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FINITE ELEMENT ANALYSIS OF PAPER COCKLING

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ABSTRACT

A cockled paper sheet has lost its planarity because small (5 - 50 mm) randomly spaced areas have bent out of the plane of the paper. The variation in surface height is usually only of the order of one millimeter. Cockling is related to the hygroexpansivity and small-scale inhomogeneity of paper, and is a problem mainly with some lightweight papers and with copying papers.

In this study, cockling is analysed theoretically using the finite element method. The results suggest that cockling is caused by local inhomogeneity in the two-sidedness of paper. Small-scale variation in the fiber orientation angle is especially detrimental in this respect. Large variations in in-plane contraction can also cause paper to buckle. The intensity of cockling increases with decreasing paper thickness. The cockles become more oval as the fibre orientation gets stronger.

The conclusions of the theoretical analysis are supported by experiment. With high basis weight papers, the local fibre orientation of the top side was found to be almost independent of that of the bottom side. Thus, two-sidedness varies considerably on the small length scale, and this allows local curling to take place. At low basis weights the top and bottom sides are no longer independent, which offsets the amplifying effect of smaller thickness.

INTRODUCTION

A cockled paper sheet has lost its planarity because small (5 - 50 mm) randomly spaced areas have bent out of the plane of the paper. The variation in surface height is usually only of the order of one millimetre. Cockling is related to the hygroexpansivity and small-scale inhomogeneity of paper. Cockling usually appears when paper is heated, which has two effects. First, water evaporates from the paper, making the paper contract. Due to the inhomogeneity of paper, the contraction is non-uniform, and the paper cockles. Second, heat softens the fibres and fibre bonds, which allows creeping to take place. As the paper cools, the lost moisture is regained, but some of the dimensional changes remain because of the creep.

Cockling is mostly a problem with some lightweight papers and with copying papers. With copying papers cockling reduces the bin capacity and makes the contact with the copying drum non-uniform, which in the worst case can prevent the toner from transferring onto the paper. Heavy paper grades are too stiff to bend under the influence of small-scale unevenness.

In this study, cockling is analyzed theoretically with the finite element method (FEM). In paper technology, the FEM has been used mainly to determine the strength of containers (1, 2), fracture of fibre networks (3), tensile strength (4), wet strain (5, 6) and tension buckling of paper (7). The FEM is most useful in analyzing problems, whose analytical handling is difficult. The heterogeneity of paper, from which cockling arises, is so complex that an analytical solution is impossible. The validity of the theoretical results is checked experimentally.

THE FINITE ELEMENT METHOD

The finite element method (FEM) is a standard method in engineering. In the FEM a continuous structure, which behaves in a complex manner in loading, is divided into elements, each of which behaves in a simple manner in loading (fig. 1). In this way an approximate solution is obtained. The elements are interconnected at nodes, where the boundary conditions as well as the loads are defined. The solution of the finite element model gives the displacements and stresses at the nodes.

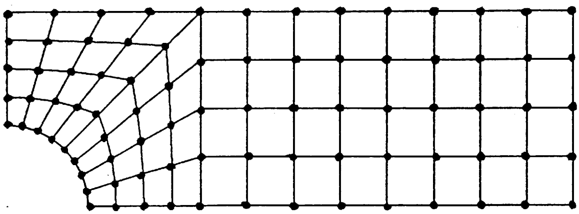

Fig. 1. Division of a continuous structure into elements.

The solution of a finite element model is based on the minimization of work done on the model. The work consists of the potential energy of the outside forces and of the internal energy arising from the deformation of the object. In a linear case the minimization leads to a matrix equation

$$\underline{\mathbf{K}}\cdot\underline{u} = \underline{p} \quad , \tag{1}$$

where $\underline{\mathbf{K}}$ is the stiffness matrix, $\underline{u}$ is a vector containing the nodal displacements and $\underline{p}$ a vector containing the load at the nodes. The load contains both the applied load and the internal stresses arising, for example, from thermal expansion. Usually the load $\underline{p}$ is known and the displacements $\underline{u}$ are solved, but the method can equally well be applied in the opposite way.

The stiffness matrix contains the geometry of the system as well as the constitutive equations, describing how the material deforms in loading. Hooke's law

$$\sigma = E\varepsilon \quad , \tag{2}$$

is an example of a constitutive equation. In eq.[2] E is the elastic modulus, σ is the stress and ε is the strain.

Linear and non-linear cases

A mechanical system behaves linearly when Hooke's law is valid and the displacements are so small that a displacement in one direction does not induce significant displacements in other directions. In other cases the system is non-linear, either materially or geometrically.

Non-linear models are solved stepwise, and at each step the solution is searched iteratively. This takes time, which is the biggest setback in non-linear analysis. Also, in some cases the iteration diverges. This is a serious problem with post-buckling analysis of internally loaded structures (i.e. thermal expansion or hygroexpansion).

APPLYING THE FINITE ELEMENT METHOD IN COCKLING

Constitutive equations

Paper is usually assumed to be orthotropic, which means that it has three planes of symmetry and nine independent mechanical constants (three elastic moduli, three Poisson ratios and three shear moduli).

The forces involved in cockling are small, and therefore the linear elastic material model can be used. Also, in cockling paper is in the state of plane stress, so the elastic constants in the thickness direction are not very important. The situation is geometrically non-linear, because the displacements are large compared to the paper thickness. However, non-linear analysis does not converge in some important cases, like in post-buckling analysis and in the two-sidedness of fibre orientation angle. Therefore, regardless of its inaccuracy, linear analysis was used. The decision is also justified by the fact that the non-linearity is not very strong, and therefore the error is small.

Elastic moduli

The elastic modulus describes how much a material strains under the influence of stress (eq.[2]). It depends on the properties of the fibres, bonding, fibre orientations and built-in stresses. The in-plane elastic moduli are usually between 1 GPa and 10 GPa and the modulus is 1 - 3 times as large in the machine direction as in the cross machine direction (8). This is the result of fibre orientation and cross-directional drying shrinkage. The out-of-plane elastic modulus is 30 - 100 times smaller than the in-plane moduli (8, 9).

Poisson ratio

The Poisson ratio v_{xy} describes how much an object contracts in the y-direction when it is strained in the x-direction

$$v_{xy} = -\frac{\varepsilon_y}{\varepsilon_x} \quad . \qquad [3]$$

The Poisson ratio of paper usually means the value $v_{md,cd} = -\varepsilon_{cd}/\varepsilon_{md}$ (Note, the notation used in this paper is x=md and y=cd). The Poisson ratio is always $0 \leq v \leq 0.5$ with isotropic homogeneous materials, usually $0.25 < v < 0.33$ (10). Orthotropic materials may have bigger values.

With ultrasound it has been found that if the tensile strength ratio is less than 3, the in-plane Poisson ratios obey the relation (8)

$$(v_{xy}v_{yx})^{1/2} = 0.3 \quad . \qquad [4]$$

The value 0.3 is the same as the Poisson ratio of isotropic laboratory sheets. The corresponding constant obtained with mechanical methods is 0.2 (11). The Poisson ratio does not depend on humidity, type of pulp or refining (8, 11). If eq. [4] is valid and the orientation ratio is 3, $v_{xy} = 0.52$. The FEM program used, however, accepts only values $0 \leq v \leq 0.5$. Therefore, the value 0.45 was used.

The out-of-plane Poisson ratios are of the order $v_{xz} \approx v_{yz} \approx 2$, measured using ultrasound (i.e. in small strains). In larger strains the Poisson ratios are negative. This is characteristic of paper, and happens because during straining the fibres straighten. The out-of-plane ratios are not very important in the present analysis, and therefore any value can be used.

Shear moduli

The shear modulus describes how the material becomes tilted in shear stress

$$\Theta = Gt \quad , \qquad [5]$$

where Θ is the shear angle in radians, G the shear modulus and t shear stress. The modulus G_{xy}, for example, indicates the effect of stress perpendicular to the x-axis and in the direction of the y-axis. The shear modulus is symmetric, i.e. $G_{xy} = G_{yx}$.

If measured values are not available, approximate values are obtained from the empirical equation (8)

$$G_{xy} = \frac{(E_x E_y)^{1/2}}{2(1 + (v_{xy} v_{yx})^{1/2})} \quad . \qquad [6]$$

The equation is valid for other directions, too, if it is substituted xy => yz => xz.

Values for the elastic constants

The elastic constants were the following, unless otherwise stated

E_x = 12 GPa v_{xy} = 0.45 G_{xy} = 3 GPa
E_y = 4 GPa v_{xz} = 0.3 G_{xz} = 0.4 GPa
E_z = 0.04 GPa v_{yz} = 0.3 G_{yz} = 0.4 GPa

β_x = 1.67E-3 $\%H^{-1}$
β_y = 5.00E-3 $\%H^{-1}$

The hygroexpansion coefficients are defined relative to the percentage moisture content of paper, instead of the usual relative humidity of air. The difference between these coefficients is tenfold in the humidity range 20%RH - 60%RH (12).

Element type and loading

From the mechanics point of view, paper is an orthotropic thin plate. The structure is layered and can be heterogeneous in the z-direction. The suitable element type is either a homogeneous thin plate or a laminated thin plate.

Cockling is caused by hygroexpansive stress, which is not included in FEM programs. However, thermal expansion is, and because of the analogy it can be used like hygroexpansion. Problems arise if both hygro- and thermal expansion have to be taken into account. With paper, thermal expansion is minimal compared to hygroexpansion, and can be neglected.

Hygroexpansive loads can be applied two ways; either the humidity changes the same amount throughout the element or there is a humidity difference between the sides.

Gravity also affects cockling, just as gravity straightens out a curled sheet. However, it has a significant effect only on the edges of a paper sheet, and therefore its effect is not studied in this paper.

THEORETICAL RESULTS

Out-of-plane deformations are caused either by local buckling or by local curl. In buckling, a plate under an increasing in-plane stress suddenly bends out of its plane. Curl is caused by in-plane bending moments induced by the two-sidedness in hygroexpansion.

THEORETICAL RESULTS - LOCAL BUCKLING

Buckling happens because a plate has two equilibrium states, straight and bent. At the critical load the bent shape becomes energetically more advantageous than the straight. The critical load is the eigenvalue and the buckled shape the eigenvector of the problem. This initial buckling is also called bifurcation buckling.

After bifurcation buckling, the structure still carries load, and the final deformation depends on the actual load. This is called post-buckling behaviour.

Buckling can induce cockling when the local contraction/expansion varies. During drying, paper buckles locally when the contraction is smaller than in the surroundings, and in moisting when the expansion is bigger than in the surroundings.

Analytical solution of bifurcation buckling

In homogeneous structures, bifurcation buckling is easiest to solve analytically. The critical load for the buckling of an uniaxially compressed isotropic, homogeneous square plate, is (13)

$$S = 4\pi^2 D/L^2 \quad , \qquad [7]$$

where S is the critical load on a unit length of the edge, L is the length of the edge and D is the flexural rigidity of a plate (10)

$$D = \frac{Et^3}{12(1-\nu^2)} \quad , \qquad [8]$$

where t is the thickness. The situation is shown in fig. 2.

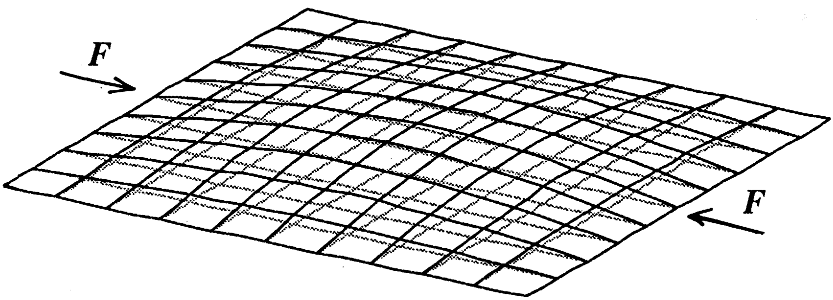

Fig. 2. Buckling of a uniaxially compressed clamped square plate.

Substituting eq. [8] into eq. [7] gives

$$S = 4\pi^2 \frac{Ed}{12(1-v^2)} \left(\frac{t}{L}\right)^2 \quad . \qquad [9]$$

In moisture expansion, when the plate edges are not allowed to move, the stress on unit edge length is

$$S = Et \cdot \beta \cdot \delta H \quad , \qquad [10]$$

which, when substituted into eq. [9] gives

$$\delta H = \frac{\pi^2}{3\beta(1-v^2)} \left(\frac{t}{L}\right)^2 \quad . \qquad [11]$$

Eq. [11] indicates that the critical load depends quadratically on the ratio of the plate thickness to the edge length, inversely on the hygroexpansivity and not at all on the elastic modulus.

It should be noted that t/L does not directly affect the strength of cockling, but only the point at which cockling begins.

The critical loads are small. Substituting into eq. [11] $v = 0.3$, $\beta = 5 \cdot 10^{-3}$ %RH^{-1}, $t = 0.1$ mm and $L = 10$ mm, we obtain $\delta H = 0.0723$%H. The corresponding change in the relative humidity of air is roughly tenfold, that is $\delta RH = 0.72$%RH. As the ratio of the thickness to the side length increases, the buckling load decreases sharply. With a threefold ratio ($L = 30$ mm and $t = 0.1$ mm) the critical load is only $\delta H = 0.008$%H.

In the case of cockling, it is important how much the factor $\beta \cdot H$ varies locally. The variation may arise from β just as well as from humidity. If it is assumed that humidity is constant (H=5%) and that β varies, eq. [11] gives $\delta\beta = 7.2 \cdot 10^{-5}$%RH^{-1}, which is roughly 1% of the average. If the actual local variations are ±10% of the average, local buckling can easily induce cockling.

Analysis using the FEM

Bifurcation buckling

Buckling of a heterogeneous plate is easiest to solve using the

FEM. The most interesting cases of heterogeneous stress are those where the stresses are highly localized or those where the expansion varies sharply from negative to positive.

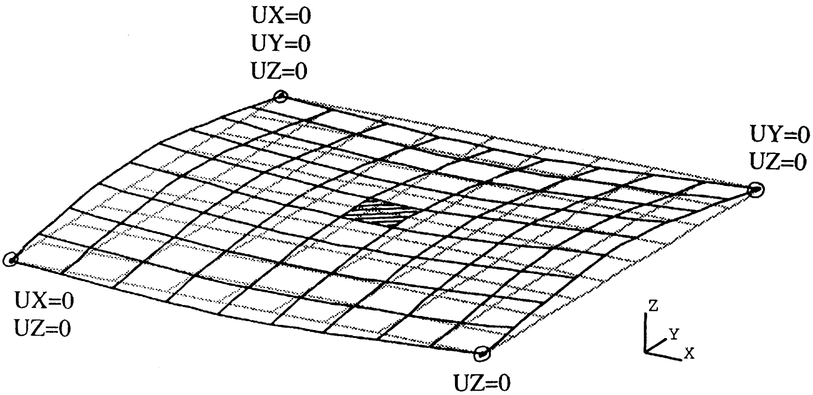

Fig. 3. Buckling of an isotropic square plate to form a saddle surface, when only the middle element has nonzero hygroexpansivity.

The effect of localized stress was examined with a model in which there is a contracting area in the middle of an otherwise stable sheet. The critical load depends on how large the dimensionally unstable area is compared to the whole area. When the contracting area is not larger than half of the whole sheet, the critical load is higher the smaller the contracting area, being approximately δH = 0.01%H - 0.001%H. When the stable area becomes only a narrow edge, the critical loads start to increase again and are inversely proportional to the width of the non-contracting edge.

Despite the localization of the stress, the eigenvector is such that the whole sheet has an even curvature. The eigenvector depends on the boundary conditions. A saddle surface is common (fig. 3). It can be seen experimentally that localized contraction makes the whole sheet evenly buckled, for example, by wetting a small area in the middle of a sheet and letting it dry freely.

With a varying load, if the net contraction is zero and the load is symmetrical so that adjacent elements have opposite hygroexpansions, the plate does not buckle at all. If the load is unsymmetrical so that the number of contracting elements is larger than the number of expanding elements (or vice versa) but the net contraction is zero, the structure in most cases buckles. Nevertheless, the eigenvector always shows a uniform curvature over the whole sheet. The actual shape depends on the distribution of the contracting and expanding elements.

Post-buckling behaviour

Post-buckling behaviour is usually analysed by placing a small initial deformation on the structure and by performing a geometrically non-linear analysis. Post-buckling analysis is also the only way to get the final deformation, because the eigenvector of bifurcation buckling gives only the shape. Nevertheless, the magnitude of the final deformation depends on the initial deformation. However, in this case post-buckling analysis is not possible, because non-linear analysis does not converge in hygroexpansive stress.

Because the element method cannot be used, post-buckling behaviour must be approximated analytically. Let the wedge of length L in fig. 4 be fixed at both ends. The length of the wedge increases in humidity expansion by dL. This makes the wedge buckle to form a cockle of height h. Let us assume that the wedge is incompressible and that the cockle is a triangle. Then

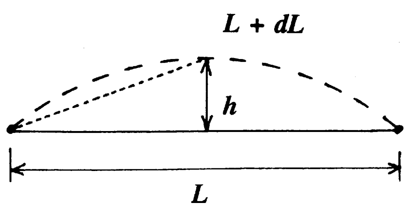

Fig. 4. The geometry of the calculations.

$$\frac{L^2}{4} + h^2 = \frac{1}{4}(L + dL)^2 \approx \frac{1}{4}L^2 + \frac{L^*dL}{2}$$

$$<=> \quad h = L^*(2\frac{dL}{L})^{1/2} \quad <=> \quad \frac{h}{L} = (\frac{\varepsilon}{2})^{1/2} \quad . \qquad [12]$$

The square root relationship means that even a small elongation makes the cockle clearly visible. For example, if ε = 0.2% and L = 40 mm, the height of the cockle is h = 1.3 mm. Copying paper contracts during heating approximately 0.1%. If the local differences are 10% of this, then the height of a 40 mm cockle is h = 0.28 mm.

It should be noted, that eq.[12] says only that reasonably high cockles can be induced through post-buckling behaviour. The actual deformation depends on the bending stiffness, which has not been taken into account in eq. [12].

Conclusions

Even if the critical loads for buckling are low, in the light of the present analysis no final conclusions can be made about the significance of buckling. The main concern is that even if the stresses are localized, or vary strongly within the sheet, the curvature of the buckled surface is uniform over the whole structure. Because of the shape of the eigenvector, it seems that instead of pure buckling, cockling can be induced so that the initial deformation is caused by local curl, for example, and the in-plane stress produces the final deformation in post-buckling behaviour. However, post-buckling behaviour cannot be analysed with the FEM when the only load is hygroexpansion.

THEORETICAL RESULTS - LOCAL CURL

Paper curls if there is two-sidedness in hygroexpansivity. The nature of curl depends on what causes the two-sidedness. The hygroexpansivity of fibres is about ten times as high in the cross direction as along their length. If the fibre orientation is stronger on the wire side, in the cross-machine direction paper contracts more and in the machine direction less on its wire side, and the sheet becomes a saddle surface. The two-sidedness of the fibre orientation angle, in turn, produces diagonal curl. The two-sidedness of bonding curls the sheet in the same direction in both the machine and cross-machine direction and the sheet becomes a cup surface.

Uniform two-sidedness produces uniform curl. If the two-sidedness varies locally, local curl occurs and paper cockles.

Analytical solution

The height of a cockle resulting from curl due to the two-sidedness in hygroexpansivity is approximately (14)

$$h = \frac{L}{t}\,\frac{L\delta\beta\delta H}{4} \quad , \qquad [13]$$

where L is the length of the curling object, t its thickness, $\delta\beta$ the two-sidedness of the hygroexpansivity coefficient and δH the change in the moisture content of paper. According to eq. [13], h is inversely proportional to the thickness of the paper, so the thinner the paper the higher the cockle.

If the ratio of the length to the thickness is constant, the rise depends linearly on the length L. As in buckling, the strength of cockling depends on the ratio of cockle size to paper thickness, but whereas it was quadratic in buckling, it is linear in curl.

Analysis with the FEM

Again, the FEM is the best approach to the analysis of heterogeneous structures. Contrary to buckling, the results yield the heights of the cockles, too.

Random models with 8x12 second-order laminate plate elements were used (fig. 5). There was inhomogeneity in the two-sidedness of fibre orientation, fibre orientation angle or paper thickness. The inhomogeneous parameter had a discrete bimodal distribution. For example, with a two-sided fibre orientation angle, the angle was either +a° on the upper side and -a° on the wire side, or vice versa. The direction of the two-sidedness among the elements was decided at random. The size of the model was 210 mm x 300 mm and the thickness 0.1 mm.

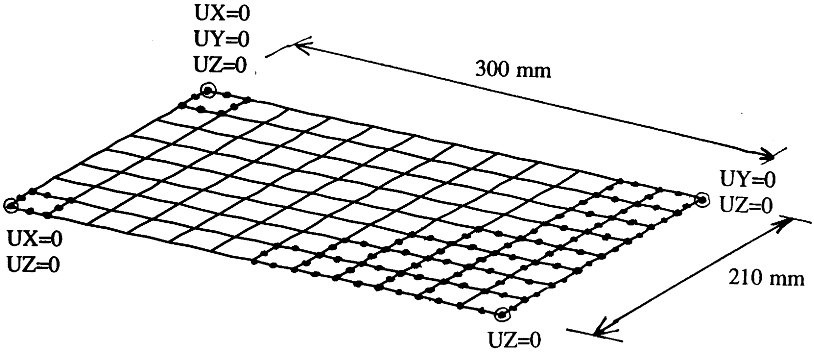

Fig. 5. The element model used. The properties of each element are random.

Cockling is described in terms of its strength and by the shape of the cockles. The strength was characterized using the difference between the highest and the lowest parts of the surface. Figures 6 - 8 show the strength with the y-axes comparable. The strength is normalized in such a way that the average of all data points is one.

The shape factor of the cockles is the ratio of the x- and y-directional characteristic lengths. Characteristic length is the average distance between the points at which the surface height moves from below to above the average surface, or vice versa. The larger the ratio, the stronger the cockles are oriented in the machine direction. In copying paper, the ratio is about two.

The shape of an individual cockle does not depend on the shape or size of the elements, but on the boundary conditions. Even if the hygroexpansivity is non-zero in cross-machine direction only, the cockles are round if all edges are restrained. If the edges are free, the cockles are oval.

Local two-sidedness of the fibre orientation angle

Local two-sidedness in the fibre orientation angle induces twisting moments in paper. If the model is otherwise homogeneous, but there is two-sidedness in only one element, the whole sheet curls diagonally. Random two-sidedness in several elements controls the effect, and the paper cockles.

Fig. 6 shows the strength of the cockling created in this way. Cockling intensifies as the two-sidedness of the fibre orientation angle and the average orientation ratio increase. The latter has the stronger effect.

The shape factor averages 1.3, so the cockles are fairly round. The shape does not depend greatly on the orientation ratio, but the shape factor decreases slightly as the two-sidedness of the fibre orientation angle increases.

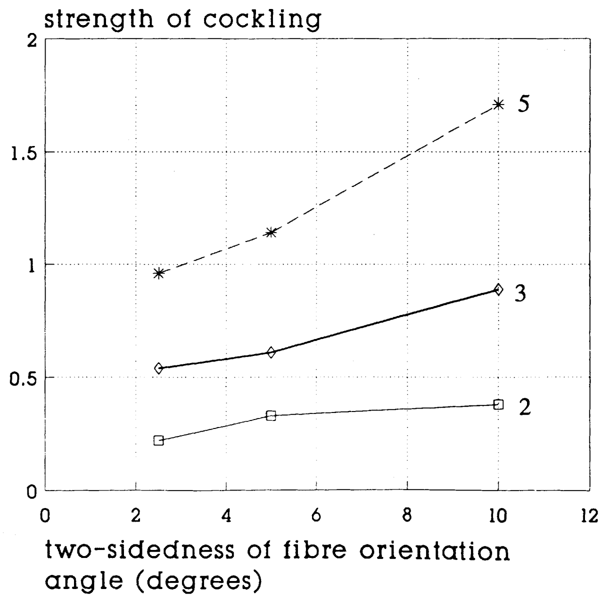

Fig. 6. Strength of cockling as a function of local two-sidedness of the fibre orientation angle. The average orientation ratio is 2, 3 or 5.

Effect of two-sidedness in the strength of orientation

In this case, the bimodal distribution was such that on the upper side the orientation was either 5% or 10% stronger than average and on the wire side 10% weaker, or vice versa. In the model, the orientation ratio and the ratio of hygroexpansivity coefficients are inversely proportional to each other. As a result, during drying, those elements that have stronger orientation on the upper side curl downwards, and vice versa.

Fig. 7 shows that the strength of cockling is about the same as in the case of the fibre orientation angle. The intensity of cockling diminishes as orientation strengthens.

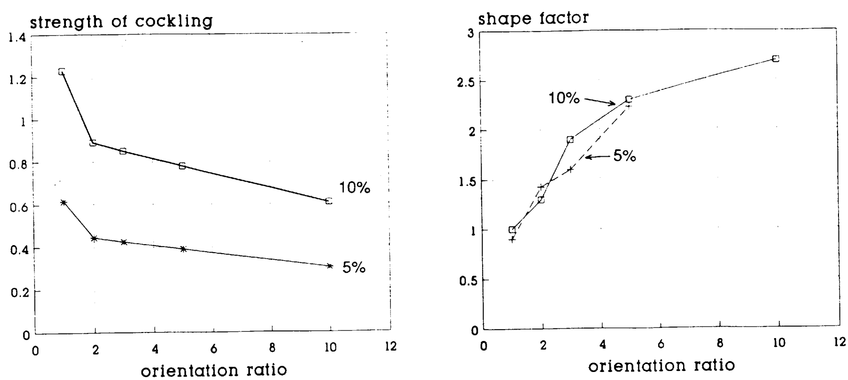

Fig. 7. Strength of cockling and the shape factor as a function of orientation ratio.

The shape factor increases radically when the orientation ratio increases from 1 to 5, but above it levels off to approximately 2.7. This is because each element affects only its near neighbours. If only one element in the middle of the model curls, the shape factor is about three. Local variation of the orientation does not affect the shape factor.

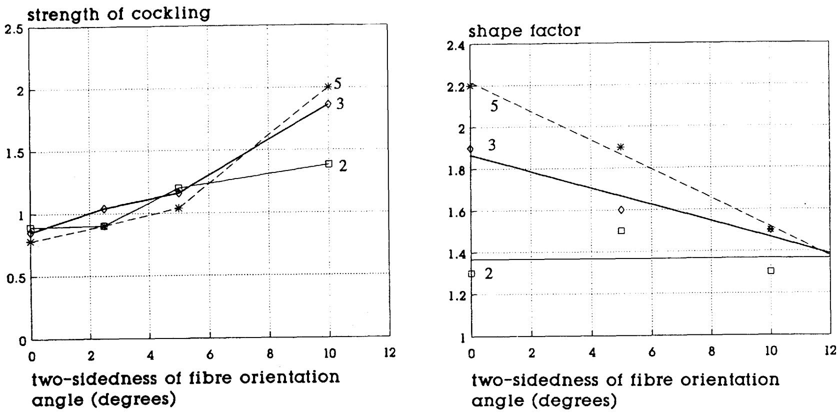

Fig. 8. Effect of local two-sidedness of the fibre orientation angle and orientation ratio on the strength and shape of cockles. The average orientation ratio is 2, 3 or 5 and the two-sidedness 10%.

Effect of fibre orientation angle and orientation ratio

When there is two-sidedness both in the orientation angle and in the orientation ratio, cockling is stronger than in either case individually (fig. 8).

Fig. 8 shows that unlike the previous cases, average orientation affects the strength of cockling only weakly. However, the average orientation affects the shape factor strongly, especially when the two-sidedness in the fibre orientation angle is low. When the two-sidedness in the orientation angle is ±10°, the orientation ratio has little effect on the shape factor. The shape factor reduces as the two-sidedness in orientation angle gets stronger. The effect is the stronger the bigger is the average orientation ratio.

The shape factors obtained with the FEM are slightly smaller than value of about two obtained in practice.

Effect of formation

In the FEM analysis, formation, i.e. the small-scale grammage variation, is simulated by thickness. Formation makes cockling stronger if it appears together with other inhomogeneity. If formation is the only inhomogeneity, cockling does not occur. The effect of formation is quite small. If the grammage varies by ±10% from the average, cockling increases by about 5%. Formation does not affect the shape of the cockles.

Conclusions

The two-sidedness of the orientation ratio affects mainly the shape of cockles, while the two-sidedness of the fibre orientation angle affects mainly the strength of cockling. The average orientation affects the shape factor strongly, and the cockles are more oval when the orientation is strong. Generally speaking, the more two-sidedness and unevenness in the paper, the stronger the cockling.

EXPERIMENTAL

Methods

Measurement and characterization of cockling

An instrument was built for measuring cockling, based on a laser displacement probe and an x-y table. A map of surface height is scanned, from which a number of parameters are calculated. The most useful parameters are the standard deviation of surface height, which shows how strong cockling is, and the characteristic lengths in the machine and the cross-machine directions. Cockling was produced by heating paper in an oven, and the intensity of cockling was the difference between the initial and the final states

$$\sigma_h = (\sigma^2_{h,final} - \sigma^2_{h,init})^{1/2} \qquad [14]$$

where σ_h is the standard deviation of surface height.

Measurement and characterization of fibre orientation on paper surfaces

Theoretical analysis demonstrated the significance of the local two-sidedness of fibre orientation. Normal methods of measuring fibre orientation give only average values over a large sheet and through the paper thickness. For this reason, a probe has been developed at the KCL, to measure fibre orientation on paper surfaces over a small area. The measurement is based on the dependence of light polarization on the orientation of the topmost fibre layer. Two parameters are calculated, one of which, the orientation index, characterizes the strength of the orientation. The other is the orientation angle.

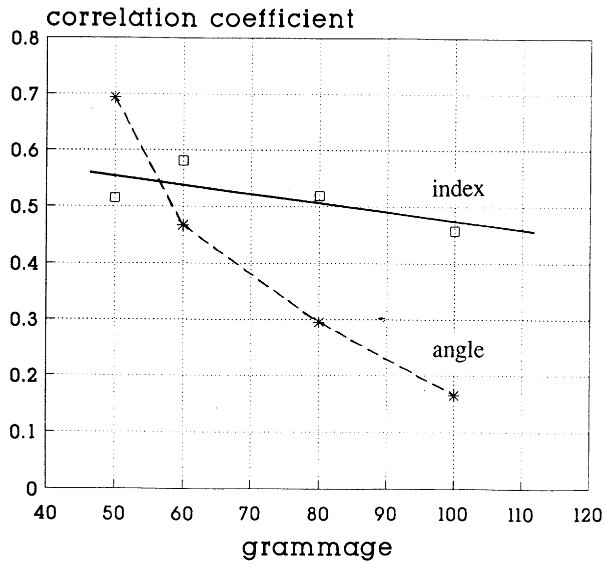

Fig. 9. Correlation of orientation between the different sides of paper as a function of grammage. All papers were fabricated on fourdrinier paper machines, either at KCL or at a paper mill.

Although no information about the orientation of the inner layers is obtained, the difference between the surfaces is the most important factor determining curl, either local or global.

The probe was placed in the x-y scanner, and the orientation was measured on both sides of the paper. From the basic quantities (orientation index and orientation angle), the standard deviation was calculated as well as the correlation between the two sides. The small-scale variation of two-sidedness is the most important factor, because it produces the local curl. The variation is characterized by the standard deviation of the difference in fibre orientation index and angle between the two sides.

Correlation of surface orientation between the sides

Local curl can occur if the surface orientations are different on different sides of the paper. According to fig. 9, the correlation between the sides is weaker the higher the grammage. The dependence is especially strong with the orientation angle, and with papers heavier than 80 g/m^2 the orientation angles are practically independent. The correlation weakens with increasing grammage, because although the adjacent fibre layers interact strongly, the interaction weakens steadily the more layers there are between the layers in question.

Results

Experiments were carried out with woodfree copying papers. Trial runs were conducted both at KCL and at paper mills. The results are compared with those obtained with local curl. With local buckling, no results were obtained that could be compared with the experiments.

Effect of formation

The unevenness of paper structure is usually characterized by its formation. Strictly, formation means the way in which the fibres and other paper raw materials are distributed in the web (15). Formation then includes the grammage variation, z-directional structure and fibre orientation. In the case of cockling, all of these are important. However, in practice formation means the grammage variation only, and the rest of the inhomogeneity is grouped together under its own name. This approach has also been adopted in the present study.

The same parameters affect all factors of formation. For example, a high jet-wire speed ratio makes formation worse, but also diminishes the standard deviation of the difference in fibre orientation angle.

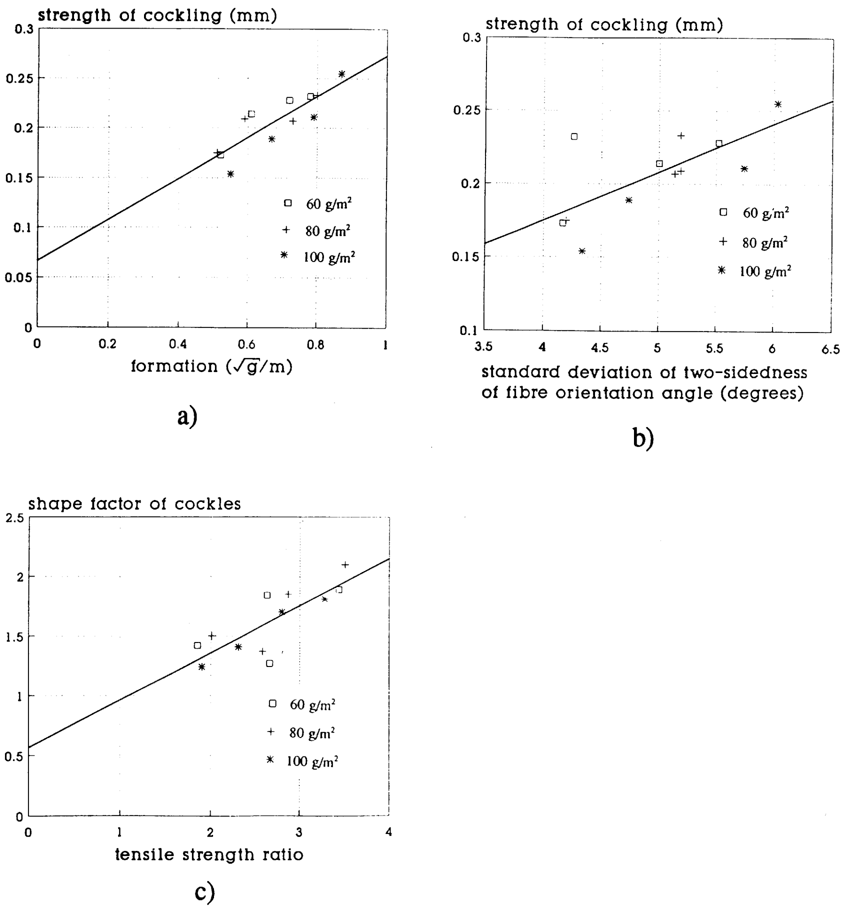

Fig. 10. Effect of a) formation and b) local two-sidedness of the fibre orientation angle on cockling. c) Effect of tensile strength ratio on the shape of the cockles. Results from a trial run at KCL.

The effect of formation was studied in two tests. In the first (fig. 10), formation was modified by varying headbox consistency. However, this approach had the side-effect that orientation got weaker as the consistency increased. There is a good correlation between cockling and formation (r = 0.85), but there is also a correlation between the local two-sidedness in fibre orientation angle and cockling (r = 0.71). Thus, it cannot be concluded which has the stronger effect. There is, however, a strong cause-effect relationship between orientation and the shape of the cockles. The correlation coefficient between the shape factor and the tensile strength ratio is r = 0.80.

The effect of formation was more clearly seen in a trial run in which formation was varied by means of wire shake (table 1). Wire shake improves formation and reduces orientation. The local two-sidedness in the fibre orientation angle changes very little, so cockling decreases mainly due to the better formation. The result is thus in accordance with the theoretical prediction. The cockles become rounder with the shake on, partly due to the reduced orientation.

wire shake	formation $\sqrt{g}/m$	orientation	std. dev. of orientation angle	cockling strength	cockling shape
no	0.57	2.51	5.77°	0.217 mm	1.81
yes	0.44	2.28	5.60°	0.161 mm	1.30
change	-23%	- 9%	- 3%	-26%	-28%

Table 1. The effect of wire shake on cockling. Formation is characterized using the normalized standard deviation of grammage (16).

The strong effect of formation in table 1 is in contrast with the theoretical prediction that formation has only a small effect. The reason for this discrepancy is that the measured formation also includes information about the general unevenness of paper, and therefore is not comparable with the formation in the theoretical analysis, which contained grammage variations only.

Effect of local two-sidedness in fibre orientation angle

Fig. 11 shows, that the experimental results for the effect of local two-sidedness in the fibre orientation angle are in accordance with the theoretical results of fig. 8. The most important parameter affecting the local two-sidedness in the fibre orientation angle is the jet-wire speed ratio (fig. 12). The larger the speed difference, the weaker the local two-sidedness of the fibre orientation angle. The local two-sidedness of the strength of fibre orientation did not affect cockling. However, the average orientation ratio affects the shape factor as in the theoretical results, that is the stronger the orientation, the more oval the cockles are.

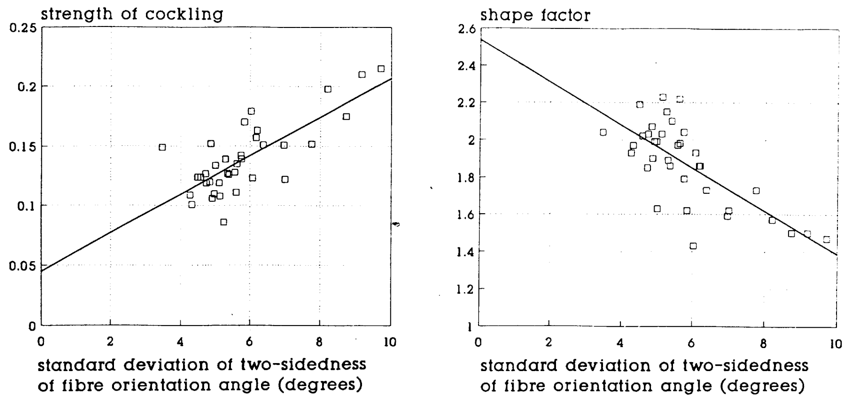

Fig. 11. Effect of local two-sidedness of the fibre orientation angle on the strength of cockling and the shape of the cockles. Results from a trial run at KCL.

The effect of the fibre orientation angle can easily override other factors. In one trial run, headbox consistency and jet-wire speed ratio were simultaneously changed. Despite the deterioration in formation due to the higher headbox consistency, cockling decreased because of the reduction in the local two-sidedness of the fibre orientation angle.

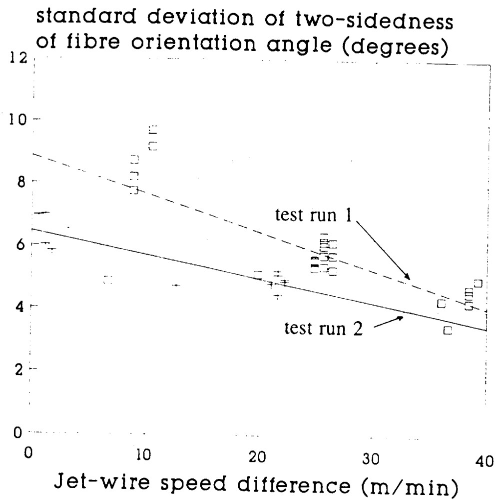

Fig. 12. Effect of the jet-wire speed difference on local two-sidedness of the fibre orientation angle. Results from a trial run at KCL.

Effect of grammage

Table 2 shows that cockling is stronger and the cockles are smaller the lower the grammage.

grammage	cockling	cockle size
60 g/m^2	0.145 mm	30.7 mm
80 g/m^2	0.136 mm	32.4 mm
100 g/m^2	0.125 mm	33.5 mm

Table 2. The dependence of cockling on average grammage. The values are calculated for a constant formation value (n = 0.6 $\sqrt{g}$/m). The cockle size is the geometric average of the characteristic lengths in the machine and cross-machine directions.

Theoretically, cockling is affected by the ratio of cockle size to paper thickness, L/t, as well as by the small-scale inhomogeneity, but above all by formation and the local two-sidedness of the fibre orientation angle. Let us concentrate on the case in which the paper gets thinner. According to table 3.9, the size of the cockles changes much less than the paper thickness. As a result, the ratio L/t increases as the paper gets thinner, and cockling thus increases. In the unevenness of paper structure, it is important that the unevenness is greater the smaller its scale. This increases cockling in thinner papers, because the cockles are smaller. Second, the local two-sidedness of the fibre orientation angle decreases, because of the correlation between the sides (fig. 9). This, in turn, reduces cockling with thin papers.

Thus, as the grammage changes, several conflicting interactions affect cockling. The net effect, though, is that cockling increases as grammage decreases.

DISCUSSION

Qualitative results have been obtained about paper cockling using the finite element method. Cockling arises either from local buckling or from local curling.

Buckling is caused by in-plane stresses arising from variations in in-plane hygroexpansivity. The critical loads for buckling are low, so in principle buckling always happens when the moisture content of paper changes. However, the eigenvectors were such that the whole structure is uniformly bent, although the stress is highly localized. Therefore, buckling may not alone cause cockling.

Curl is caused by in-plane twisting moments arising from variations in the two-sidedness of hygroexpansivity. In the light of the present analysis, the most important factor is the two-sidedness of the fibre orientation angle. Formation also has a significant effect. The shape of the cockles is affected most by the average orientation; the stronger the orientation, the more oval the cockles are.

Both in buckling and in curling, cockling occurs more easily the smaller the ratio of paper thickness to cockle size. The main implication of this is that cockling intensifies as paper gets thinner. However, if the main factor causing cockling is the local two-sidedness, this effect is balanced because the paper surfaces correlate more with each other in the case of thin papers.

The present work shows the importance of local curl. Buckling could not be studied thoroughly, and further experimental work is needed to reveal the relative importances of buckling and curl.

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Transcription of Discussion

FINITE ELEMENT ANALYSIS OF PAPER COCKLING

I Kajanto

J F Waterhouse, Institute of Paper Science & Technology, USA
A nice piece of work. One question and an observation – what relationship do the variables you have looked at, have to the periodicity of the cockling? What is controlling the wavelength?

I Kajanto
I did not notice any periodicity in cockling. It is a random structure. The average wavelength was controlled by the grammage. The higher the grammage the longer the cockles.

J Waterhouse
Observation – did you consider the role of internal stresses or the effect of drying stresses on cockling?

I Kajanto
I did not analyse the drying stresses.

J Waterhouse
I suppose you could think about cockling as really a local variation of the residual or internal stress distribution, varying from point to point in the sheet.

Prof J Lindsay, Institute Paper Science Technology, USA
I am not sure I necessarily agree with ruling out buckling as a cause

for cockling. As I understand it, (the relation) in the buckling problem, the final shape you get ought to be very strongly affected by the boundary conditions of the sheet, restraint and the possibility of shrinkage, those types of things. Would you explain about the boundary conditions and the conditions of restraint you applied in your model. (Fig. 3).

I Kajanto

The boundary conditions were such that only the movement of the model was prevented so that it corresponds to a freely dried sheet.

J Lindsay

So, I take it the fours corners are held in place, the middles of the four sides can all deform though. Is that correct?

I Kajanto

One corner is held exactly in place and the others are allowed to move in one direction but not in the other so the model cannot rotate.

J Lindsay

The cockling problem I would be interested in seeing run would be trying to predict cockling coming off of the machine or forming in the dryer section where we have significant restraint especially in the machine direction and seeing if you could then get oval shape cockles from the buckling.

I Kajanto

But that is a different situation because I analysed the free sheet and not the cockling on the dryer section and that effect may be different in this case.

Dr T Uesaka, Pulp & Paper Research Institute, Canada
A very interesting result. Is your finite element analysis based on so called large deformation theory. Is it non-linear plate analysis type?

I Kajanto
No, I also made some non-linear runs but most were linear. Non-linearity was not very significant in this case.

T Uesaka
Some of the non-linear plate analyses clearly showed that such local buckling – local non-homogeneity of elastic property certainly produced such cockling. So there seems to be some difference in the result in your case.

I Kajanto
The result depends on the boundary conditions so with certain boundary conditions that may well happen.

Prof M Kortschot, University of Toronto, Canada
Would it be possible to distinguish between buckling and a two-sided effect experimentally by seeing whether the cockles popped through, that is whether they have two stable positions or just one?

I Kajanto
In principle yes, but I doubt whether it works in practice. If a cockle has formed through buckling, after a while the paper will have

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I Kajanto
In principle yes, but I doubt whether it works in practice. If a cockle has formed through buckling, after a while the paper will have crept

and the cockle will stay on one side of paper, just like a cockle formed through localised curl.