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STOCHASTIC ANALYSIS OF THE CRITICAL STABLE VELOCITY OF A MOVING PAPER WEB IN THE PRESENCE OF A CRACK

Maria Tirronen, Tero Tuovinen,
Juha Jeronen and Tytti Saksa

Department of Mathematical Information Technology, P.O. Box 35 (Agora), FI-40014 University of Jyväskylä, Finland, maria.j.e.tirronen@jyu.fi, tero.tuovinen@jyu.fi, juha.jeronen@jyu.fi, tytti.saksa@jyu.fi

ABSTRACT

In this study, we present a probabilistic approach for analysing runnability of a moving paper web with random defects. The paper web is modelled in the case of an open draw as an axially moving elastic plate that has an initial crack of random length. We derive a formula for the optimal velocity, at which the probability of fracture is limited. We study edge and central cracks perpendicular to machine direction (mode *I* cracks) and oblique central cracks (mixed mode). The crack length is modelled with the Weibull distribution. The effect of changing the value of distribution parameters and the probability of fracture on the optimal velocity is illustrated. It is found that the optimal velocity decreases when the expected value and variance of the crack length increase. The results also show the relation between the effect of edge and central cracks on the optimal velocity. The study is fundamental for rigorous analysis of the paper making process.

INTRODUCTION

Mathematical modelling is often a low-cost tool to enhance industrial processes. By simulating the real systems it may be possible to obtain information of phenomena that cannot be measured directly. In this paper, we will look into the possibility of predicting a maximum safe running velocity for a paper web when elastic stability and fracture considerations are taken into account.

When modelling a moving paper web, the usual choice is to model it as an axially moving material, e.g., a moving plate. Axially moving materials have also many other applications in industry, and thus their mechanics have been studied widely. Traditionally, the studies are based on a deterministic approach but, in practice, the values of different parameters are not known precisely and the process to be modelled may also include random factors. For example, Salminen found the strength of paper to obey the Weibull and Duxbury distributions [1]. Other examples of uncertainty in paper making are given by variation of tension, in space and time, in the press system and defects, which vary in their location, size, shape and orientation (see studies by Björklund and Svedjebrant [2]).

Several researchers (e.g., Schuëller [3]) point out that, in order for the model to reflect physical phenomena, it should also include the uncertainties. When considering paper production, it is easy to agree with the statement. According to Uesaka [4], the majority of web breaks in paper production are caused by tension variations, combined with strength variations of the paper web. Wathén [5] discusses the effect of flaws of paper on web breaks and, according to him, even a seemingly flawless paper can fail at very low tensions due to stress concentrations caused by discontinuities, e.g. cuts and shives, in the structure. Tackling a more realistic model of the paper making process, we include a random length initial crack in the model of the travelling paper web, and study the problem of finding the optimal value of velocity from a probabilistic point of view.

In the manufacturing process, there may occur many kinds of defects in the paper web, such as edge cracks and blister and fiber cuts. Smith [6] classifies edge cracks as edge cuts or nicks that usually extend only a short distance. Typical reasons for edge cracks include dry edges, high sheet caliper at the edge, and web overlapping. In the same book, a blister cut is defined as a cut in the web that is usually at an angle to the machine direction, and is normally a result of excess paper accumulating as a 'blister' at the entrance of a nip. A blister cut may reach across entire deckle or be only in a localized area. A fiber cut in the web is defined as a typically short and straight cut that is located randomly and is usually at an approximately right angle at the edge. A fiber cut is caused, when a pulp fiber or shive, that is less compactible than the rest of the web, passes through a high pressure nip. This study considers two different locations of the crack: the web edge and the center of the web. In the first case, the crack is assumed to be perpendicular to the machine direction (MD) and it is of mode *I*. In the latter case, we consider different orientations. This case includes cracks of mode *I* and mixed mode cracks.

In this study, we model a travelling paper web in an open draw as an axially moving elastic plate. Finding the optimal value of velocity is essential for efficiency in many applications of axially moving materials. In this kind of systems, the most serious threats to good runnability are instability and material fracture, and on these phenomena a change in tension magnitude has opposite effects. An increase in tension has a stabilizing effect, but high tension may lead to growing or arising of cracks. Web tension too high may cause material fracture, which deteriorates production efficiency and the strength properties of the processed material (see e.g. studies by Banichuk et al. [7], Robertson [8], Sanborn [9] and Skowronski and Robertson [10]). In this paper, we seek the maximal tension under the constraint that the probability of fracture is limited. Binding this to the results on stability by Banichuk et al. [11], we obtain the optimal velocity. The obtained theoretical limits provide us an insight of failure and possible recommendations for the solution in process control.

The history of modelling vibrations of travelling elastic material began in 1897, when Skutch published a paper [12] concerning the axially moving string. The first papers in English date from the 1950's, when Sack [13] and Archibald and Emslie [14] studied the axially moving string model. Many researhers (e.g., Miranker [15], Swope and Ames [16], Mote [17, 18, 19], and Simpson [20]) have continued these studies concentrating mainly on various aspects of free and forced transverse vibrations of beams and strings. The stability of travelling two-dimensional rectangular membranes and plates has been studied by Ulsoy and Mote [21], Lin and Mote [22, 23], Lin [24] and Banichuk et al. [25, 11]. In their papers, Lin and Mote predicted the equilibrium displacement and stress distributions under transverse loading. Later on, they continued studing the wrinkling of axially moving rectangular webs with a small flexural stiffness. They predicted the critical value of the non-linear component of the edge loading after which the web wrinkles, and the corresponding wrinkled shape of the web. Banichuk et al. [25] studied stability characteristics of axially moving isotropic plates. They continued their studies on orthotropic membranes and plates in [11].

The field of fracture mechanics was developed by Irwin [26], based on the early papers of Inglis [27], Griffith [28] and Westergaard [29]. Linear elastic fracture mechanics (LEFM) was first applied to paper materials by Seth and Page [30], who measured fracture toughness for different paper materials. The fracture toughness of paper, using both the stress intensity factor and the strain energy release rate, was determined by Swinehart and Broek [31]. They found the measured crack length and fracture toughness to agree with the LEFM theory. Lately, critiques for using LEFM with paper materials have been presented, and the J-integral has displaced LEFM in many studies. However, we are interested in analytical solutions to obtain extremely efficient solvers for future purposes, and LEFM provides a solid foundation for this study.

In the literature, there exist various analyses of vibrations and stability of stationary beams and plates containing cracks. For an extensive review on fracture of cracked materials, we refer to Dimarogonas [32]. To analyse the vibrations and stability of cracked rectangular plates containing centre or edge located cracks, finite element analysis has often been applied. Bachene et al. [33] used the extended finite element method and Liew et al. [34] developed an efficient decomposition method to study vibrations of cracked plates. Brighenti [35] examined buckling failure of cracked plates for different crack orientations with the help of finite element analysis. Prabhakara and Datta [36, 37] covered both buckling and vibration analysis in the finite element studies of cracked plates. Stahl and Keer [38] studied vibrations and stability of rectangular plates with the help of dual series equations. Vafai et al. [39] studied parametric instability of plates having one crack at an edge. They considered simply supported rectangular plates under periodic loadings using an integral equation method. Effect of cracks on the eigenfrequencies and eigenmodes of axially moving beams at sub-critical transport speeds was studied by Murphy and Zhang [40]. However, the effect of the cracks on the results was found to be small.

Crack growth evolution in paper material has been studied by Tryding [41], who used experiments and a cohesive crack model with finite element analysis in his study. Hamad [42, 43] investigated fatigue of wood-pulp fibres on a microstructural level. Experimental studies for mode *I* cracks with long initial edge notch have been made by Rosti et al. [44]. In their studies, they have used statistical analysis based on acoustic emissions measurement. The strength properties of paper and other fiber like materials have been studied by Cox [45]. The interplay of structural disorder and strength properties was investigated by Hristopulos and Uesaka [46].

When modelling moving materials, the most common assumption in literature is the use of isotropic or orthotropic material properties (see, e.g., studies by Thorpe [47] and Baum et al. [48]). For example, Banichuk et al. [25, 11] and Tuovinen [49] studied the stability of an axially moving plate using Bolotin's approach in isotropic and orthotropic cases. In these studies, the effect of orthotropicity was found small, and, by using the Huber value for the shear modulus (the geometric average), one can reduce the orthotropic model into an isotropic one. Hence, we have chosen the isotropic assumption in this study. However, in a rigorous analysis of a paper web, the reader should note that the material is orthotropic and viscoelastic, nearly plastic, and moreover, measured values of the shear modulus typically do not match the Huber value. In this research, excluding

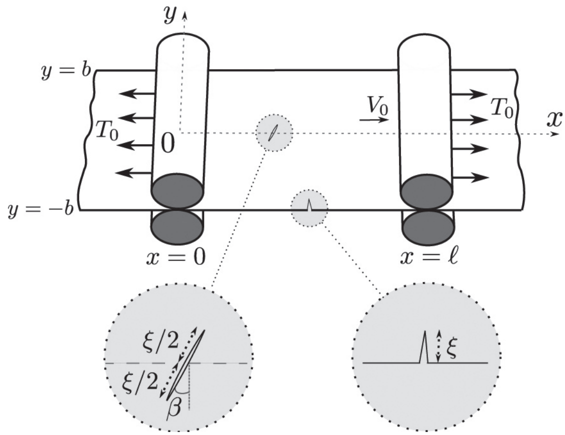

Figure 1. A paper web with initial cracks.

viscoelasticity should not be a major problem, as the effect of damping has found to be minor on the critical velocity in studies by Saksa et al. [50] and Marynowski [51]. We have also excluded the interaction between the travelling web and the surrounding air, although the presence of air was found to influence the critical velocity (see studies by Pramila [52] and Frondelius et al. [53]) and the dynamical response (Kulachenko et al. [54, 55]), possibly also affecting the buckling shape. For an extensive study and literature review on travelling webs interacting with flowing fluid, we refer to Jeronen [56].

In this study, we formulate analytical expressions for the optimal tension and velocity of the plate. The obtained analytical expressions are used for computing the optimal tension and the corresponding optimal velocity numerically. To do this, we use the Weibull distribution for the crack length. The effect of changing the values of distribution parameters is illustrated. We also illustrate the effect of changing the value of the admissible probabilites in the constraints. The results of the studied crack geometries are compared.

PROBLEM SETUP

We consider a rectangular part of an elastic web, which is moving at a constant velocity V_0 between supporting rollers. Denoting the part as

$$\Omega = \{(x, y): 0 < x < \ell, -b < y < b\}, \tag{1}$$

where l (m) and b (m) are prescribed parameters of length and width, we assume it to travel in the x direction (also called as machine direction, MD). The supporting rollers are located at $x = 0$ and $x = l$. See Figure 1.

The considered part Ω is represented as a thin elastic plate having constant thickness h (m), Poisson ratio v, Young modulus E (Pa), and bending rigidity

$$D = \frac{Eh^3}{12\,(1 - \nu^2)}. \tag{2}$$

The mass of the plate per unit area (of the middle surface of the plate) is denoted by m (kg/m^2). We further assume that the plate is subjected to homogeneous tension T_0 (N/m) acting in the x direction.

The critical velocity of instability of the travelling plate is given by [11]

$$V_0^{\mathrm{cr}} = \sqrt{\frac{T_0}{m} + \gamma_*^2 \frac{\pi^2 D}{m\ell^2}}, \tag{3}$$

where γ_* is the root of the equation

$$\Phi(\gamma, \mu) - \Psi(\gamma, \nu) = 0\,, \quad \mu = \frac{\ell}{2b}, \tag{4}$$

with

$$\Phi(\gamma, \mu) = \tanh\left(\frac{\sqrt{1-\gamma}}{\mu}\right) \coth\left(\frac{\sqrt{1+\gamma}}{\mu}\right) \tag{5}$$

and

$$\Psi(\gamma, \nu) = \frac{\sqrt{1+\gamma}\,(\gamma + \nu - 1)^2}{\sqrt{1-\gamma}\,(\gamma - \nu + 1)^2}\,. \tag{6}$$

In Figure 2, the behaviour of the functions Ψ and Φ is presented in a schematic manner.

We consider the case in which there is an initial crack of random length in the plate. The following two cases are studied: In the first case, the crack is assumed to be situated at the edge of the plate (slit edge crack). In the second case, the crack is located centrally in the plate. In the first case, the crack is assumed to be perpendicular to the machine direction. In the second case, we consider different orientations of the crack.

The value of γ_* in (3) does not depend on the value of tension T_0, and hence the limit of stable velocity (3) is increased when increasing the tension T_0. But, as increasing

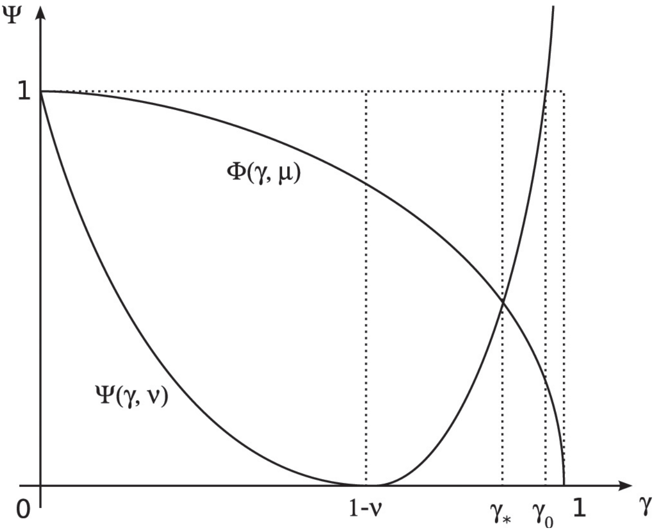

Figure 2. Schematic figure of behaviour of Ψ and Φ.

the tension may lead to growing or arising of cracks, the tension cannot be increased infinitely. A constraint for increasing the tension is given by a fracture criterion.

Optimization problem

To formulate the above as an optimization problem, we take a similar approach as presented in [57]. We seek the maximal magnitude of stable velocity under the constraint that the probability of fracture is small. Cracks that are perpendicular to the machine direction are of mode *I*, so that, when the edge crack or the central crack with $\beta = 0$ is considered, our optimization problem reads as

$$\max_{T_0} V_0^{\mathrm{cr}}(T_0), \quad \text{such that} \tag{7}$$

$$P(K_I \geq K_{\mathrm{C}}) \leq p, \tag{8}$$

where K_I (Pa $\sqrt{\mathrm{m}}$) is the stress intensity factor of mode *I*, K_{C} is the critical fracture toughness of the considered material and $p \in]0, 1[$ denotes the probability of fracture. In the case of an oblique central crack ($\beta > 0$), the crack is of modes *I* and *II*, and instead of (8) we take the constraint

$$P(K_I^2 + 1.56K_{II}^2 \geq K_{\mathrm{C}}^2) \leq p, \tag{9}$$

where K_{II} is the stress intensity factor related to mode *II*. For failure criteria for a mixed mode crack, see [58, 59].

Denoting the length of the crack with a positive valued random variable ξ (m), it holds for the stress intensity factors

$$K_j = \frac{\alpha(\xi)\, T_0\sqrt{\pi\xi}}{h}, \quad j = I, II, \tag{10}$$

where α is a function that depends on the crack geometry. The constraint (9) is equal to

$$P\left(\frac{\alpha(\xi)T_0\sqrt{\pi\xi}}{h} \geq K_C\right) \leq p \tag{11}$$

with

$$\alpha(\xi) = \sqrt{(\alpha_I(\xi))^2 + 1.56(\alpha_{II}(\xi))^2}, \tag{12}$$

where α_I and α_{II} are geometric functions related to modes *I* and *II* respectively.

The formula of α in some crack geometries is given, e.g., in [60] and [61]. In [62] and [63] a more general formula for α is given. We assume α to be an increasing positive function of ξ and, in this research, approximate α with a constant function. Note that, if in (12), the functions α_I and α_{II} are positive and increasing, also α is increasing.

To solve (7)–(8), we first look for the maximal value of the tension T_0 that satisfies (8) with (10). The inequality (8) is equivalent with

$$P\left(\xi \geq g^{-1}\left(\frac{K_C h}{T_0\sqrt{\pi}}\right)\right) \leq p, \tag{13}$$

where g^{-1} is the inverse function of the function $g(\xi) := \alpha(\xi)\sqrt{\xi}$. The inverse function exists, since g is strictly increasing due to the assumptions on α. Further, the inequality (13) is equal to

$$F_\xi\left(g^{-1}\left(\frac{K_C h}{T_0\sqrt{\pi}}\right)\right) \geq 1 - p, \tag{14}$$

where F_ξ is the cumulative distribution function of ξ. Assuming ξ to have a continuous probability density function, the function F_ξ is strictly increasing and hence the inverse function $F^{-1}{}_\xi$ exists. Denote

$$\xi_C = F_\xi^{-1}(1 - p). \tag{15}$$

The value ξ_C is the minimum of the set

$$\{x: F_\xi(x) \geq 1 - p\}. \tag{16}$$

See Figure 3.

Take another value of crack length, $\xi_C^* > \xi_C$, and consider the values of tension T_C and T^*_C that satisfy

$$\xi_C = g^{-1}\left(\frac{K_C h}{T_C\sqrt{\pi}}\right), \qquad \xi_C^* = g^{-1}\left(\frac{K_C h}{T_C^*\sqrt{\pi}}\right). \tag{17}$$

It holds that

$$T_C > T_C^*. \tag{18}$$

Thus, the maximal value of T_0 satisfying (8) is found by the equation

$$P(K_I \geq K_C) = p, \tag{19}$$

and can be expressed as

$$T^{\max} = \frac{K_C h}{\alpha(F_\xi^{-1}(1-p))\sqrt{\pi F_\xi^{-1}(1-p)}}. \tag{20}$$

The solution of the optimization problem (7)–(8) is

$$(V_0)_{\text{opt}} = V_0^{\text{cr}}(T^{\max}). \tag{21}$$

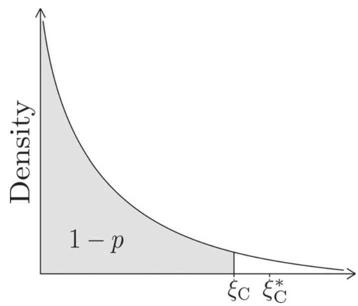

Figure 3. Two values of crack length, ξ_C and ξ_C^*, that satisfy $F_\xi(\xi_C) \geq 1 - p$ and $F_\xi(\xi_C^*) \geq 1 - p$.

Table 1. Formulas of probability density function, cumulative distribution function, expected value and variance for a Weibull distributed random variable

Probability density function	$f_\xi(x) = \frac{s}{c}\left(\frac{x}{c}\right)^{s-1} \exp(-\left(\frac{x}{c}\right)^s), \ x \geq 0,$ $s > 0, \ c > 0$
Cumulative distribution function	$F_\xi(x) = 1 - \exp(-\left(\frac{x}{c}\right)^s)$
Expected value	$\mathbb{E}[\xi] = c\Gamma_f(1 + 1/s),$ $\Gamma_f(x) = \int_0^\infty e^{-t}t^{x-1}dt$
Variance	$\mathrm{Var}[\xi] = c^2(\Gamma_f(1 + 2/s)$ $-(\Gamma_f(1 + 1/s))^2)$

The optimal tension $T^{\max}$ and the optimal velocity are computed numerically presuming a distribution for the crack length. For numerical examples, we chose the Weibull distribution. The parametrization we used is shown in Table 1. In an actual industrial application, the choice of distribution should of course depend on the data of crack lengths.

Note that, inversely, one may calculate the probability of fracture, if the fracture parameters, parameters for the crack length distribution, paper geometry and value of tension are known. Indeed, the left-hand side of the inequality (14) gives the probability of a web break with a given value of tension T_0 and a known distribution of the crack length ξ. In this research, our focus is on finding the critical value of tension.

NUMERICAL EXAMPLES

In this section, we illustrate numerically the results obtained in the previous section. The chosen parameter values are shown in Table 2. The paper fracture

Table 2. Physical and geometrical parameters

Parameter	*Value*
v	0.3
E	10^9 Pa
m	0.08 kg/m^2
h	10^{-4} m
l	0.1 m
b	5 m
G_C/ρ	10 Jm/kg

Table 3. Fracture parameters

	α	
Crack geometry	K_I	K_{II}
Edge crack, CD	1.12	–
Central crack, at an angle β from CD	$\cos^2 \beta$	$\sin \beta \cos \beta$

toughness K_C was calculated from the equation $K_C = \sqrt{G_C E}$. The value of the weight function α is approximated as presented in Table 3.

Figure 4 shows the effect of changing the value of the distribution parameters s and c on the probability density function. As Figure 4 demonstrates, the probability density is concentrated close to the origin (corresponding to a small length of the crack) as the parameter c is decreased and s increased. This can be seen also in Figure 5, in which the expected values and variance for the crack length are illustrated for the parameter ranges $s \in [0.5, 0.8]$ and $c \in [0.001, 0.008]$. Note that the probability densities corresponding to the corner points in Figures 5a and 5b are plotted in Figure 4. In Figure 6, we illustrate the effect of changing the value of s and c on the optimal values of tension and velocity for the parameter ranges $s \in [0.5, 0.8]$ and $c \in [0.001, 0.008]$. For this, it was set $p = 0.001$. Figures 5 and 6 show that the optimal values are decreased when the expected value and variance of the crack length increase.

In Figure 7, we illustrate the effect of increasing the value of p from $p = 0.001$ to $p = 0.1$ with $s = 0.5$ and $c = 0.005$. The corresponding probability density function is shown in Figure 4. As expected, the optimal values increase when the admissible probability of fracture p is increased.

With fixed s and c, the lowest optimal value is found by considering edge cracks. However, in Figure 6, we see that if the distributions of central and edge cracks are not the same, the optimal value of tension related to central cracks may give the minimum.

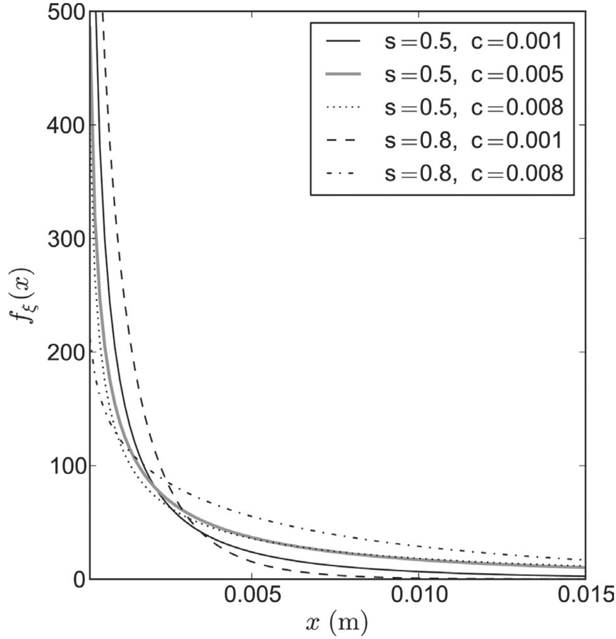

Figure 4. Some probability density functions of ξ. Weibull distribution.

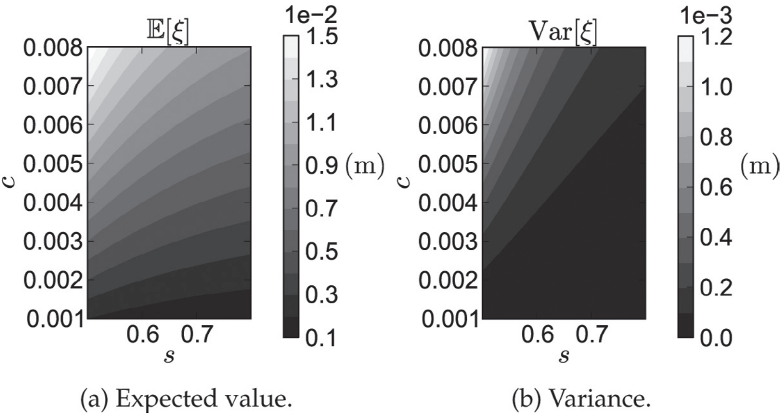

Figure 5. The expected values and variances of crack length corresponding to Figure 6. Weibull distribution.

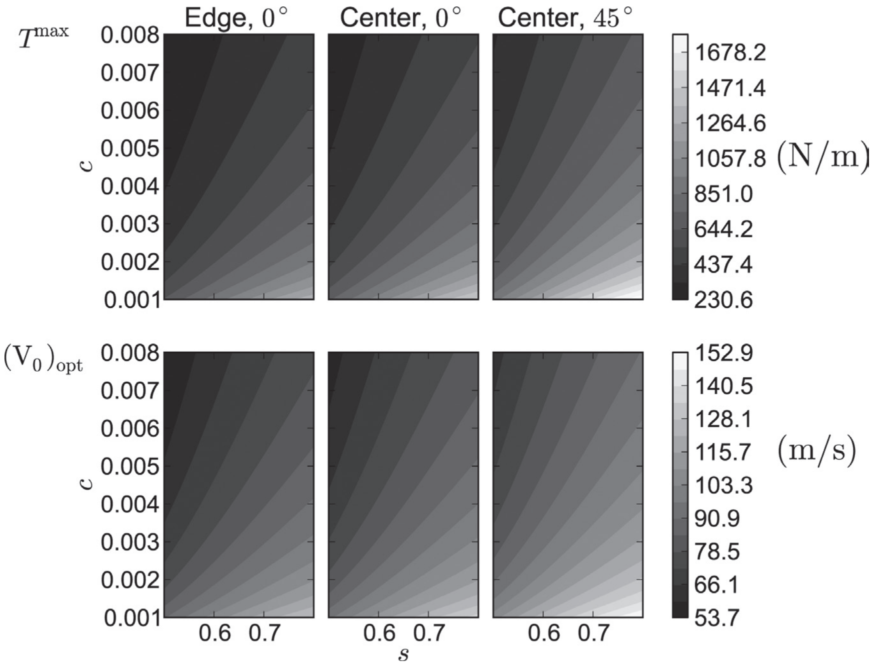

Figure 6. The effect of changing the value of the distribution parameters s and c on the optimal tension $T^{\max}$ (N/m) and the optimal velocity $(V_0)_{\text{opt}}$ (m/s) with the admissible probability of fracture $p = 0.001$. Weibull distribution.

Some of the optimal values are gathered in Table 4. The obtained critical velocities are higher than the running speeds of current paper machines, and a reason for this is that the interaction between the travelling web and the surrounding air is excluded in our model. According to the results by Pramila [52], the critical velocity obtained with the vacuum model may be even four times the value predicted by the model in which the surrounding air is present.

CONCLUSIONS

In this research, we modelled a travelling paper web in an open draw as an axially moving elastic plate and studied the problem of finding the optimal velocity for the plate. The plate was assumed to have an initial central or edge crack of random length and the optimal velocity was investigated under the constraint that the

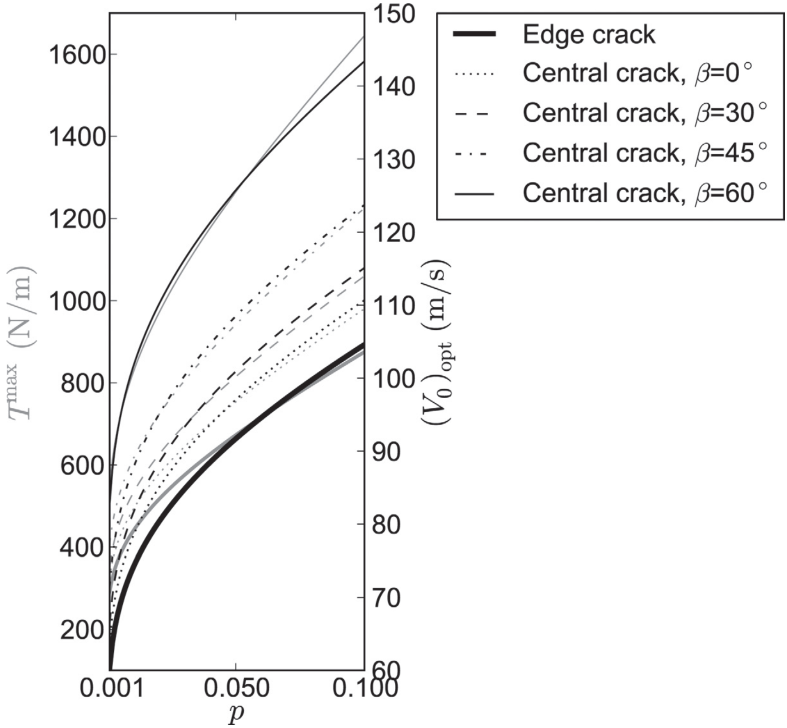

Figure 7. The effect of increasing the admissible probability of fracture on the optimal tension T^{max} (N/m) and the optimal velocity $(V_0)_{opt}$ (m/s) with $s = 0.5$ and $c = 0.005$.

Table 4. The optimal tension (upper value, N/m) and velocity (*lower value*, m/s) with $s = 0.5$ and $c = 0.005$ with respect to *p*.

Crack geometry	0.001	0.005	0.01
Edge crack	291.7	380.3	437.5
	60.4	*69.0*	*74.0*
Central crack, $\beta = 0°$	326.7	425.9	490.0
	63.9	*73.0*	*78.3*
$\beta = 30°$	353.3	460.6	530.0
	66.5	*75.9*	*81.4*
$\beta = 45°$	408.4	532.4	612.6
	71.5	*81.6*	*87.5*
$\beta = 60°$	548.3	714.9	822.5
	82.8	*94.5*	*101.4*

probability of fracture is limited. The optimal velocity was found by first formulating an analytical expression for the optimal tension. By combining this to previous results of stable velocity, a formula for the optimal velocity was found. With the formulae derived in the paper, one may also, inversely, calculate the probability of fracture, if the parameters for the crack length distribution and the value of tension are known.

Modelling the crack length with the Weibull distribution, the effect of changing the probability of fracture on the optimal values was numerically investigated. Increasing the probability of fracture increases the optimal tension and the corresponding optimal velocity. Compared with the same distribution for the considered crack geometries, the optimal values were the lowest for the edge crack. In the case of the central crack, when the angle from the cross machine direction was increased, the optimal values increased.

The optimal velocity was also numerically computed with different values of distribution parameters. The values were chosen such that the distributions were close to the presumable crack length distribution of a paper web. As the expected value and variance of the crack length were increased, the optimal values of tension and velocity decreased. It was also seen that if the distributions of central and edge cracks are not the same, the minimum of optimal values may be given by central cracks.

In this research, we have used simple models for analysis of optimal tension and velocity. However, this approach gives the first solutions for direct optimization of this problem. Later on, it will be an interesting task to study different defect geometries and multiple cracks in this framework. This study also opens the possibility to efficient solving processes for industry with realtime data sources.

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Transcription of Discussion

STOCHASTIC ANALYSIS OF THE CRITICAL STABLE VELOCITY OF A MOVING PAPER WEB IN THE PRESENCE OF A CRACK

Maria Tirronen, *Tero Tuovinen, Juha Jeronen and Tytti Saksa*

Department of Mathematical Information Technology, P.O. Box 35 (Agora), FI- 40014 University of Jyväskylä, Finland

Daniel Söderberg KTH and Innventia (from the chair)

So, you indicated in the paper that there is an effect of surrounding air. So, what is the effect of air? So, if you looked at what people have done and their results, what would be the influence if you added that?

Maria Tirronen

It lowers the optimal values. This was shown in a study by Pramila[2].

Juha Jeronen University of Jyväskylä (co-author)

It depends on what model you use for the surrounding air, because the studies do not really agree on how much it affects the critical velocity. What they all agree on is that it reduces the eigen-frequencies — the frequencies of natural vibration — into something like a third or a quarter of the vacuum values, but there is no consensus on whether it has the same effect on the critical velocity, if it affects it at all. So, it depends on the model. But this study from the 1980s that Maria

[2]A. Pramila, "Sheet flutter and the interaction between sheet and air", TAPPI Journal, 69(7):70–74, 1986.

mentioned, by Antti Pramila, who later became a professor at Oulu University, used the potential flow model for the surrounding air. In that case, depending on whether you assume the surrounding air remains stationary or moves with the web, you get two different results. Either it will reduce the critical velocity to about a quarter of the vacuum value, or it won't affect it at all. There have been later studies with other models. For example, one study in 2006 by Frondelius *et al.*[3], where the result was that the critical velocity is expected to be about 90% of its vacuum value. They used boundary layer theory in that study, so it really depends on the fluid model and the assumptions concerning the flow.

Daniel Söderberg

Work on liquid-plane jets in air, which are similar, suggests it is also dependent on the wavelength of the wave that travels in the web. Is this true here?

Juha Jeronen

Yes, probably. It will depend on the assumptions of what kind of flow you have, of course.

[3]T. Frondelius, H. Koivurova, and A. Pramila, "Interaction of an axially moving band and surrounding fluid by boundary layer theory", Journal of Fluids and Structures, 22(8):1047–1056, 2006.