through the nip and I agree with him.

Chapman long ago showed that what is important in printability is the printing smoothness, but these incompressible, heavily coated papers are so smooth anyway that they become no smoother when compressed. We are still aware, however, of the importance of printing smoothness. There is a relationship between the compressibility and the printability of a compressible paper like newsprint and I point out again what is stated on page 603 that the force has to be transmitted through the surface to the body of the paper, then through the body of the paper to the packing underneath. Basically, I agree with Parker and thought we had said so.