THROUGHFLOW ACROSS MOIST AND DRY PAPER

O. Polat, R. H. Crotogino and W. J. M. Douglas
Pulp and Paper Research Centre, Department of Chemical Engineering, McGill University, Montreal, Quebec, Canada, H3A 2A7

Pressure drop for air flow through dry and moist paper has previously been expressed in terms of permeability as determined by Darcy's law, an approximation now demonstrated to be substantially in error at throughflow rates relevant to through drying. The more fundamental, nondimensional treatment using the Reynolds number-friction factor model has never been applied for paper because of the need for characteristic dimension in Reynolds number. The use of various assumptions for this characteristic dimension in terms of permeability, specific surface, and Hagen-Poiseuille equivalent capillary diameter are now shown to be in substantial disagreement with the pore structure of paper as examined by scanning electron microscopy.

A new characteristic dimension for flow through paper has been determined by application of fundamental principles of momentum transport. This characteristic dimension was determined for kraft paper over a wide range of basis weight, 25-250 g/m^2, and over the full range of moisture content from wet to dry. With this characteristic dimension, Reynolds number is rigorously the ratio of the inertial to the viscous contribution to momentum transport. With variation in moisture content, the value of this characteristic dimension changes between two asymtotic limits which differ by a factor of about 2.5. The limits of these asymptotic regions correspond to known water-fiber relations. The values of the characteristic dimension agree with measurements by scanning electron microscopy.

A theoretical relationship between Reynolds number and friction factor is shown to fit a set of about 3000 measurements of pressure drop taken with about 150 sheets of kraft paper over a wide range of air throughflow rate, paper moisture content and basis weight. This successful treatment, based on momentum transport theory, not only eliminates the need for the Darcy law permeability approximation, which leads to errors up to 600% for throughflow rates used industrially in through drying, but also provids the basis for theoretical analysis of heat and mass transport phenomena during through drying.

1. Introduction

For the flow of air through paper, the present study presents a new method which is more powerful than the customary permeability-based approach. Flow through paper has traditionally been treated in terms of the physical property, permeability, k, for which the units are m^2. For air flow through dry paper we have previously presented the first measurements of permeability of paper made according to a rigorous momentum transport procedure, i.e. without the substantial error which can be associated with the Darcy's law approximation, Polat, 1989. That work included the extension of this method from dry paper to moist paper, as well as a correlation for the permeability of moist kraft paper as a function of basis weight and moisture content, based on a unique set of such measurements.

While the above represents a significant advance over all previous work, the permeability of paper, even determined without intrinsic error by this new procedure, remains a very specific physical property, one without generality. The fundamental treatment would be in terms of Reynolds number, Re, and friction factor, f, a nondimensional model widely used for flow through simpler porous media but never for a structure as complex as paper. The obstacle is quite evident. The Re-f model requires, for Reynolds number, specification of a characteristic dimension, but for a porous structure of the complexity of paper there is no obvious basis for definition of this key parameter. For kraft paper, used in the present study, the porous structure is determined by the bonding of flattened ribbon-like fibers, 4-8 μm thick, 20-40 μm wide, up to 3 mm long. For the range of basis weight used, 25-150 g/m^2, paper thickness was about 50-300 μm, and the sheets correspondingly about 10 to 70 fibers thick.

Previous workers have attempted to overcome this dilemma by using various arbitrary bases for the characteristic dimension in flow through paper. Three alternatives for characteristic dimension are those based on permeability, k, on specific surface, a_p, and on equivalent capillary diameter, d_{eq}, as determined by the Hagen-Poiseuille equation for viscous flow. In order to obtain the required dimension, i.e. length, the form of characteristic dimension based on permeability is $\sqrt{k}$, and that based on specific surface is $1/a_p$. Specific surface may be obtained from permeability by use of the Kozeny-Carman equation, as detailed by Polat, 1989.

These alternate proposals for the characteristic dimension, $\sqrt{k}$, $1/a_p$, and d_{eq}, were determined and were compared with an examination of the pore structure of this paper using scanning electron microscopy. The results of these comparisons, detailed by Polat, 1989, establish that none of these measures correspond to the pore size range as determined by S.E.M. It is therefore concluded that neither $\sqrt{k}$, $1/a_p$ or d_{eq} provide an acceptable basis for the characteristic dimension of paper.

2. Characteristic dimension from momentum transport analysis: Theory

The approach adopted here is that the appropriate characteristic dimension should derive directly from the flow phenomena through paper. The Reynolds number, $Re = d_p G/\mu$, where d_p is the characteristic dimension of the porous media, is by definition the ratio of inertial to viscous forces. The inertial and viscous contributions appear explicitly in the momentum transport for flow through porous media,

$$\frac{\Delta P}{L} = \alpha \mu u + \beta \rho u^2 \tag{1}$$

where

$$\alpha = \frac{1}{D^2 V} \iiint \nabla^2 v \, dV \quad ; \quad \beta = \frac{1}{DV}\left[\iiint v \, \nabla v \, dV + \iint \frac{P}{\rho u^2} \, n \, dA\right]$$

In the terms of this momentum transport equation, Reynolds number must be

$$Re = \beta \rho u^2 / \alpha \mu u = \beta \rho u / \alpha \mu \tag{2}$$

Therefore the correct, theoretically based characteristic dimension for flow through porous media is

$$d_p = \beta / \alpha$$

a characteristic dimension derived directly from application of the momentum transport equation which yields the parameters α and β.

This definition of characteristic dimension, $d_p = \beta/\alpha$, has never been proposed or tested for paper, although it has been used successfully for flow through beds of simple rigid particles, as detailed by Polat, 1989.

3. Characteristic dimension from momentum transport analysis: Experimental results for dry paper

The experimental technique was as described by Polat, Douglas and Crotogino, 1987. Table 1 gives the average values of characteristic dimension, $d_p = \beta/\alpha$, for each basis weight of kraft paper as determined from the momentum transport parameters, α and β, given by Polat, 1989. N indicates the number of sheets of paper on which these values are based. Although the type of pulp and the sheet forming techniques are different, results for the 250 g/m^2 blotter paper and 45 g/m^2 Papriformer newsprint are also included in Table 1 in order to extend the pulp type and basis weight range.

These values of characteristic dimension are from 4 to 16 times higher than those of $1/a_p$ and $\sqrt{k}$ measured here, are 1/4-1/3 the Hagen-Poiseuille equivalent capillary diameter as determined by Gummel (1977), and are 1/3-1/2 the measurements of Bliesner (1964) for paper of basis weight 50 g/m^2 and heavier. Reference to the scanning electron micrographs, Polat (1989), indicated that the values of the $d_p = \beta/\alpha$ characteristic dimension are of a realistic magnitude, in contrast to $1/a_p$ and $\sqrt{k}$ as frequently used, which are much too small, and to the d_{eq} of Gummel, which is much too large.

TABLE 1

Value of characteristic dimension of dry paper based on fundamental momentum transport relations

Basis weight g/m^2	Before a wetting-drying cycle		After a wetting-drying cycle	
	N	$d_p = \beta/\alpha, \mu m$	N	$d_p = \beta/\alpha, \mu m$
25	3	21.6	65	16.8
50	3	5.5	61	5.4
100	3	6.4	5	5.0
150	3	3.1	5	4.7
250	3	4.2	-	-
45	-	-	9	3.6

The increase in the $d_p = \beta/\alpha$ characteristic dimension by a factor of 3-4 when basis weight is reduced from 50 to 25 g/m^2 is believed to reflect the corresponding large increase in pin holes at low basis weight.

4. Effect on characteristic dimension of pulp type

The characteristic dimension of the 45 g/m^2 Papriformer newsprint, Table 1, is about 2/3 of that of kraft handsheets of comparable basis weight. Relative to kraft paper, the S.E.M. evidence showed that groundwood based newsprint has more fibrillated fibers, in a web of higher bonded area, with more fines present in the interfiber pores, all factors which are consistent with the smaller value measured for d_p in newsprint.

5. Reynolds number-friction factor-characteristic dimension relations

The momentum balance, Eq. (1), rewritten in terms of mass velocity $G = u\rho$, may be rearranged to

$$\frac{\Delta P/L}{\beta G^2/\rho} = \frac{\alpha\mu}{\beta G} + 1 \qquad (3)$$

or, in terms of the new characteristic dimension for Reynolds number,

$$\frac{\Delta P/L}{\beta G^2/\rho} = 1/Re + 1 \tag{4}$$

The left hand side of Eqs. (3) and (4), $(\Delta P/L)/(\beta G^2/\rho)$, is the ratio of total energy loss to the kinetic energy loss, which is precisely the friction factor, f, as expressed by Bird et al. (1960). Thus Eq. (4) can be written more simply as

$$f = 1/Re + 1 \tag{5}$$

It is important to note that this friction factor is not one of the several forms of the Fanning friction factor. The adaptations of the Fanning friction factor from flow through conduits to flow through packed beds is a much inferior treatment in that it requires the use of some complex function of porosity, avoided in the present treatment.

Definition here of a characteristic dimension, $d_p = \beta/\alpha$, based on basic transport phenomena relations eliminates the need for the various porosity, pore size distribution, particle or pore shape empirical factors of earlier approaches, as is detailed by Polat (1989). This nondimensional form, Eq. (4), of the fundamental momentum transport equation, Eq. (1), is tested in the present study for the first time for flow through paper of variable moisture content, a porous media of much greater complexity than any previously tested for this Reynolds number-friction factor.

6. Reynolds number-friction factor experimental results: Dry paper

The ΔP measurements for dry kraft paper, calculated in terms of Reynolds number-friction factor with the characteristic dimension $d_p = \beta/\alpha$, are shown in Fig. 1. The friction factor data on Fig. 1 derive from 120 ΔP measurements for 15 sheets made before a wetting-drying cycle with both helium and air flow, and from approximately 100 measurements of ΔP with 20 sheets after wetting-drying. For Fig. 1 the Reynolds number value associated with each friction factor measurement was determined using an individual value of d_p for each sheet of paper tested.

For the Re-f results shown on Fig. 2, by contrast, the values of Reynolds number were calculated using the average value of $d_p = \beta/\alpha$ for all the sheets of each basis weight. The Fig. 2 results are for about 500 ΔP-G measurements made after a wetting-drying cycle with 126 sheets, 65 sheets of basis weight 25 g/m^2, 61 of 50 g/m^2, using only the two values of average $d_p = \beta/\alpha$ given in Table 1.

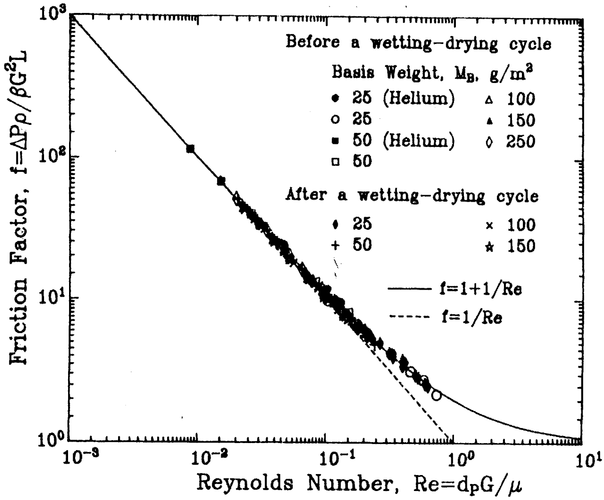

Fig. 1 Pressure drop across dry paper: d_p for individual sheets.

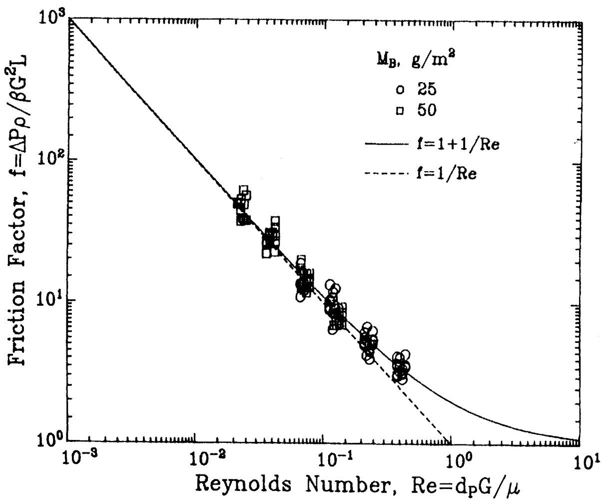

Fig. 2 Pressure drop across dry paper: average d_p for each basis weight.

Figs. 1 and 2 show all the ΔP data for dry paper obtained in the present study, some 700 measurements. The amount of experimental scatter around the theoretical line $f = 1/Re + 1$ is greater for Fig. 2 because of the use of only two values of d_p, an average value for each basis weight, compared to an individual d_p value for each of the 35 sheets of the Fig. 1 results. The excellent agreement of the experimental results with the theoretical $f = 1/Re + 1$ line, with surprisingly small scatter of the extensive data, Figs. 1 and 2, confirms the approriateness of this new approach to the analysis of throughflow phenomena for dry paper. Moreover, this characteristic dimension for paper is the first to give values of d_p consistent with observations from scanning electron microscopy of paper, Polat (1989). These two types of confirmation validate the central feature of the analysis, definition of the characteristic dimension of paper, d_p, as the ratio of appropriate parameters from the momentum transport equation.

7. Characteristic dimension from momentum transport analysis: Experimental results for moist paper

The success achieved with this Re-f-d_p approach for dry paper suggests testing it for moist paper as well. Application of the momentum transport equation, Eq. (1), to the case for moist paper leads to the definition of the characteristic dimension as $d_p = \beta'/\alpha'$, equivalent to the $d_p = \beta/\alpha$ relation for dry paper. Table 2 shows the values of d_p for moist paper corresponding to the detailed listing of momentum transport parameters, Polat (1989). Again N represents the number of sheets tested, on each of which normally 4 ΔP-G measurements are made at each moisture content, X. Data are not available for basis weight greater than $M_B > 100$ g/m^2 because the throughflow rates used were not sufficiently high to give a measurable inertial contribution to pressure drop, hence the characteristic dimension $d_p = \beta'/\alpha'$ could not be determined.

TABLE 2

Values of characteristic dimension of moist paper based on fundamental momentum transport relations

Moisture Content X, kg/kg	Basis Weight M_B = 25 g/m^2		Basis Weight M_B = 50 g/m^2	
	N	$d_p=\beta'/\alpha'$	N	$d_p=\beta'/\alpha'$
2.5	35	43.8	33	9.5
2.0	55	42.3	57	13.2
1.5	63	26.6	58	13.0
1.0	65	16.2	61	8.5
0.5	65	16.6	61	5.0
0.1	65	16.6	61	4.0

As moisture content decreases, the Table 2 values of the characteristic dimension, $d_p = \beta'/\alpha'$, approach the corresponding values for dry paper after a wetting-drying cycle, Table 1, which come from the same complete set of ΔP-G-X measurements.

8. Characteristic dimension: Paper moisture content and basis weight effects

The effects of basis weight on the characteristic dimension of dry paper, Table 1, and of moisture content on d_p for wet paper of two basis weights, Table 2, are shown on Figs. 3 and 4.

For a dry sheet, the characteristic dimension is essentially independent of paper thickness for basis weight above 50 g/m^2 (paper thickness ≈ 100 μm), but increases by a factor of 3 when basis weight is decreased from 50 to 25 g/m^2. The latter effect is no doubt because of pin holes, discussed by Polat, 1989. Bliesner (1964) also observed little effect of basis weight on mean pore size of dry paper for $M_B > 100$ g/m^2.

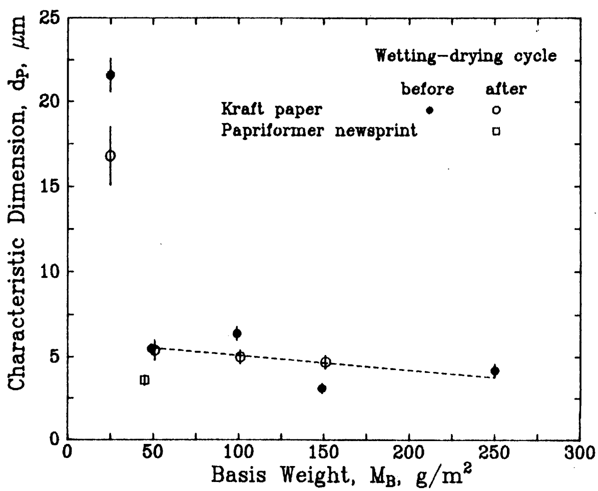

Fig. 3 Effect of basis weight on characteristic dimension for dry paper.

Fig. 4 shows the first measurement of effect of paper moisture content on the characteristic dimension for paper with air throughflow. One typically associates a decreasing ΔP with an increasing size of flow channels, d_p. However the value of d_p for wet paper is 2-3 times that for the same paper when dry. For moist paper, as X decreases the number of pores participating in the flow increases, increasing the fraction of area open to air flow, decreasing the interstitial velocity in the pores as both ΔP and d_p decrease. For wet paper, although the size of pores through which air flows is large, the fraction of the area open to throughflow is small, resulting in high ΔP for the high velocity flow in these channels.

For basis weight of 25 and 50 g/m^2, the effective pore size for throughflow, $d_p = \beta/\alpha$, becomes independent of moisture content below some value in the range 0.6-0.9 kg/kg, while d_p reaches its maximum value at around X = 2 kg/kg. The plateau in d_p at low moisture content corresponds to water being present only in intra-fiber pores and in the smallest inter-fiber pores, through which there is negligible flow.

Electron micrographs of the paper used indicate a maximum pore size of about 50 µm and 10-20 µm, respectively, for the 25 and 50 g/m^2 basis weight paper, Polat (1989). Thus it is clear that the maximum value of d_p found for wet paper, about 44 and 14 µm at basis weights of 25 and 50 g/m^2, is the limit imposed by the size of the largest pores in the sheet. The lower d_p values for 50 g/m^2 paper at $X > 2.3$ kg/kg may be due to compressibility of the sheet at the high ΔP values at these conditions. Testing this hypothesis would require a further study.

Thus the effect of moisture content on characteristic dimension reflects three zones for which the d_p-X relationship between the minimum and maximum values may be represented as

$$d_p = 1/[c_1 + c_2 e^{-X}] \tag{6}$$

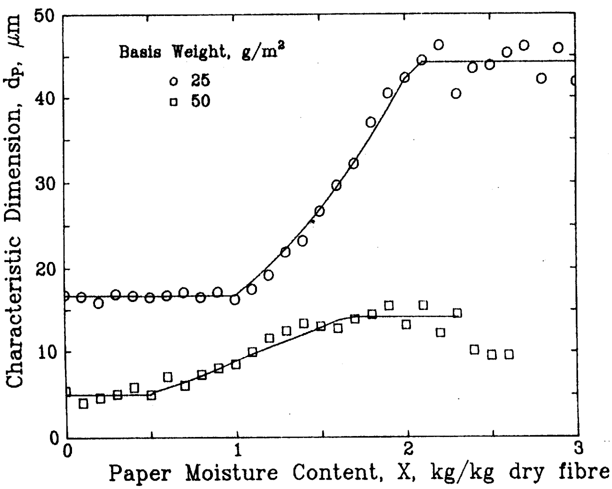

Fig. 4 Effect of moisture content on characteristic dimension.

The results with the two parameters of Eq. (6) determined by non-linear regression of the Fig. 4 data are given in Table 3.

TABLE 3

Parameters for the regression equations for effect of moisture content on characteristic dimension

Zone I: $d_p = A$			
	$M_B = 25\ g/m^2$	$A = 44 \pm 2\ \mu m$	$X > 2.1$
	$M_B = 50\ g/m^2$	$A = 14 \pm 1\ \mu m$	$1.7 > X > 2.3$
Zone II: $d_p = 1/[c_1 + c_2e^{-X}]$			
	$M_B = 25\ g/m^2$	$c_1 = 0.0027$ $c_2 = 0.16$	$0.9 < X < 2.1$
	$M_B = 50\ g/m^2$	$c_1 = 0.026$ $c_2 = 0.23$	$0.6 < X < 1.7$
Zone III: $d_p = B$			
	$M_B = 25\ g/m^2$	$B = 16.7 \pm 0.3\ \mu m$	$X < 0.9$
	$M_B = 50\ g/m^2$	$B = 5.0 \pm 0.5\ \mu m$	$X < 0.6$

The moisture limit between the second and third zones would be the point where essentially all pores that participate in flow through dry paper are free of water and open to flow. As the remaining moisture should be absorbed in fibers and fiber agglomerates through which no flow occurs, this moisture content should be very close to the fiber saturation point (FSP) of the rewetted, unbeaten softwood kraft fibers of the present study. The fiber saturation point of the present kraft pulp, determined by the solute exclusion technique using 2% dextran solution, is 0.81 kg H_2O/kg fiber and that of the paper after a wetting-drying cycle is 0.72 kg/kg. Thus these limits in Table 3, 0.9 for 25 g/m^2 paper and 0.7 for 50 g/m^2, agree remarkably well with the fiber saturation point.

The d_p-X relation for 100 and 150 g/m^2 paper is expected to be similar to that for 50 g/m^2 because d_p for dry paper is effectively independent of basis weight over the range 50-150 g/m^2, Table 1, Fig. 3.

9. Reynolds number-friction factor experimental results: Moist paper

Fig. 5 is based on all the ΔP measurements of the present study for basis weight of 25 and 50 g/m^2 over the wide range of moisture content, $2.5 > X > 0.1$ kg/kg, at intervals of $X = 0.5$. The ΔP-G-X data are converted to the Re-f form using an average value of d_p for each of the 12 M_B-X combinations listed in Table 3. This data set, constituting about

3000 ΔP measurements on the 126 rewetted sheets, i.e. 65 sheets of 25 g/m^2, 61 sheets of 50 g/m^2 paper, falls quite well around the theoretical line, f = 1/Re + 1. Comparison of Fig. 5 with Fig. 1 shows that the experimental scatter is larger for moist than dry paper, reflecting introduction of the additional key variable, paper moisture content. From wet to dry paper, ΔP varies by up to a factor of ~ 20, d_p by up to a factor of ~ 2.5. For moist paper no such comprehensive treatment of pressure drop has previously been available.

In industrial practice of through drying paper, throughflow rates may go up to G = 4 kg/m^2s for tissue paper of 25 g/m^2. With the characteristic dimension $d_p = \beta/\alpha$, of the present study, the Reynolds number corresponding to this maximum throughflow rate would be about Re = 6, i.e. to a flow condition for which we are now able to fix the error in ΔP from use of the Darcy's law approximation at about 600%.

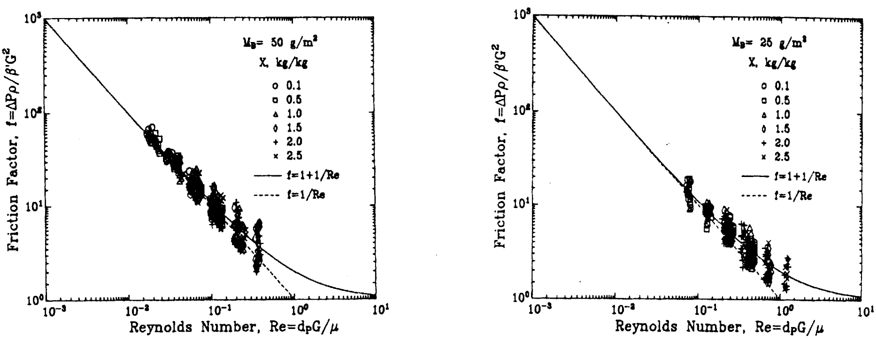

Fig. 5 Pressure drop across moist paper: average d_p for each basis weight.

10. Application of the new method

For the case of kraft paper over the range of basis weight 25-150 g/m^2 and moisture content 2.5 kg/kg to dry, it is possible with the results of the present study to predict ΔP for any throughflow rate, moisture content and basis weight. The characteristic dimension, d_p, required for Reynolds number is obtained from Eq. (6) with the parameters listed in Table 3. For the Reynolds number corresponding to the throughflow rate with that value of d_p, the friction factor is obtained from Eq. (5), and ΔP from Eq. (4). As this value of pressure drop applies for paper formed in a standard laboratory handsheet former it would differ from that for commercial paper of the same basis weight and moisture content made from the same pulp. For each different type of pulp and method of sheet forming it is necessary to determine the $d_p = f(M_B,X)$ relation analogous to Eq. (6), using the general procedure outlined below.

For any paper, in principle only two independent pieces of ΔP-G information, i.e. not both in the region of negligible inertial effects, are needed in order to use the present Re-f-d_p method. With this minimum of two such values of ΔP-G one may determine the characteristic dimension, d_p, for a particular pulp type, formation technique, basis weight and moisture content. With that d_p from two ΔP-G measurements, and the theoretical Re-f relation, Eq. (5), one may calculate from Eq. (4) the ΔP for any throughflow rate, G.

Moreover, the characteristic dimension thus determined would apply for all nondimensional variables, i.e. Reynolds, Nusselt, Sherwood and Peclet numbers concerning the various transport phenomena involved for flow through that paper. The Re-f method thus constitutes a much more powerful treatment than the traditional method involving permeability. The characteristic dimension-Reynolds number analysis development here for moist and dry paper is therefore recommended due to its simplicity, its sound theoretical basis, and its applicablity to related transport phenomena in through drying.

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Nomenclature

a_p - specific surface, m^2/m^3.
d_p - characteristic dimension, m.
f - friction factor.
G - mass flow rate, kg/m^2s.
k - permeability, m^2.
L - paper thickness, m.
M_B - basis weight, g/m^2.
N - number of sheets of paper.
ΔP - pressure drop, Pa.
u - superficial velocity, m/s.
X - moisture content of paper, kg/kg dry fiber.
α - viscous effects parameter, $1/m^2$. ($\alpha' = \alpha L$).
β - inertial effects parameter, 1/m. ($\beta' = \beta L$).
μ - viscosity, Pa.
ρ - density, kg/m^3.
Re - Reynolds number.

Transcription of Discussion

THROUGHFLOW ACROSS MOIST AND DRY PAPER

O. Polat, R. H. Crotogino and W. J. M. Douglas

Mr. I.K. Kartovaara FPPRI Finland

If you calculate pore diameter from permeability data you get something like 1 micron, but you can also determine the pore size by liquid intrusion and extrusion techniques and these all yield about the same value. I think it is evident that if you have a pore system having expanses and bottlenecks all these techniques give you the value for the bottleneck. How does your characteristic dimension account for a pore system having bottlenecks and expanses?

Prof. W.J.M. Douglas

I would say it averages it because the flow passes through both the easy passages and the bottlenecks, there is momentum transfer occurring in all the passages and overall this gives the characteristic dimension.This comes from the momentum transfer equation - the momentum transfer processes have done the averaging for you. You have not imposed any arbitrary averaging procedures. The paper has done the averaging for you and you have read what the paper is doing.

Prof. J. Silvy EFP France

You present a result based on the flow through in the transverse direction of the paper. However, the micrographs we have seen are all of transverse cross sections - would it not have been better to use planer view micrographs?

Prof. W.J.M. Douglas

You must remember that we are interested in throughflow drying and for that process we are interested in the momentum transfer and flow behaviour across the sheet. I would not therefore claim that this is the universal characteristic dimension for paper. I believe that our characteristic dimension does adequately describe the paper for the process of flow through the paper. This may not be the appropriate method for characterising other properties of the paper.