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A NEW THEORY FOR THE LOAD/ELONGATION PROPERTIES OF PAPER

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Synopsis—The new theory of the load/elongation properties of paper, in essence, sums the loads developed in the fibres intersecting a line at rightangles to the direction of straining. Fibre nonlinearity and the intermittent bond failure that occurs before the final gross rupture of the sheet are taken into account. The hypothesis is proposed that gross rupture is triggered by one of two mechanisms—bond failure in sheets of strong fibres and fibre failure in sheets of weak fibres. The theory does not consider the phenomena occurring during gross rupture. Measured stress/strain curves of many handsheets of five pulps of springwood fibres were found to be in good agreement with calculated curves.

Nomenclature

a	= axial deflection of a fibre, cm
b	= deflection of fibre at rightangles to its axis, cm
E	= axial elastic modulus of fibre, g/cm^2
$E_{\perp}$	= elastic modulus of fibre at rightangles to its axis, g/cm^2
e	= eccentricity of non-randomly oriented network
e_f	= unit fibre strain
e_x	= unit strain of sheet in x-direction
e'_x	= maximum unit strain of sheet in x-direction
e_y	= unit strain of a sheet in y-direction
f_c	= probability that a fibre does not develop load because of its non-linearity
f_e	= probability that a fibre does not develop load because of end bond failures
F_T	= x-direction component of load developed by a fibre, g
$F(\theta)$	= axial load developed by a fibre at θ, g
$N(\theta)$	= number of points at which fibres between θ and $(\theta+d\theta)$ intersect a scan line
$PL(\theta)$	= probability that a fibre intersecting a line at θ carries load
$P(\theta)$	= probability density function for fibre orientation
R	= width of sheet, cm
RBA	= relative bonded area

RBA_{crit} = relative bonded area at which fibre failure first triggers sheet rupture
s = shearing resistance of bonded area, g/cm^2
t = failure load of a fibre, g
T = load developed by a sheet, g
T_{fibre} = tensile strength of a sheet whose failure was triggered by fibre failure, g
T_{bond} = tensile strength of a sheet whose failure was triggered by bond failure, g
w = weight per unit length of fibre, g/cm
W = substance of sheet, g/cm^2
Y = true elastic modulus of a sheet per unit width of sheet, g/cm
Y' = elastic modulus of a sheet at a small finite strain, per unit width of sheet, g/cm
α = cross-sectional area of fibre, cm^2
α_{fe} = bonded area at end of a fibre that failed, cm^2
γ = density of cellulose, g/cm^3
λ = fibre length, cm
θ = orientation of fibre with respect to a line at rightangles to direction of straining
θ_c = critical angle at which fibres do not develop load
ν_{xy} = Poisson's ratio of a sheet
ω = fibre width, cm
δ = fibre thickness, cm

Basic considerations

THIS NEW THEORY of the load/elongation behaviour of paper is a quantitative description of the phenomena that we believe to occur during its straining. It determines the stress/strain curve by calculating and summing the fibres' loads developed across a transverse line of a sheet at successive increments of elongation. It is assumed that the deformation of one end of every fibre relative to the other conforms to the uniform deformation of the sheet, but the deformations and loads in specific parts of each fibre are determined from other considerations such as its orientation, degree of bonding and intrinsic bond strength. The contribution of each fibre is determined from its stress/strain properties, its shape and its geometry within the network. Even in its rudimentary form, the theory's predictions are remarkably close to measured values.

When the sheet shown in Fig. 1 is elongated at AA, the magnitude of the load developed across transverse lines such as CC and DD depends only on the number of fibres that cross a given line and the forces in each of them. The bonded areas between the fibres are the only means by which loads are transmitted between fibres.

Only a fraction of all of the fibres crossing a line develop a load and this fraction diminishes with increasing elongation of the sheet. We believe two mechanisms prevent the development of an appreciable force in all the fibres.

One is caused by the non-linearity of the fibres, the other by the failure of bonded areas during straining.

It can be readily seen from surface photomicrographs and cross-sectional views of paper that fibres exhibit three types of nonlinearity. One, that within the plane of the sheet, is shown in Fig. 2.* A second is the gross in-and-out-of-plane weaving that exists, because every fibre must accommodate the others in its immediate vicinity (Fig. 3). A third is an in-and-out-of-plane weaving that occurs on a microscale (microcompressions); these are shown in Fig. 4.* (See Page & Tydeman[1] for a discussion of their origin.) We believe that, when paper is strained to the point of failure, a proportion of the fibres are partially straightened out rather than strained axially. The forces arising from

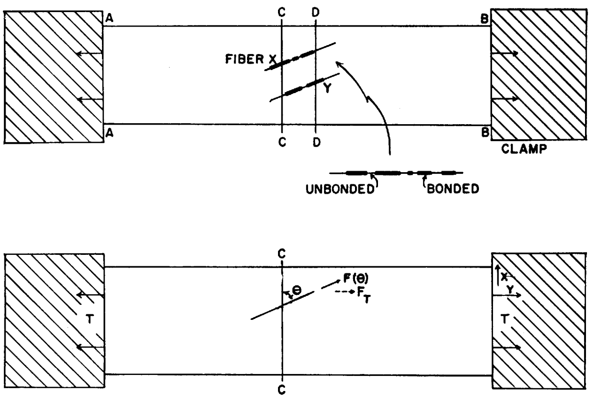

Fig. 1—Pictorial representation of fibres taking load across a line

straightening are undoubtedly negligible relative to those caused by axial straining of fibres. (On straightening curled, individual fibres on the Instron machine before testing, we never observed the development of a load.) Although the contributions of each of the three types of non-linearity cannot be assessed at present, the fraction of fibres across a line of a sheet developing a load because of axial straining can be estimated. (The subject is discussed briefly below and at length elsewhere.[2])

It is well known that the bonded area of a sheet diminishes throughout the straining cycle at an ever-increasing rate.[3,4] This prerupture bond failure is readily incorporated in the picture by a model based on end bond failures. When a sheet is elongated, the load in most fibres—hence, across most bonded

* Fig. 2 and 4 were taken by A. Rezanowich of the Pulp and Paper Research Institute of Canada

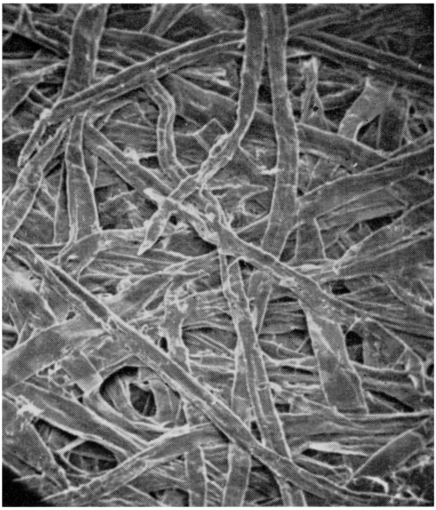

Fig. 2

areas—increases. Of all the bonded areas along a fibre, it would appear that those at the ends develop the largest forces[5] and, therefore, fail first. The bonded area that has failed at the two ends of every fibre *from the inception* of straining can be estimated at every interval of straining from the load in each fibre.

Let us now examine two fibres of the above model, X and Y of Fig. 1, to see the effect that bond failure has on their contributions to the loads across CC and DD. Since both were straight and partially bonded along almost their entire length before straining, both initially supported loads across CC and DD. After an increment of straining, the end lengths of both fibres (shown as light lines) became unbonded, so that fibre X no longer contributed to the

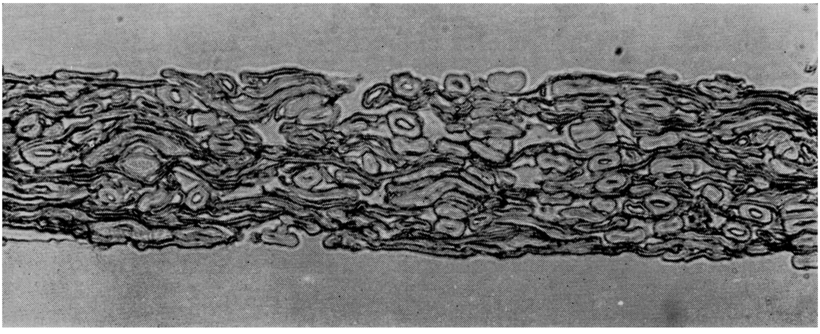

Fig. 3

force across DD and fibre Y no longer contributed to the force across CC. In other words, as a sheet is strained further and further, the number of fibres contributing to the load across any given line diminishes step by step. At an elongation yet to be defined, the number of load-bearing fibres across a line decreases to the point where gross rupture (the well-known tensile failure) of the sheet occurs. Although the theory must specify the conditions for the inception of gross rupture, it need not—and, in its present form, does not predict—the manner in which the failure line propagates after its initiation.

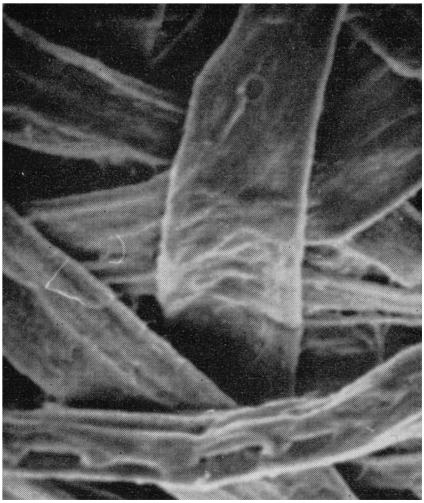

Fig. 4

We have no evidence at present that the bonded areas at the ends of fibres actually fail first: in fact, one might argue that they rarely or never fail. Bonded areas near the ends of fibres are often restrained on one side only, the fibre tips being unattached, hence they have a greater freedom of movement than bonded areas along the central portion of fibres that are clamped on both sides. The matter is not crucial at this time for the theory. One could as well picture that central portions of fibres become relaxed by bond failure, thus removing parts of some fibres from the picture as load-bearing elements. Such a model is probably closer to the actual case, but its quantification is far more difficult than one based on the failure of bonded areas at the ends of fibres. This is the primary reason we used the end bond failure model initially.*

* Since presentation, we have concluded that the mathematical model used might be used to describe both mechanisms

We believe that gross rupture is initiated by one of two mechanisms, whichever occurs first: either *1* or *2*—

1. The point is reached at which the failure strength of a fibre of average properties is exceeded. Fibres with slightly smaller loads in the proximity to the one that failed must then assume higher loads, thereby reaching their failure strength; this chain reaction initiates gross rupture.

2. The point is reached at which bonded areas are lost at such a rate per unit of sheet elongation that the load borne by the fibres *removed* as load-bearing elements begins to exceed the increase in the load of the fibres still taking load—that is, a point of maximum stress on the calculated stress/strain curve is the start of gross rupture.

In the continuation of gross rupture initiated by either of the above mechanisms, both fibre and bond failure occur. The proportion of the two is a function of many factors and is unknown as yet.

The power of this stress/strain theory lies in the fact that, as far as we are aware, it has the capacity to describe the important phenomena that occur during the elongation of a sheet; that all of these mechanisms can be quantified (at least approximately); that many of the unique properties of paper can be explained naturally in terms of the fibre properties described in our first contribution to this symposium. We believe these to be the primary reasons for the close similarity between the measured and calculated curves. The differences observed are probably due to (*1*) the approximate nature of the model, (*2*) the low precision of much of the data used in the calculations, (*3*) the use, in many cases, of average properties rather than distributions and (*4*) the omission of phenomena of which we are either unaware or that we believed to be of negligible importance and therefore omitted. We see no reason that improvements in all of these factors will in time greatly diminish the gap between theory and reality.

A basic shortcoming of earlier theories is that they predict smooth stress/strain curves without a final failure point.[6] We know from listening to paper fail,[4] from studies by Page & Tydeman[3] on bond failure during straining and from work with thin sheets that the failure of paper is a step-by-step breakdown of its structure—that is, failure of bonded areas. It begins at a slow, but steadily increasing rate, then suddenly accelerates to an extremely high rate. Thus, the part of the stress/strain curve before gross rupture should have the appearance of that in Fig. 5. Measured curves are smooth only because recorders cannot discriminate the stress drops caused by the failure of a few bonded areas, a few grams at most, at a total load of many lb.

The theory predicts the initial slope of the stress/strain curve (it is essentially the same as that predicted earlier[5,7] with $PL=(1-f_c)$; see below), its changing slope because of the failure of bonded areas (the so-called *plastic* *zone*) and the inception of the final failure. A discontinuous curve as in Fig. 5 would be constructed as follows—

1. Calculate the elastic modulus of the sheet structure at A from equation (*20*) below; this is part Y of the curve in Fig. 5.
2. Calculate the load at a small increment of strain from equation (*13*) below, point B. The strain increment is what would give a change in bonded area equal to a few fibre overlap areas; an exact means of making this calculation is yet to be determined.
3. Draw a vertical line from point B to curve Y.
4. Calculate the modulus of the structure responsible for the load at point B. Its slope is slightly lower than that of the structure at A because a few load-bearing fibres were removed by bond failure in the strain increment AB.
5. Repeat steps 2–5 up to the initiation of gross rupture as specified by equations (*14*) and (*15*) below.

We have not actually applied this rigorous technique of constructing stress/strain curves. It is presented only to emphasise the discontinuous nature of the stress/strain curve and to demonstrate how this theory can describe it. In the mathematical section to follow, it is assumed that bond failure is a continuous

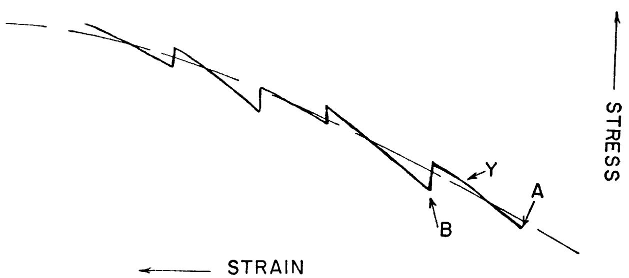

Fig. 5—Pictorial representation of true stress/strain curve of paper

function, equation (*10*) and that the load across the sheet is calculated from equation (*14*). This gives a continuous function shown by the dotted curve of Fig. 5 and is an excellent approximation.

The straightening out of the fibres could be incorporated as part of their stress/strain curve instead of as a separate factor. The initial part of the fibres' curve would then have an extremely low slope and the development of load in a fibre due to axial straining would begin at a finite elongation rather than at zero strain. We have included the straightening out of fibres as a loading factor rather than as part of the fibres' properties because of the virtual impossibility of obtaining and testing isolated fibres with the shape they take in paper.

Basic assumptions

THE following are the seven premises on which reasoning is based—

1. The *average* strain of one end of every fibre with respect to the other conforms to the macroscopic uniform strain of the web. *This is the basic definition of a uniform strain theory*, but this assumption does not mean that each fibre is strained uniformly; on the contrary, conceptual difficulties arise unless one considers that the strain along a fibre varies. If a fibre's strain were uniform, the stress along its length would be uniform and all its bonded areas would fail simultaneously. This is obviously not the case. Moreover, the assumption of uniform sheet strain does not require every fibre to develop an appreciable load; in many cases, elongation may cause only a straightening out of the fibres, resulting in negligible loads.

2. The fibres are ribbon-like with their surfaces virtually parallel to that of the sheet. For computational purposes, their cross-sectional shape is assumed to be a rectangle of area $\delta\omega$.

3. The structure of the sheet is described by the bonding state model depicted earlier.[8] In the present study, we assumed the fibres were positioned and oriented randomly. (Means of introducing non-random functions are mentioned when appropriate.)

4. The forces developed within a fibre caused by bending and shearing are so small relative to the tensile forces that they are neglected; the basis for this assumption was presented in our first contribution. As a consequence, the theory considers that fibres fail in tension only.

5. Bond failure is due to shear forces acting parallel to the fibre axis—rotational forces on the bonded areas are neglected. We believe this to be a reasonable approximation by virtue of the relatively small loads developed by shearing and bending.

6. The fibres are Hookean up to the point of their failure. Techniques of introducing any stress/strain curve of a fibre are mentioned when appropriate.

7. Although every fibre property has a distribution, mean values were generally used in the equations. This was done so that the final expression could be integrated in closed form. The use of distributions throughout would have required numerical integration on a computer.

The theory

THE load developed across the line CC of Fig. 1 is a function of three quantities—(*1*) the load developed per fibre at θ, $F(\theta)\sin\theta$, (*2*) the number of points at which fibre axes intersect CC between θ and $\theta+d\theta$, $N(\theta)\ d\theta$ and (*3*) probability that a fibre at θ develops a load $PL(\theta)$, then—

$$T = \int [F(\theta) \sin \theta][PL(\theta)]N(\theta)\ d\theta \qquad (1)$$

Each of these three variables will now be evaluated in turn.

1. The tensile force developed in a fibre of length λ oriented at $(\pi/2-\theta)$ to the direction of straining $F(\theta)$ is determined by the same considerations as used elsewhere.[5,7] Assuming that the average deflection per fibre is proportional to the strain of the sheet in the x-direction e_x, the deflections a and b defined in Fig. 6 are—

$$a = e_x\lambda(\sin^2 \theta - \nu_{xy} \cos^2 \theta) \qquad (2)$$

$$b = e_x\lambda(1+\nu_{xy}) \sin \theta \cos \theta \qquad (3)$$

where ν_{xy}, the Poisson ratio for a sheet strained in the x-direction is equal to $-e_y/e_x$; e_y is the strain in the y-direction of a sheet elongated in the x-direction.

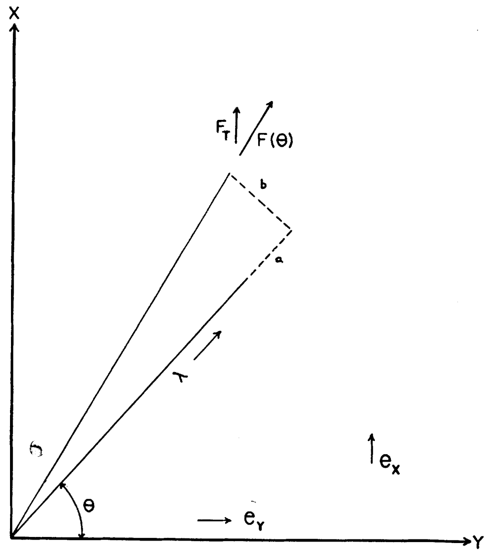

Fig. 6—Schematic representation of deformation of a fibre

The extension a is related to the tensile force developed in the fibre by—

$$a = [F(\theta)]\lambda/E\alpha \qquad (4)$$

where $\alpha=\omega\delta$. In the case of non-Hookean fibres, a different expression, one based on experimental data, would be used in place of equation (*4*).

The deflection b is associated with the negligible forces of bending and shearing and the free rotation of the fibres; it is of no further concern now. Neglecting bending and shearing relegates this term to pure rotation.

Combining equations (*2*) and (*4*) yields—

$$F(\theta) = E\alpha e_x(\sin^2\theta - \nu_{xy}\cos^2\theta) \quad . \quad . \quad . \quad (5)$$

The contribution per fibre to the load in the x-direction F_T is—

$$\begin{aligned} F_T &= F(\theta)\sin\theta \\ &= E\alpha e_x(\sin^3\theta - \nu_{xy}\cos^2\theta\sin\theta) \quad . \quad . \quad . \quad (6) \end{aligned}$$

2. In a random sheet, the number of points at which fibre axes intersect a line of length R between θ and $(\theta + d\theta)$ was shown in equation (*2*) in the 1963 paper[7] to be—

$$N(\theta)\,d\theta = \frac{WR\sin\theta}{\pi w}\,d\theta \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where W is the sheet substance and w the weight per unit length of fibre. Being the equation for a randomly oriented network, equation (*7*) is based on the probability density function $P_\theta = 1/\pi$ (note: $\int_0^\pi(1/\pi)\,d\theta = 1$), but one could work with a network of any orientation distribution by using, instead of $P_\theta = 1/\pi$, the appropriate non-random probability density function—for example, earlier,[9,10] $P_\theta = 1/\pi + e\cos 2\theta$ was used.

The integration of equation (*1*) after the incorporation of equation (*6*) has been limited to fibres under tension, because we believe those under compression make a negligible contribution; in fact, many buckle and are removed from consideration entirely (see Ranger & Hopkins[11] for a discussion of this matter). Mathematically, the integration of equation (*1*) was carried out between $\pi/2$ and the angle θ_c, below which angle fibres are in compression. The value θ_c is calculated from equation (*5*) equated to 0 as follows—

$$F(\theta) = 0 = E\alpha e_x(\sin^2\theta_c - \nu_{xy}\cos^2\theta_c)$$

$$\theta_c = \tan^{-1}\sqrt{\nu_{xy}}$$

Poisson's ratio ν_{xy} can be found from the forces developed in the sheet at rightangles to the applied strain. By neglecting bending and shear, this results in $\nu_{xy} = \frac{1}{3}$. Hence, $\theta_c = \pi/6$.

3. The probability that a fibre at θ carries load $PL(\theta)$ is defined as—

$$PL(\theta) = (1 - f_c)(1 - f_e) \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where f_c, assumed a constant here, is the fraction of fibres across a line that develop no load because of their non-linearity and f_e is the fraction of fibres that develop no load because of the failure of bonded areas at their ends. The term $(1 - f_c)$ is equal to the ratio of the measured to theoretical maximum load developed in the zero-span test.[2] A true evaluation of f_e is impossible to make at this time. For the present, we are assuming that the bonded area that has failed at the end of a fibre at the angle θ since the inception of straining α_{fe} is—

$$\alpha_{fe} = \frac{\text{load developed in a fibre at } \theta}{\text{failure sheer stress of bonded area}}$$

That is,
$$\alpha_{fe} = \frac{F(\theta)}{s} \qquad (9)$$

If RBA is the fraction of the fibres' external surface area bonded, the total bonded area per fibre is $2\lambda(\omega+\delta)RBA \approx 2\lambda\omega RBA$. Since there are two ends per fibre, the fraction of the bonded area per fibre that has failed f_e is—

$$f_e = \frac{2\alpha_{fe}}{2\bar{\lambda}\varpi RBA} = \frac{F(\theta)}{s\bar{\lambda}\varpi RBA} \qquad (10)$$

Then,
$$(1-f_e) = 1-\frac{F(\theta)}{s\bar{\lambda}\varpi RBA}$$
$$= 1-\frac{E\alpha e_x(\sin^2\theta-\nu_{xy}\cos^2\theta)}{s\bar{\lambda}\varpi RBA} \qquad (11)$$

Substituting equations (*6*), (*7*) and (*11*) into equation (*1*) gives the tension developed across the line CC of a randomly formed sheet T as—

$$T = \frac{2WRE\alpha e_x}{\pi w}(1-f_c)\int_{\theta=\pi/6}^{\pi/2}(\sin^2\theta-\nu_{xy}\cos^2\theta)$$
$$\left[1-\frac{E\alpha e_x(\sin^2\theta-\nu_{xy}\cos^2\theta)}{s\bar{\lambda}\varpi RBA}\right]\sin^2\theta\,d\theta \qquad (12)$$

$$T = \frac{WRE\alpha e_x}{w}(1-f_c)\left(0.34-0.30\frac{E\alpha e_x}{s\bar{\lambda}\varpi RBA}\right) \qquad (13)$$

In formulating equation (*13*), mean values rather than distributions were used for all of the fibre dimensions because the use of distributions would have required a numerical integration. By setting E constant, we assume the fibres to be Hookean up to the beginning of gross sheet rupture. In a more comprehensive version of the theory, the measured stress/strain curves of the fibres will be used in place of equation (*4*). It is this technique that will make it possible to account for effects such as tension drying, shrinkage, mechanical conditioning, etc., on paper properties (see section on future work).

The two criteria for gross sheet failure—(*1*) the maximum of the stress/strain curve and (*2*) the sheet extension at which the first fibre of average properties fails—are calculated as follows—

1. The maximum of the stress/strain curve is given by—

$$\frac{dT}{de_x} = 0 \qquad (14)$$

which, when operated on equation (*14*), gives the extension at the maximum load developed e_x' as—

$$e_x' = \frac{0.58 s\bar{\lambda}\varpi RBA}{E\alpha} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

2. The first fibre to fail is the one under the largest load; it is clear from equation (*6*) that such a fibre is parallel to the direction of straining ($\theta = \pi/2$). Hence, if—

$$E\alpha e_x' > t \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where t is the fibre failure load, gross sheet rupture begins when the extension of the sheet reaches—

$$e_x = \frac{t}{E\alpha} = e_f \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Two expressions then arise for the tensile strength of handsheets. One, for sheets whose gross rupture is initiated by bond failure, is found by substituting equation (*15*) into equation (*13*)—

$$T_{\text{bond}} = 0.097(1-f_c)\frac{WRs\bar{\lambda}\varpi RBA}{w} \quad . \quad . \quad . \quad (18)$$

The tensile strength of handsheets whose rupture is induced by fibre failure is found by substituting equation (*17*) into equation (*13*) or—

$$T_{\text{fibre}} = \frac{WRt}{w}(1-f_c)\left(0.34 - \frac{0.30t}{s\bar{\lambda}\varpi RBA}\right) \quad . \quad . \quad (19)$$

Equation (*18*) states that, other factors being equal, T_{bond} is directly proportional to RBA, s and the fibre dimensions; equation (*19*) states that T_{fibre} is in addition a function of the fibre strength t. The importance of this finding is discussed later.

The stretch of handsheets at failure depends on the same criteria as the tensile strength. When gross rupture is initiated by bond failure, stretch is proportional to RBA as given by equation (*15*); when induced by fibre failure, it is a constant equal to the mean failure stretch of individual fibres, equation (*17*). (In a more comprehensive version of the theory using distributions, sheet stretch would be equal to the somewhat lower failure stretch of a fibre weaker than the mean.)

The elastic modulus of handsheets Y is defined to be the initial slope of the stress/strain curve per unit width of sheet (as $e_x \to 0$) or—

$$Y = \frac{1}{R}\frac{dT}{de_x}\bigg]_{e_x \to 0} = (1-f_c)\frac{0.34WE\alpha}{w} \quad . \quad . \quad (20)$$

Equation (*21*) is more conveniently expressed in the dimensionless form of equation (*22*), which we have called the *modulus ratio*—

$$\frac{Y\gamma}{WE(1-f_c)} = 0.34 \quad . \quad . \quad . \quad . \quad . \quad (21)$$

where $w/\alpha = \gamma$, the density of cellulose (1.55 g/cm^3). Equation (*21*) states that for a random sheet, the ratio of the sheet-to-fibre modulus, corrected for density, is a constant; it is assumed of course that f_c is constant for a given type of fibre. This conclusion differs from the one[3] in which it was concluded that Y is affected to a small extent also by variations in *RBA*. The dependence of *RBA* arose out of the contributions of bending and shearing that are now known to be quite small (see our first contribution).

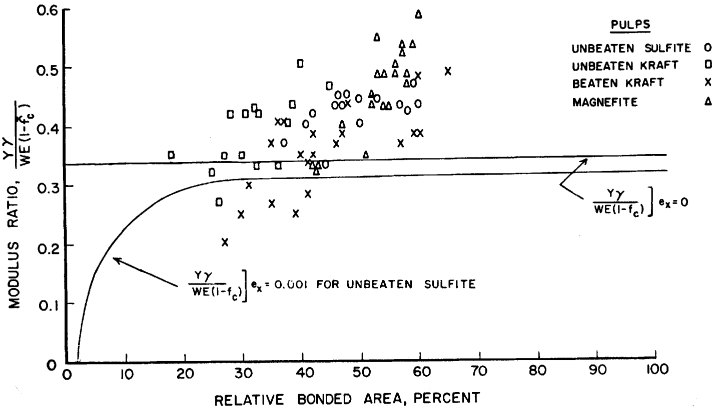

Fig. 7—Modulus ratio as a function of relative bonded area

We have reason to suspect that the measured modulus ratio can be affected to some extent by experimental conditions. It is particularly difficult to measure Y of poorly bonded, weak sheets (those whose $RBA < 30$ per cent). Quite likely the measured modulus is the slope of the stress/strain curve at a small finite strain ($e_x > 0$), a modulus we have called Y'. It is readily seen from the form of equation (*13*) differentiated with respect to e_x that Y' at a finite value of e_x is a function of *RBA*.

$$\frac{Y'\gamma}{(1-f_c)WE} = \frac{\gamma}{(1-f_c)RWE}\left.\frac{dT}{de_x}\right]_{e_x>0} = 0.34 - \frac{0.60E\alpha e_x}{s\lambda\varpi RBA} \quad . \quad (22)$$

Equation (*22*) has been plotted in Fig. 7 for $e_x = 0.001$ and some typical fibre properties ($E = 35\times10^7$ g/cm^2; $\varpi = 3.7\times10^{-3}$ cm; $\lambda = 0.22$ cm; $s = 3.0\times10^4$ g/cm^2;[12] $f_c = 0.33$[2]). In a general way, the curve accounts for the deviation of the modulus ratio of poorly bonded sheets from 0.34.

Theoretical aspects of stress/strain theory

ON PLOTTING theoretical stress/strain curves as a function of *RBA*, many interesting features of the theory become evident. These are discussed in this section with numerical conclusions that refer only to the sheets used as examples. At the time this paper was submitted for publication, we lacked experimental verification of some of the points, which are therefore presented primarily as a matter of theoretical interest. We plan to publish experimental verification later.

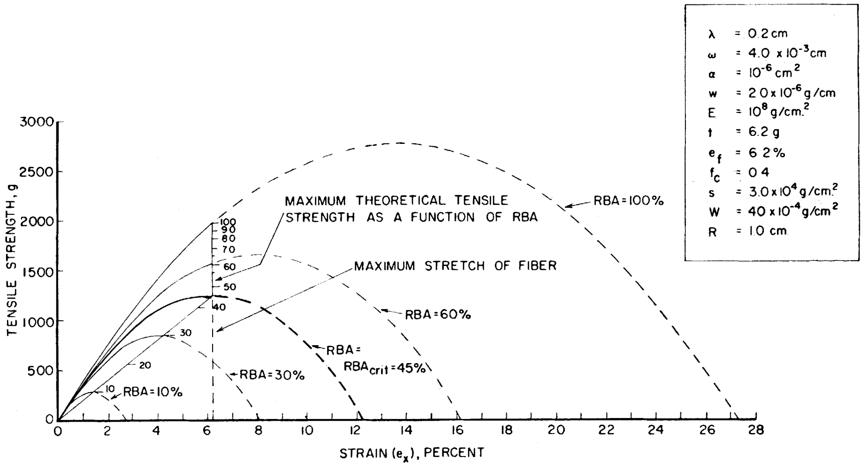

Fig. 8—Theoretical stress/strain curves for a typical pulp

Five curves of a pulp with typical fibre properties are shown in Fig. 8. These curves were constructed without considerations of the fibre strength t or time effects and, hence, are parabolic. The part of the curves up to sheet rupture for typical fibres whose $t=6.2$ g and whose $e_f=6.2$ per cent are shown as solid lines; the remainder of the curves as broken lines. The broken-line part of the $RBA=60$ per cent and 100 per cent curves before rupture up to their maximum stress represent the curves that would be obtained by sheets of fibres whose $t>8$ g and >15 g and whose $e_f>8$ per cent and >15 per cent, respectively, but otherwise had the same dimensions.

The part of the curves beyond the maximum stress represent the gross rupture of sheets in which only bond failure occurs. We have produced roughly parabolic load/elongation curves in the laboratory by straining poorly bonded sheets of very strong fibres at extremely low rates of straining (Fig. 9). In ordinary testing, the part of the curve at which the stress drops is vertical because of the relatively high rates of straining used.

Gross sheet rupture is initiated by bond failure when $RBA > 45$ per cent. Up to this RBA, the failure strength, equation (*18*) and stretch, equation (*15*), are *linear* functions of RBA and E, variables over which the papermaker has some control; f_c, s and δ, variables that are essentially constant for a given pulp. In other words, at RBA values below 45 per cent, the failure strength and stretch depend on the fibres' dimensions and elastic modulus, as well as the ability of the bonded areas to transmit loads between fibres. The fibres' t and e_f *per se* do not enter into the picture.

At $RBA > 45$ per cent, sheet failure is initiated by fibre failure as equation

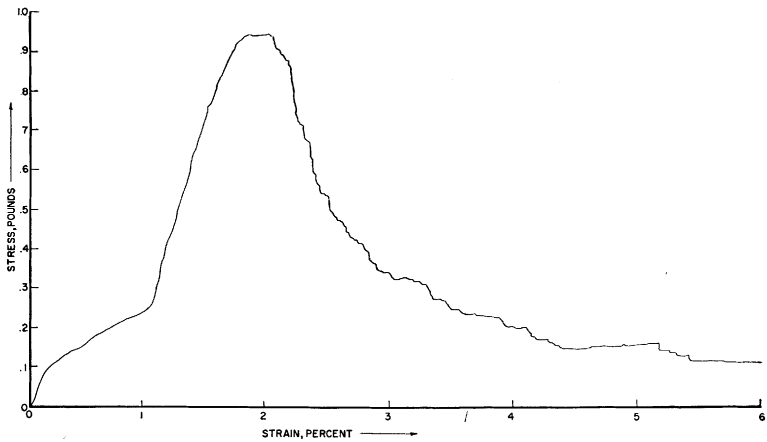

Fig. 9—'Parabolic' stress/strain curve

(*16*) is no longer satisfied. This is because fibres aligned in the direction of sheet elongation fail before the maximum of the curve is reached. The failure strength of such sheets is a function of t as well as the variables affecting T_{bond}—see equation (*19*). The failure stretch above $RBA = 45$ per cent is a constant equal to $e_f = 6.2$ per cent.

In Fig. 8, the line between the origin and the maximum of the $RBA = 45$ per cent curve passes through the maximum of all curves with intermediate RBA values. This line represents the *linear* relationship between the tensile strength and stretch and RBA of sheets whose rupture is initiated by bond failure. The vertical line between the maximum of the $RBA = 45$ per cent curve and the $RBA = 100$ per cent curve gives the parabolic relationship between the tensile strength and RBA of sheets whose rupture is induced by fibre failure. These relationships are plotted separately in Fig. 10.

It is evident from the above discussion that there is a critical RBA value equal to 45 per cent *for this particular pulp*. Below it, the tensile strength and stretch are linear functions of *RBA* and fibre strength does not enter into the picture; only the ability of the bonds to transmit loads is important. Above RBA_{crit} the stretch is constant and the tensile strength is a parabolic function of fibre strength and *RBA*.

RBA_{crit} is found from equation (*15*)—

$$RBA_{\text{crit}} = \frac{1.7t}{s\bar{\lambda}\varpi} = \frac{2.7t\bar{\delta}}{s\bar{\lambda}w} \qquad . \quad . \quad . \quad (23)$$

It is evident from equation (*23*) that RBA_{crit} is a function of both fibre and bond strength; the greater the fibre strength, the larger the *RBA* at which *t* enters the picture. This might be expected intuitively. The failure strength of sheets with extremely strong fibres is governed by bond strength over most or all of the *RBA* range; conversely, the strength of sheets of weak fibres is

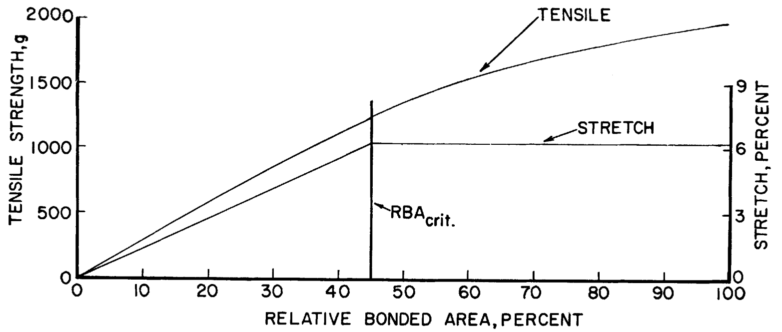

Fig. 10—Tensile strength and stretch of a pulp in Fig. 8 as a function of RBA

generally governed by fibre strength. The same considerations apply to the bond strength *s*: as it increases, the *RBA* value at which fibre strength enters the picture decreases and vice versa.

Substituting equation (*23*) into equation (*18*) gives the tensile strength of a handsheet at RBA_{crit}—

$$T_{RBA_{\text{crit}}} = 1.65(1-f_c)_{RBA_{\text{crit}}} \frac{WRt}{w} \qquad . \quad . \quad (24)$$

Dividing equation (*24*) into equation (*19*), with $RBA = 100$ per cent in equation (*19*), gives the ratio of the maximum tensile strength of a handsheet to that at RBA_{crit} or—

$$\frac{T_{RBA=100\%}}{T_{RBA_{\text{crit}}}} = \frac{\left(0.34 - \frac{0.30t}{s\bar{\lambda}\varpi}\right)(1-f_c)_{RBA=100\%}}{0.165(1-f_c)_{RBA_{\text{crit}}}} \qquad . \quad (25)$$

Substituting typical values for a sulphite woodpulp into equation (*25*) ($t = 6$ g; $s = 3 \times 10^4$ g/cm^2; $\lambda = 0.22$ cm; $\omega = 3.7 \times 10^{-3}$ cm; $f_{c(RBA=100\%)} = 0.25$; $f_{c(RBA_{crit})} = 0.5$[2]) gives—

$$\frac{T_{RBA=100\%}}{T_{RBA_{crit}}} \approx 2.4 \qquad . \quad . \quad . \quad . \quad . \quad (26)$$

The value of equations (*25*) and (*26*) is that they make it possible to predict the maximum tensile strength of a sheet from measurements on a number of partially bonded sheets wet pressed to different levels.

The predicted role of fibre length in the picture is consistent with experience. Both of the equations for the sheet's tensile strength show that T increases with increasing λ.

Experimental programme, results and discussion

LITTLE new experimental work was performed for this study. Instead, stress/strain curves were calculated for the sheets prepared for an earlier study.[13] This was done because almost all of the information required for a comparison of calculated to-measured curves was available; the values of s came from another article[12] and that of f_c from Perez & Kallmes.[2] Most of the results are presented in Fig. 7 and 11–13.

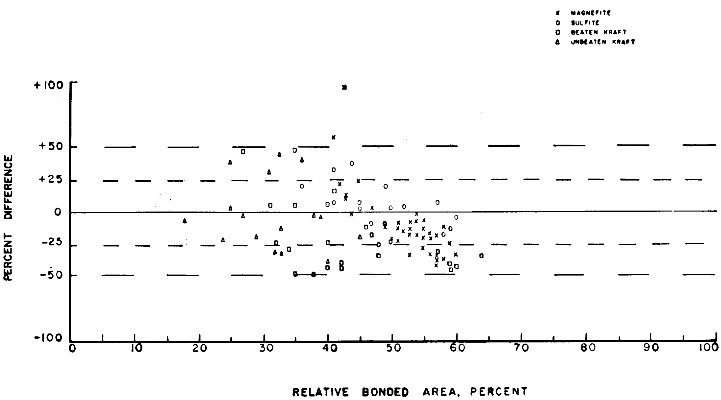

Fig. 11—Percentage difference between measured and calculated tensile strength—$(T_{meas} - T_{calc}) \times 100/T_{meas}$

It is evident from an inspection of the graphs that the data scattered appreciably—that is, the calculated values often disagreed by as much as 50 per cent from the measured ones. There are several possible causes of this disagreement—the approximate nature of the model; the use of mean values rather than distributions for the fibre properties; low precision of the measurements, particularly those made under the microscope, etc. We do not

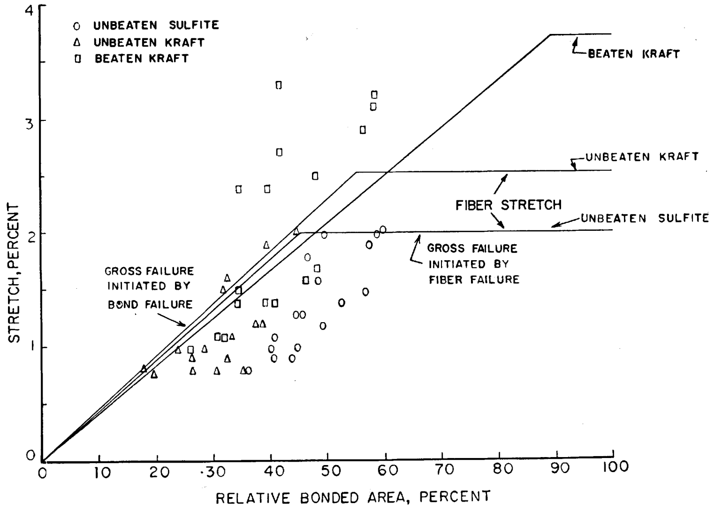

Fig. 12—Relationship between stretch and relative bonded area

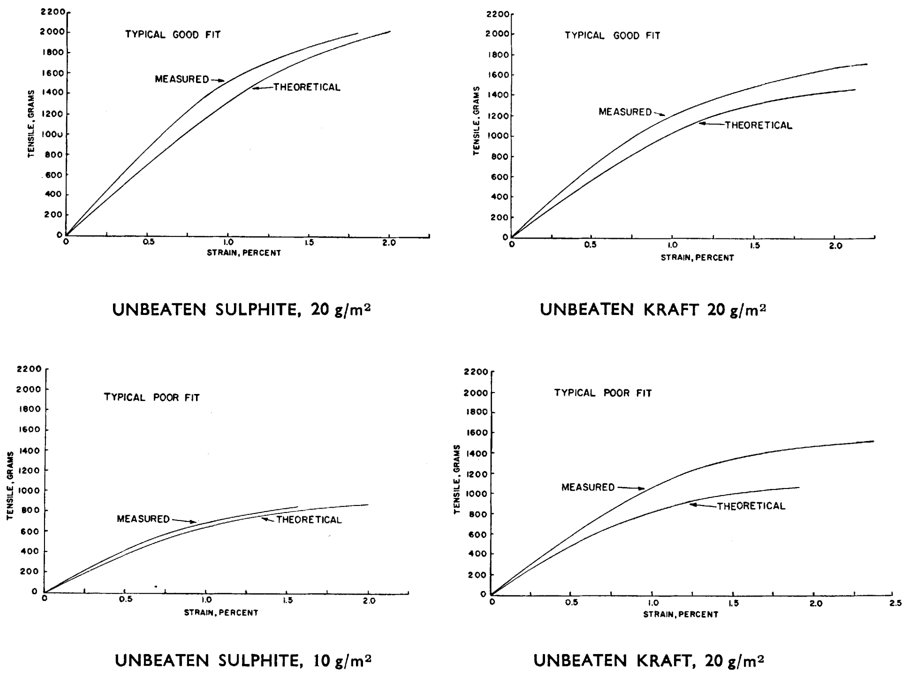

Fig. 13—Typical measured and calculated stress/strain curves

know at this time which factor or factors are primarily responsible for the scatter. The important point to bear in mind is that the values calculated for three paper properties—elastic modulus, tensile strength and stretch—using an entirely analytical approach, *agree approximately with measured values*. To our knowledge, this is the first time such predictions have been made and we are very satisfied with the level of agreement obtained.

In calculating the stress/strain curves, the fibres were assumed to be Hookean and their E was calculated from their t and e_f. A value of $f_c = 0.4$ was used as it was found[2] to be the average for handsheets of the type used in 1961.[13]

Fig. 7 is reproduced from another article.[2] It shows that the modulus ratios are within 50 per cent of 0.34; the difference between the calculated and measured ratios of the lighter sheet was discussed in the last section. Fig. 11 shows the percentage difference between the measured and theoretical tensile strengths, based on measured values; the data scatter about the line of agreement $(T_m - T_c)/T_m \times 100 = 0$ per cent and, in all but a few cases, lie within 50 per cent of it.

The measured and calculated stretch data in Fig. 12 agree to about the same extent as the modulus and tensile data. One interesting point to be noted is that none of the measured stretch values exceed the calculated maxima (indicated by the horizontal lines).

Fig. 13 shows four typical stress/strain curves of handsheets, two for a sulphite woodpulp (whose gross rupture was triggered off by fibre failure) and two for kraft woodpulp (whose gross rupture was triggered off by bond failure). In each case, a typical good and poor fit is shown.

It seems to us that, even in its present rudimentary form, the new stress/strain theory accounts for the load/elongation behaviour of handsheets quite well. The general shape of the calculated curves are remarkably close to the measured ones. The maximum disagreements that occur are about 50 per cent of the measured values. Considering the simplicity of the model, the fact that it was developed analytically and the low precision of much of the data that went into the calculations, such agreement is very satisfactory.

Future work

WHEN stripped of its ornamental factors, the new stress/strain theory boils down to a summation of the forces developed in the fibres across a given line. It will be easily seen from the discussion to follow that, when viewed thus, the theory is able to account for many of the unique properties of paper. The quantitative inclusion of arbitrary fibre stress/strain characteristics and orientation distributions in the theory is at present underway and will be reported on later; here, we present only the approach being taken.

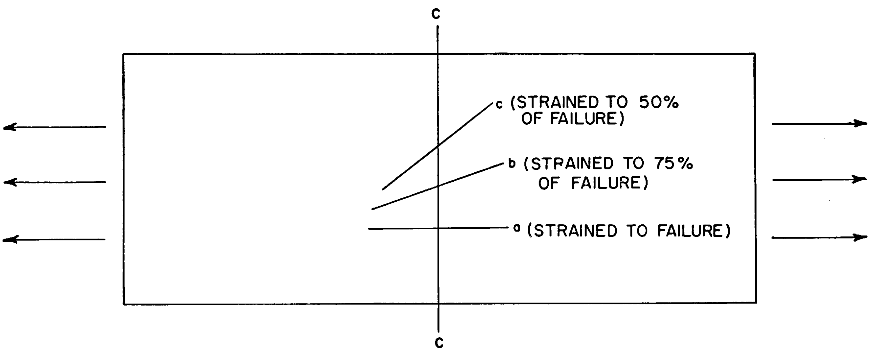

Fig. 14

Consider the three fibres *a*, *b* and *c* of Fig. 14 as representative of those developing a load across the line CC of the sheet. The three stress/strain curves shown in Fig. 15 represent the properties of all the fibres of a sheet constrained *isotropically* in some manner under three sets of drying conditions. Curve I represents fibres *extended* during drying; curve II, fibres dried under restraint; curve III, fibres shrunk. (All three curves are exaggerated for purposes of illustration.)

We shall consider that the gross rupture of the sheet is triggered off by fibre failure, keeping in mind that the same considerations hold for rupture initiated

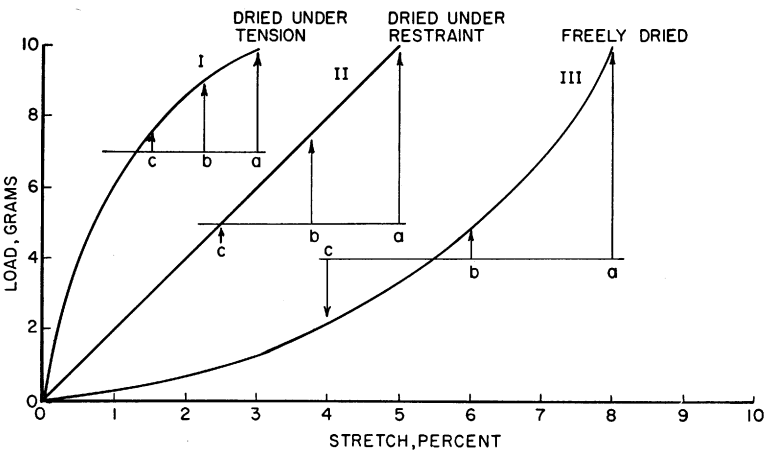

Fig. 15—Stress/strain curves of fibres of sheet in Fig. 14 dried under three different conditions

by bond failure. The points *a*, *b* and *c* on the three curves of Fig. 15 are representative of the strain of the fibres *a*, *b* and *c* in the sheet shown in Fig. 14. Because it is assumed that the sheet elongates uniformly, the fibres *a*, *b* and *c* will be strained on the average to the same percentage of their failure stretch in the sheets dried under the three sets of conditions: *a* is strained to failure; *b* to 75 per cent of failure; *c* to 50 per cent. Note that the failure strength of the fibres dried under all three sets of conditions is assumed to be the same—10 g.

Let us now sum up the loads in the fibres *a*, *b* and *c* at failure in sheets dried under the three sets of conditions. The load in the fibres extended during drying, curve I, is 26.9 g (7.2+9.2+10.0); in the fibres restrained during drying, 22.5 g (5.0+7.5+10.0); in the shrunken fibres, 17.1 g (2.2+4.9+10.0). The corresponding failure stretch of the three sheets was 2.0 per cent, 5.0 per cent and 8.0 per cent, respectively.

It is apparent from these simple calculations that the new stress/strain theory will eventually be able to account quantitatively for the well-known relationships between the tensile strength and stretch of papers—that the tensile strength decreases as the stretch increases. In a similar manner, many other paper properties such as strain-hardening and creep should be accountable through their corresponding fibre properties.

Acknowledgements

We are indebted to Dr Mervyn Stone of the University of Wales for mathematical assistance; to Dr Ivar Stockel, of St. Regis Paper Co. for critically reviewing the paper; in particular to Dr K. A. Arnold, St. Regis Paper Co., for permission to publish the findings.

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Discussion

Dr J. Kubát—The theory of stress/strain behaviour proposed by Kallmes is based on the concept of bond breaking as a flow mechanism. An experimental fact that should be mentioned in this connection is the variation of the modulus of elasticity with strain. It is always found that, on straining, the modulus either increases or remains unchanged. This phenomenon, known as strain hardening, occurs in paper as well as in other materials. Under exceptional conditions—as on repeated loading of paper with high strain amplitudes—a softening may take place, that is, a breakdown of the sheet structure. This effect is of no importance, however, when ordinary stress/strain curves are discussed. The increase in modulus with strain can be demonstrated both in creep and stress/strain processes on which a sinusoidal deformation of comparatively small amplitude is superimposed.* Such a procedure permits an evaluation of the elastic response of the sheet during the various phases of the flow process to be made. Needless to say, the slope of the plastic region of stress/strain curves does not represent any measure of the elastic response of the sheet.

The theory presented here is based on the assumption that the number of fibres engaged in the loading process of the stress/strain curve decreases. Such a theory of plastic flow could formally be correct, if any one of the following additional assumptions were to be taken into account—for instance, compensation for the effect of bond breaking by strain hardening; the formation of new bonds; even redistribution of the stress in the sheet. Such assumptions are not contained in the theory being discussed. The question is how can the decreasing number of fibres engaged in the straining process be brought into agreement with the constancy of the modulus or, what is more likely to occur, its increase with strain. The second question is more general and relates simply to the fact that no attention has been paid to the accepted theories of solid state flow.

* Kubát, J., Nyborg, L. and Steenberg, B., *Svensk Papperstidn.*, 1963, **66** (19), 754–764
Kubát, J. and Lindbergson, B., *Svensk Papperstidn.*, 1965, **68** (21), 743–756

Dr O. J. Kallmes—In order to answer these questions, we have to describe the theory in more detail than we did in the paper. To begin with, Fig. 5 illustrates the way in which the stress/strain curve ought to be constructed according to the basic concepts of the theory; in the paper, we used the approximate method indicated on page 785.

The first step in constructing the curve is the calculation of the elastic modulus of the unstrained structure by equation (*21*); note that bond failure *per se* —in the form of the term $(1 - f_e)$ of equation (*9*)—does not enter into the picture. After a small finite elongation, the first modulus curve is terminated.

As a result of the first elongation (no matter how small, even infinitesimal), a few fibres are eliminated as load-bearing elements because of bond failure. Thus, we have a new structure, one that is slightly different from the original unstrained one. The load developed per unit of elongation e, as $e \rightarrow 0$, of this new structure is lower than what would have been developed by the original one at the end of the first elongation if no bond failure had occurred; for the new structure, f_c of equation (*21*) is increased slightly. Thus, the elastic modulus curve of the second structure is displaced downwards and to the left (in Fig. 5) of the first curve. The two modulus curves are connected by a vertical line from the termination point of the first curve to the second; the latter intersection is the beginning of the second curve. The vertical line represents the stress drop caused by the bond failures that occurred during the first elongation.

The second modulus curve is terminated after an elongation equal to the first. The modulus of the third structure developed as a result of bond failure in the second elongation is then calculated. The second and third modulus curves are connected in the same way as the first pair. By proceeding in the same manner, the sawtooth stress/strain curve depicted in Fig. 5 is constructed. The entire load/elongation curve is terminated when the conditions representing sheet failure, equations (*18*) and (*19*), are met.

If it is assumed that the modulus of the fibres remains constant during straining, the slope of every modulus curve is lower than that of the preceding one, because of the reduction in the number of load-bearing fibres (that is, an increase in f_c). We know, however, from the work of Taylor & Craver presented at this symposium that the modulus of the fibres increases during the straining of the sheet, owing to a strain-hardening effect. Therefore, the small loss in sheet modulus from the removal of fibres as load-bearing elements is probably compensated for by an increase in the modulus of the fibres still taking load. Hence, the elastic modulus of the sheet remains constant during straining.

The above concept is supported further by another observation of Taylor & Craver. They found that the modulus of a sheet after straining is lower than that before straining and attribute this loss to bond failure. We concur with this concept in the sense that we believe that the number of fibres participating in the modulus measurement of the relaxed sheet is fewer than in the original because of bond failure. It should be noted, however, that the drop in sonic modulus is not only a measure of the number of fibres removed as load-bearing elements, but also represents the small irreversible change in the fibres' modulus caused by straining and relaxing (see pp. 518–519 of our first contribution).

In the case of thin 2-D sheets, the picture is slightly different than for an ordinary sheet. The fraction of fibres acting as load-bearing elements is reduced markedly by the failure of every crossing. Thus, the increase modulus of the strain-hardened fibres still developing a load after a given strain cannot compensate for the loss in the number of fibres participating. As yet, we have not attempted to include creep phenomena in the theory. All we have done is to calculate the modulus of structures resulting from given strains, though there is no reason that you could not substitute time for elongation and develop a picture for creep.

Mr D. H. Page—My point is complementary to that made by Kubát. Tydeman and I did an experiment some time ago when we had ideas more or less along the lines expressed by Kallmes. As a result of our thoughts following this experiment, we abandoned those ideas.

We made some idealised models of paper using strips of a clear plastic (cellulose acetate I believe). The strips, several inches long, were laid down at random positions, but with two preferred orientations for convenience. The strips were glued together at points where they crossed. We then put this structure under tensile strain. The stress/strain curve had an initial elastic linear region. Then, as the bonds began to break, just as Kallmes has indicated, the stress/strain curve turned over exactly as the stress/strain curve of paper. When we had broken about half the bonds, we lowered the stress and found that the curve returned more or less linearly *to the origin*. There is no permanent set in such a structure—and there cannot be, because all our elements are elastic; while there is still one elastic path remaining between the jaws of the tester, there will be a restoring force returning the structure to zero strain.

Now, the stress/strain behaviour of paper is not like this. Permanent set is an intrinsic part of it. In an attempt to create a structure that would exhibit set, we remade the model putting in a number of kinked segments. We hoped that during straining bonds would break and kinks would be pulled out, but again this did not happen. The result was very similar to that obtained previously. The same argument holds. To achieve permanent set, every original elastic path must be broken and this would require a degree of bond breakage that would imply complete disruption of the sheet.

Discussion

The experiments here are not proofs, of course, but rather guides to thought. They lead to the important principle that it is impossible to produce a theory that incorporates permanent set simply on the assumption of bond breakage in a random structure of elastic fibres, even if some of these are kinked.

These thoughts led us to the view that the stress/strain curve was not so much a property of the gross sheet structure, but was more the property of the fibres in the regions of the fibre-to-fibre crossings.

I query the suggestion that the stress/strain curve of microcompressed fibres is concave towards the stress axis. Under the usual testing procedure, the rather gross curls and kinks that are usually present on microcompressed fibres may produce this effect, but the microcompressed regions themselves may not display such a curve. I have been able to produce sheets of paper in which every fibre is microcompressed down its entire length and they exhibit quite normal stress/strain curves.

Dr Kallmes—This is a model of plastic that contains many features not in paper. When a bond fails in this thin, rigid model, there is no opportunity for the type of stress redistribution inherent in our theory. For example, crossing failures do not eliminate fibres as load-bearing elements. As you well know, a microcompressed part of a fibre often does not develop a load after bond failure.

As far as the stress/strain curve of microcompressed fibres is concerned, I do not know its exact shape any better than you do; we have never measured

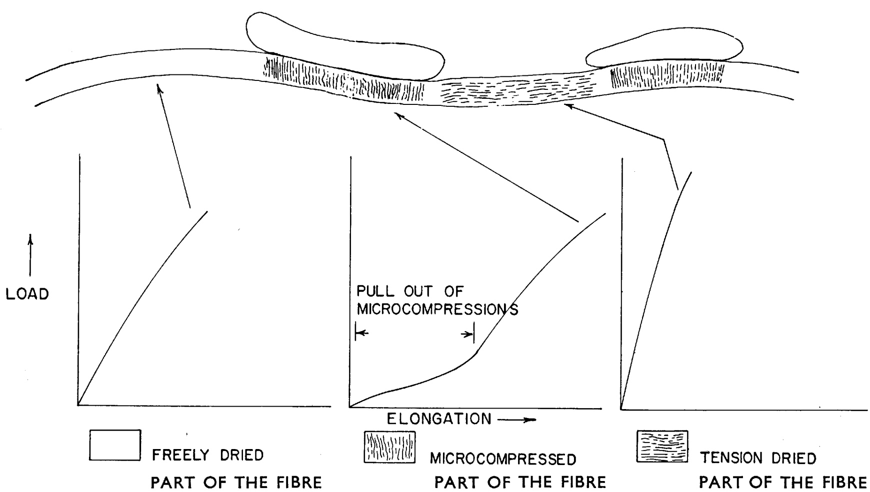

Fig. N—Load/elongation properties of parts of fibres under different conditions of restraint within a sheet it successfully. The picture I presented yesterday (illustrated in Fig. N) is purely speculative.

Mr P. A. Tydeman—It is of further interest to note that, in this plastic model, a fibre broke very early on in the stress/strain curve by shear, yet this still did not give the structure any significant permanent set.

Some points about the theory and its derivation and, firstly, on the basic assumptions. It is assumed that the strain in fibre segments can be based on the average or sheet strain, hence the paper ruptures when its extension is the same as the extensibility of a fibre. This must be seen in the light of the variability of heterogeneity of paper on a macroscopic scale. We have observed appreciable variation from point to point of the strain of paper under tension. This variability should be taken into account when calculating the value of the average strain when rupture will occur.

On another assumption, that the transverse modulus of fibres is negligible, I should like to see this more firmly established. I discussed one of your pieces of evidence after presentation of the paper by yourself and Perez on fibre properties; the other evidence is based on sonic moduli experiments of the nature described by Taylor & Craver. How conclusive are these experiments?

Fig. 7 is claimed to show the constancy of the modulus ratio (vertical axis) at a value of 0.34 for all values of RBA and the curve drawn is supposed to be a better fit, but I suggest that this graph indicates a high dependence of the ratio on RBA and that a line through zero at roughly 45° would not be a bad fit.

I want to make the plea that it is essential when comparing theoretically derived results with experimental data that some indication is given of what magnitude of the discrepancy between experiment and theory would cause one to re-examine and reformulate the theory.

Dr Kallmes—We fully realise that the stresses and strains within a sheet are not uniform. We have developed a technique for measuring the variations in strain along fibres within a sheet and found them to be quite large. For example, in one sheet strained 3 per cent, the strain along a fibre oriented in the direction of elongation varied 1.26–4.06 per cent. It would be possible to write equations for these strain variations and calculate stresses on the basis of them. The resulting equations for the modulus of a sheet at any given elongation would be extremely complex and would be of no value to the development of an understanding of the mechanisms that occur when a sheet is strained.

You must realise that our goal in this work is *not* the development of exact equations with perfect predictive powers. Rather, we are trying to develop a relatively simple picture of the stress/strain behaviour of paper and incorporate in it all the important phenomena in an approximate way. Our variables all represent easily pictured fibre properties and structural elements. These are assembled in a model representing a point on the stress/strain curve. The manner in which you go from one point on the curve to the next is specified somewhat arbitrarily. This is the only way we know of to describe a discontinuous mechanism such as the straining of paper to failure.

Measuring the modulus of fibres in its two principal directions (parallel and at rightangles to the fibre axis) by the sonic technique applied to highly oriented handsheets gives a relative rather than an absolute measure. We found a 10:1 ratio, which, incidentally, is about the same as the ratio of the amount of swelling a fibre undergoes in these two directions upon wetting. I believe you found that a fibre swells 20–25 per cent at rightangles to the fibre axis,* although it is well known that it swells only 1–2 per cent axially. These swelling values are an indication of the strength of a fibre in these two directions.

I have already commented on the precision of the data. We are quite pleased to have a model based on the known mechanisms of straining and the measurable fibre and structural properties giving predicted values within 100 per cent of the measured ones.

Mr P. E. Wrist—Kallmes' theory is based on the assumption of uniform strain in the sheet. Van den Akker used this assumption in his 1961 Oxford symposium paper and concluded that fibres lying approximately at rightangles to the strain direction were under compressive strain. In a well-formed sheet, the frequency of fibre bonding along a fibre is such that this compressible stress cannot be readily relieved by fibre buckling. Bond failure under these compressive strains is therefore probable. It is not clear whether Kallmes has taken such failure into account in his paper or only considered failure from tensional stresses.

My second comment is addressed to Page, who has cited his model experiment with bonded plastic strips to show that bond breakage between elastic fibres is not able to explain permanent set in a network. In a dry paper web, the fibres themselves are made of elastic fibrils, bonded together. Each fibre has dried-in stresses and, on straining a web, these internal bonds can break in addition to intrafibre bond failure. Relaxing such a structure will lead to a permanent change in dimension, since removing the external stress will not recreate the internal fibre stresses of the original sheet. I do not agree, therefore, that we must assume visco-elastic behaviour on the part of the fibres to

* *Nature*, 1963, **199** (4 892), 471–472

explain permanent set in a strain/relaxation cycle; internal bond breakage within the fibre is a sufficient mechanism.

Dr Kallmes—We neglected fibres in compression, because we believe they buckle and so contribute negligibly to the load developed by the sheet. Therefore, we integrated equation (*12*) from 30° to 90° instead of from 0° to 90°. I agree that buckling can cause bond failure, but we have not included the mechanism in the theory as yet—of course, it can be done.

Failures within fibres would be incorporated in the theory by changes in the fibre properties as a function of time or elongation. For example, instead of the modulus E, one could use the modulus $(E_0+E_1\, e_x+E_2\, e_x{}^2+\ldots)$ in equation (*5*). We did not do this in the first version of the theory, because such complications add little to the basic concepts we tried to emphasise in the paper.

Prof. B. Steenberg—The purpose of any of the many models we have is a practical one: to condense our knowledge into a simple picture comprehensible to anyone and of use both qualitatively (for example, atomic models in wood) and quantitatively for calculations. Consequently, a new model should explain more details than do earlier models, so that an acid test is not only that it explains what is already known, but predicts something new.

The mechanical model with dashpots and springs is a useful way of depicting a complicated series of equations for the behaviour of many materials under stress and strain, taking time-dependent properties into consideration. A system made up of electrical elements can be used to depict the same equations, but we are not used to electrical analog components today in such a way that an electrical model is useful.

The important point is that a series of empirical equations depicted to the layman by, for instance, a model of springs and dashpots describes remarkably accurately the stress/strain properties, including loops, creep and relaxation curves for paper. It was four parameters, two moduli of elasticity and two parameters that describe the viscous properties of an Eyring fluid.

If the Kallmes *et al.* theory is converted to a mechanical model, it consists of a series of identical springs coupled in parallel. These springs break one after another during straining: to describe this, he has to use the complete stress/strain curve. I cannot see that it has fewer parameters than the old model, nor that it can depict more than the 'plain' stress/strain curve—obviously, not the permanent set, creep or relaxation. Is there an unknown feature of paper that can be predicted by it? If so, the model is useful with all its shortcomings.

Mr Tydeman—There is a contradiction in the theory. As the bond is broken, there are fewer fibre elements under strain, therefore the modulus is lower: it does not remain the same.

Dr Kallmes—I agree that the modulus of a sheet does not necessarily remain constant while a sheet is being strained; Taylor & Craver (at this symposium) have shown that it rises. The modulus depends on the number of fibres developing load $(1-f_c)$ and on their modulus E. This matter is discussed more fully in my first reply (page 805).

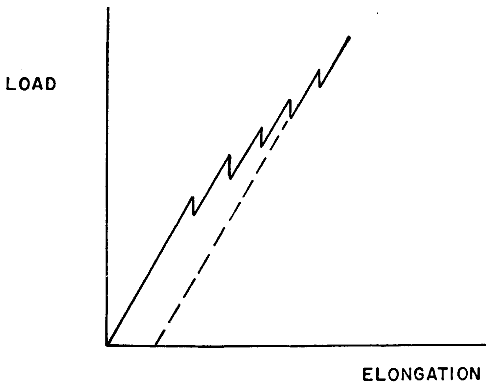

Fig. O—Mechanism by which theory accounts for permanent set

The elastic modulus and the entire slope of the stress/strain curve of a sheet at a given elongation are different items. Referring to Fig. 5, the former is represented by the solid slope ν and the latter by the dotted curve drawn through the sawtooth curve. I have already referred to the factors on which the modulus depends (page 805). The slope of the stress/strain curve as a whole at an elongation e is a function of the elastic modulus of the sheet at e and the size of the stress drops caused by bond failure (elongation 9). The smooth stress/strain curve with which we are all so familiar is, in fact, the dotted curve in Fig. 5. It comes about because stress/strain recorders do not have the sensitivity to construct the true sawtooth stress/strain curve of paper.

For a sheet whose elastic modulus remains constant during straining, the load/relaxation curves do not go through the origin (see Fig. O).

Dr J. A. Van den Akker—It seems to me that Kubát's remark is very cogent: the visco-elastic properties of the fibres must be brought into the model. This (it seems to me) is extremely important for the theoretical explanation of the known tensile stress/strain characteristics of paper.

Dr Kallmes—As is shown on page 810, the visco-elastic properties of fibres can be taken into account quite readily.

Dr Van den Akker—This was indeed brought out in the discussions at the 1961 symposium, which were the basis for the work that we and others are doing today on the visco-elastic properties of individual fibres.

Dr A. H. Nissan—I would like to point out one thing. There are explicit and implicit ways of introducing arbitrary parameters. When one makes something known to vary strongly into a constant, a whole battery of arbitrary parameters have been introduced, because of the contra-acting parameters brought in to eliminate what is known to be variable and to make it constant. Therefore, the statement that your theory is nothing but observables is not strictly correct: you are introducing unobservables by omission. This, in itself, is not uncommon in theoretical work nor is it bad—indeed, it is essential for progress—but we ought to be aware of it.

Dr Kallmes—We made numerous simplifications of the theory in the paper, because we did not want to complicate the basic concepts with many mathematical embellishments. The basic concepts would get lost in a swamp of equations. Since this paper was submitted, we have incorporated many refinements and the only way that we can integrate the final equation is numerically on a computer. Although these improvements in the theory have produced more accurate stress/strain curves, they have added relatively little to our storehouse of basic knowledge.

Dr H. K. Corte—I would like to reply to Nissan's last remark and support Kallmes. It is one thing to simplify what in reality is fibres bonded together and so avoid complicated statistical-geometrical calculations. It is quite another thing to analyse a completely artificial model, even though it might have fewer constants, which behaves as if it were similar to a piece of paper, but with no element that can be traced or identified in a sheet of paper, because of which it was eventually abandoned. Although simplifications were made in this work, Kallmes is speaking of fibres and, of course, fibres are elastic, they are kinked, they are microcompressed and plastic. To take all these features into account in a random or non-random network is extremely complicated arithmetically, but not conceptually.

Mr P. E. Wrist (*in a written contribution*)—Support for Giertz' proposition that the hemicellulose present in the fibre may be soluble and therefore sufficiently mobile to provide the bonding adhesive both within and between fibres is supported by some recent studies of McIntosh of our laboratories on high-yield holocellulose fibres prepared by the peracetic acid method from loblolly pine. Fig. P shows several fibres that have been beaten in a Waring blender for 10 min to a freeness of 310 CSF, then dried down to on a glass slide. The sample was metal shadowcast for improved contrast. The photograph shows

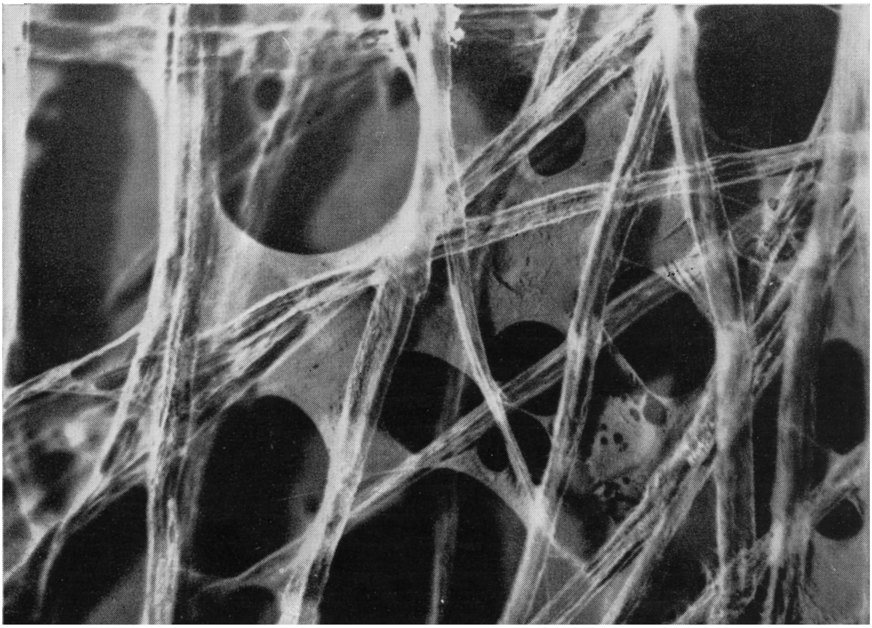

Fig. P—Loblolly pine summerwood holocellulose fibres (peracetic acid method), refined for 10 min in Waring blender with distilled water to 310 CSF [× 147]

the presence of an unoriented film of hemicellulose, which formed as the water was evaporated. The film is seen to span quite large voids between the fibres and to be drawn also around points of fibre interaction. Although much of this hemicellulose would be removed in a conventional cook, some hemicellulose remains and it is not unreasonable to expect it to behave as Giertz proposes.