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REVISITING THE RANDOM DISK MODEL: DECOMPOSITION OF PAPER STRUCTURE USING RANDOMLY DEPOSITED DISKS WITH ARBITRARY SIZE DISTRIBUTIONS

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ABSTRACT

In this study, a numerical algorithm was developed to decompose the planar mass structure of paper into a random array of grey disks with a discrete size distribution. The optimum size and the frequency of these disks were determined such that the second order statistics of the corresponding random disk structure resembled that of the paper sample. Using this method, eighty two (82) commercial and laboratory-made samples were analyzed. It was found that; independent of the forming conditions, the average disk size was proportional to the standard deviation of the disk size distribution. The utility of this new tool in analyzing the effect of papermaking conditions on paper formation is illustrated.

INTRODUCTION

Formation remains to be a key factor to achieve desirable optical, mechanical, and end use properties for various grades of paper. Several methods have been developed to characterize paper formation, among which statistical geometry is a powerful tool that directly relates formation to furnish; i.e. fibres and flocs [1–4].

Earlier work has shown that paper formation may be simulated by the random deposition of grey disks representing flocs [4–7]. Disk size and disk grammage; hence, were used to quantify formation. In those studies, a lognormal disk size distribution was employed to model the mass structure of paper. However, this is a stiff assumption since the actual form of disk size distribution *a priori* is unknown.

To address the above shortcoming, a numerical algorithm is developed to decompose the planar mass structure of paper into a random array of grey disks with an arbitrary size distribution. This distribution is discretized into an arbitrary number of bins, where each bin consists of disks of the same size and grammage. The optimum disk size and the mass frequency of each bin are determined using an optimization procedure. The utility of this new tool in analyzing the effect of papermaking conditions on paper formation will be illustrated.

THEORY

Random structure of disks

It has been shown that the variance of local grammage of a random disk structure with a square inspection zone of side length ‘x’ is given by [4]:

$$Var(x) = a_d \bar{\beta} \rho_{disk}(x, D) \tag{1}$$

where a_d and $\bar{\beta}$ are the disk grammage and sheet grammage, respectively. In the above equation, ρ_{disk} is the dimensionless variance that can be estimated from [4]:

$$\begin{aligned} \rho_{disk}(x, D) &\approx 1 - 0.592x/D \qquad && x \le 0.843D \\ &\approx D^2/x^2(0.71 - 0.30D/x) \qquad && x > 0.843D \end{aligned} \tag{2}$$

Here D is the diameter of the disk. From (1) and (2), the variance of local grammage of the paper can be predicted as a function of the zone size and the disk diameters.

According to (2), as the zone size approaches 0, the dimensionless variance approaches unity, hence:

$$Var(0) = a_d \bar{\beta} \tag{3}$$

or,

$$a_d = \frac{Var(0)}{\bar{\beta}} \tag{4}$$

Since the sheet grammage is typically known and given that the point variance can be approximated by the plot of variance vs. zone size, the average disk grammage can then be determined from (4).

For a random structure of disks with a discrete size distribution consisting of N bins, the variance then becomes:

$$Var(x) = \sum_{i=1}^{N} m_i Var(D_i, x) \tag{5}$$

where m_i is the mass frequency of each bin that must satisfy the following relationships: $0 \le m_i \le 1$ & $\sum_{i=1}^{N} m_i = 1.$

The inverse problem of finding the optimum distribution of disks that produces a mass structure statistically similar to that of a paper sample is highly non-linear [6]. To solve such a problem, an optimization procedure was developed in this study. Using this approach, the disk size distribution and the disk grammage that best predict the variance of grammage of paper were determined.

Numerical Approach

A computer program, written in C++, was developed to determine the optimum size distribution of disks using the LJ optimization procedure [8] by minimizing the following performance index:

$$Index = \sum_{i=1}^{N} \frac{|Var_i^{\exp} - Var_i^{predicted}|}{Var_i^{\exp}} \tag{6}$$

Here $Var^{\text{predicted}}$ and $Var^{\exp}$ are the predicted variance and the experimental variance of local grammage at a given inspection zone size.

To initiate the optimization procedure, the number of bins that is used to discretize the disk size distribution should be selected. A disk size and a mass frequency (both chosen randomly within the search space) are then assigned to each bin. This procedure is repeated for roughly 1000 randomly selected points within the search space and the point with the minimum performance index is identified. The search space is then reduced in size around the above point and the previous steps are repeated until the performance index is below a target threshold value.

Based on the minimum size of the inspection zone (0.1 mm) and the dimensions of the grammage maps (50mm × 50mm) in this study, the search space for disk diameter was restricted to 0.1 mm –25 mm.

Experimental data

The experimental data consisted of 86 paper samples that included variance of local grammage as a function of zone size as well as forming conditions [4]. These samples included 23 laboratory handsheets, all of which were made of the same furnish. Among these handsheets, seven samples were prepared under varying settling times (i.e. degree of flocculation), while the remaining sixteen handsheets had varying grammages. The experimental data also comprised of sixty three commercial samples tested, including those reported by Corte [1] (22 samples) and 41 samples reported by Farnood [4]. In addition to these experimental data, variance-zone size for several simulated random disk structures with uniform disk diameters (as reported in [6]) were analyzed to validate the optimization procedure.

PLS Analysis

Projection to latent structures (PLS) is a multivariate statistical technique that helps to reveal correlation amongst input-variables or predictors (X space) and also their impact on several responses (Y space). Unlike multi-linear regression, PLS can handle data with strong collinearity, noise and missing values in both the X- and Y-spaces. This tool reduces the dimension of the system to fewer "latent variables" (referred to as principal components or scores) that can simultaneously explain the significant variance in X and predict Y. The scores are independent of each other and are a linear combination of the original predictors. The weight of each "x"-variable on the scores is directly related to their level of influence on the measured "y"-properties. An important aspect of PLS is the ability to show – in a single plot – the interrelationships between all predictors, the relationships between all responses, and simultaneously the predictors' influence on the measured "y"-responses. Details of the PLS calculations are described elsewhere [9].

In the present work, PLS analysis was conducted using SIMCA-P software (Umetrics). This tool was used to examine the relationship between disk size, disk grammage and the forming conditions.

RESULTS

Optimum Number of Bins

As discussed earlier, the optimization algorithm required that the number of bins to be specified as input parameter. As seen in Figure 1, the error between the predicted

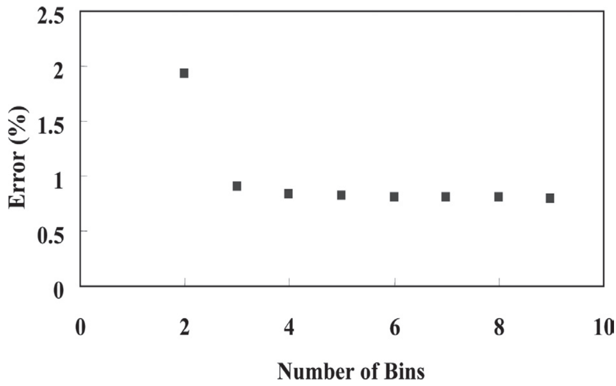

Figure 1. The average % relative error of the predicted variance for the commercial samples examined in this study.

variance and the measured variance decreased as the number of bins increased and eventually approached a plateau. Accordingly, a maximum of 5 bins were used in the optimization procedure for subsequent analyses. It is worth noting that the actual number of bins required to achieve the best 'fit' to the experimental data is often less than five, with the remainder of bins having a frequency of zero.

Validation

To examine the validity of the optimization procedure, computer-generated structures of randomly deposited unisize disks reported previously [6] were examined (Table 1). The predicted average disk diameter correlated well with the actual values with a correlation coefficient of 0.999 (Figure 2).

Paper samples

The parity plot for the predicted and experimental values of grammage variance for all paper samples is given in Figure 3. This figure shows that the optimum disk size distributions; determined using the optimization procedure described earlier, provided excellent fit ($R^2 = 0.999$) for the second order statistics of both commercial papers and handsheets.

Figure 4, illustrates the application of proposed method for a typical commercial paper (sample C1) and for a handsheet (sample HS1). The predicted grammage variance closely followed the actual values with a relative error better than 1%. The corresponding disk size distributions for these samples are given in

Table 1. Comparison of *the actual disk diameter and predicted values for random structures of unisize disks.*

Sample	*Actual Disk Diameter (mm)*	*Predicted Mean Disk Diameter (mm)*	*Variance Error (%)*
R1	0.97	0.98	2.07
R2	3.09	3.08	1.29
R3	3.94	3.99	0.22
R4	4.75	4.73	0.34
R5	3.12	3.10	0.14
R6	3.56	3.57	0.48
R7	4.09	4.09	1.40
R8	0.28	0.28	6.71

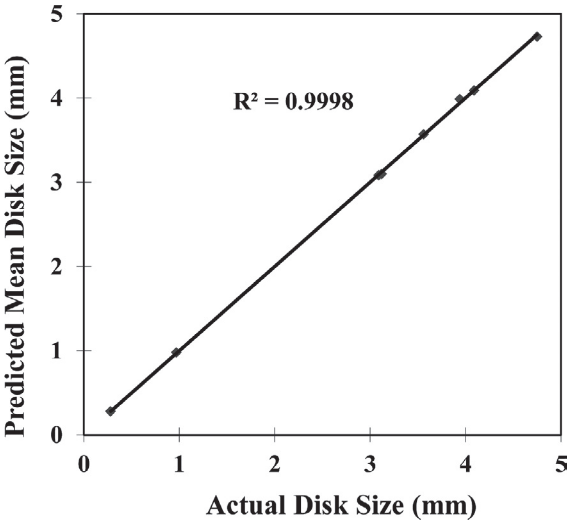

Figure 2. The predicted mean disk diameter and actual disk diameter of the simulated random disk structures with unisize disks.

Figures 5 and 6. The average disk diameters for the commercial paper and for the handsheet were 3.83 mm and 4.42 mm and the disk grammages were 0.76 g/m^2 and 0.32 g/m^2, respectively. In both cases, the mode disk size was in the order of fibre length (about 1~2 mm); however, some large disks (10~20 mm) also existed that stemmed from the presence of flocs in the sheet.

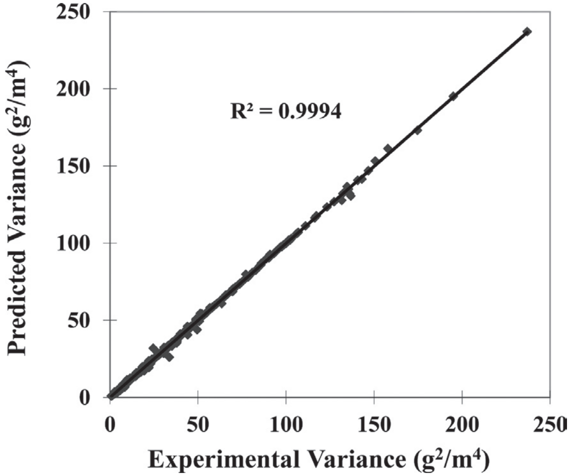

Figure 3. The predicted variance and actual variance of all the commercial and handsheet paper samples.

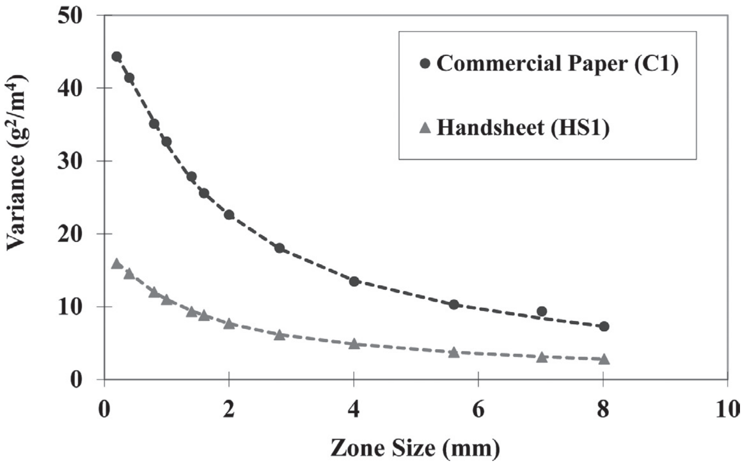

Figure 4. The experimental variance (symbols) for a commercial sample (C1) and a handsheet sample (HS1) along with the predicted variance values (dashed lines).

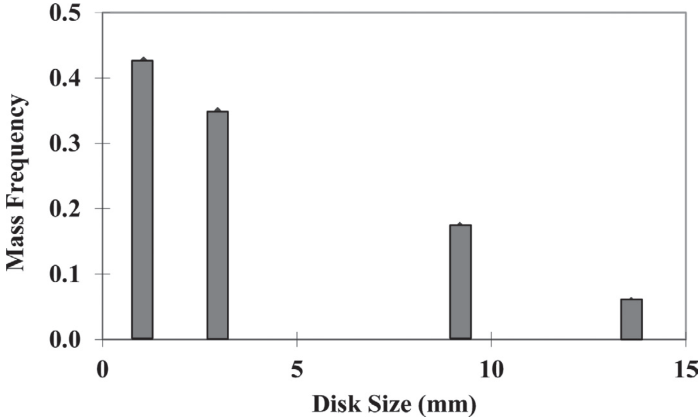

Figure 5. The disk size distribution for the commercial paper sample C1.

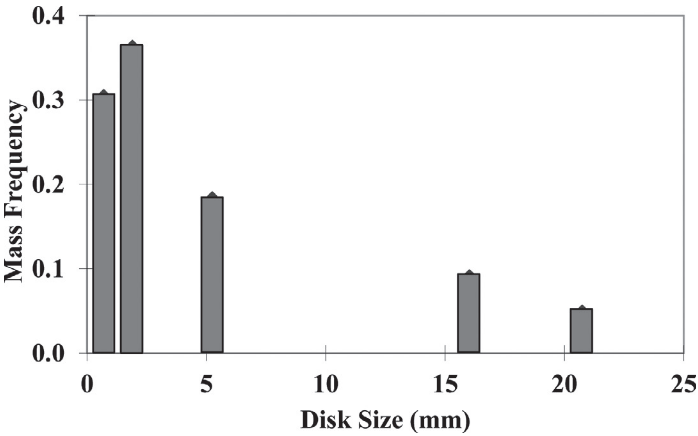

Figure 6. The disk size distribution for handsheet sample HS1.

Figure 7 represents the average of disk size distributions for all 82 commercial and laboratory made sheets used in this study. It is interesting to note that the average disk size distribution for the papers samples resembled a lognormal distribution; hence supporting the use lognormal function in our earlier work [4]. Once again, the mode of this distribution was around 1–2 mm, while large disks (~10 mm and larger) were also observed.

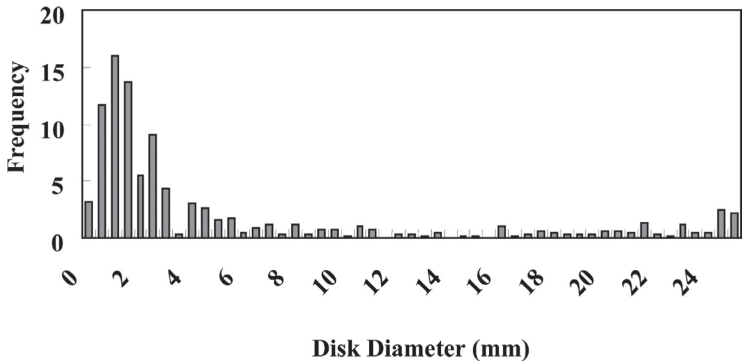

Figure 7. The distribution of the random disks for all samples tested.

Degree of Flocculation

To examine the effect of flocculation on the predicted disk size, handsheets prepared under varying settling times were analyzed. Handsheet properties together with the predicted disk size, disk grammage, and mean disk mass are given in Table 2. The mean disk mass was determined by multiplying the average disk area by the disk grammage.

As seen in Figure 8, the mean mass of the disks increased linearly by increasing the settling time. This increase is indicative of formation of larger flocs at increased settling times.

Table 2. The properties of handsheets made with the same fibres and under the same conditions but with different settling time.

Sample	*Sheet Grammage* (g/m²)*	*Settling Time* (s)*	*Disk Grammage (g/m²)*	*Mean Disk Size (mm)*	*Standard Deviation of Disk Size (mm)*	*Mean Disk Mass (μg)*
HS1	54.3	0	0.32	4.42	5.72	0.11
HS2	51.8	15	0.33	3.81	6.11	0.10
HS3	51.6	30	0.38	6.21	7.58	0.22
HS4	51.8	45	0.5	4.69	5.94	0.29
HS5	51.6	60	0.58	6.77	7.81	0.57
HS6	52.5	90	0.84	6.10	4.88	1.08
HS7	52.8	120	1.08	6.03	2.52	1.76

*Data from Farnood [4].

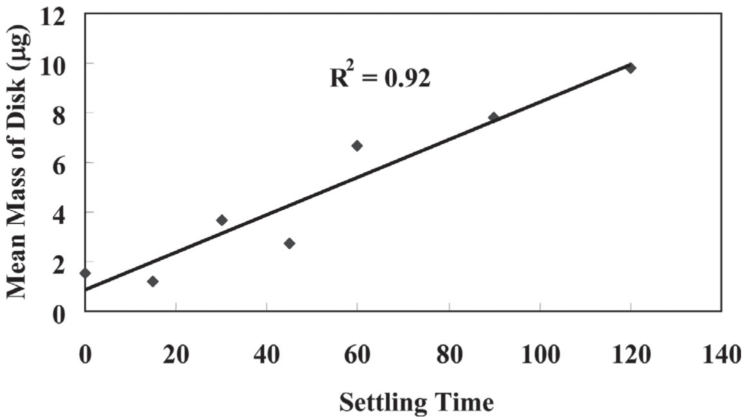

Figure 8. The mean mass of the flocculations versus the settling time of each of the 7 paper samples tested from Table 2.

Forming Conditions

In this section, the effect of forming conditions on the disk size and disk grammage is examined. Given the complexity and inter-dependency of this data, PLS analysis was employed. The forming conditions and sheet properties for commercial samples used in this analysis are listed in Table 3. The loading plot (seen in Figure 9) shows that mean disk diameter, standard deviation of disk diameter, and disk grammage were negatively related to the machine speed, while headbox consistency was positively correlated with the above parameters. This result suggests that increasing the machine speed or reducing the headbox consistency resulted in smaller and looser flocs. This is expected since the machine speed and headbox consistency are known to have significant effect on the sheet formation.

The Similarity Relationship

In an earlier work [6], a similarity relationship has been reported that related the mean disk size to the standard deviation of disk size distribution. However, this finding was based on the assumption that disk size was lognormally distributed, covering a semi-infinite domain from zero to infinity. As discussed in the previous section, the functionality of disk size distribution is not known *a priori*. Furthermore, since disks represent the presence of flocs in paper, physically disk size cannot be infinity small or infinitely large.

Table 3. The different conditions that the commercial samples were made and their properties (J/F: jet to fabric speed ratio, H.B. Cons: headbox consistency, F.: Fourdrinier, S.: Symformer, C.: "C" Former).

Sample	*Sheet Grammage * (g/m²)*	*Machine Speed * (m/min)*	*J/F**	*H.B. Cons. * (%)*	*Former Type**	*Mean Disk Size (mm)*	*Standard Deviation of Disk Size (mm)*	*Disk Grammage (g/m²)*	*Mean Mass of Disk (mg)*
C1	62.5	502	0.97	0.66	F.+S.	3.83	3.72	0.76	2.78
C2	62.4	503	1.01	0.66	F.+S.	3.96	5.91	0.54	2.11
C3	51.5	634	0.99	0.61	F.+S.	3.75	3.38	0.46	1.62
C4	51.5	634	0.99	0.52	F.+S.	7.66	9.55	0.56	8.22
C5	48.8	1000	1.01	0.78	S.	3.48	3.98	0.37	1.12
C6	45	887	0.96	N/A	S.	5.30	8.30	0.42	2.95
C7	86.3	284	1.01	1.031	F.+C.	4.60	7.10	0.45	2.38
C8	86.3	284	0.97	1.031	F.+C.	8.73	9.85	0.57	10.86
C9	95.1	284	0.97	1.031	F.+C.	6.30	9.56	1.15	11.40
C10	95.1	284	1.02	1.031	F.+C.	7.67	9.68	0.8	11.78
C11	48.8	494	0.97	0.61	F.	4.92	6.93	0.47	2.85
C12	48.8	494	0.95	0.61	F.	5.65	8.23	0.57	4.55

*Data from Farnood [4]

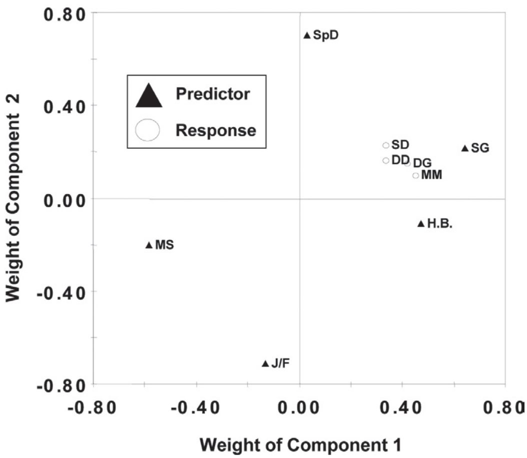

Figure 9. The loading plot for the commercial papers. (HB: headbox consistency (%), DD: mean disk diameter, DG: disk grammage, J/F: jet to fabric ratio, MM: mean mass of disk, MS: machine speed, SD: standard deviation of disk diameter, SG: sheet grammage, SpD: speed differential between jet and fabric).

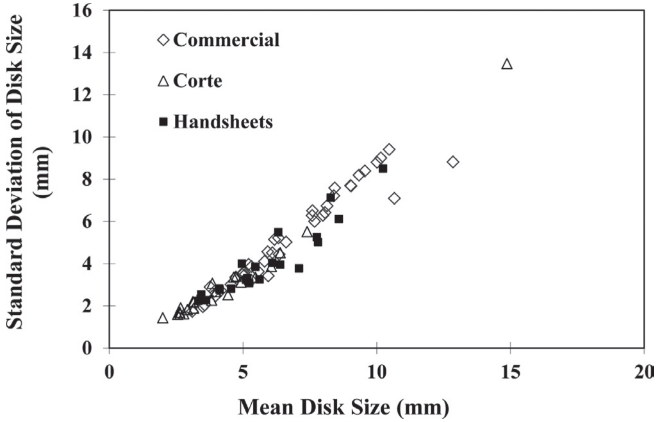

Figure 10. The plot of standard deviation versus mean disk diameter for the commercial papers and handsheet samples.

To address the above concerns, the existence of such a similarity relationship is re-examined using the method developed in this study. Figure 10 shows that for paper samples used in this work, the standard deviation of disk diameter (σ) was proportional to the mean disk diameter ($\bar{D}$) with a coefficient of determination of $R^2 = 0.91$:

$$\sigma = 0.76\,\bar{D} \tag{7}$$

It is useful to note that the above similarity relationship holds both for newer commercial paper samples and the ones reported by Corte in 1970.

CONCLUSIONS

The procedure developed in this work offers a new tool for the characterization of paper mass structure. The average disk size and disk grammage obtained in this way are expected to correlate with the size and density of flocs in the sheet of paper and hence can be used to characterize paper formation. Unlike spectral analysis techniques that supply information regarding the variance as a function of wavelength, the proposed method directly provides information concerning floc dimension. Moreover, this method does not require subjective thresholding of the grammage map of paper that is typically required in other image-based formation analysis procedures.

The proportional relationship between the average disk size and the standard deviation of disk size confirms the presence of a similarity relationship both for commercial and laboratory paper samples. It is noteworthy that despite technological advancements in papermaking process, the above relationship holds for all commercial samples covering a large range of forming conditions spanning over forty years.

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Transcription of Discussion

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ERRATA

The final column was incorrect in tables 2 and 3. Corrected versions are below:

Table 2. The properties of handsheets made with the same fibres and under the same conditions but with different settling time.

Sample	*Sheet Grammage* (g/m^2)*	*Settling Time* (s)*	*Disk Grammage (g/m^2)*	*Mean Disk Size (mm)*	*Standard Deviation of Disk Size (mm)*	*Mean Disk Mass (μg)*
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HS3	51.6	30	0.38	6.21	7.58	11.4
HS4	51.8	45	0.5	4.69	5.94	8.64
HS5	51.6	60	0.58	6.77	7.81	20.9
HS6	52.5	90	0.84	6.10	4.88	24.6
HS7	52.8	120	1.08	6.03	2.52	30.8

* Data from Farnood [4].

Table 3. The different conditions that the commercial samples were made and their properties (J/F: jet to fabric speed ratio, H.B. Cons: headbox consistency, F.: Fourdrinier, S.: Symformer, C.: "C" Former).

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C3	51.5	634	0.99	0.61	F.+S.	3.75	3.38	0.46	5.08
C4	51.5	634	0.99	0.52	F.+S.	7.66	9.55	0.56	25.8
C5	48.8	1000	1.01	0.78	S.	3.48	3.98	0.37	3.52
C6	45	887	0.96	N/A	S.	5.30	8.30	0.42	9.27
C7	86.3	284	1.01	1.031	F.+C.	4.60	7.10	0.45	7.49
C8	86.3	284	0.97	1.031	F.+C.	8.73	9.85	0.57	34.1
C9	95.1	284	0.97	1.031	F.+C.	6.30	9.56	1.15	35.8
C10	95.1	284	1.02	1.031	F.+C.	7.67	9.68	0.8	37.0
C11	48.8	494	0.97	0.61	F.	4.92	6.93	0.47	8.93
C12	48.8	494	0.95	0.61	F.	5.65	8.23	0.57	14.3

* Data from Farnood [4]

Figure 8 was also incorrect, see below:

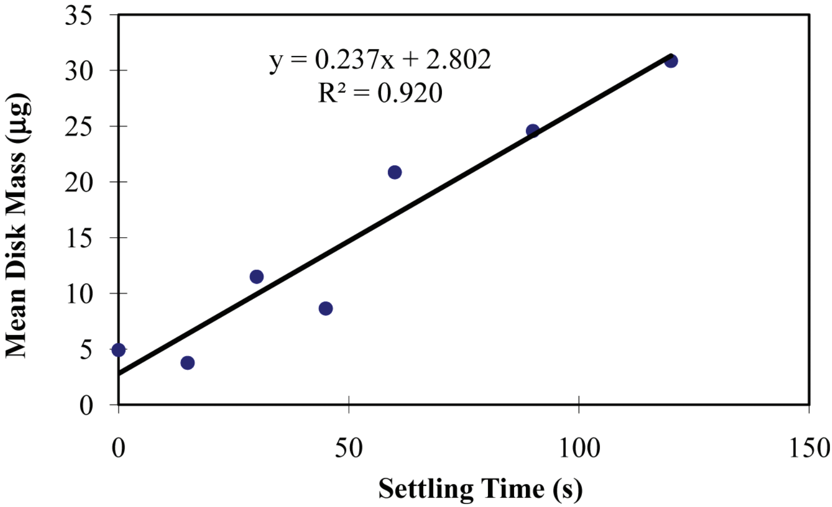

Figure 8. The mean mass of the flocculations versus the settling time of each of the 7 paper samples tested from Table 2.

DISCUSSION CONTRIBUTIONS

Steve I'Anson FRC Chairman (from the chair)

You have two tables of data, Ramin, for the handsheets and the machine-made paper. You've got very different mean disc mass for the two papers: the one for the handsheets is of the order of a tenth of a microgram and the other, for the machine-made paper, is of the order of a few milligrams,. That's a huge difference, around four orders of magnitude; is there any particular significance to that?

Ramin Farnood (after the session)

The value for mean disk mass should be in the order of micrograms. There appears to be an error in the reported values of mean disk mass in Tables 2 and 3. The revised values are given in Errata, above, and Figure 8 is also revised accordingly. These corrections, however, do not change the qualitative arguments presented in the manuscript and presentation.

Discussion

Kit Dodson University of Manchester

Ramin, it is very nice to see this work extended. I think the reason that we originally used the log normal distribution was because it was convenient and it looked reasonable. This most recent work is an optimisation method that is effectively a spectral decomposition. Now, the nature of the spectral decomposition that comes out is surprising, but only a little bit surprising. The fact that the standard deviation comes out as proportional to the mean, and not a power of the mean, is really significant. This means that the spectral decomposition is of necessity, a truncated gamma distribution, which is very exciting. I think, there is lot of work to do, but where I would like you to elaborate a little more is on the optimisation of the number of bins: there look to be very few of them to a mathematician. I do not see why you could not truncate a gamma distribution and use all of the bins. How do you eliminate the bins because they are all there to start with?

Ramin Farnood

So, I would like to make a comment about your first point, and then I will answer your questions. You mentioned truncation of the gamma distribution. Those who have worked with truncation of a distribution are aware of the challenges and opportunities of such things. So the real question is: where we are going to truncate, what should be the criteria? Now, in fact, the same challenge exists when you are dealing with a discrete size distribution. In this argument, we assume that that the disc size cannot be larger than the area of the image, as an upper limit, and we have also a lower limit, which is the pixel size, so that we also only allow the bin size to vary between these limits. The number of bins being so few is really the result of the optimisation algorithm, with the maximum number of the bins allocated being 10, but we found, through the optimisation, that, really, we needed only 4 or 5 bins in most cases to be able to achieve the same second order statistics that we would expect to see from paper itself. Now "why is that the case?" is an interesting question, but we did not restrict the number and could have got 10 bins in each case. In fact some bins don't show up on the diagrams because they may have a very small frequency. How the optimisation algorithm functions is that, for example for bin number 1, the location of the bin (disc diameter) and the frequency (number of discs) are both optimised and, in some cases, frequency will be close to zero and the bin will not be significant in the distribution. I have found that, in some cases, the distribution it is not unique, which is not surprising, given the fact that this is a highly complex nonlinear problem. In some cases, we actually observed that slightly different distributions would give an equally good fit. So, one possible approach, following on from what you have just said, is to take all the solutions and, since they all give us the same variance versus zone size behaviour, use an average of all the distributions. Would that resemble better the truncated gamma distribution? I will try it and will let you know how well it worked.

Jim De Witt Sappi Fine Paper

Ramin, you know, if I think about the way we make paper and the way we try to improve formation, our gold standard measurement is beta radiography. One of the things it allows us to do is compare across platforms and basis weights. We can normalize the number we get by dividing by the square root of basis weight, allowing comparison between forming techniques on different machines for achieving the best formation. Now in this method, presumably getting smaller and smaller discs, that indicates more and more uniform paper, but how would you correct for basis weight?

Ramin Farnood

In an earlier piece of work, we reported using this technique and mapping the state of formation[3]. We concluded that two parameters matter and that, with these, we can fully characterise the state of formation: they are average disc size (simulating floc size), and, as the other parameter, disc grammage (simulating floc density). If we consider papers that have low disc grammage and low disc diameter, they have a rather uniform structure. If you still have a low disc diameter but a high disc grammage, you will get a spotty formation with high variability. So the state of formation is really a balance of these two parameters: floc density and floc size. I hope that I answered your question. In my opinion, you cannot just look at one parameter.

Ulrich Hirn Graz University of Technology

I was wondering, did you look at the FFT spectra of your simulated formation images? Because what I would expect, if you have only a few bins, is that you will see spikes in the spectrum at the wavelengths corresponding to the disc diameters.

Ramin Farnood

Yes, we did that 18 years ago. I did not present it here, but it was presented at the Niagara-on-the-Lake Paper Physics Conference[4]. You are right. We compared the

[3]R.R. Farnood and C.T.J. Dodson. "The similarity law of formation". *In proc.Tappi 1995 International Paper Physics Conference*, pp5–12, Niagara-on-the-Lake, Canada. Tappi Press Atlanta, 1995.
[4]R.R. Farnood and C.T.J. Dodson. "The similarity law of formation". *In proc.Tappi 1995 International Paper Physics Conference*, pp5–12, Niagara-on-the-Lake, Canada. Tappi Press Atlanta, 1995.

power spectra of the simulated structures with power spectra of the actual paper. Not only did the coefficients of variation match, but also the power spectra matched, within a reasonable error.

Ulrich Hirn

Yes, and if you brought in more bins, you would probably smooth it out more and make the spectrum more continuous, making it look more like a real paper spectrum.

Ramin Farnood

Yes, I'm sure that you are right.

Li Yang Innventia

My question is about basis weight. Imagine your process for a very low basis weight, meaning that you have a very shallow structure, which I think means that you would have very small discs. If the basis weight is increased, I think that the disc size would be increased. Is this correct? I was really wondering how the grammage will impact on the disc distribution, for example, when it goes from the very low to rather high.

Ramin Farnood

I would be concerned about using this technique for very low basis weights like the tissue papers that Steve Keller talked about in the last paper. The reason I would be concerned is because of the basic assumption that you can replace the distribution with discrete blocks, meaning that you need to have a certain minimum coverage for this to work properly. Having said that, in the data that I presented, some of the handsheets were made at a basis weight of 12 g m^{-2}; that is as low as we got.

Li Yang

If we increase the grammage, the same paper, same machine making, all the process the same, just increasing the basis weight, will there be more larger discs in the disc distribution just simply because the basis weight is higher? For example, if you put two of your paper samples together, how will that work?

Ramin Farnood

Assuming that representation of flocs by an average disc is a good assumption, then anything that can change flocculation we would expect to affect the disc size and density. In the paper, we have changed settling time, and we see the effect on the disc size and on the grammage of the disc. What we found is that the mass of the disc increases with increasing settling time. So going back to your question, anything that you do to change the basis weight will, in some way, change the state of flocculation. If it is a machine made paper, you may have a slightly different consistency, or maybe you have changed the slice opening; these will change the state of flocculation. If it is a handsheet that you are making, you change the concentration and that also can affect the state of flocculation, and as a result it will affect the size of the disc.

Steve I'Anson FRC Chairman (from the chair)

You could actually test this without a change in flocculation state by taking one of your handsheet grammage images, rotating it by 90 or 180 degrees and then adding it to the original. You would see whether the model still worked because then you have effectively a double grammage sheet without a change of flocculation. It would be interesting to see whether you still got the same bins and the same distribution. In this way you could actually test this without doing more scanning.

Ramin Farnood

Okay, so perhaps I misunderstood the question. The question is about the analytical technique and the algorithm that we are using, not so much about how the sheet is formed.

Steve I'Anson

You could physically stick two sheets together and re-scan, but alternatively ,with the handsheet samples, you could just take two copies of the image, rotate one and then add them together digitally, couldn't you?

Ramin Farnood

Yes, that is one way to look at it. The other way is that we have analysed paper with a coverage of 1 and with a coverage of 50, and both have gone through the procedure to give us the same disks size of 3 mm.