Preferred citation: J. Woodhouse. Elastic properties of sheet and materials from vibration testing. In **Advances in Pulp and Paper Research, Cambridge 2013**, *Trans. of the XVth Fund. Res. Symp. Cambridge, 2013*, (S.J. I'Anson, ed.), pp 995–1006, FRC, Manchester, 2018.
DOI: 10.15376/frc.2013.3.995.

ELASTIC PROPERTIES OF SHEET MATERIALS FROM VIBRATION TESTING

Jim Woodhouse

Cambridge University Engineering Department, Trumpington St, Cambridge CB2 1PZ jw12@cam.ac.uk

ABSTRACT

Many materials, including paper products, come in sheet form and exhibit orthotropic symmetry. Information about the elastic stiffnesses of such materials can often be obtained quickly and accurately using a measurement method based on the vibration modes and natural frequencies of rectangular panels. The method is outlined and illustrated, and some case studies discussed in which the method is applied to fibre-reinforced composite materials and to the selection of wood for musical instruments.

INTRODUCTION

Many structural materials are used in sheet form: examples range from packaging cardboard, wood and plywood to fibre-reinforced printed circuit board material, and other composite structures like honeycomb-core sandwich panels. At least approximately, most such materials share the property of orthotropic symmetry: symmetry under reflection in two particular axis directions that are mutually perpendicular. These would be the machine direction and cross direction in paper, along the grain and across the grain in a wood panel, or the directions of the reinforcing fibres in circuit board material.

It is frequently necessary to measure and control the mechanical properties of such materials: properties like density, stiffness and toughness, for example. This paper will concentrate specifically on elastic stiffnesses. For a sheet of isotropic material like steel, two independent elastic constants are needed for a full description: for example, Young's modulus and Poisson's ratio. For an orthotropic sheet material, the number grows to four: for example, Young's modulus along the two principal directions, the in-plane shear modulus (which governs stiffness to twisting motion), and the in-plane Poisson's ratio (which will be discussed in more detail below).

There are several reasons why stiffness information might be important. Most obviously, it may be directly relevant to the application: in a musical instrument body, for example, vibration resonance frequencies are central to the acoustical response and controlling these will be a primary objective of design and material selection. But there are other possible reasons for monitoring stiffness. One is for quality control in manufacture: a change in stiffness, or perhaps in the associated vibration damping (which can be measured by the same kind of method to be described here), may point to a change in raw material quality, or a change in manufacturing conditions. It may also point to the presence of localised flaws in the produced material, for example associated with bonding failure and delamination. A third reason for looking at stiffnesses is to give some insight into the micro-scale mechanics of the material: we will see an example later from the behaviour of softwood.

VIBRATION MEASUREMENT METHOD

Background

The approach to measuring stiffness via vibration is best introduced through the simplest case. If the only quantity of interest is Young's modulus in a particular direction, it can be found by cutting a long, thin strip from the sheet in the direction of interest, then making it vibrate like a xylophone bar. Such a bar, provided it is thin in comparison to its length, will have a series of resonant vibration modes as shown in Fig. 1. It is a standard calculation to find these mode shapes and their corresponding natural frequencies (see for example Rayleigh [1]). For the first mode, the frequency in Hz is given by

$$f_1 = 1.03 \frac{h}{L^2} \sqrt{\frac{E}{\rho}} \tag{1}$$

where L is the length of the beam, h is the thickness, ρ is the density and E is Young's modulus. There are similar formulae for the second and third modes, with different numerical constants in place of the factor 1.03.

The geometric dimensions of the beam are easily measured, and the density determined by weighing. If the frequency can be measured, for example by using FFT software of some kind, the formula (1) can be inverted to give the value of

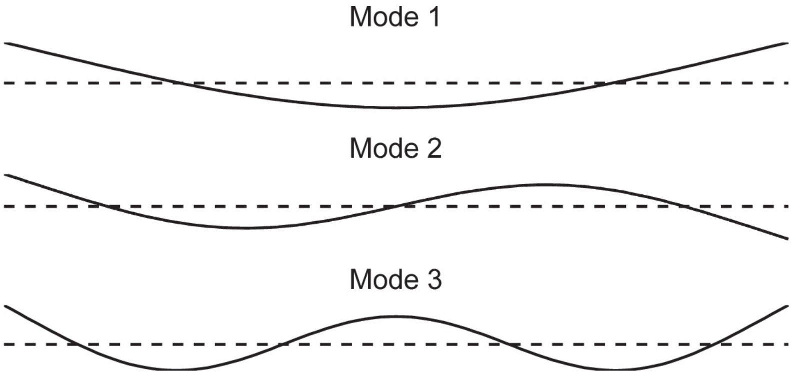

Figure 1. The first three free-free mode shapes of a thin elastic beam

E. But there is more. The formula depends on the theory used when deriving it: how can we be sure of the accuracy of the approximations underlying that theory? If the frequencies of the second and third modes can also be measured, the formulae corresponding to (1) can be used to give additional estimates of *E*. If the three estimates are in good agreement, we can have confidence that the theory is valid for the particular beam being tested.

Plate modes and Chladni patterns

Unfortunately, it is not possible to find all four of the elastic stiffnesses of an orthotropic sheet by doing one-dimensional strip tests like this. Instead, it is convenient to apply the same logic to the vibration modes and natural frequencies of a rectangular plate of the material. The theory is a little more complicated for this case, and numerical calculations are required in place of the explicit formula (1). However, it is still quite simple to carry the process through. The full details are described elsewhere [2], but the steps in the process will be outlined here. Among the low vibration modes of a rectangular plate there will (almost always: see next subsection) be found three that look more or less like the plots in Fig. 2. What is plotted here is the pattern of nodal lines: the lines where the plates are *not* vibrating. The motion can then be visualised by noting that the sign reverses whenever you cross a nodal line: alternate regions can be labelled with + and – signs. To a useful first approximation, these modes consist respectively of simple twisting, and one-dimensional bending in the two principal directions. Approximate formulae can be used to convert the measured frequencies of these three modes into first estimates of three of the four stiffnesses.

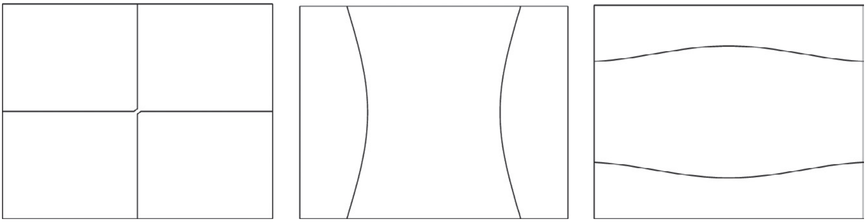

Figure 2. Nodal line patterns of three low modes of a rectangular plate of orthotropic material

How should these frequencies be determined? Unlike the beam case, we need to see the shapes of the vibration modes to be sure that the correct ones have been identified. The simplest approach is the classical method of Chladni patterns: the plate is supported on small pieces of soft foam, and set into vibration by a loudspeaker driven by a sine-wave oscillator. Powder of some kind is sprinkled on the plate: tea leaves work well. The oscillator frequency is adjusted until a resonance is located, whereupon the powder will jump into activity, and collect along the nodal lines of the particular vibration mode found. A little care is needed over details, but with practice this method can be used to locate several vibration modes in a short time. The frequency can be simply read from a frequency meter attached to the oscillator.

Some typical Chladni patterns are shown in Fig. 3. Three modes of a thin panel of Norway spruce, corresponding to the three shown in Fig. 2, are illustrated. The shapes are clearly identifiable, but notice that the second shape in this figure shows a somewhat asymmetric and distorted pattern of nodal lines. This is a result of non-uniformity of the growth pattern of this particular wood specimen. The mode shape is clear enough to allow the measurement procedure to be carried through, but the distorted shape also gives an immediate warning that significant non-uniformity is present.

Figure 3. Chladni patterns illustrating three mode shapes of a thin wooden plate

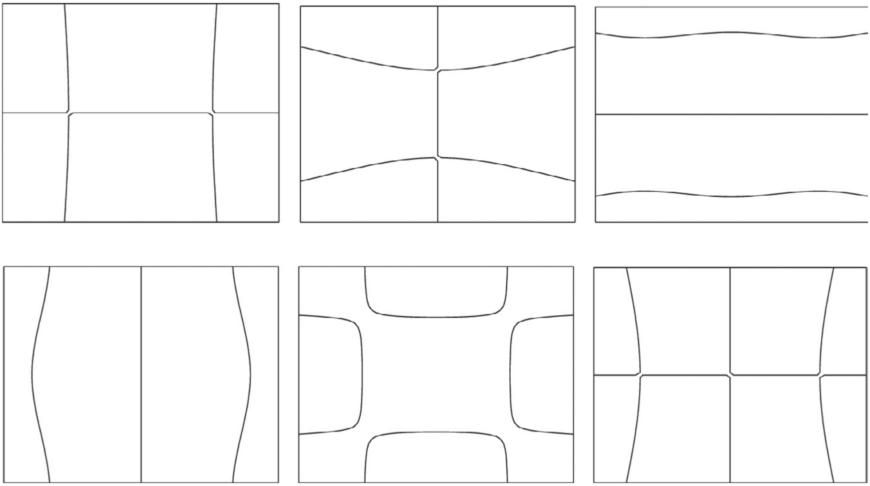

Figure 4. Typical higher modes of rectangular thin plate. The sequence of these frequencies will vary depending on the material properties and the aspect ratio of the plate.

Just as in the beam case, it is possible to measure additional resonance frequencies and use these to give a check on the accuracy of the theory being used to interpret the results and deduce stiffness values. Shapes such as those shown in Fig. 4 can be found by persevering with the Chladni pattern search. A simple computer program can then be run, taking the stiffnesses from the first approximation described above and predicting the frequencies of these higher modes. The predictions can be compared with the measured frequencies, and if the agreement is acceptably close the stiffness values can be trusted. In practice, the computer program would be used iteratively to arrive at a best set of stiffnesses to match the pattern of all resonance frequencies measured. Examples of the process are described in ref. [2].

Measuring Poisson's ratio

So far, three of the four elastic stiffnesses have been examined. The fourth one is a Poisson's ratio, and needs a little more care to obtain a reliable measurement. It is first necessary to understand how Poisson's ratio influences bending vibration of a plate. Suppose the plate is bent along the x-axis. Fibres on one side will be stretched, and those on the other side compressed. The stretched fibres will tend to shrink in the perpendicular direction from the Poisson effect. Conversely, the fibres that are compressed will tend to expand in the perpendicular direction. The result is a tendency towards *anticlastic curvature*, bending along the y-axis with the opposite sign to the imposed x-axis bend.

We now adjust the aspect ratio of our plate to maximise the influence of this effect. When the plate is made “effectively square” by scaling the aspect ratio to counteract the difference of stiffness in the two principal directions, the second and third modes from Fig. 2 combine into a pair of shapes like those shown in Fig. 5. To clarify the associated deformation, these modes are shown both as nodal line patterns and as 3D wire-frame plots. The first of the modes, the X-mode, shows strong anticlastic curvature, so it is “going with the flow” of the Poisson effect. The second mode, by contrast, shows curvature with the same sign in the two directions. This is fighting against the Poisson effect, with the result that the effective stiffness of the plate in this mode is higher, and so the frequency is higher. If the two frequencies are measured and the ratio calculated, the value of Poisson’s ratio can be read from a pre-computed design plot. The simplest version is for isotropic material, and is shown in Fig. 6. Note the sensitivity: over the usual range of values of Poisson’s ratio, the frequency ratio varies from 1 to almost 1.5. Since resonant frequencies can usually be measured to 1% accuracy or better, Poisson’s ratio can be obtained to similar accuracy, a tall order by other methods. The orthotropic case is a little more complicated: details, and a corresponding design plot, are given in ref. [2].

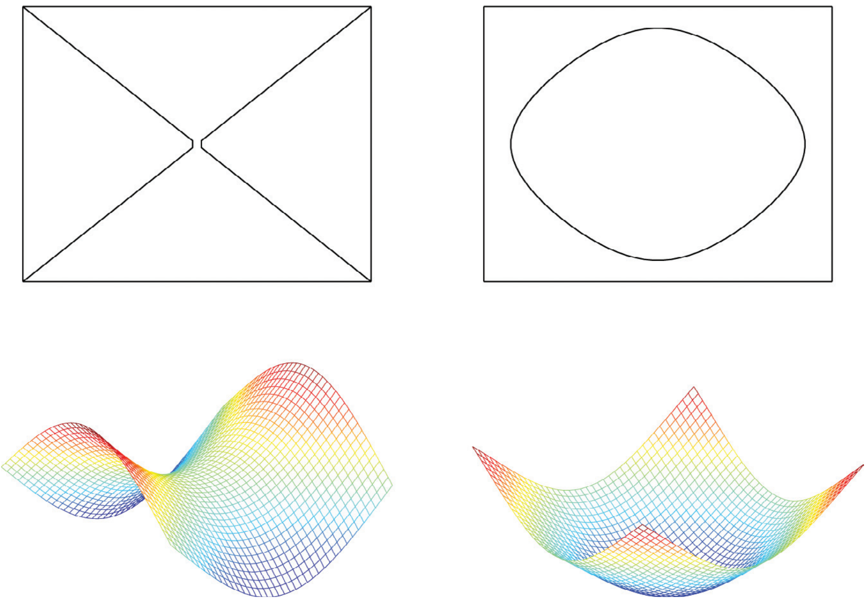

Figure 5. The X-mode and O-mode of a plate with adjusted aspect ratio to make it effectively square for bending vibration.

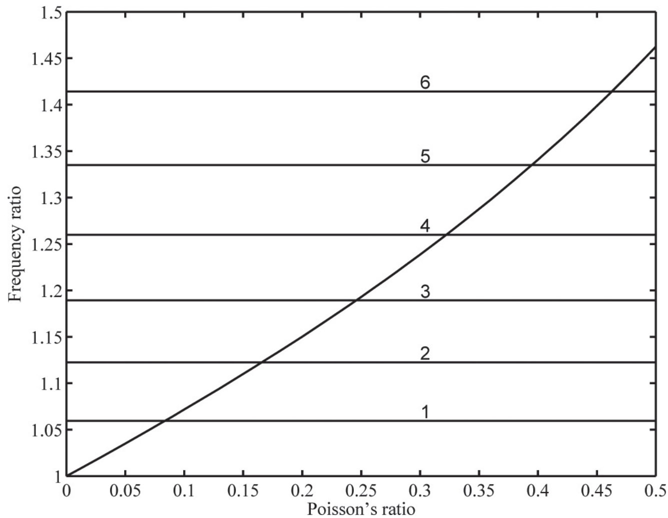

Figure 6. Frequency ratio of O-mode to X-mode as a function of Poisson's ratio, for an isotropic square plate. Horizontal lines show the ratio in terms of the number of musical semitones: Poisson's ratio can be found by listening alone, with a good musical ear.

CASE STUDIES

Printed circuit board material

One common type of printed circuit board material consists of a resin matrix, usually green in colour, reinforced by approximately equal numbers of glass fibres running in two perpendicular directions. This material gives an illustration of the pitfalls of carelessness in the measurement and interpretation of stiffness values. It is natural to cut a rectangular plate of such material with sides oriented parallel to the two sets of reinforcing fibres. Measurement of such a plate reveals, as one would expect, that the Young's modulus is roughly equal in these two directions. It is then easy to jump to the bogus conclusion that the plate is isotropic. However, a check can be made. For an isotropic material there is a formula for the shear modulus in terms of the Young's modulus and the Poisson's ratio (see e.g. [3]). This can be applied to the Young's modulus determined for the two principal directions, together with Poisson's ratio measured as just described. The shear modulus thus calculated turns out to be very different from the value determined from the frequency of the twisting mode, the first of the set in Fig. 2.

This disparity is revealing that the material is not in fact isotropic at all, and serious errors in predicting dynamic performance can arise if that assumption is made. It is not hard to see the physical reason. Consider stretching or bending the plate in a direction at 45° to the reinforcing fibres. In this orientation the fibres provide no additional stiffness: the effect is similar to stretching a woven fabric in the "bias" direction. The stiffness in that direction is determined almost entirely by the resin matrix, and will be much lower than the value in the fibre directions.

Wood for musical instruments

A second case study relates to the choice of material to make musical instrument bodies: why is wood still used, now that we have such a wide range of alternative materials available? This question illustrates yet another reason why measured stiffnesses can be important, in addition to those listed earlier: the information may be needed for design optimisation. Figure 7, reproduced from Ashby [4], summarises the stiffness and density information for a very wide range of materials. Young's modulus and density are plotted on the two axes, both with logarithmic scales because the range is so large. Most individual materials would appear as a single point on this plot, since they have particular values of both parameters. Wood appears twice, because of its anisotropy: there is only one value of density for a given sample of wood, but there are two different values of Young's modulus, along and across the grain. Materials in the same general class are grouped together within large islands: metals, polymers, ceramics and so on.

Charts like this, or the corresponding software implementation of the same database of information [5], can be used to facilitate a wide range of design calculations involving material choice. Musical instrument design gives an example, but many other examples are described by Ashby [4]. To get a first idea of why one material might be preferred to another for making a guitar soundboard, say, we can do a simple and crude calculation. The purpose of the soundboard is to take some of the energy from the vibrating string, and radiate it as sound into the surrounding air. It is acting as a kind of mechanical amplifier, and a simple preliminary design objective might be to make the sound as loud as possible. But the amplifier works by virtue of the vibration resonances of the guitar body, so if different materials are to be tested for the purpose it is important to adjust the detailed design to keep those resonance frequencies in the same place relative to the range of notes a guitar is expected to play. This can be achieved by suitable scaling of the plate thickness, keeping the plan dimensions fixed.

This leads to a simple optimisation problem: find the material that makes the loudest possible guitar, with given strings and keeping the resonance frequencies in the normal place. An elementary calculation (see for example [6]) shows that what is required is to maximise the parameter combination E/ρ^3: in other words, you want a light but stiff material, with more emphasis on the “light” than the “stiff” elements because of the fact that density appears cubed. With the logarithmic scales of Fig. 7, this has a simple geometric interpretation. Any two materials lying along a straight line with a slope of 3 would make equally loud guitars. An example of such a line is plotted: it shows that a silicone rubber guitar and a lead guitar would be equally loud.

But it is intuitively clear that neither of these materials would be a good choice. For the loudest guitar, we need to push the line as far to the left as possible, indicated by the arrow. It is immediately apparent that the most favourable materials

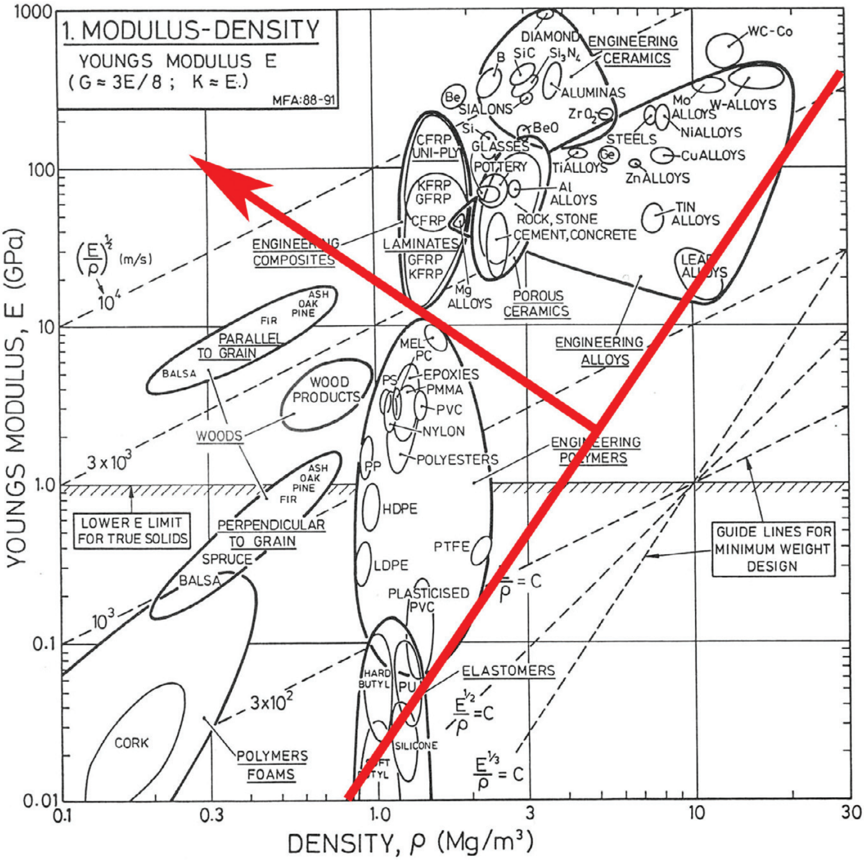

Figure 7. Summary of Young’s modulus and density for “all” materials. Superimposed line and arrow illustrate the material selection task of designing a loud guitar, see text. Plot reproduced, by permission, from Ashby [4].

are woods, especially woods at the low-density end of the range. Most musical instruments indeed use spruce or cedar species for soundboards. Note that the most extreme of all is low-density balsa wood. Balsa poses other design challenges, but some interesting experiments have been done with balsa-wood violins: see for example [7]. Note also that the material that comes closest to the performance of wood is carbon-fibre reinforced plastic (CFRP). It is no coincidence that this is the other material used for some commercially-produced high-end guitars.

One might ask why wood performs so well, and the answer to this question illustrates another of the reasons described earlier why stiffness measurement can be useful. The extreme performance of low-density wood has its origin in the microstructure of the cell arrangement. The cell-wall material of which wood is built does not itself have extreme performance: it would probably fall in the general region of the "Polymers" island in Fig. 7. But this cell-wall material is arranged in a particular way, illustrated by the micrographs in Fig. 8. The majority

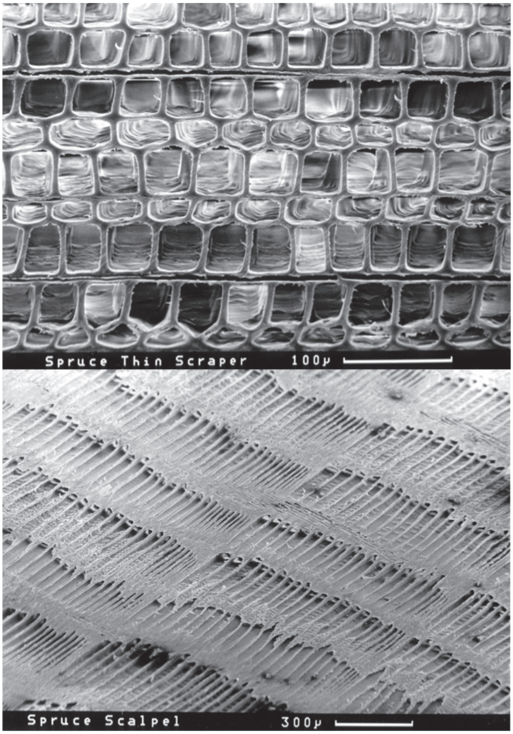

Figure 8. Two scanning electron microscope images of the cellular structure of Norway spruce (Picea abies), the commonest choice for musical instrument soundboards.

of cells in softwood are tracheids, thin-walled tubes running vertically in the tree. Once the wood is well dried, these tubes are essentially empty. From a mechanical standpoint, the result is a structure rather like a box of drinking straws. This is indeed an excellent structure for providing high stiffness-to-weight ratio [8]. The same idea is used, on a larger scale than wood cells, in the various honeycomb materials used as core material in sandwich construction for interior doors or aircraft floors.

SUMMARY

For any sheet material that shows sufficiently low vibration damping that it is possible to find resonances, vibration testing can provide a fast, accurate and appealing method to measure elastic stiffnesses. The majority of sheet materials have orthotropic symmetry, and for these there are four independent elastic constants to find. Three can be found quite quickly from the frequencies of three particular vibration modes of a rectangular plate. The fourth constant, a Poisson's ratio, needs a little more ingenuity. By adjusting the aspect ratio of the plate so that it behaves as if it were effectively square, a pair of modes are seen whose frequency ratio is very sensitive to Poisson's ratio. This provides an efficient route for measuring what is otherwise a difficult quantity to estimate accurately.

Stiffness measurement, perhaps with the additional step of measuring the associated vibration damping, can give valuable diagnostic information about a material. The method has been successfully applied to a wide range of materials, including paper products (see e.g. [9]). The results can be sensitive to imperfections of manufacture, offering a non-destructive route to quality control. They can also give information about the micro-mechanics of the material, and provide the data for optimisation of design, as has been illustrated with two brief case studies here.

REFERENCES

1. Lord Rayleigh, 1877 *Theory of Sound*. Reprinted Dover, New York, 1945
2. McIntyre ME, Woodhouse J, 1988 *On measuring the elastic and damping constants of orthotropic sheet materials*. Acta Metall. 36, 6, 1397–1416
3. Landau LD, Lifshitz EM, 1970 *Theory of Elasticity*. Pergamon, Oxford
4. Ashby MF, 1992 *Materials Selection in Mechanical Design*. Pergamon, Oxford
5. CES Selector, Granta Design, http://www.grantadesign.com
6. Schelleng JC, 1963 *The violin as a circuit*. J. Acoust. Soc. America 35, 3, 326–338
7. Revkin AC, 2006. *String theory: new approaches to instrument design*. New York Times November 28. See http://www.nytimes.com/2006/11/28/science/28acou.html

8. Gibson LJ, Ashby MF, 1988 *Cellular Solids*. Pergamon, Oxford
9. Sato J, Hutchings IM, Woodhouse J, 2008 *Determination of the dynamic elastic properties of paper and paperboard from the low-frequency vibration modes of rectangular plates*. Appita J. 61, 291–296

DISCUSSION CONTRIBUTIONS

Chair: Stephen I'Anson

FRC Chairman

Steve I'Anson FRC Chairman (from the chair)

Jim, I heard a piece on BBC Radio 4 (the UK's adult, educated radio station, for those of you who do not know) suggesting that the Stradivarius violins were so fantastic because of a period of particularly bad weather, that gave rise to very thin tree rings. Is that right?

Jim Woodhouse

I think this is nonsense. It is a typical example of a little bit of information being a dangerous thing. Violinmakers and instrument makers typically buy their spruce from mountainous places. As you walk up the mountain side, the tree rings get closer, as you approach the tree line, so you can choose whether you want wide but sparse rings or closer rings or really close indicating that the tree is hardly growing at all. The bad weather meant that the average moved down the hill a bit, but that did not change what was available; it just meant you made your choice slightly differently. So a period of cold weather did not create a kind of wood that wasn't already there, they just had to look in a slightly different place to find what they needed, and this is well understood by the people who go and harvest those trees. Not only that, but you count the rings in a typical front of a cello and there may be 250 years of growth, with a minimum of around 50 years, so I think the cold weather would produce just a little stripe in the middle. So I think that this explanation does not have much substance.

Steve I'Anson

So is this just an excuse from the modern instrument makers?

Jim Woodhouse

Oh no, it wasn't the instrument makers that proposed this, it was just somebody wanting to get in the press by discovering the secret of Stradivarius. There is a long history of people who know very little claiming "I have discovered the secret of Stradivari".

Roger Gaudreault Cascades

What is the material that is used for the electric violin or non-wood violin? Can you comment on the differences?

Jim Woodhouse

Electric violins are different because, actually with an electric violin, you do not want mechanical sounds; it is just a way of holding the strings as with an electric guitar. The sound from the vibration of the strings is amplified electronically rather than mechanically. So for that kind of electric violin, you do not want a light and stiff material, you want something heavy and stiff. You want it not to vibrate, although the player grumbles if it's too heavy; you want to make the structure that has the least vibration rather than the most. So the design is not so much of a material choice, although that's why they are usually skeleton shaped, so that they do not have big vibrating areas. Even if it is made to look like a violin, it may be filled with rubber or sand to stop it from vibrating.

There are musical instruments using composite materials and, if you look back at figure 7 in my paper, the last materials you passed by before you got to the woods were carbon fibre reinforced polymers which are the closest you can get. The Ovation guitars pioneered their use a long time ago. Carbon fibre laminate with a light core material can get close to wood and it is somewhat more repeatable than wood.

Steve I'Anson FRC Chairman (from the chair)

Am I correct that these instruments have shaped backs?

Jim Woodhouse

You can make shapes easily but it is not so easy to perform some of the instrument makers' tasks. You have to use different glues that they'd rather not use and so on. You can just about get there but you still can't really beat the best wood, you can just about equal it. There are very good carbon fibre violin bows because the problems are slightly different there. So new materials are creeping in. Instruments makers are actually very keen on light and stiff properties, so they do all kinds of tricks with composite structures, including mixtures of woods, to push further and further towards the limit. They want to improve the instruments to make them more saleable and profitable.

Hahn-Ning Chou Thepharak

Just a quick question: from a cost standpoint, what is the difference between natural materials and composite materials, with regard to musical instruments?

Jim Woodhouse

That is a quick question, but it does not have a really quick answer. The cost of the materials instrument makers would most like to use is going up, partly because of their scarcity and partly because some of the traditional tropical hardwoods are on the Convention on International Trade in Endangered Species of Wild Fauna and Flora (CITES) list. There are issues with ebony fingerboards and that kind of thing, where it is becoming impossible for makers to buy the traditional materials that were used. Not because the forests have been denuded to make musical instruments, they've been denuded to panel boardrooms, but, nevertheless, CITES is an ingredient there, as the restrictions on exporting some materials are getting understandably more severe. I don't think that the composite materials that can come near the same performance are getting cheaper but the woods are definitely getting more expensive, and so the switchover point is getting closer, although this is not so much the case for the sound board materials that I've been talking about, because these are temperate woods. Mountain grown softwoods still exist and are not so difficult to find, although some of the ones they most like to use often have shrapnel embedded in them. It lasts a long time and sawmills don't like trees with bits of metal in them. All the same, on the softwood side of things, there's not a big supply problem, although makers are very fussy; they like to convert the tree in a way which is rather wasteful. They prefer to have it radial-sawn rather than planked because they want each piece to be quarter-cut, so they waste quite a lot, and that pushes the price up quite a bit.