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THE CALENDERING CREEP EQUATION – A PHYSICAL MODEL

R. E. Popil
MacMillan Bloedel Research, 3350 East Broadway,
Vancouver, British Columbia, Canada, V5M 4E6.

ABSTRACT

The modelling of the calendering process has been largely empirical, resulting in "creep" equations which relate the finished paper properties to calender parameters. Such modelling has the utility of optimizing calendering configurations for the attainment of a desired paper finish. This approach demonstrates the limitations of machine calendering and other alternatives to reach higher levels of surface finish are suggested.

This study endeavours to establish an understanding of the physical basis for the form of the calendering creep equation. A simple physical model of calendering has been developed which allows at least the qualitative prediction of various calendering effects. The physical model of paper compression is based on the non-Hookean behaviour of paper under compression which is known to arise from the statistical distribution of the number of fibers in a paper web. Elastic constants associated with the exponential stress-strain relation for paper determine the dependencies of caliper reduction on moisture, temperature and fiber processing. A simple viscoelastic model suggests that

the dependencies on moisture and temperature may not be as autonomous as they appear in the usual forms of the creep equation. This observation is corroborated by experimental data obtained on pilot laboratory calenders.

INTRODUCTION

A great deal of effort is presently being concentrated throughout the paper industry towards the development of high printing performance, high gloss groundwood printing papers. The most common method for the production of high gloss paper has been multi-nip off-line supercalendering which is costly in terms of productivity per unit time. The search for more attractive alternatives has been evidenced in recent years by the proliferation of combinations of on-machine soft-nip and temperature gradient calenders. Variables such as the roll temperature, paper elasticity, paper moisture all play a part in the paper finishing process. In order to understand how these variables affect the paper finish, a mathematical model of the paper surface finishing process is beneficial.

Background

The effects of compression on paper have been previously quantified through experiment by Colley and Peel (1) with a subsequent adaptation by Kerekes (2) to a calender nip. Crotogino (3) has generalized the empirical creep equations to include all the parameters of a conventional mill calender stack. The equation has been applied to several mill situations (4) to predict the final permanent relative compression ε, which is defined as:

$$\varepsilon = B_i - B_f / B_i \tag{1}$$

where B_i , B_f are the initial and final bulks respectively. The calendering equation takes the form of:

$$\varepsilon = A + \mu B_i \tag{2}$$

where μ is referred as the "nip intensity factor" and is in turn:

$$\mu = a_1 + a_L \log L + a_S \log S + a_R \log R + a_\theta \theta + a_M M \tag{3}$$

The quantities A, a_1, a_L, a_S, a_R, a_θ, and a_M are all furnish dependent constants but average typical values for newsprint groundwood furnishes (5) can be ascribed to be:

$$\begin{aligned} A &= -0.422 \pm 0.031 \\ a_1 &= 0.0283 \pm 0.010 \\ a_L &= 0.096 \pm 0.002 \\ a_S &= 0.0228 \pm 0.0038 \\ a_R &= -0.0366 \pm 0.0009 \\ a_\theta &= 0.00089 \pm 0.000068 \\ a_M &= 0.00645 \pm 0.000632 \end{aligned} \tag{4}$$

The calender parameters are:

L(kN/m) = nip load
S(m/min) = machine speed
R(m) = roll radius
θ ($^\circ$C) = paper temperature
M (%) = paper percentage moisture

The calendering equation (2) is used iteratively for each nip in the calender stack to determine the final bulk. The logarithmic and linear dependencies originally arose from graphical analysis of the data by Colley and Peel. A primary difficulty and inconvenience in applying the calendering equation in optimizing a calender stack is that the furnish dependent constants have to be determined by calendering trials and fitting the equation to the experimental results. However, once the constants are determined for a particular furnish by calendering on a small, single nip pilot calender, the calendering equation can then be applied to any machine calendering configuration. A labor saving fact pointed out by Kerekes (2) is that the dependence on calender roll radius arises from the dependence of paper compression on the nip dwell time which in turn is related linearly to the nip load and roll speed:

$$a_R = -(a_S + a_L)/2 \tag{5}$$

so that a_R does not have to be determined independently. The "a_i" coefficients are all determined by single nip calendering, varying one parameter at a time while keeping others fixed. The coefficients are then calculated by a linear regression fit of the data. The form of the calendering equation presented above places a lot of faith in the independence of the variables, for instance, that the final reduced bulk remains proportional to the log of the applied load L irrespective of the moisture content or temperature of the paper. There is no **a priori** reason why the calendering equation should take the form suggested by the data of Colley and Peel. It is the purpose of this paper to explore the nature of the regression equation for paper compression and to suggest a physical basis for it.

Limits of machine calendering

The objectives of calendering are also to attain a desired level of surface smoothness suitable for printing as well as a pleasing level of gloss. In machine calendering, both these paper properties are completely determined by the amount of densification effected by the calender stack. Thus, the smoothness and gloss are found to scale in the same fashion as the bulk reduction. Stated simply, the Parker Print-Surf S-10 roughness (PPS) and the Hunter gloss (Gloss) take the form (4):

$$PPS = A_1 \, (B_f) - A_2 \tag{6}$$

$$Gloss = A_3 \, (B_f)^{-\lambda} \tag{7}$$

where the coefficients A_1, A_2, A_3 and λ are all furnish dependent coefficients and must once again be determined experimentally along with the calendering equation coefficients. For North American west coast furnishes, $A_1 = 3.8$, $A_2 = 2.5$, $A_3 = 28$ and $\lambda = 2.0$ approximately for a 75% groundwood (GWD) and 25% semi-bleached kraft (SBK) furnish.

It can very well be argued that the objective of calendering is entirely to achieve a smoothness suitable for the printing process for which the product is intended. Bulk reduction is merely the simplest and the most conventional means to achieve the required degree of smoothness. However, the limitation on this process is that machine calendering breaks fiber bonds upon compaction thus weakening the press runnability of the web. Moreover, a detrimental effect that usually precedes decrease of strength is the loss of opacity due to the disappearance of light scattering surfaces upon compaction. The latter phenomenon is commonly referred to as calender blackening which is evidenced by the calendered paper web becoming speckled with small translucent zones that seriously interfere with good print quality.

Calender blackening

Paper moisture level when calendered appears to have the most significant effect on calender blackening. The bulk at which calender blackening starts to occur was examined in detail by using a single nip of a pilot calender to densify single sheets of standard newsprint consisting of 25% SBK and 75% GWD. The samples were conditioned in rooms of various humidities and kept in plastic bags prior to and immediately after calendering. The moisture content determined by oven drying and weighing of samples varied from 1% to 16% by weight. Calender loads were varied to extreme levels (9000 pli or 1576 kN/m) to obtain pronounced levels of blackening. The results are summarized in Fig. 1, where a curve is shown which separates blackened samples from unblackened ones.

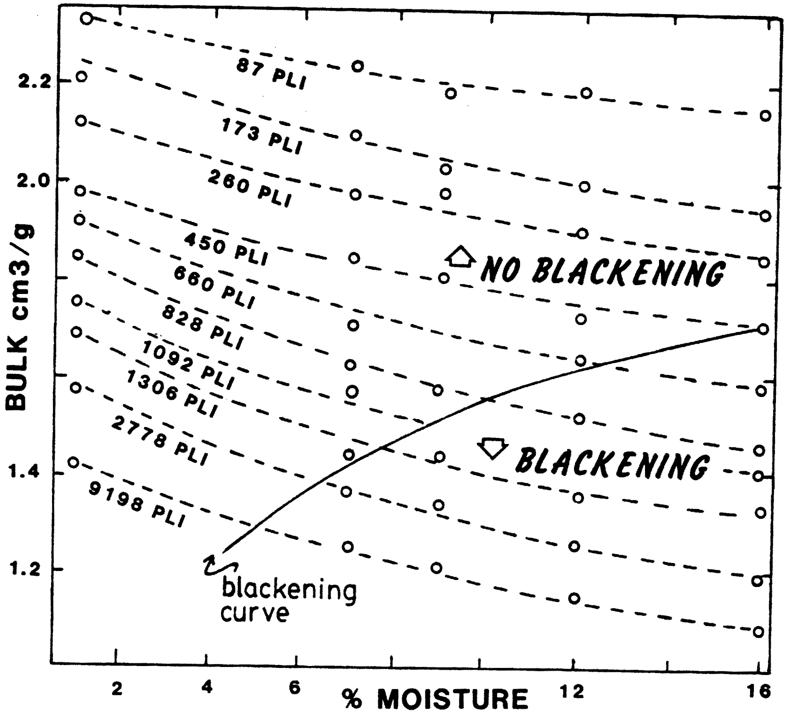

Fig. 1. Results of single nip calendering to determine blackening conditions.

The onset of blackening was determined by close visual inspection of each series of calendered sheets. It is easy to pinpoint the critical density at which blackening just starts to occur. A criterion relation can now be established between the final reduced bulk of a sheet and its moisture content at the time of calendering to that final bulk:

$$B_f \geq 0.95\ M^{0.212} \qquad (8)$$

The final bulk of the sheet after calendering at a given moisture level must be above the value given by equation (8). This blackening criterion was found to agree quite closely with newsprint mill observations of blackening in connection with cross machine moisture profiles.

The blackening criterion establishes the limitation of machine calendering. For example, to achieve a 40 point Hunter gloss or a PPS roughness of 2.5 microns the density of the compacted paper by equations (6) and (7) would have to be greater than that of crystalline cellulose. Given a limiting bulk of 1.5cm^3/g for a typical finishing moisture level of 7% by virtue of equation (8), the maximum attainable surface roughness and gloss would then be 3.2 microns and 12 Hunter points respectively. Attempts at exceeding these limitations by further densification by machine calendering would lead to unacceptable levels of calender blackening followed by pronounced losses in tensile strength. It is expected that smoothness and gloss will not be dependent on bulk reduction alone when processes other than machine calendering are used. Supercalendering and temperature gradient calendering, which preferentially act on the surface of the paper rather than the entire bulk, can be expected to result in a different dependence of gloss on other variables besides bulk alone.

ALTERNATIVES TO MACHINE CALENDERING

Supercalendering

Supercalendering is the conventional method of increasing surface gloss without a deleterious decrease in bulk. A supercalender nip differs from a machine calender nip in that shear stresses are present along with normal stresses. Driving forces are transmitted to the supercalender soft roll and friction opposes the relative movement of the surfaces. Both these effects cause an observable difference in the surface velocities of the rolls measured away from the nip which is called "creep" in distinction from paper compression creep. The action of a supercalender is that paper is subjected to normal vertical and horizontal compression and also to shear arising from the difference in the elastic properties of the rolls. This amount of shear, which manifests itself as creep, can be calculated from calender parameters and roll material elastic constants. This sort calculation allows a judgement of the effectiveness of a particular supercalender nip.

Peel and Hudson (6) summarized the theory of elasticity as applied to a supercalender nip. They quote an expression for the creep ratio, i.e., the relative speed difference between a driven roll and a filled roll. The expression for the creep ratio was found to agree well with their experimental results with an eight roll supercalender. Elasticity theory of an elastomeric roll driven by an iron roll allows the calculation of the creep ratio using Young's elastic modulus along with the Poisson's ratio of the rolls. The results of elasticity theory may be applied to the nip of a given on-line soft calender once its elastic constants are known, to calculate a creep ratio which would be indicative of the potential effectiveness of the nip in simulating supercalender action.

Supercalendering action is also thought to arise from "microslipping" where the nip is comprised of locked strained surfaces and partly of a microslipping region. Slip of the elastic roll against the rigid roll will occur when the shear stress exceeds the frictional force that arises from normal loading. Calculations show that this condition is usually satisfied in a supercalender nip. The resulting shearing action is thought to enhance the glaze of the paper surface when it is placed in a nip where such slip occurs by a burnishing sort of action.

The role of microslip in calender action has been proposed and supported by Peel and collaborators in several other publications (7,8,9). These investigations were carried out on casein bonded clay coated paper since it is most significantly affected by supercalendering. Increases in gloss and roughness occur for clay coated papers most because the flat clay platelets are thought to become aligned parallel to the paper surface under the influence of the cyclic shear and microslipping in a supercalender nip. The implication of these studies is that supercalendering action may be enhanced or even emulated by the application of braking to an elastomeric driven roll.

Temperature gradient calendering

Recent results (10 - 12), indicate the pronounced advantage in single nip temperature gradient calendering. The idea here is to pass the paper between rolls heated to a high temperature such that heat transfer allows plasticization of only the surface of the paper. It is thougt that there exists an optimum critical strata beneath the surface of the paper which must be molded flat to get smoothness and gloss. The sudden improvement in gloss and smoothness that is observe to occur with increase in roll temperature is believed to be due to a critical portion of the web beneath the surface being heated to its glass transition temperature, T_g. According to Vreeland (11), the approximate location of the critical substrate is believed to be about 8 microns.

The essential equations for temperature gradient calendering are as follows. The space-time heat conduction across a boundary is given by the expression (11):

$$\frac{T(x,t) - T_o}{T_i - T_o} = \operatorname{erf}\left[\frac{x}{2}\frac{1}{\sqrt{at}}\right] \qquad (9)$$

where $T(x,t)$ = temperature in Celsius degrees at distance x into the web at time t, T_o = surface temperature of the heated calender roll, T_i = initial temperature of the paper entering the nip, a = heat conductivity of the paper taken to be 0.005 ft^2/hr, t = calculated as the time of the web in the nip in hours. The nip width is best estimated from measurements of the pressurized nip impression on pressure sensitive paper of the same caliper as the uncalendered paper. The glass transition temperature, corrected for the dynamic conditions existing in a calender nip, is given to a good approximation by (11):

$$T = 234.2 \times e^{-0.131M} \qquad (10)$$

where M is the percentage moisture content by weight. The experimental conditions of Crotogino (12), Kerekes and Pye (13) were used to solve the heat conduction equation for the depth of the plasticization temperature contour. The depth turns out to be 12 and 9 microns respectively, close to the 8 microns of Vreeland's patent. This means that only the top layer of fibers are heated beyond their plasticization temperature in temperature gradient calendering.

Equa'tions (9) and (10) can be used to evaluate the potential effectiveness of any given temperature gradient calender configuration. The key issue here is to ensure that the nip dwell time and temperature differences are such that only the top layer of fibers is heated to the plasticization temperature.

The temperature gradient calendering data of Crotogino and Gratton (10) were analyzed to generalize the results to include the effect of roll temperature in a single temperature gradient calender nip. The data from Figure 11 of reference 19 was linearly interpolated to obtain the result:

$$\text{T-G Gloss} = (0.251 - 0.108\, B_f)\, \theta + (11.54 - 2.86\, B_f) \qquad (11)$$

where T-G Gloss is the gloss achieved with a single nip calender that has reduced the paper to a final bulk B_f at a roll temperature of θ.

Equation (11) shows that it is possible to attain high gloss levels at higher bulks than by machine calendering alone. For example, if a roll temperature of 300 ^{o}C were possible, then the T-G gloss equation predicts that 40 point gloss can be obtained at a bulk of 1.3 cm^3/g which may easily be in a regime where calender blackening may be avoided by keeping the moisture levels during calendering below 6%.

PILOT CALENDER TRIALS

Calendering investigations at MacMillan Bloedel have shown that the conventional form of the calendering equation (2) cannot be used to describe the effect of temperature. Mitchell (14) calendered rolls of MacMillan Bloedel newsprint containing up to 40% TMP on a 5 nip Beloit laboratory pilot calender stack. Empirical regression equations were derived from the results, with moisture, temperature, roll speed and nip load as the variables. The study was initially undertaken to examine the effect of TMP content in the furnish, however, caliper was found to be dependent only on the amount of SBK in the furnish. Two separate sets of equations were necessary to describe the data at the two different roll temperatures 54 ^{o}C and 77 ^{o}C. Since the effect of roll temperature is expected to be linear, the results can be summarized into the form:

$$\begin{aligned} T'' = & - (68.4 - 0.674\,\theta) \log L \\ & - (0.7 - 0.00565\,\theta)\, S \\ & - (0.82 + 0.0348\,\theta)\, M \\ & + 17 \log V \\ & + (190 - 1.43\,\theta) \end{aligned} \qquad (12)$$

where θ = roll temperature, ^{o}C, L = nip load in kN/m, S = % SBK content, M = % moisture content, V = roll speed, m/min, and T" = final calendered caliper normalized to a bulk of 48.8 g/cm^2 i.e.,

$$T'' = \frac{T_{actual}}{(\text{Actual Basis Weight})} \times 48.8\ g/m^2 \qquad (13)$$

where T_{actual} = the actual measured final caliper in microns. This normalization accounts for the fact that smaller calipers are achieved for papers of greater bulks for the same calendering conditions. This is simply because greater bulks are more easily compressed due to a smaller solid fraction. Equation (12) is the preferred form of the calendering equation in industrial circles as caliper is more readily measured than bulk.

The striking feature of equation (12) is that the coefficients of the calender parameter variables are functions of the roll temperature. This is a departure from the usual regression equation (2) for calendering which has the dependence on temperature as an isolated linear term.

Further calendering trials on standard west coast newsprint were done using single nips of pilot calenders. Roll diameters were 36 cm and the length 84 cm. Calender variables were the load, roll speed and the moisture content. Calender nip load was varied from 90 to 9300 lb/in (15.8 - 1628 kN/m) and the final reduced caliper was found to have a logarithmic dependence on the nip load with a correlation coefficient of $r^2 = 0.97$. Calender roll speed was varied form 15 to 366 m/min (48 - 1200 fpm) and a logarithmic dependence on roll speed was found with an average $r^2 = 0.86$. Moisture was varied from 1% to 16% by storage of PM6 sheets in conditioned rooms of different humidities for a twelve hour duration prior to calendering. Moisture content of the samples was determined by oven drying and weighing the samples. The dependence of the final caliper on moisture was found to be linear and the average r^2 to be 0.94. The resulting calendering regression equation is :

$$T'' = -25.3 \log L + 5.7 \log V - 1.03 M + 134 \quad (14)$$

where T" is the normalized final caliper in microns.

Comparison can be made with other previously obtained equations if the relative compression ε is defined as follows:

$$\varepsilon = (T''_f - T''_i)/T''_i \quad (15)$$

where T''_i and T''_f are the initial and final normalized calipers. The regression equation then takes the form:

$$\varepsilon = 0.22 \log L - 0.049 \log V - 0.088 \qquad (16)$$

and is in good agreement with the single nip calendering equation of Kerekes (2) :

$$\varepsilon = 0.25 \log L - 0.052 \log V - 0.090 \qquad (17)$$

If the appropriate calender variable substitutions are made into the equations of reference (5) such as moisture to be 6.5 %, the roll radius to be 0.178 m, the temperature to be 23 ^{o}C, then the resulting equations are:

$$\varepsilon = 0.257 \log L - 0.0617 \log V - 0.127 \qquad (18)$$

for a west coast furnish of 76% GWD and 24% SBK and:

$$\varepsilon = 0.245 \log L - 0.0701 \log V - 0.167 \qquad (19)$$

for the other west coast furnish consisting of 49% GWD, 32% TMP and 19% SBK. Thus, there is a consistency amongst all the obtained calendering equations at room temperature.

However, a different regression equation appears to be necessary as the roll temperature increases from room temperature. Indeed, it is generally found in our laboratory trials that at least the log L coefficient of equation (14) increases with the roll temperature. This indicates that the calender load is less effective in densifying the sheet as the roll temperature increases. This may be an expected result if the dominant mechanism taking place were temperature gradient calendering whereupon the calender load was being expended on the top layers of the sheet thereby making bulk compression comparatively ineffective. However, calculations using equations (9) and (10) show that for the low roll speeds used, the plasticization isotherm is 21 microns into the sheet, about a factor of two greater than the plasticizing depths of other temperature gradient investigations.

Experiments were also repeated at slower speeds on a smaller laboratory calender consisting of a single heated nip of roll diameter of 11.4 cm and roll length of 38 cm. In this case the penetration depth of the plasticization isotherm contour is calculated to be 27 microns. An increase of the log L coefficient with increase in temperature was consistently observed for both 116 micron 49 g/m^2 paper and 65 micron 31 g/m^2 paper. A study of the log L coefficient as a function of roll temperature was attempted. The exact dependence is unknown at this time as there is much scatter in the data. A linear approximation to the temperature dependence of the log L coefficient is sufficient at this point. A summary of the data is presented in Table 1 below:

Basis wt. g/m^2	Roll Temp ^oC	Roll Speed m/min	Log L coeff.	Intercept
31	150	156	-37.4	125
31	25	156	-21.5	145
49	122	49	-37.2	142
49	18	49	-30.7	147
52	122	49	-33.3	145
52	18	49	-28.7	136
52	112	49	-34.5	137
52	18	49	-27.7	141
49	18	49	-30.0	145
49	112	49	-41.2	148
31	18	49	-19.4	120
31	112	49	-27.8	119
49	21	136	-30.9	153
49	120	136	-34.0	150

Table 1. Summary of results of single nip calendering

Inspection of Table I. shows that the coefficient of the log P term in the regression equation invariably increases with an increase in roll temperature.

MODELLING PAPER COMPRESSION

The behaviour of paper under compression may be explained at least in part by analogy. This is commonly done in visco-elasticity theory to describe the effects of forces and stresses on polymer materials (15). Some applications of viscoelastic rheology to wood deformation have been published (16) along with studies on loudspeaker papers (17),and static tests on compressed paper layers (18). However, there is no satisfactory model of the dynamic compression that occurs in a calender nip.

Viscoelastic models consist of various combinations of springs and dashpots. The springs are idealized Hookean springs obeying the law:

$$F = -kx \tag{20}$$

where F = force, k = the spring constant, x = displacement from equilibrium. The springs are used to describe the elastic part of the deformation of materials, namely their ability to recover their initial shape after the stress is removed. This elastic response to stress is instantaneous, however, for most materials one observes both an elastic and a retarding viscous response. The viscous response is a function of time and is modelled by a dashpot - a loose fitting piston surrounded by an energy dissipating fluid of viscosity η. The rate of displacement is inversely related to the viscosity and causes a permanent nonrecoverable deformation after the force on the piston has stopped.

A model that can be used to describe paper compression is known as the Burgess model shown diagrammatically in Figure 2.

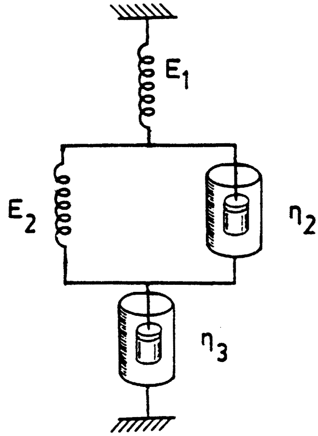

Fig. 2. Diagram of the Burgess model of viscoelasticity.

It consists of Newtonian dashpot element 3 and Hookean spring element 1. Together these two elements comprise the Maxwell model of viscoelasticity. The parallel combination of spring and dashpot of element 2 is known as the Voigt model. The time retarding effects of response due to viscosity are described by the Voigt model. When the Burgess model is subjected to stress σ, the displacement from equilibrium position is given by the following expression:

$$\varepsilon(t) = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \frac{\sigma}{E_1} + \frac{\sigma}{E_2}(1 - \exp -tE_2\eta_2) + t\sigma/\eta_3 \qquad (21)$$

The subscripts refer to the numbered elements in the model. The elastic term σ/E_1 as well as the term ε_2 are both recoverable once the stress σ is relieved. However, the viscous term ε_3 leads to a permanent deformation of:

$$\varepsilon(t) = t_1\sigma/\eta_3 \tag{22}$$

where t_1 is the time duration of application of a constant stress σ. For a compressive applied stress $-\sigma$, the sign must be altered for the relative displacement. This is the situation for a Burgess model consisting of Hookean springs and Newtonian dashpots. However, there is considerable evidence that paper cannot be modelled by Hookean springs.

Pfeiffer (18) for example studied the velocity of sound through paper under various applied pressures and deduced an exponential stress-strain curve relation for paper given by:

$$\sigma = - K_1 \exp [K_2 \varepsilon] \tag{23}$$

For σ in units of psi, the constant K_2 takes on values from 40 or lower for compressible papers such as newsprint. Differentiation of equation (23) of σ with respect to the strain ε defines the usual form of the compressive modulus of elasticity namely that

$$d\sigma/d\varepsilon = E_z = K_2\sigma \tag{24}$$

which shows that the modulus of elasticity increases linearly with the applied stress σ.

The dependence of the compressive elastic modulus on the compressive strain has been derived theoretically by Osaki et al (19). Analysis of their experimental curve for the compressive modulus as a function of the applied stress at 35 Hz frequency gives the expression:

$$E_z = 1.1 \times 10^2\sigma^{0.89} \quad \text{psi} \tag{25}$$

which is in agreement with equation (24). Exponential stress-strain relations have been observed for single fibers by Nyren (20) and Balodis et al. (21) using microscopic techniques. Certainly, if the stress-strain relationship for a single fiber is non-linear then it must be so for a paper sheet as well.

Bleisner (22) followed the consensus that compressibilities measured at stressing times of about 1/10 second are sufficiently rapid to characterize the dynamic compression properties of paper. Analysis of a sample oscilloscope trace from reference (22) displaying the loading cycle of a paper sample produces a stress-strain relation of the form:

$$\sigma = 54\, e^{9.47\varepsilon} \tag{26}$$

which is once again, in agreement with equation (23). Bleisner investigated the compressibility of single laboratory handsheets while Pfeiffer and Osaki et al., used stacks of single sheets, hence the difference in the numerical constants.

Theoretical considerations by Ionides et al. (23), also lead to the conclusion that the compressive strain-strain relation for paper is exponential. The derivation is based on the fact that on a microscopic scale the number of fibers per unit area is described by a Poisson distribution. The well known Hertzian expression for the force required to deform two cylinders in contact is also invoked:

$$f = (2/3)\, h^{3/2} \left[\frac{E}{1 - \nu^2}\right] (R/2)^{1/2} \tag{27}$$

where the compressive elastic modulus is E and Poisson's ratio is ν. The total required force to compress a paper web is then taken as the summation of the product of the Hertzian force and Poisson distribution for the number of fibers in a pile. The result is graphically plotted as a series of parametric curves. Analysis of the n_f vs ln σ/C graph of Figure 5 of reference (23) shows that the relationship between the applied stress and fiber deformation is:

$$\sigma = 619 \times C \exp -1.43\, n\, (1 - \varepsilon/d) \qquad (28)$$

where ε is the deformation of a single fiber of initial diameter d in a pile of n fibers. The numerical constants here are those that produce the best agreement with empirical data. The constant C takes the form:

$$C = (2/3)\,(d/2)^{1/2}\, e^{\lambda}\, \frac{E}{(1 - \nu^2)}\, d^m \qquad (29)$$

where λ is the mean number of fibers and m is the power law dependence of the force. Equation (28) shows that the form of the stress strain relationship for paper is exponential.

Therefore, there is considerable experimental data and theoretical work that supports an exponential stress-strain relationship for paper. This non-linear form of Hooke's law can be incorporated into a viscoelastic model for the compression of paper. For simplicity, the Voigt terms are ignored and only the Maxwell model considered of a spring and dashpot in series. Assume that the viscosity is non-Newtonian and is of the form:

$$\eta = \frac{\sigma}{K_3\, d/dt(\exp[K_2\varepsilon])} \qquad (30)$$

If we assume application of a constant stress σ_o for duration of time t_1 then the displacement is the sum of the now non-linear elastic and plastic components:

$$\varepsilon(t) = \varepsilon_1 + \varepsilon_2 = \frac{1}{K_2} \ln\left(\frac{\sigma_o}{K_1}\right) + \frac{1}{K_2} \ln\left(\frac{t\sigma_o}{\eta K_3}\right) \qquad (31)$$

using equations (23) and (30).

The permanent deformation after application of the stress for duration t_1 is given by the viscosity term:

$$\varepsilon_f = -\frac{1}{K_2} \ln (\sigma_o t_1/K_3 \eta]$$
$$= - 1/K_2 \ln (\sigma_o t_1/K_3 \tau E_z) \tag{32}$$

where $\eta = \tau E_z$ with τ being the characteristic relaxation time of the viscous flow. The above equation (32) is the general form of the calendering equation. It shows that the final caliper reduction after application of the step load is proportional to the logarithm of the applied compressional stress and to the logarithm of the time duration of that stress. Thus, calender parameters that prolong the duration that the paper spends in the nip such as roll diameter and roll speed contribute positively to the reduction of caliper. Examination of equations (3), (12 -19) all show that caliper or bulk reduction is proportional to the logarithm of the nip load while the logarithm of the roll speed reduces the bulk reduction. The number of nips also affect t_1 so that to a first approximation it may be expected that caliper reduction is proportional to the logarithm of the number of nips in a stack. The data of Howe and Lambert (24) of the successive caliper reduction in a calender stack was examined. It was found that the caliper reduction is indeed proportional to the logarithm of the number of nips with a regression coefficient of r^2 of 0.98.

Inspection of the general calendering equation shows that it is inversely proportional to the compressional constants E_z and K_3. The Ionides' expression for the stress-strain relationship (28, 29) of paper indicates that K_1 and consequently K_2 is proportional to the elastic modulus. Thus any factors that contribute to diminishing the compressional elastic modulus would result in additional caliper reduction.

Recently, Batten and Nissan (25) have written a series of articles where they theoretically derive a temperature and moisture dependent form of the elastic modulus for an ideal, isotropic cellulosic paper. The derivation treats paper as a continuum of H-bond dominated isotropic structures meaning that the mechanical properties of the structure are primarily governed by the characteristics of the hydrogen bond. Although the theory is probably more suited for the in-plane modulus for paper, the dependencies on moisture and temperature should be expected to hold for the out-of-plane compressional modulus as well.

The results of the Batten Nissan work can be summarized by their equation for the moisture and temperature dependent elastic modulus:

$$E = E_o \exp \left[- 2.4 \times 10^{-3} (\theta - 25) - 6.407M + 0.2433 \right] \qquad (33)$$

where E_o is the elastic modulus of dry paper at 25 ^oC and 0% moisture content.

The significance of this result on the final caliper is realized upon substitution of the elastic modulus expression above into that of E_z in the final caliper:

$$\varepsilon_f = - 1/K_2(\ln \sigma_o + \ln t_1 - \ln \Upsilon K_3 - \ln E_o + \alpha\theta + \beta M + K) \qquad (34)$$

where α, β, and K are all constants.

The form of the Batten and Nissan elastic modulus has produced the result that caliper reduction in calendering is linearly proportional to the temperature θ and to the percentage moisture content M. These are the forms of the dependencies of the caliper creep equation on moisture and temperature. A major point to be noted here is that the influence of moisture is much greater than that of temperature on the elastic modulus and hence paper compression. For example, increasing the moisture content from 4.5% to 9% has the same effect as increasing the temperature from 25 to 150 ^oC.

The form of the final caliper equation (34) has the coefficient $1/K_2$ for all the variable terms. It is expected that this term would also be dependent on the elastic modulus of paper. If this is the case, then all of the constants in front of the variable terms in the calendering equation would vary with moisture and temperature. The work presented by Mitchell as well as the calendering results presented herein indicate that the constant K_2 is dependent on the temperature. Indeed, if it is proportional to the elastic modulus as well, an increase in temperature would lower the modulus thereby increasing the coefficient of the log P term as observed and noted in Table 1.

It is expected that concentrations of additives to paper that effect the modulus should effect the final caliper as the logarithm of the concentration of the particular additive. For instance, gums or starch in the furnish would increase the modulus and the final caliper for a given set of calender parameters. Conversely, the concentration of clay decreases the modulus and a corresponding higher paper deformation would result. Hartler and Nyren (26), observing that the collapse force of single fibers varies in the same fashion as the transverse modulus, have found that the modulus of kraft pulp decreases with decreasing yield. This is probably because lignin is the stiffest wood polymer under wet conditions over a wide range of temperatures. The beating of fibers generally lowered the modulus.

CONCLUSIONS

Machine calendering has severe limitations on the best available surface finish which necessitates other alternatives such as temperature gradient calendering. The available data on temperature gradient calendering can be generalized and combined with calendering regression equations to indicate that desirable surface finishes are feasible without unacceptable levels of calender blackening. However, the form of the calendering equation appears to change when heated rolls are used. Pilot calendering trials indicated that the usual calendering regression equation holds for a variety of moistures and roll speeds as expected but the coefficient constants appear to change with roll temperature. Calculations indicate that the experimental conditions of the pilot calendering trials are not representative of temperature gradient calendering. A viscoelastic model of paper compression implies that the observed behaviour is a consequence of the exponential stress-strain relation for paper. Calendering effects can be at least qualitatively predicted from the examination of parameter dependences of the elastic constants of the paper web.

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THE CALENDERING CREEP EQUATION – A PHYSICAL MODEL

R. E. Popil

ERRATA

Page 1087, line 7 from bottom "normalized to bulk" should read "normalized to basis weight" - this is done to facilitate immediate comparison with the calendering equations of Mitchell (14). As shown by Crotogino (3), the relative bulk reduction is linearly proportional to the initial bulk of the paper going into the nip. The initial bulk for all the single nip calendering data presented is the same, 2.4cm^3/g to within a few percent.

Page 1091, bottom line and Fig.2 caption, Page 1092 "Burgess" should read "Burgers"

Page 1092, equation 21 should read as follows:

$$(t) = {}_1 + {}_2 + {}_3 = \frac{\sigma}{E_1} + \frac{\sigma}{E_2} \cdot \left(1 - \exp \frac{tE_2}{{}_2} \right) + \frac{t\sigma}{{}_3} \qquad (21)$$

I.K. Kartavaara The Finnish Pulp and Paper Research Institute

I think that there is one fundamental problem in developing these calendering equations that has not been properly addressed. It should be very straightforward to develop an equation for the compression stage. The equations do not, however, describe only the compression. The equations are for the final calliper after springback. When doing this you make the implicit assumption that there is a unique relationship between compression and springback. In my opinion such a relationship does not exist - can you comment on this?

Dr. R.E. Popil MacMillan Bloedel Research

Are you saying that the expression does not include springback?

I.K. Kartaarva

I am saying that there is not a unique relationship between maximum compression and springback.

Dr. R.E. Popil

I deliberately avoided the Kelvin-Voight terms for describing relaxation or springback behaviour in paper compression.I am presenting an over simplification to describe the final calliper reduction. However, if one were to make assumptions about the nonlinearity of the viscosity and stress/strain curve, as I have done and probably solve the whole system of the Burger's model and incorporate various relaxation time constants; in other words, expand the model and do further work on it you may be able to derive an expression of springback as well. However, springback is a temporal thing with a time constant of hours. Measurements done by the Paprican group, as well as ourselves, have usually been 24 hours after calendering. So we are concerned here really with the final calliper reduction but I do believe that our model can be adjusted to include springback if one were to generalise the Burger's model with a distributed network of springs and dashpots.

Dr. J.D. Peel Kusters

I would like to refer to your comments on blackening. Firstly, would the blackening criteria not depend on the formation of the paper so that your graph would have to be unique to one sort of paper. Secondly, there was an interesting observation made in 1961, at one of those conferences. It had been observed at Appleton that if you pressed paper between glass slabs you could see water menisci forming between the fibres and this occurred when the paper was at equilibrium with air at about 60-70% relative humidity. That is the paper was dry in our terms but at 10-12% moisture. I wondered if this was a phenomenon associated with the little blackening points that you get in the high spots of the calender. Do you have any comments on this?

Dr. R.E. Popil

Please realise that this study is very specific to our paper grades and furnishes which may be quite unique in themselves. The blackening criteria was specifically developed for the furnish mentioned here, which was a 25% kraft and 75% groundwood. Secondly, the formation was unique for all the samples used. One grade of newsprint from the same papermachine was studied extensively. So there was no variation in filler content or formation throughout the study. However, the blackening criteria was applied to a completely different machine with a different grade and was found to corroborate the findings at another mill. So I think that I would be careful when applying the blackening criterion that I have specified as a general equation, but it could be used as an idea for what limits one can expect for a given calender configuration to avoid blackening. So it is just suggested at a guide.

Dr. E.L. Back Stockholm

You defined a regime of moisture and high pressure with respect to blackening. Was this low temperature calendering or can you answer an question as to how temperature increase would affect this regime?

Dr. R.E. Popil

I have not studied this experimentally, the blackening criterion was done from room temperature calendering only. It was done in this way for convenience but the application to the mill used ambient conditions which was a temperature of about 40°C. I cannot really offer a suggestion as to the temperature effect in that blackening criterion but I can only speculate on the dependency. I would say that blackening would worsen with increased temperature.

Dr. H. Baumgarten PTS Munich

Did you measure the specific volume of the paper areas which turned black? You mention a specific volume of 1.5 cm^3/g for the paper after calendering,but what was the specific volume of the areas which turned black in the calender nip?

Dr. R.E. Popil

The blackening criterion is rather subjective ie. when is blackening actually blackening? However, it is not as arbitrary a decision as one might think. It was taken in conjunction with a panel of experts who felt that they knew the criterion for blackening in terms of what is acceptable or commercially viable. In terms of numbers the threshold of blackening is a few specks per square centimetre, with these being about 0.5 mm in diameter.

Dr. H. Baumgarten

So you were not able to measure the specific volume of these specks.

Dr. R.E. Popil

We never really tried to.