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MODELLING THE STATE OF STRESS AND STRAIN IN SOFT-NIP CALENDERING

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Calendering, Contact, Finishing, Friction, Supercalendering

ABSTRACT

A brief review of pertinent works in the field of contact mechanics is made, with emphasis placed on works related to the calendering process, starting with Hertz' assumptions and ending with non-Hertzian effects, including thin-layer covered rolls, sliding, sticking, "micro-slip", and friction effects.

Emphasis is placed on the effect that changing Poisson's ratio has on the behavior of stress and strain in the nip and how this is related to soft-nip calendering parameters (the relative rotational velocities of soft-covered and mating rolls, for example).

High resolution color graphics of the state of strain in the nip zone are presented, as modelled by finite-element analysis of the contact problem in soft-nip calendering.

INTRODUCTION

The literature in the field of Contact Mechanics is so extensive that the papers surveying this field have had to concentrate on narrower aspects of the discipline. Some outstanding examples are the relatively recent surveys by Kalker (1), Goodman (2) and Johnson (3). Although the main purpose of these papers was to review the field of Contact

Mechanics, these authors (who are authorities in the field) have had to exclude topics like lubrication (elastohydrodynamic, etc.), thermal deformation, detailed material behavior in contact (tribology), and experimental mechanics, in order to be able to do a concise job of reviewing the field. Therefore, we will only attempt a cursory review of the field placing particular emphasis on topics that are closely related to the calendering process.

Hertz' theory

The Hertz (4) theory of contact rests on the following assumptions: 1) the geometry of the contacting curved surfaces (at least one of the bodies must have a curved surface) can be accurately described by quadratic terms in the spatial coordinates (this is actually a good approximation as long as the contact lengths are smaller than the relative radii of curvature of the surfaces), 2) the bodies deform as (infinitely deep) half-spaces, 3) the (small-deformation) linear theory of elasticity applies, and 4) there is no friction present at the contacting surfaces.

This theory has been quite successfully applied in engineering practice. Its success is due to the fact that it rests on a consistent set of assumptions. Fessler and Ollerton (5) measured the contact dimensions of photoelastic models, and found that the measurements did not depart significantly from Hertz' theory up to the limit of the material strength of the models they used. Furthermore, the agreement between Hertz' theory and these experiments was good even when the ratio of the significant dimensions of contact area to the radius of curvature of the surfaces reached the surprisingly high value of 0.3.

Most publications dealing with contact problems retain at least some of the assumptions of Hertz' theory of contact. For example, when friction forces are included, most works still keep the assumption of linear elastic half-spaces in contact and the assumption that the dimensions of the contact area are smaller than the principal radii of curvature of the contacting surfaces.

While the assumption of half-spaces in contact is a good one for cotton-filled rolls, as employed in supercalendering of paper, and for metal rollers used in calendering, the synthetic covers used for on-machine soft-nip calendering of paper (and more recently, for supercalendering of paper) can be so thin (in comparison with the contact length) that the half-space assumption has to be discarded.

Of course, doing away with Hertz' assumptions causes major complications in obtaining closed-form solutions, and hence, for complicated problems, numerical rather than analytical solutions may have to be employed.

Classification of contact problems affected by friction

Most analyses of friction effects on contact problems use Amonton's (Coulomb's) law of friction.

In order to discuss friction effects, it is useful to adopt a classification similar to the one used by Johnson (3) and divide the types of contact problems affected by friction into those where there is a resultant tangential traction acting on the contacting bodies (as well as a force normal to the contacting surface) and those types of contact problems where there is only a resultant normal load acting on the surface.

Contact problems affected by friction

- 1) Resultant tangential traction at contact interface
  - a) Sliding contact
  - b) Static contact under tangential load
  - c) Tractive rolling
- 2) No resultant tangential traction at contact interface
  - d) Static contact of dissimilar solids under no tangential load
  - e) Free rolling of dissimilar solids

It is important to make a distinction between the two types (b and d) of static contact problems (that is, with and without a tangential load). It is also important to distinguish between the two types (c and e) of rolling contact (tractive rolling, and free rolling).

Static contact

Static contact under a tangential traction is that type of contact where the tangential traction is not large enough to produce overall sliding of the bodies, but where there are still regions of slip present in the contact area. Static contact of dissimilar solids under no tangential load takes place when only normal loads are acting on the contacting surfaces. There are regions of slip present in the contact area due to the fact that bodies with different elastic properties will experience different tangential displacements under a normal load.

In the condition of static contact under a normal load, as well as in the free rolling of dissimilar solids, only purely normal loads (to the contacting surfaces) are acting and hence the resultant tangential load is zero. In the conditions of sliding, static contact with a resultant tangential load, and in tractive rolling, a non-zero resultant tangential load is acting on the surface in addition to the normal load.

Dissimilar materials

Only when differences in tangential displacements of the contacting bodies may arise (whether as a result of a resultant tangential traction or from the dissimilar elastic characteristics of the contacting bodies), will friction result in a solution of a different nature than Hertz's theory.

Indeed, it is interesting to observe that Hertz' theory of contact will still apply when friction is present at the interface if: a) both bodies experience identical tangential displacements at the contacting surface, or if b) the tangential displacements of both bodies at the contacting surface are zero. Under these conditions the contact process, including free rolling, is completely reversible in the thermodynamic sense, since there is no slip, and hence there is no heat dissipated at the interface. The contact stresses and deformations are still given by Hertz' theory of contact.

If the contacting bodies can be regarded as half spaces, then the condition under which both bodies will experience identical tangential displacements at the contacting surface (when no resultant tangential traction is present) is when both bodies have identical elastic properties (identical Young's modulus and Poisson's ratio), or, more precisely, when bodies have identical bulk moduli, (see Dundurs (6) and Comninou and Dundurs (7), for a definition of bulk moduli under conditions of plane strain or plane stress). The tangential displacements of both bodies (here regarded as isotropic elastic half-spaces) are zero (again, when no resultant tangential traction is present) when both contacting bodies have Poisson's ratios exactly equal to 0.5, or when one body is rigid (infinitely large Young's modulus) and the other one has a Poisson's ratio exactly equal to 0.5. The condition that the tangential displacements of both contacting (half-space) bodies are zero can, of course, be more simply stated to be have been met when the bulk moduli of both bodies are infinitely large.

Sliding, sticking and slipping

Dealing now in more detail with the types of contact problems where a resultant tangential traction is present at the contact interface, sliding contact is defined as the case where one body slides with respect to the other, so that the whole contact area is under a state of slip. Under this condition, the tangential force is equal to the coefficient of friction times the normal force (according to Amonton's law of friction). Static contact (under the presence of a resultant tangential traction) takes place when the resultant tangential traction is smaller than the normal force times the oefficient of friction. It is interesting to observe that the absence of overall (gross) sliding does not imply that there is no slip over some part of the contact area. According to Amonton's law of friction, the coefficient of friction is defined as the ratio of the tangential traction to the normal load, under sliding conditions. Since the normal (compressive) stress goes to zero at the perimeter of the contact region faster than the shear (tangential) stress, one would need an infinitely large coefficient of friction in order to prevent slip at this boundary. Therefore, some slip is inevitable, for any finite amount of friction, under the action of the smallest tangential traction. When the region in which slip takes place is very small with respect to the total contact area, this slip has been called "microslip" by some authors. As one increases the tangential traction, (keeping the same coefficient of friction) the slip zone(s) becomes larger and larger, at the expense of the sticking zones, until when the overall tangential traction becomes equal to the overall normal load times the coefficient of friction, there is complete sliding of both bodies.

Rolling contact

Since friction is a mechanism by which heat gets dissipated, and dissipative processes are history-dependent, the state of contact becomes history-dependent when slip is present. Hence, it is not surprising that the state of contact while rolling under a tangential load (tractive rolling) is different than under static contact. Examples of tractive rolling conditions are a driving wheel, and a wheel on which brakes are applied, since these wheels must exert a tractive load at the contacting surface when friction is present. Under tractive rolling conditions the regions of stick and slip are (unlike the situation under static contact) no longer symmetrically located. One would expect this, since as previously mentioned, slip produces a history dependence and therefore the history of load application in a rolling process results in a lack of symmetry around the center of contact. When both bodies have identical material properties (that is, when they have the same Dundurs' bulk moduli) under tractive rolling conditions, there is a stick region located at the leading edge of contact and a region of slip located at the trailing edge. The strain in the stick region has different (constant) values in each body. For example, the stick region of a driving wheel experiences circumferential shrinking, while the stick region of the mating rail experiences stretching. The driving wheel behaves as if its circumference were decreased in length, while the rail behaves as if its length had been increased. As a result of this the driving wheel will roll forward in one revolution a distance that is less than its undeformed perimeter.

Rolling Creep

The fractional difference between the forward speed of the wheel and its peripheral (surface) speed is known as the "creep ratio". Free rolling bodies having identical material properties experience no rolling creep, since there is no tangential load or slip in this case. Also, if both contacting bodies were perfectly rigid the creep ratio would be zero. Indeed, it is important to note that it is due to the fact that the contacting bodies do deform under load, that rolling creep takes place.

This creep, due to the difference between the tangential deformations of the contacting bodies in the stick zone, has (in the trade literature) sometimes been confused with sliding. The reason for this confusion is that two rolling bodies that experience complete sliding at their interface will still experience a differential velocity and, hence, their creep ratio will not be zero. Since people can more intuitively relate to differences in speed between rolling bodies by attributing this speed differential to sliding effects, rolling creep has sometimes been attributed to sliding rather than to differences in tangential strains between the contacting bodies. Roll to roll slip was postulated more than forty years ago (8,9 and 10) as the mechanism that produces a polishing effect on supercalendered paper. This belief has continued to be widespread among papermakers. This is possibly due to an extrapolation made from watching the action of brush calenders and friction calenders. In these types of calenders slip between one roll and the paper is deliberate and pronounced. Furthermore, it has long been known that a peripheral speed differential exists between the rolls in any supercalender stack. Schacht and Kirchner (11) measured a peripheral-speed differential between the top and bottom rolls of a nine roll supercalender. The fractional speed difference (or creep) between these rolls was 0.00408 when externally loaded. Under no external loading (just under the action of the roll weights) the measured creep was 0.00285. Papermakers naturally assumed that this speed differential was due to sliding between the rollers. However, it should be clear to the analyst that the speed differential cannot be due to gross sliding between the rollers, since the speed differential increases with increasing load. This speed differential is actually due to the deformation of the bodies in contact. Since the deformation increases with increasing load, the creep increases with increasing load as well. Howe and Lambert (15), among others, have measured the speed differential of metal calender stacks (where all the rolls are made of the same material). Their precise measurements showed that no speed differential existed. This confirmed that rolling creep is due to the differential deformation of the rollers in contact and that it was not due to sliding. Since the rolls in a (hard nip) calender are all made of metal, there is no differential speed and hence no creep is present.

As remarked by Johnson (3), nearly a century had to elapse after Hertz published his theory (4), until a complete theory of rolling contact of dissimilar solids appeared in 1967 (12 and 13). Bentall and Johnson (12 and 13) explained the nature of the stresses and strains, as well as the extent of sticking and slipping zones in the nip of dissimilar rolling bodies. The frictional properties of paper (14) are such that it is actually not physically possible to have overall sliding of the rollers involved in the supercalendering process, unless sufficient braking force would be deliberately applied to one of these rollers. Indeed, the shear stress due to friction is an order of magnitude smaller than the normal stress throughout most (but not all) of the contact area. Therefore, most of the contact area "sticks" rather than "slips".

Micro-slip

The ratio of shear stress to normal stress rises to infinity at certain locations in the contact zone unless slip is allowed to take place at these locations. Therefore, slip does take place at these locations, for any finite coefficient of friction. The extent of these slip zones is minute (compared to the total extent of the contact zone) under free rolling conditions, and therefore the slip in these zones is referred to as "microslip". As described at length by Johnson (16), and by Bentall and Johnson (12), the shear stresses and tangential (or hoop) strains at the surface depend on the conditions of stick or slip at the point in question. At a given space point of contact, there are two material points that instantly mate at that contact point. Each of these material points belong to one of the contacting bodies. One of these bodies will be denoted by the suffix "1", while the other body will be denoted by the suffix "2". At the contact point the two bodies have local instantaneous velocities v_1 and v_2. Far away from the contact points, the instantaneous deformation of the bodies is zero, and the surface velocities are the peripheral speeds V_1 and V_2. Bentall and Johnson (12) derived the following formula for the creep of both bodies, assuming that the difference between the peripheral speeds was very small:

$$\xi = \frac{V_2 - V_1}{V_2} = \frac{\Delta v}{V_2} + \varepsilon_1 - \varepsilon_2 \qquad (1)$$

where ξ = creep ratio

$$\frac{\Delta v}{V_2} = v_2 - v_1 = \text{micro-slip velocity}$$

ε_1 = circumferential (hoop) strain of body 1

ε_2 = circumferential (hoop) strain of body 2

This equation is consistent with the assumptions of the theory of linear elasticity and with the condition of no (gross) sliding. Similar formulae were previously derived by other authors (17). Soong and Li (18) recently published a proof of a similar relationship between the speed differential and the local strain. Where the two bodies stick, the micro-slip velocity is zero and hence the creep ratio is exactly equal to the circumferential strain difference.

As previously noted, unless there are significant tangential forces (for example when brakes are applied to one roller) the micro-slip regions are very small. Bentall and Johnson (12 and 13) show that two dissimilar freely rolling elastic cylinders have three regions of microslip: one at each edge of their contact zone and a third near the trailing end. Bentall and Johnson draw the general conclusion that, in most practical circumstances, the amount of micro-slip will be very small and the stress distribution will be close to the one predicted by neglecting all micro-slip, that is, by assuming that the coefficient of friction is infinitely high. Tabor (19 and 20) concluded from his experiments that the energy losses due to micro-slip are negligible in the calculation of rolling resistance.

The elastic strip in plane rolling contact

Bentall and Johnson (13) provide details of the normal and shear stresses at the contact surface, the nip width, and the regions of stick and slip of a sheet of elastic material as it passes through the nip of a pair of rollers. The analysis assumes plane strain deformation. The usual Hertz' assumptions are obeyed with the exceptions that friction effects and that the finite thickness of the strip are taken into account. The numerical analysis technique used by Bentall and Johnson approximates the surface stress distribution by overlapping triangular elements. This results in a piecewise-linear stress distribution. The solution of the problem is shown to depend on the following non-dimensional parameters:

- -friction coefficient
- -strip-thickness to nip-width ratio
- -Poisson's ratio of the strip
- -ratio of plane-strain elastic moduli between the strip and the rollers
- -Dundurs' (6 and 8) parameter for the mismatch in plane-strain bulk modulus between the strip and the rollers

The Poisson's ratio of the strip only influences (as an independent parameter) the amount of the indentation. Since for practical values of the coefficient of friction the extent of the slip regions is small, the computations concentrated on solutions that assumed the coefficient of friction to be infinitely large. The thickness of calendered paper is much smaller than the nip width. Also, since paper is a porous material, its Poisson's ratio is very small. Therefore, the solution of interest is the case of very thin strips with low values of Poisson's ratio. If the rollers are much stiffer than the paper a rigid roller solution is approached where the normal stress has a parabolic distribution, rather than the semi-elliptical distribution found in Hertz' solution. This is the case in calendering of paper with metal rollers (also called hard-nip calendering). In the limit, as the paper becomes infinitesimally thin, the deformation of the rollers should be taken into account, since, however large the ratio of the roller to paper stiffness, the deformation of the roller must predominate over that of the paper. The point at which the assumption of rigid rollers should be abandoned was determined by Bentall and Johnson (13). The non-dimensional parameter that governs whether the deformation of the strip is negligible, or whether the deformation of the rollers is negligible, is simply the ratio between the deflexion of the paper and the deflexion of the rollers. If the deflexion of the rollers is significantly less than the deflexion of the paper (say, by a factor of 10), then one can assume the rollers as being rigid. Conversely, if the deflexion of the rollers is significantly more than the deflexion of the paper then the normal stress distribution becomes Hertzian. One can show that the deflexion of the synthetic covers and the deflexion of the cotton-filled rolls used in soft-nip calendering is as large or larger than the deflexion of the paper. Hence the stress distribution is approximately Hertzian.

Based on Bentall and Johnson's analysis (13) and the values of paper's coefficient of friction as measured by Jones and Peel (14) one expects the micro-slip regions to occupy a very small portion of the contact area. A braking force would need to be applied, deliberately, to one of the rollers, in order to produce a slip zone of considerable size. Three micro-slip regions exist, one at each edge of the contact zone, and another slip region near the trailing edge, in which the slip direction is opposite in sign to the other two. Micro-slip takes place due to the different elastic properties of the paper and of the rollers.

Contact analysis of thin covered rollers

As remarked by Johnson (3), elastic contact stress analysis of layered solids, such as rubber covered rollers, is well developed by now. The works of Soong and Li (18, 21 and 22), and of Wong (23) are noteworthy in that they include the effect of a sheet in the nip, in contact with soft-covered rollers. However, these authors do not take into account, in detail, the transverse compressibility of the sheet. Instead, they model the sheet as a flexible beam. Their work is applicable to the case when the nipped sheet deforms much less than the roll covers. For example, when relatively stiff paper is nipped by very soft rubber covers.

Calendering of paper: plastic deformation of a porous medium

Surface finish characteristics produced by the calendering process are caused (24) by the replication of the surfaces of the mating rollers. This replication arises from the thermoviscoplastic compaction of the paper. Plastic deformation of the paper can be produced by plastic collapse of the fiber network, by damage of the bonded sites, or by both. In order to obtain the desired uniform high gloss and smoothness properties, the calendering operation should operate (by applying the required load) in the region of the stress-strain curve where the tangent modulus of the paper is at a minimum. Since paper is an anisotropic material that has very different properties in-plane as opposed to out-of-plane, it should be pointed out that the stress-strain properties that are relevant are the through-the-thickness (out-of-plane) properties. The tangent modulus of paper can be decreased by increasing the applied temperature or by increasing the water content, or both. When the tangent modulus has a small value, the strains, rather than the stresses, are of primary interest and significance. This is evident since the tangent modulus is the derivative of stress with respect to strain. Therefore a small value of tangent modulus means that a small change in stress corresponds to a large change in strain. A small error in the strain will produce a smaller error in the stress; whereas, a small error in the stress will produce a much larger error in the strain. For this reason, strain based criteria for plastic processes are more sensitive and more reliable than are stress-based corresponding criteria. Furthermore, if there is a material instability, it is the strain and not the stress that will be able to uniquely describe the problem. For example, in unstable buckling, three different levels of strain may correspond to the same level of stress. Therefore, these nonlinear material processes can be better described in terms of strain than in terms of stress. This is opposite to what one finds in linear elastic analysis. These problems are usually described in terms of stress, since in the elastic region the tangent modulus is Young's modulus, which has a large value.

FINITE ELEMENT ANALYSIS OF SOFT-NIP CALENDERING WITH SYNTHETIC COVERED ROLLS

A finite element analysis of the contact problem was carried out, in order to obtain the complete strain (and stress) distribution of a covered roll in a soft-nip calender nip. This finite element analysis computer program (including the theoretical foundation, the computer programming, and the postprocessing) was written by Dr. Kenji Kubomura based on his Ph.D. work at the Massachusetts Institute of Technology (25). Virtually the same contact scheme was utilized as in (25), but this time assumed-displacement 8-noded isoparametric finite elements were used to model the continuum. The cylindrical geometrical shape of the rollers is exactly modeled (instead of assuming the Hertzian quadratic approximation for the geometry of the curved bodies). The contact conditions at the interface between the rollers and the paper are such that micro-slip is disregarded. This is in agreement with the results of Bentall and Johnson (13) that indicate that the effect of micro-slip should be negligible. Free rolling conditions are assumed. The paper is assumed to stick to both rollers: in this example, a (synthetic) covered roll and a (metal) mating roll. Since the paper being calendered is deformed to large plastic strains, in the stress-strain region of low tangent modulus, it opposes practically no tangential resistance to the deformation. This follows from the fact that the paper may have a tangent modulus of 10,000 to 30,000 psi, while the (synthetic) covered roll may have a cover with a Young's modulus (at the operating temperature) of 200,000 to 2,000,000 psi. The mating roll has an elastic stiffness value of about 30,000,000 psi. This value is one to two orders of magnitude higher than the stiffness of the roll cover. Therefore, the mating roll acts as a rigid material relative to the covered roll. On the other hand, the roll cover has an elastic stiffness value that is one to two orders of magnitude higher than the paper's tangent modulus. Therefore the paper practically acts as a "lubricant" between the two rollers (like a "porous lubricant" that has a Poisson's ratio close to zero rather than like an incompressible fluid).

Poisson's ratio effect on the state of strain

In order to display the state of strain in the covered roll, the following example has been chosen:

- -line load: 2000 pounds per linear inch
- -diameter of the covered roll: 23.6 inches
- -diameter of the mating roll: 24.7 inches
- -total cover thickness: 0.5 inches
- -Young's modulus of cover (at operating temperature): 200,000 psi.

On Figure 1 there is a schematic representation of the polymer cover as it experiences a normal contact load in the free rolling, soft-nip calendering of paper. In the rectangular section is indicated the section of the polymer cover that suffers the most stress and strain, since the stresses and strains die out at locations that are far removed from the contact zone. The rest of the figures show the state of strain in the cover in that rectangular section that is adjacent to the contact interface. Figures 2 to 12 display the state of hoop (also called circumferential or tangential) strain in the cover, for different values of Poisson's ratio.

Figure 2 displays the state of hoop strain for a Poisson's ratio of zero, that is, for a very compressible, porous material. The state of strain on the surface of the covered roll is negative in the contact zone. Negative values of strain mean that shrinking is occurring in the circumferential direction. The maximum negative value of strain (-1.22%) occurs at the surface, in the middle of the contact zone. The hoop strains decrease in value towards the inside of the cover. Adjacent to the edge of the contact zone, there is a region of positive values of hoop strain at the surface of the covered roll. These positive strains mean that the cover is experiencing stretching. The maximum value of positive strain is 0.51% and it occurs at the surface of the cover. Since the paper sticks (due to friction) to the cover only in the normally loaded contact zone, and the surface strain at the surface contact zone is negative, this means that the paper is actually experiencing shrinking in the machine direction when calendered with this cover.

Figure 3 shows a cover with a Poisson's ratio value of 0.10 compressed under the same conditions. The maximum negative value of hoop strain (-1.01%) still occurs at the surface, but it is lower than in the previous case. There is a zone of positive circumferential strain that appears in the middle of the cover, underneath the zone of surface negative strains. Since the hoop strain at the surface is still negative, the paper being calendered with this cover will also experience shrinking in the machine direction, but this shrinking will be less than with the cover that had a Poisson's ratio of zero.

Figure 4 displays the circumferential strain pattern for the case of Poisson's ratio of .20. This is close to the Poisson's ratio of a cotton filled roll. In this case, the contact zone surface strain is still negative, but the maximum (negative) value has diminished (in comparison with the previous cases) to -0.075%. The maximum positive value of strain occurs now in the interior zone of positive strains that lies beneath the surface, at the center of the cover. This positive strain is now higher than the positive strains that occur at the edges of the contact zone.

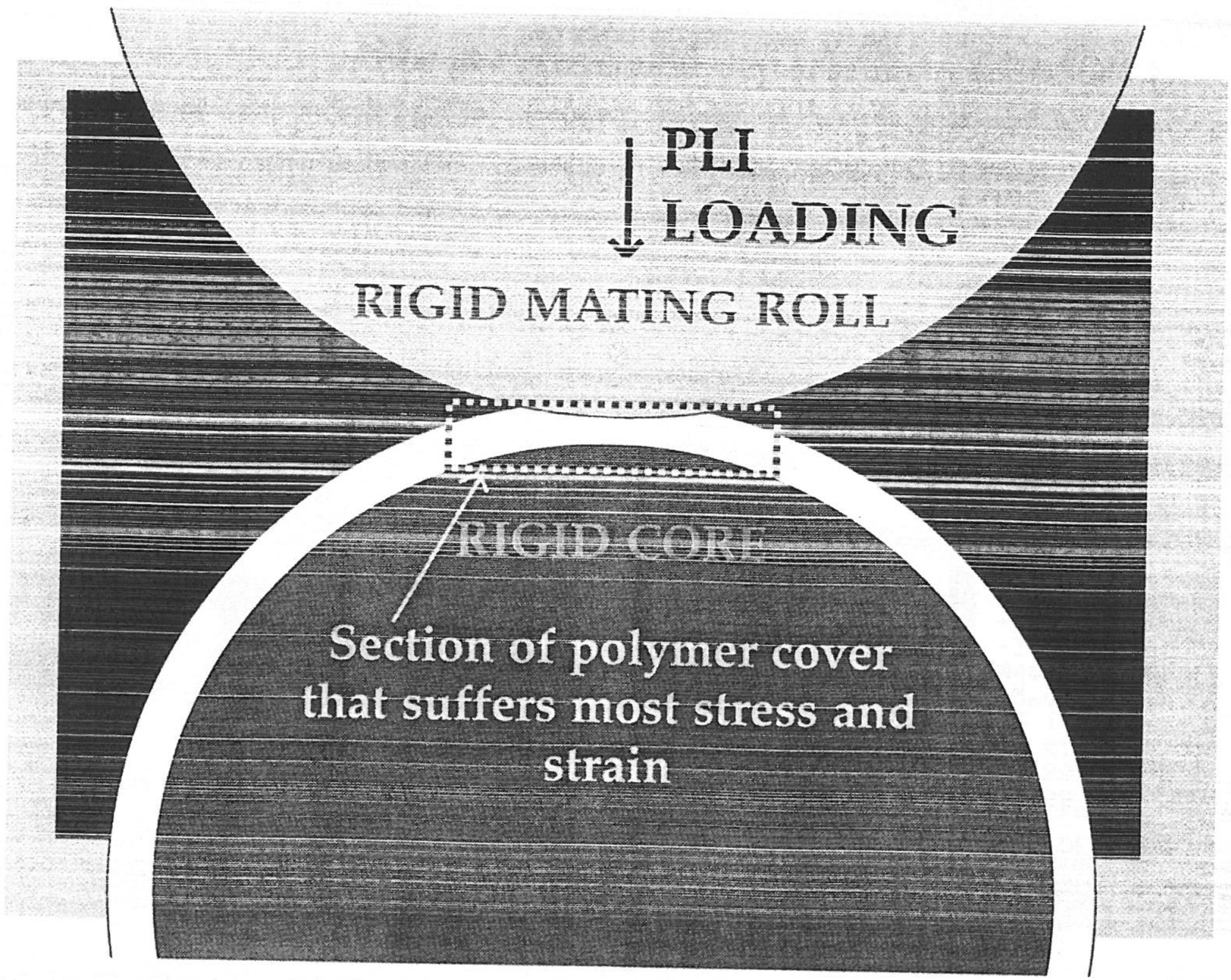

Figure 1 - Section of cover that experiences the most stress and strain

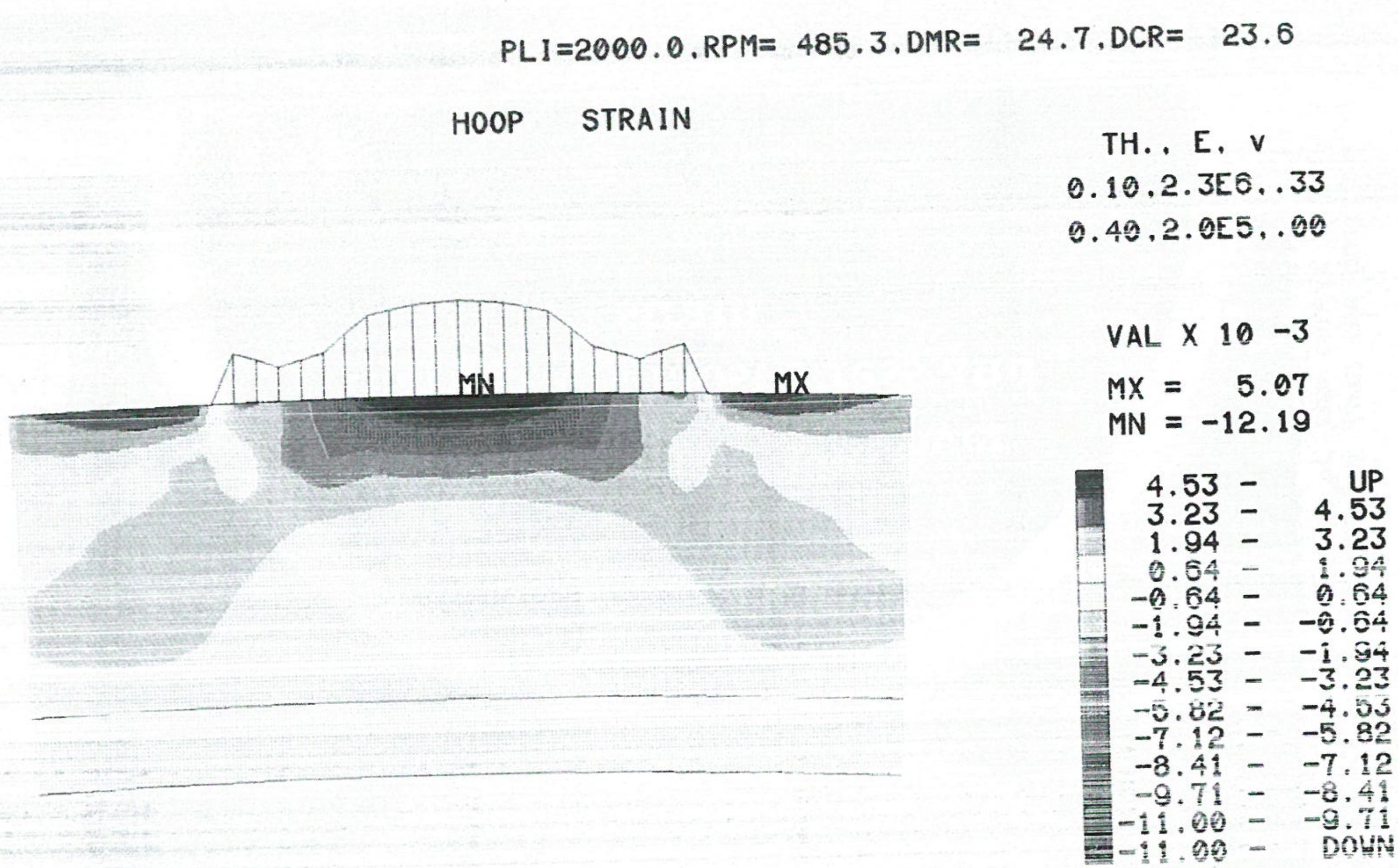

Figure 2 - Circumferential strain
Poisson's ratio = 0.00

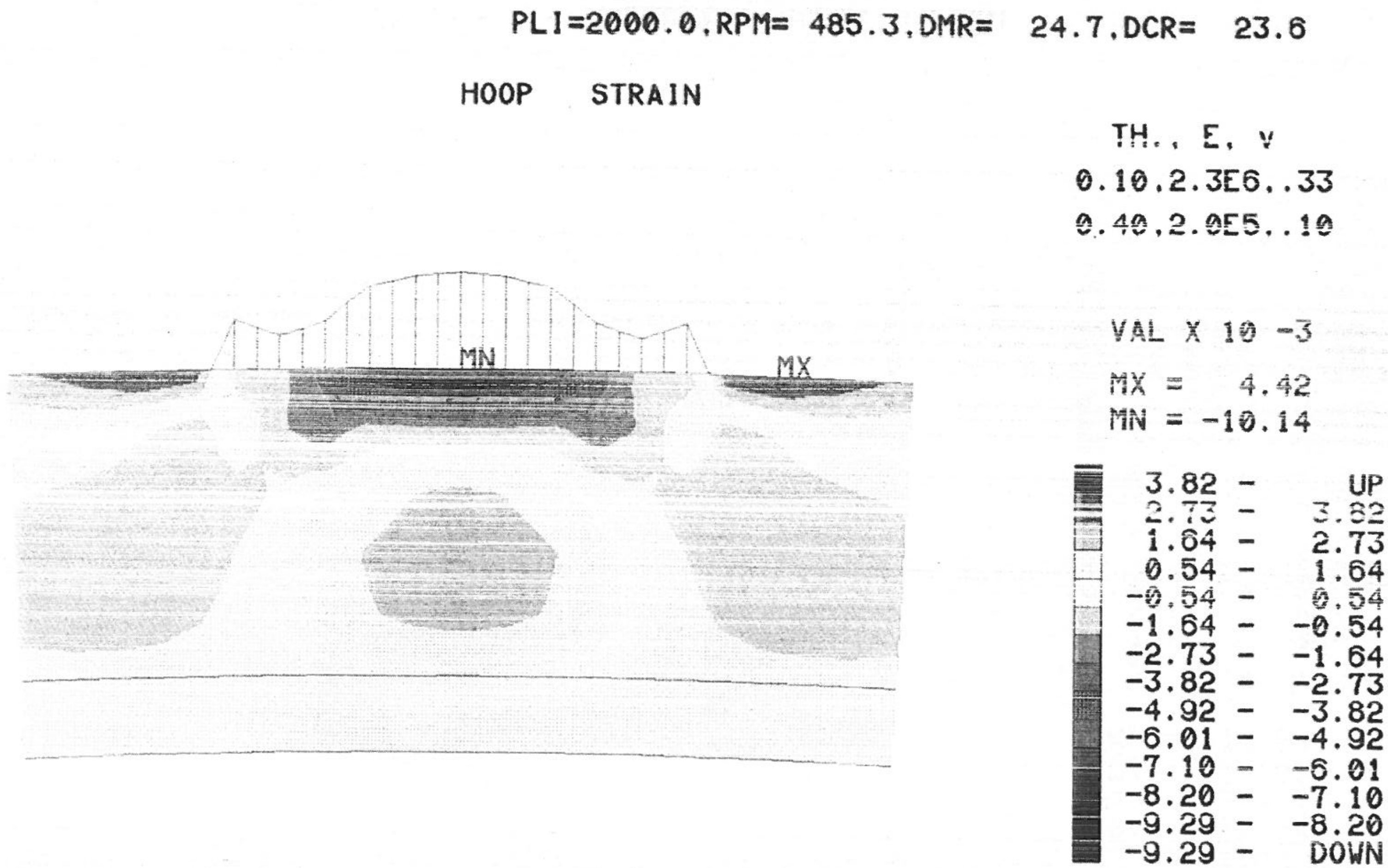

Figure 3 - Circumferential strain
Poisson's ratio = 0.10

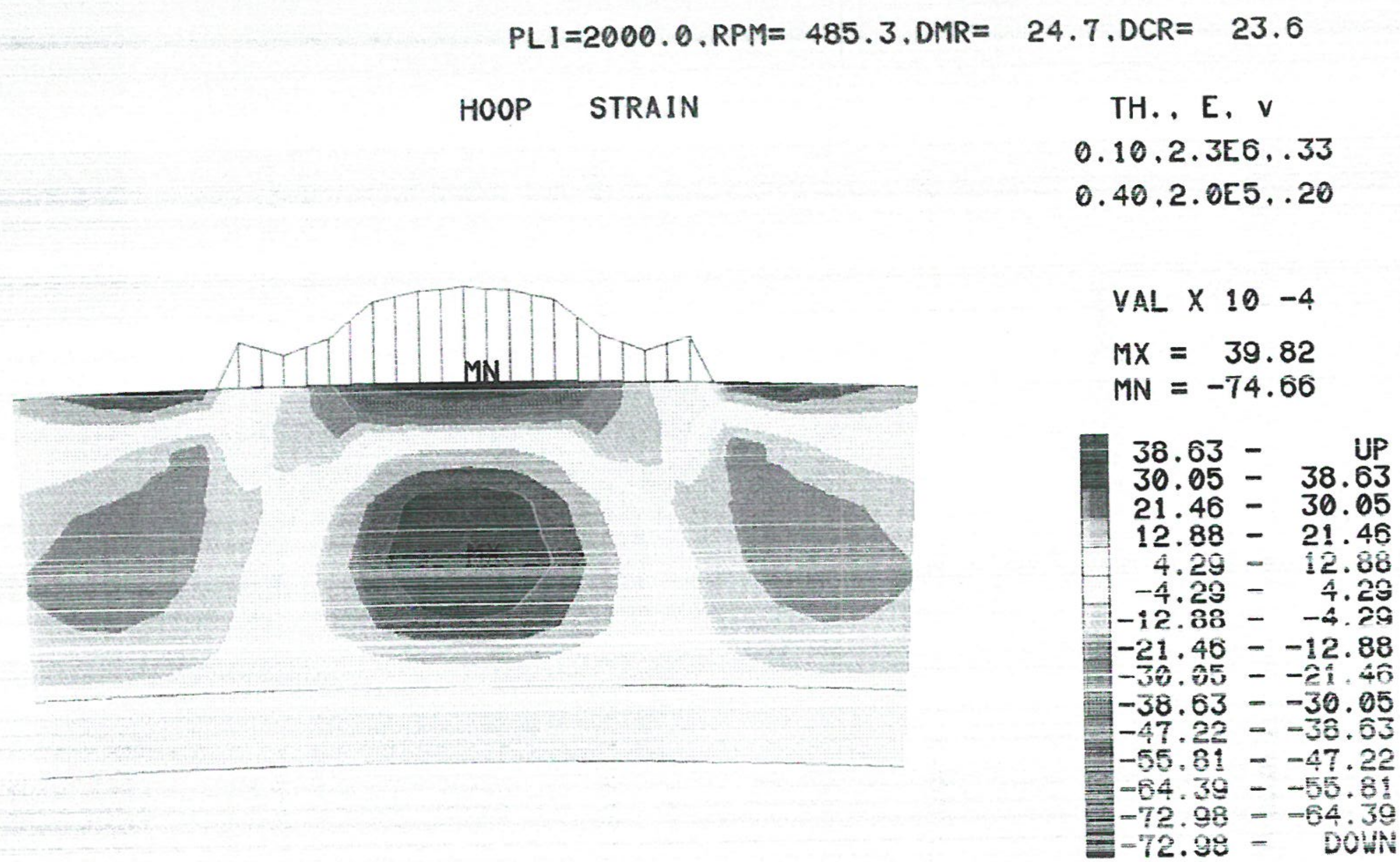

Figure 4 - Circumferential strain
Poisson's ratio = 0.20

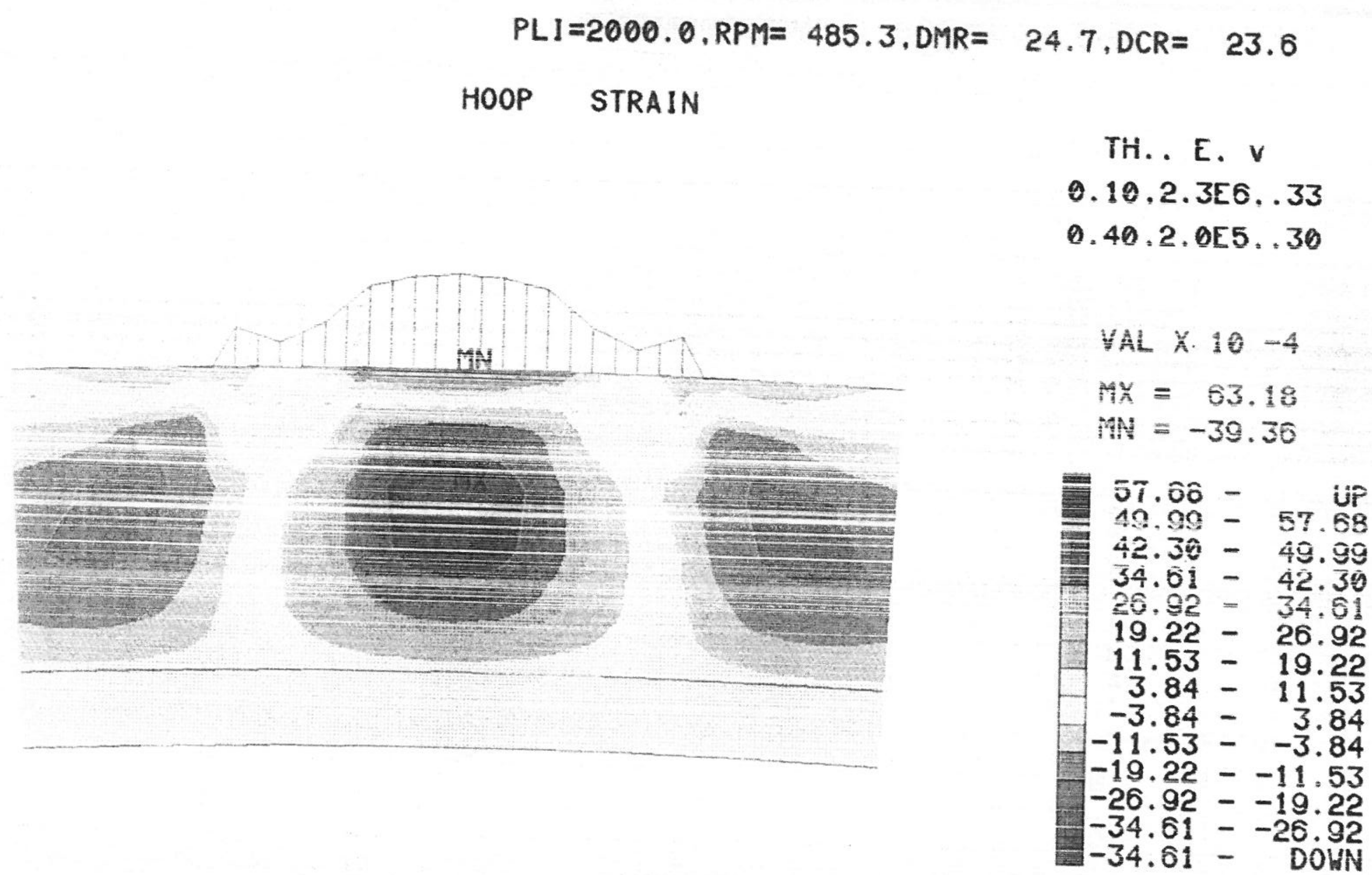

Figure 5 - Circumferential strain
Poisson's ratio = 0.30

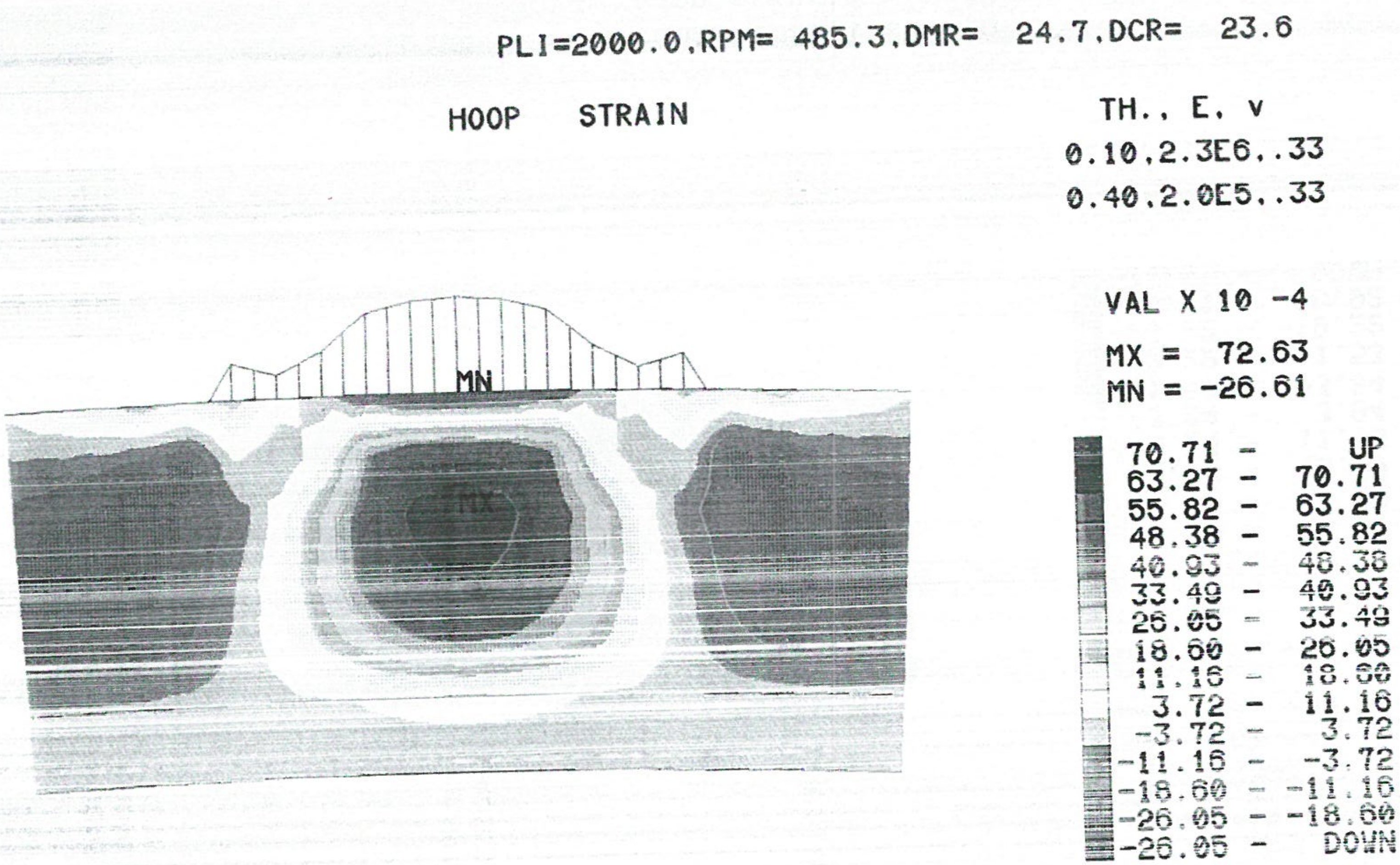

Figure 6 - Circumferential strain
Poisson's ratio = 0.33

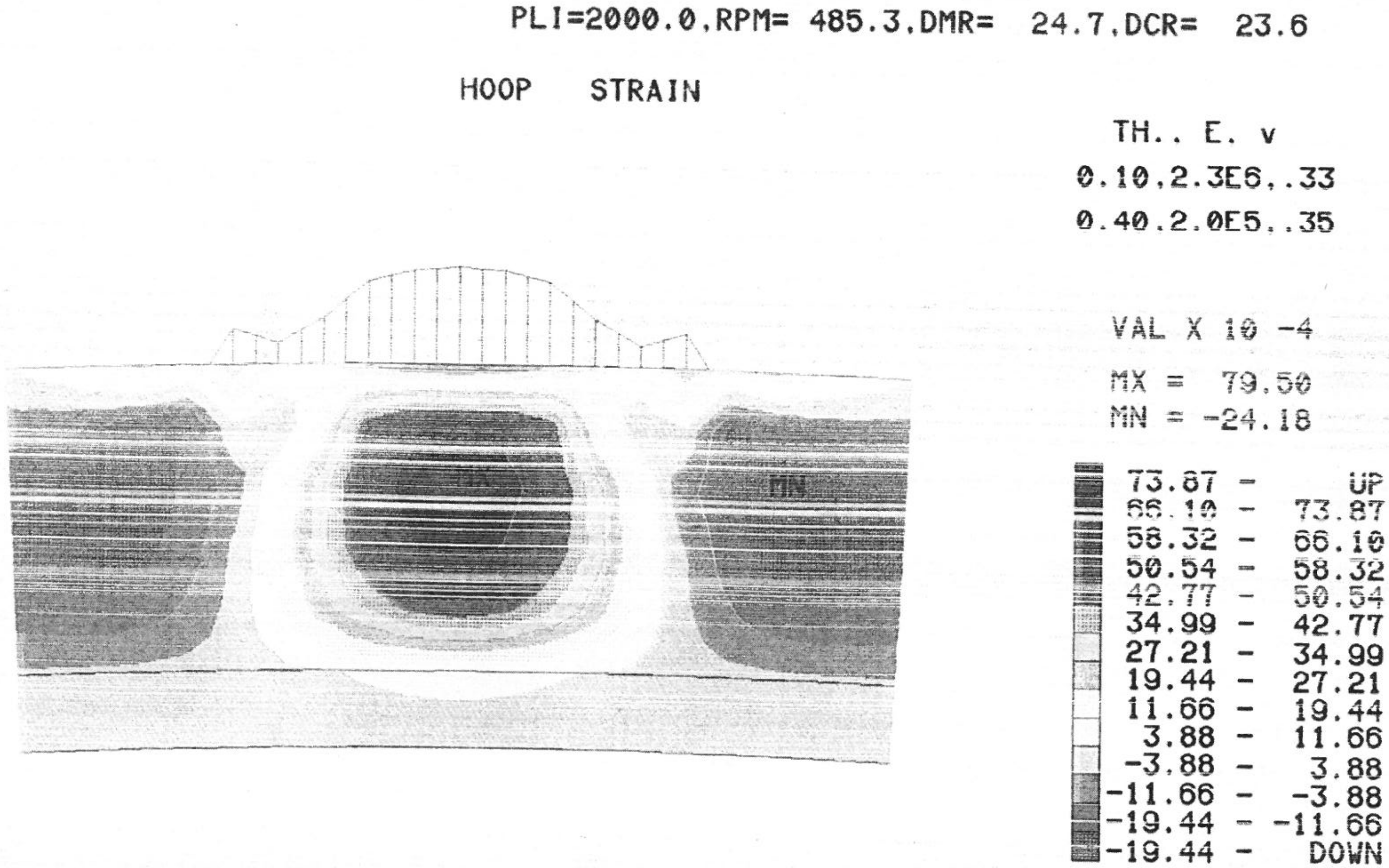

Figure 7 - Circumferential strain
Poisson's ratio = 0.35

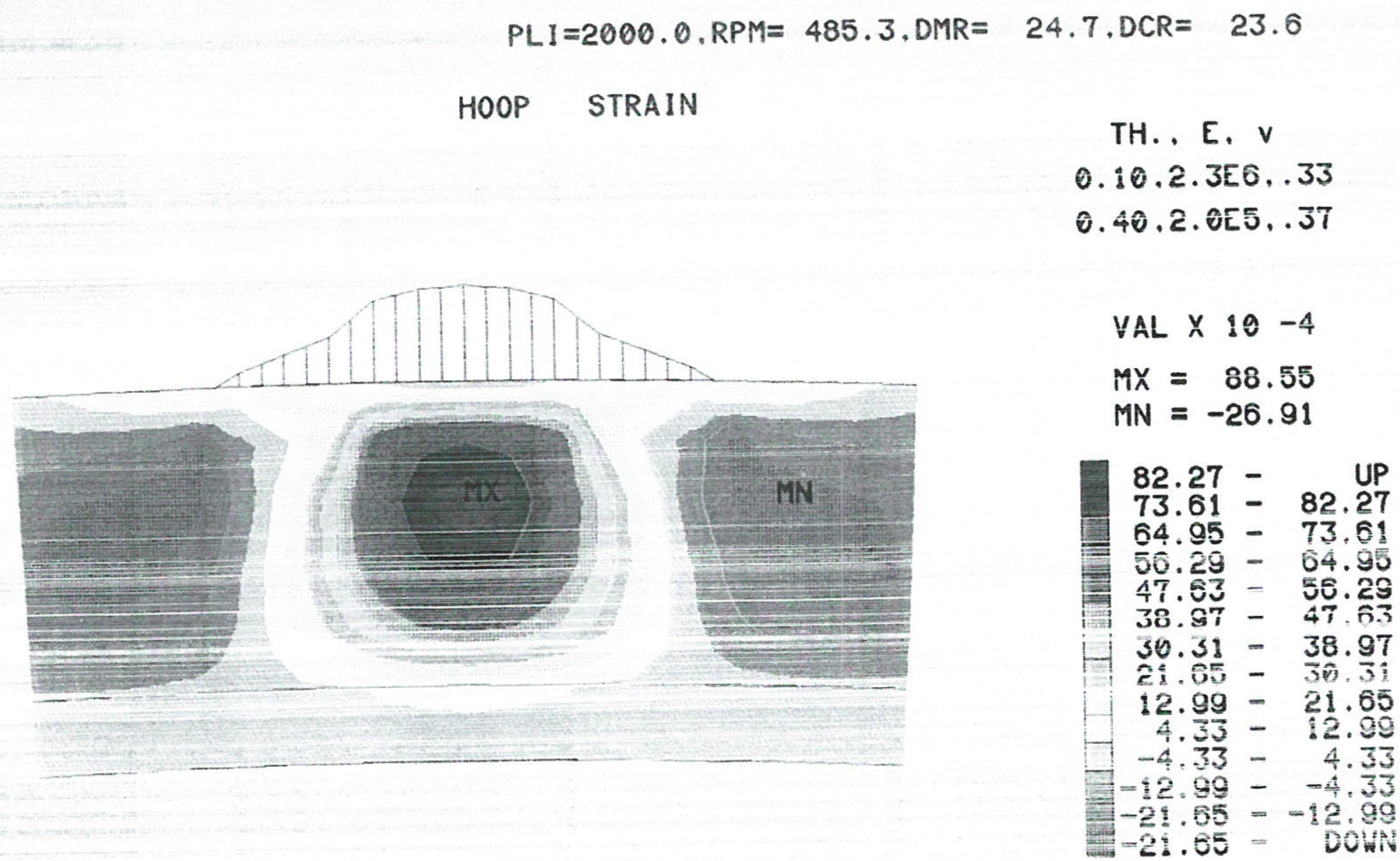

**Figure 8 – Circumferential strain
Poisson's ratio = 0.37**

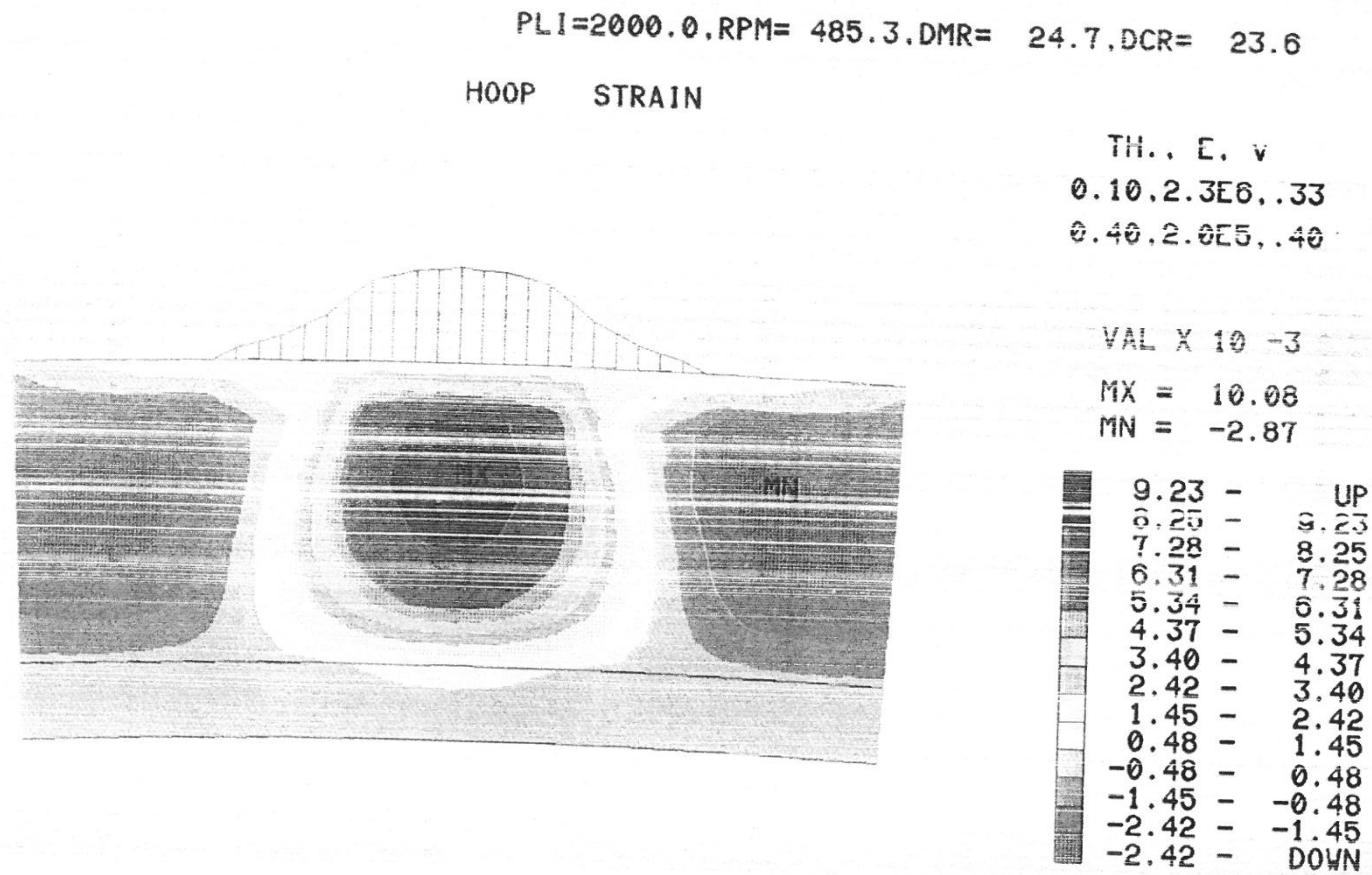

Figure 9 - Circumferential strain
Poisson's ratio = 0.40

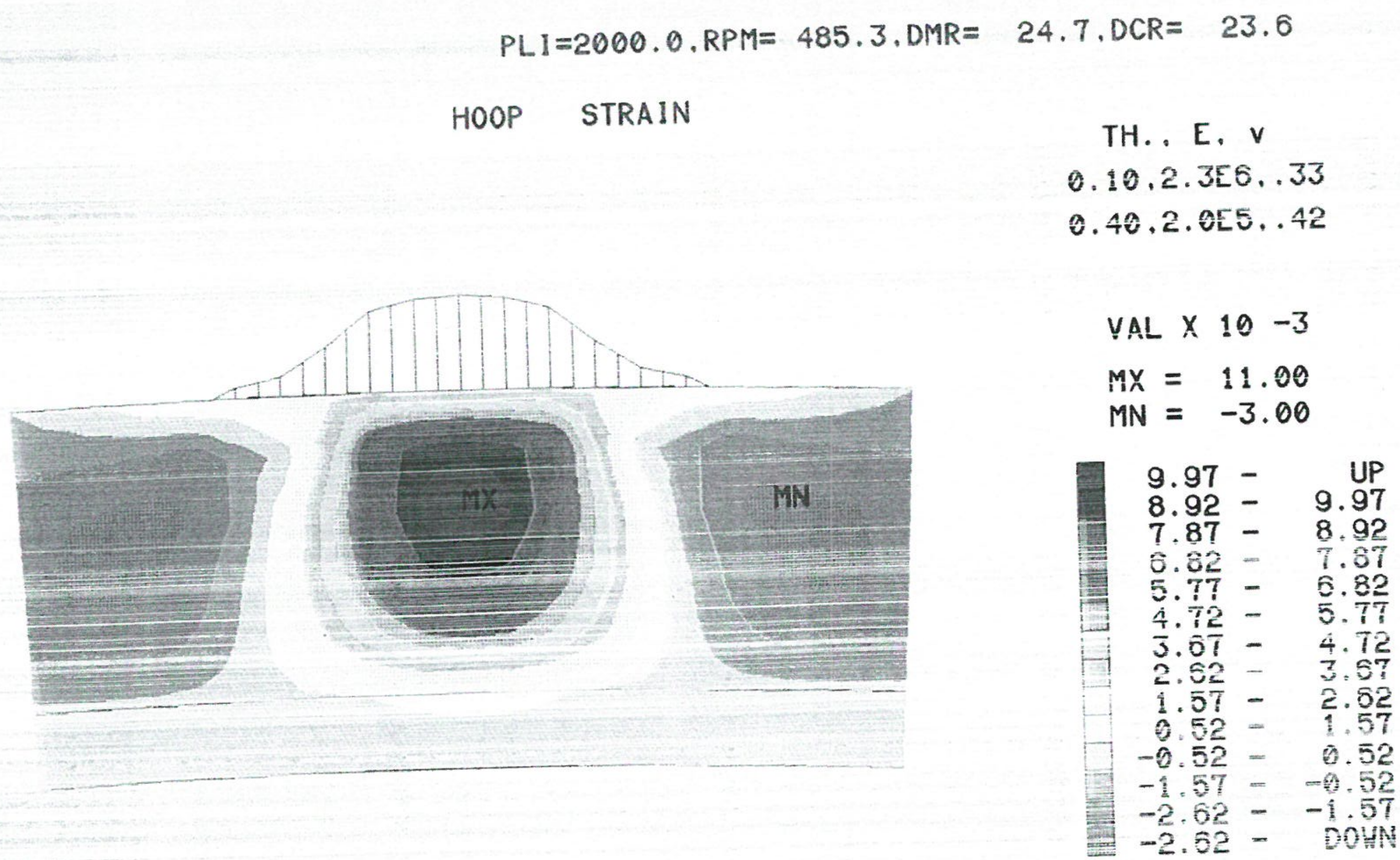

Figure 10 – Circumferential strain
Poisson's ratio = 0.42

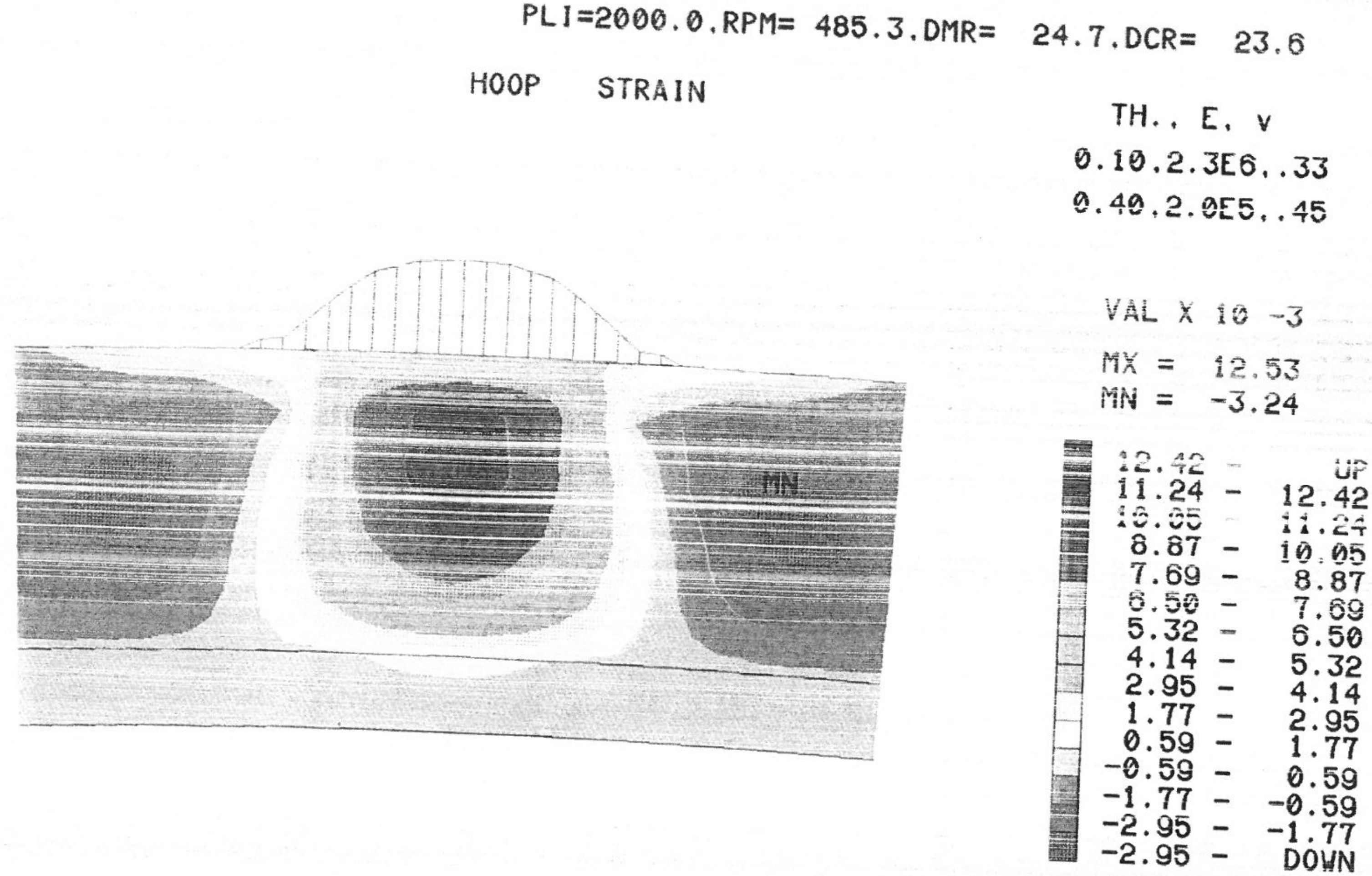

Figure 11 - Circumferential strain
Poisson's ratio = 0.45

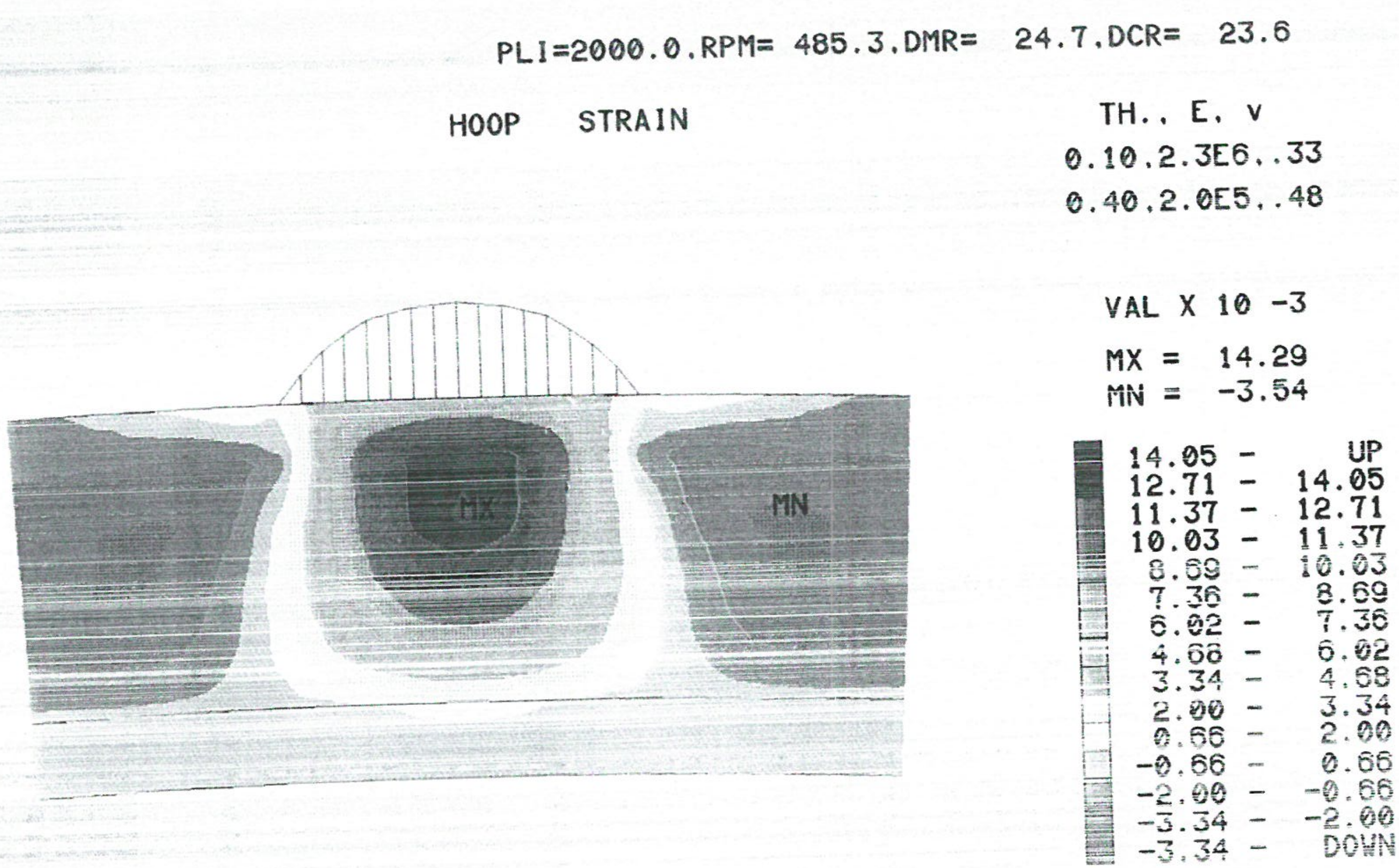

Figure 12 - Circumferential strain
Poisson's ratio = 0.48

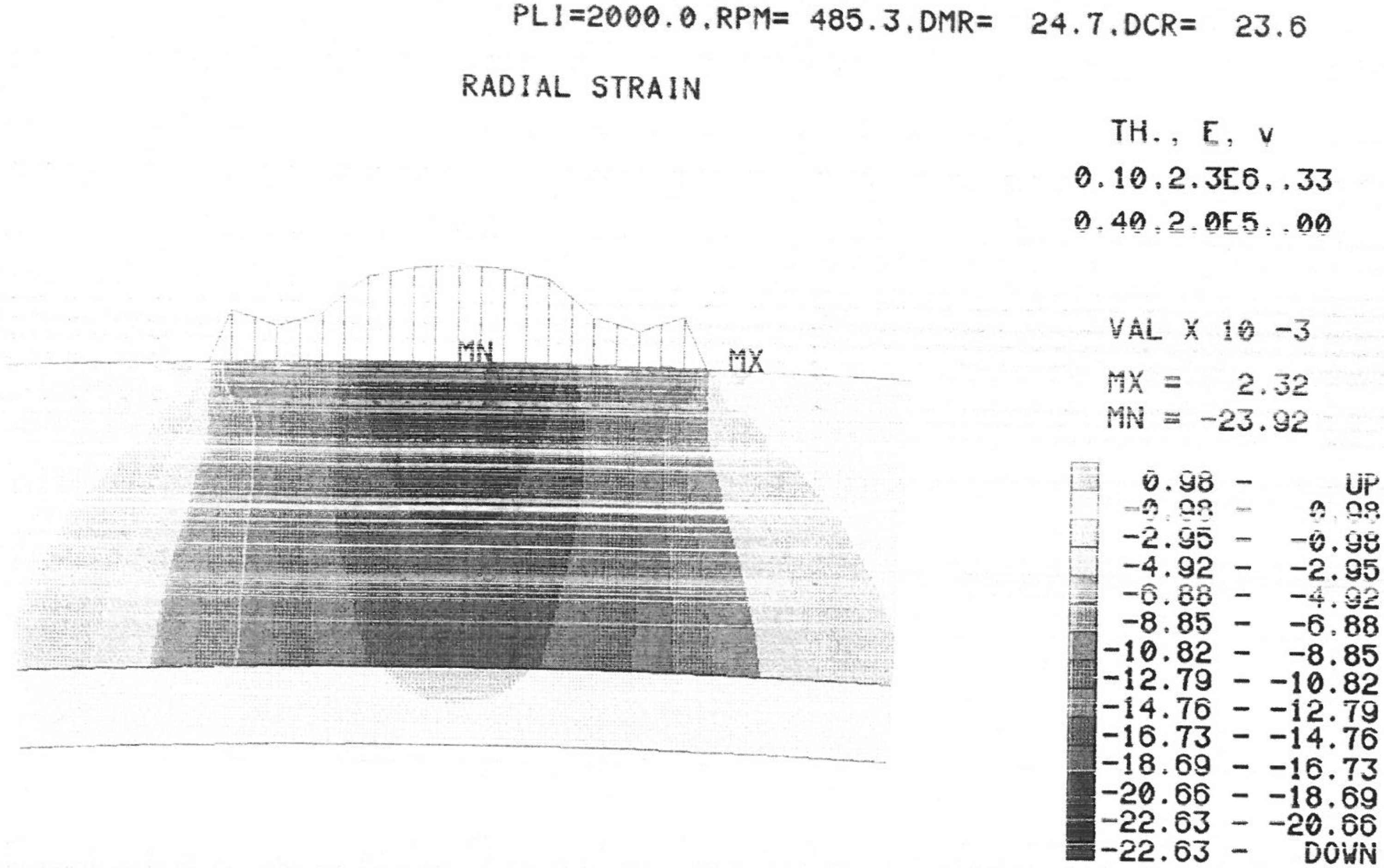

Figure 13 - Radial strain
Poisson's ratio = 0.00

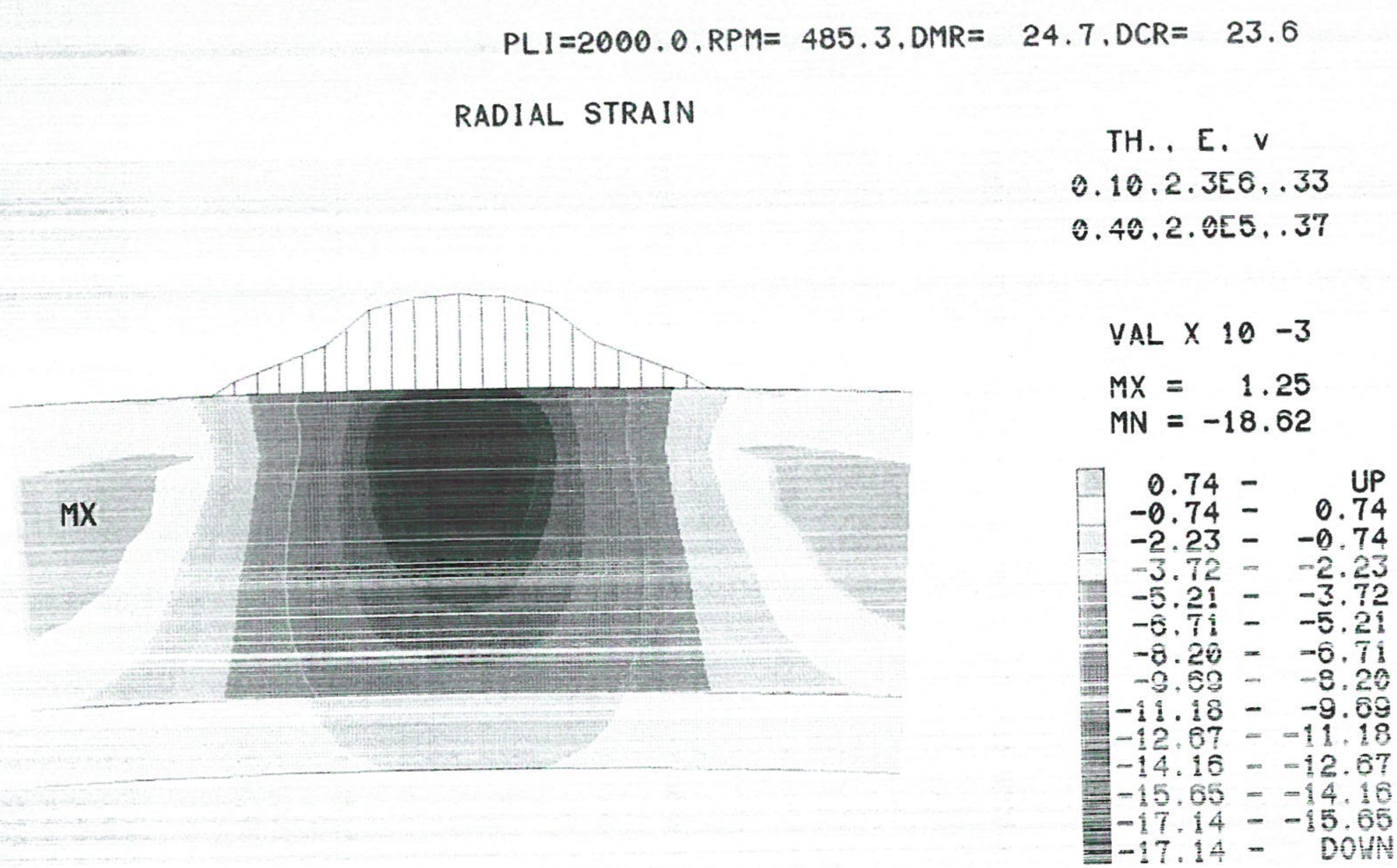

Figure 14 - Radial strain
Poisson's ratio = 0.37

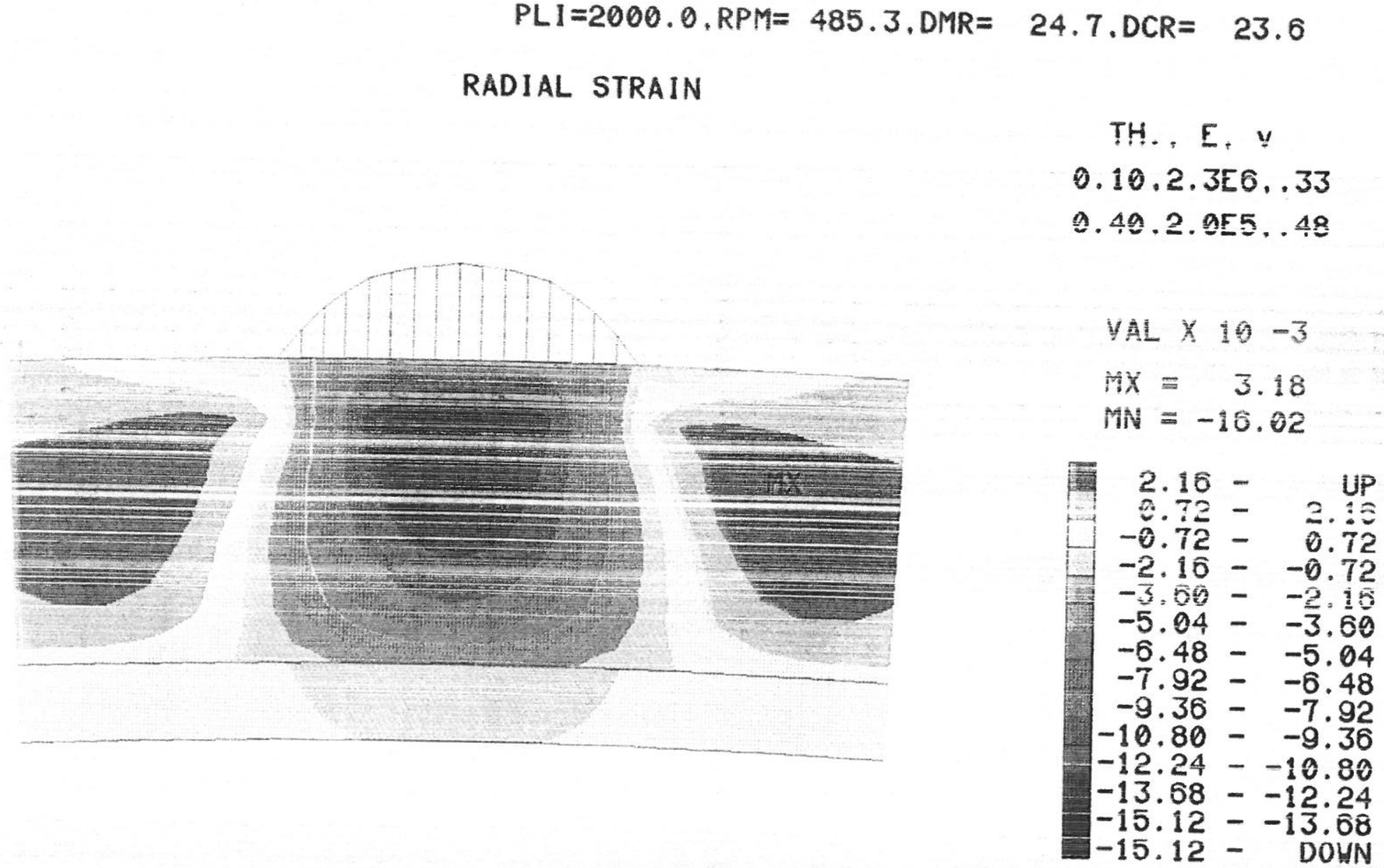

Figure 15 - Radial strain
Poisson's ratio = 0.48

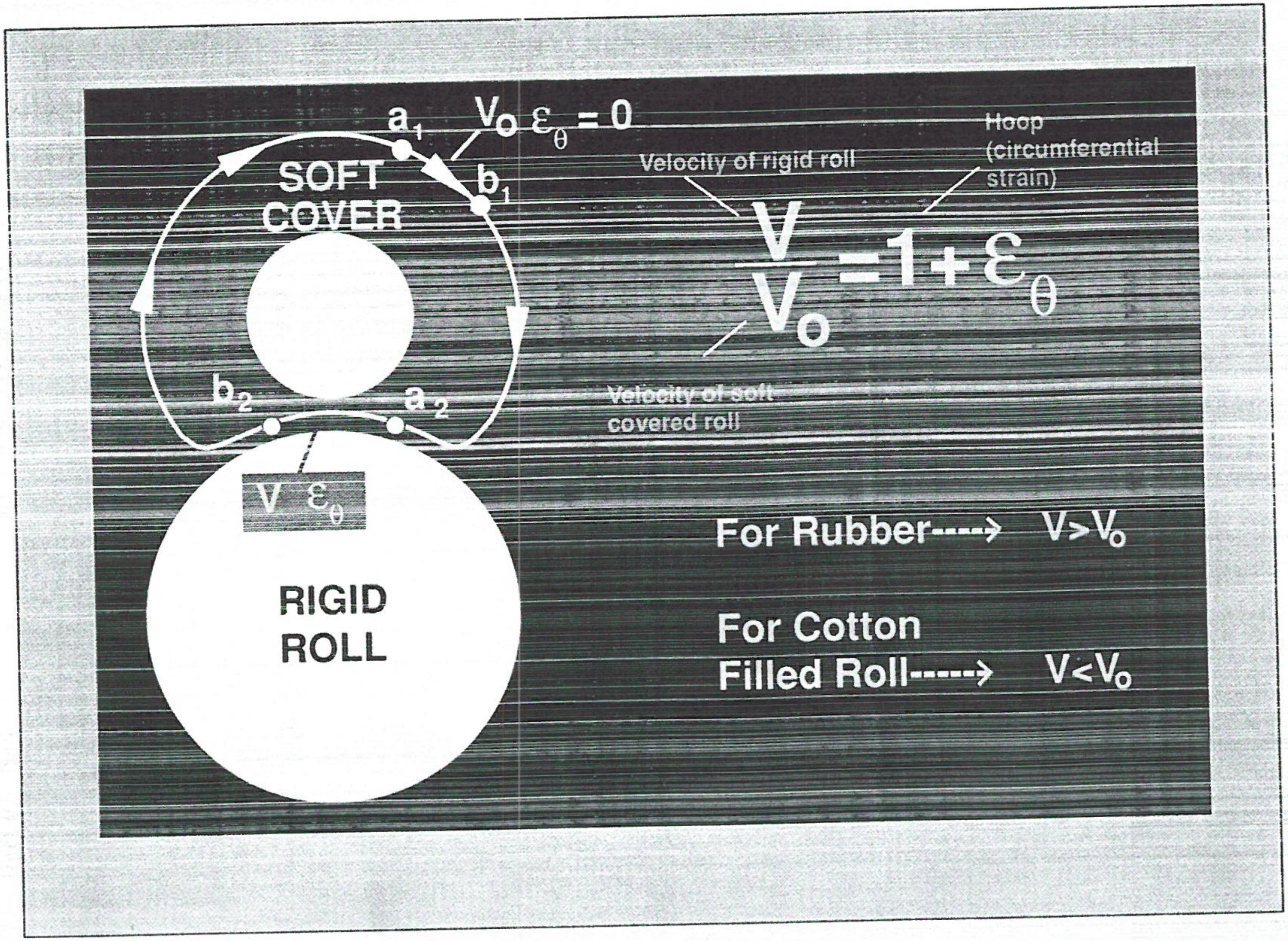

Figure 16·– Peripheral speed differential

Figure 5 shows the hoop strain distribution for a cover with a Poisson's ratio of .30. It is evident that the zone of positive circumferential strains located in the center of the cover has increased in size so much that the maximum absolute value of strain (0.63%) occurs there, rather than on the surface of the cover. At this value of Poisson's ratio the surface strains are considerably smaller than at the lower values of Poisson's ratio. Most of the contact area experiences negative strains, the maximum value of negative strain being -0.39%. There are some very low positive values of strain now occurring on the inside of the edges of the contact interface. Since most of the contact area is experiencing compressive strains, and these values of negative strain are larger than the positive values on the surface, the paper being calendered with a cover with these properties will still experience shrinking in the machine direction.

Figure 6 displays the strain state for a Poisson's ratio of 0.33. At this value of Poisson's ratio the situation shows the same pattern. The area of positive strains located at the center of the cover continues expanding at the expense of the other areas. The maximum value of positive strain (occurring at the center of the cover) is now 0.73%. The maximum value of negative strain (-.27%) occurs at the center of the contact surface.

Figure 7 (Poisson's ratio of 0.35) continues with the same pattern of strain in the cover. The center positive strain continues its increase, by reaching the value of 0.80%. The strains at the surface are now much smaller, so much so, that the maximum negative value of strain does not occur at the surface but it occurs now inside the cover, in areas that are symmetrically located from the center of the cover. The paper that is calendered with this cover that has a Poisson's ratio of 0.35 will experience a much smaller value of machine direction shrinking than with covers of lower Poisson's ratios.

The Poisson's ratio of plastic covers that have been used to replace cotton-filled rolls, in supercalendering as well as in on-machine soft-nip calendering, have Poisson's ratios of 0.35 to 0.40. Therefore, Figures 7, 8 and 9 display the circumferential strains that these covers experience in the calendering operation.

Figures 8 and 9 display the results for Poisson's ratios of 0.37 and 0.40 respectively. At these values of Poisson's ratios there is practically no circumferential strain occurring at the surface of the covered roll. The maximum hoop strains occur inside the cover.

Figures 10, 11 and 12 show the circumferential strain results for Poisson's ratios of 0.42, 0.45 and 0.48, respectively. One can see that the surface strain now becomes positive. The central region of positive strains has now grown so much that it has reached the surface of the covered roll. The higher the Poisson's ratio, the higher the tensile value of the surface hoop strain. At a Poisson's ratio of 0.48 the maximum tensile strain reaches a value of 1.43% at the center of the cover.

The explanation for the severe changes in strain distribution, going from a Poisson's ratio of 0.00 to a Poisson's ratio of 0.48 is due to the compressibility of the cover material. A cover with a Poisson's ratio of 0.48 (rubber or urethane, for example) is almost incompressible. Due to the severe constraint imposed by the finite thickness of the cover, the cover material being compressed has no alternative but to flow sideways, in order to be able to deform under load. The flow of material to the sides of the center of contact results in circumferential tensile strains. When the Poisson's ratio is 0.00 the whole strain field is negative, since the compressive state of stress imposed on the cover by the contact load can be supported by the volumetric compression of the cover. The volumetric compression of the cover results in circumferential shrinking of the cover surface.

Figures 13, 14 and 15 display the strain in the radial direction of the cover. These figures show that for a zero value of the Poisson's ratio the strain field is negative, with the maximum negative strain occurring at the cover surface. As the Poisson's ratio increases, two symmetric areas of positive strain develop and the maximum strain moves inside the cover. At a Poisson's ratio of 0.48 it is evident that the deformation under the contact zone is compressive, but that significant areas of positive, tensile strains occur adjacent to the contact area. These areas of positive strain are also due to the relative incompressibility of a cover when the Poisson's ratio is close to 0.5. When the cover material is relatively incompressible, the total volume of the material will remain almost the same. The material will "flow" to the sides such that it will bulge on the sides of the contact area to accommodate for the central indentation produced by the contact load.

The pressure distribution in the contact area of the cover is also indicated in Figures 2 through 12, by means of a load diagram on the top border of the cover's section. The non-Hertzian nature of the pressure distribution for low values of Poisson's ratio is typical of what one would expect for a thin layer (the paper being calendered) at the contact interface that has relatively stiffer in-plane (machine direction) properties. This layer at the interface effectively acts as a plate deformed in a plane-strain mode at the contact interface, that is supported by a foundation (the cover) that is compressible due to its low Poisson's ratio. This kind of non-Hertzian pressure distribution is discussed, for example, in references (26), (27) and (28). At a Poisson's ratio of 0.48 the pressure distribution becomes semi-Hertzian. This is due to the relative incompressibility of the cover, which acts as a stiffer foundation. The pressure distribution for this Poisson's ratio can actually be accurately represented (27) as a weighted sum of parabolic and elliptic distribution functions.

Figure 16 displays the differential velocity (creep) relationship (see equation 1) for a soft cover mating against a rigid roll, with no slip in the nip. It is clear from the previous discussion that for a Poisson's ratio smaller than 0.37 the circumferential strain at the contact surface of the covered roll is negative, and therefore the covered roll peripheral speed will be higher than the peripheral speed of the (rigid) mating roll. Conversely, for a Poisson's ratio larger than 0.40 the peripheral speed of the covered roll will be lower than the peripheral speed of the (rigid) mating roll, since the hoop strain at the contact surface is positive for these values of Poisson's ratio. A cover with a Poisson's ratio in the range of 0.37 to 0.40 will experience very little or no peripheral speed difference with respect to the mating roll. This is of importance for technological reasons. For example, the tension controls in a supercalender have to be adjusted for a "reverse nip" where two cotton-filled rolls mate against each other, since there is no peripheral speed differential between the cotton-filled rolls, (since they are made of the same material), while there is a speed differential between the cotton filled rolls and the (metal) mating rolls. On the other hand paper calendered with a synthetic cover with a Poisson's ratio of 0.37, for example, will experience practically the same speed at the "reverse nip" and all the other nips. Also, the machine direction deformation of the paper may be different when calendered with a cover that has a low Poisson's ratio and when calendered with a cover that has a high Poisson's ratio. The paper will experience shrinking in the machine direction when calendered with a cover that has a low Poisson's ratio (close to 0.00), and it will experience extension when calendered with a cover that has high Poisson's ratio (close to 0.50).

SUMMARY

Due to the severe constraint imposed by the finite thickness of the cover of a covered roll, the volumetric compressibility properties of the cover have a very important effect on the deformation experienced by a covered roll in the soft-nip calendering operation. Due to friction, the paper sticks to the mating rolls, except at some minute regions of the contact area. These regions of micro-slip have been previously shown to be too small (for practical values of the material properties and of the coefficient of friction) to affect the stress-strain distribution of the cover. Since the paper being calendered is deformed to large plastic strains in the calendering operation, its tangent modulus is relatively low. In the contact area, the paper will experience virtually the same surface strain of the cover. In the example shown, the surface hoop strain of the cover is negative (that is, shrinking) for Poisson's ratio values of the cover's material that are below 0.37. The surface hoop strain is positive (extension) for Poisson's ratio values that are higher than 0.40. For Poisson's ratio values in the range of 0.37 the cover experiences virtually no surface deformation. This phenomenon should affect the (machine-direction) deformation of the paper being calendered, as well as the peripheral speed differential of the rolls in the calendering operation.

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Transcription of Discussion

MODELLING THE STATE OF STRESS AND STRAIN IN SOFT NIP CALENDERING

Dr. J. J. A. Rodal

Dr. R. Mark ESPRI

The figures that you have just showed do not appear in your paper and I am not sure if they are in your references either. Will these be available for study by interested parties?

Dr. J.J.A. Rodal

See my reference number 24: they do appear in my article published in the May 1989 issue of the TAPPI Journal (Vol.72, No.5, pp 177-186). That article discusses these figures (the stress - strain properties of paper) in more depth than I did here.

I would also like to add a few remarks to the ongoing discussion of whether slip or shear deformation takes place in the nip of a soft - nip calender. This phenomenon has been quite well analyzed in the literature of (Rolling) Contact Mechanics. Professor K.L. Johnson (Cambridge University) for example, has done quite a lot of work in this field.

I think it is unequivocally distinguish between slip and shear deformation in order to have a clear understanding of what is happening. From elementary physics we know that Amonton's (or Coulomb's) law of friction states that the sliding force F_s equals the coefficient of friction μ times the normal load $\ddot{P}$.

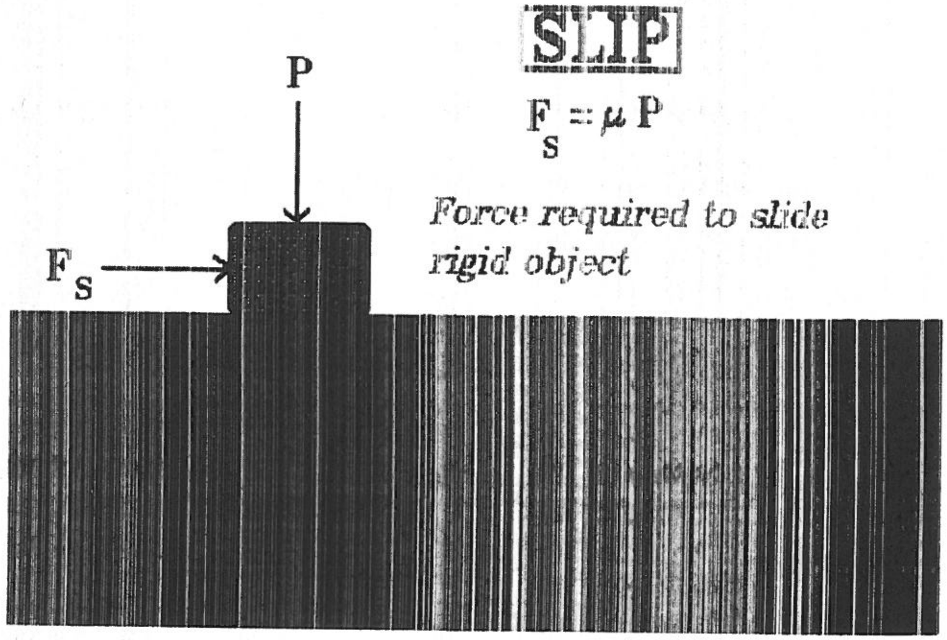
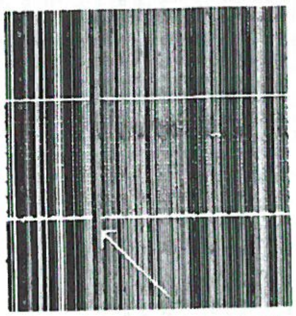


Fig. 1

Sliding occurs if the lateral force F_s is greater than the product of the coefficient of friction μ times the normal load P. If the lateral force F_s is smaller than this , there is no sliding. For deformable (**that is, non-rigid**) bodies there still will be shearing deformation of the materials under a lateral force F_s even when this force is not large enough to produce sliding. Whether sliding or just shearing deformation occurs in a given situation is governed by: the applied force, the coefficient of friction, **and the material properties of the bodies** involved. Since we are dealing with deformable, non-rigid bodies (the paper and the soft cover) **one has to take their compliance into account.** Just because one has shearing force at the nip, one cannot immediately conclude that one has sliding. For illustration purposes, consider the following. One knows from fluid mechanics that the boundary conditions that apply to a fluid are the no-slip boundary conditions (even for flow of viscous fluids).

SHEAR DEFORMATION



Fluid flow

No slip boundary condition

Fig. 2

So you see that in the case of a fluid, in which extremely large displacements occur, there is no slip present at the boundaries. This is so because the resistance to shear deformation (rate) of a fluid is extremely small. Of course, **paper** is not a fluid, but it **is not a rigid body** either. Paper suffers large plastic deformations in the calendering process. To ascertain whether there is sliding or just shearing deformation involved, one should study the mathematical formulation of the problem and analyze whether sliding or shearing deformation is occurring. If one does this, the answer is that, for normal industrial process conditions (with parameters like the nip width to paper thickness ratio and nip width to roller radius with usual values), for coefficients of friction that are higher than $\mu = 0.1$, **unless one deliberately brakes** one of the rolls, one will not have a significant amount of slip. This has been shown mathematically (and experimentally).

Even if you change the coefficient of friction of the paper to be calendered, and obtain a different gloss as a result of calendering this paper, one should not conclude that this change in gloss is just due to a change in sliding conditions (brought about by a change in the coefficient of friction). We know from the theory of friction (explained by adhesive junctions) due to Bowden and Tabor that the coefficient of friction is related to the material's strength in plastic flow. More precisely stated, it is related to the ratio of the material's value of "shear yield stress" divided by the material's value of "normal yield stress." A change in coefficient of friction may reflect a change in this ratio. For example, the low coefficient of friction of Teflon ($\mu = 0.05$) is due to the alignment of the polymer chains parallel to the surface, in the direction of shear. This alignment can be produced by plastic flow due to sliding. Due to this orientation the polymer is anisotropic and the value of the ratio of yield stress in shear to plastic yield under normal stress is much smaller than it is for isotropic materials. So, once again, before jumping to conclusions of whether sliding or shearing deformations are involved, one should analyze the problem mathematically **and take the compliance of the bodies into account.**

Dr. J.D. Peel Kusters

If we use the Hertzian formula which is the old one for calculating stresses and strains in rolls we all know the formula to use. If we wish to be more sophisticated we can use K.L. Johnson or Bufler's work in the 1950's and 1960's. Of the references you give which do you think are the more useful ones for modern theoretical calculations of nip widths and stress/strains in covers? I realise that we would need computers to make the calculations.

Dr.J.J.A. Rodal

The calculation of the nip width for thin-covered rolls is a complex problem since it involves integral equations with complicated kernels. However, there is an asymptotic solution obtained by P. Meyers which is referenced in your paper, (Ref. 119) that can be programmed in a hand-held calculator (Like a Hewlett-Packard HP_28S) and hence it does not require a computer. There is a paper in the TAPPI Journal by Desphande (from Xerox), which is also referenced in your paper (Ref. 120), in which Meyers' formula is used for interpolation in order to obtain a simpler, empirical formula. This (empirical interpolation) formula by Desphande has been used by people in the industry, due to its simplicity. Sometimes this empirical formula by Desphande, which is much simpler than Meyers' asymptotic series, has been (wrongly) referred to as "Meyers' formula." Again, since nowadays one can actually obtain the nip width and pressure distribution given by Meyers' original (asymptotic series) formula with a hand-held calculator, I do not see the need for the empirical formula of Desphande. Meyers' solution has a fairly rigorous mathematical definition, and his assumptions are well known (linear elasticity, small deformations, dimensions of contact being much smaller than the radius of curvature and no friction being present at the interface). Therefore one can obtain its range of validity. On the other hand, I cannot on the range of validity and accuracy of Desphande's approximation, since, again, it does not rest on as firm a mathematical foundation. Later on, Alblas and Kuipers (Acta Mechanica, 9, 292, (1970)) also obtained a mathematical solution to this problem, for the case of very thin nip covers (that is very small ratio of thickness to nip-width). Alblas and Kuipers' solution is more mathematically rigorous than Meyers' but the difference between them is minor. Furthermore, Meyers' obtained a solution for layers where the nip width is of the same order as the thickness, in addition to the solution for very thin layers. Hence, Meyers' solutions cover a larger range of validity. I should also mention the very important (and voluminous) Russian literature on this subject. V.M.Aleksandrov, for example, obtained quite a number of solutions to contact problems for layered bodies since the 1950's and continues to be active in this area in the 1980's.

I should also like to say that the solution of Meyers, Alblas et. al. and Aleksandrov are valid for situations where the nip width is of the same order as the layer thickness, or larger than the layer thickness. This is the controlling factor. For problems for which the layer thickness is larger than the nip width, Hertz's solution is fine. For example, for cotton-filled rolls you have a layer thickness that is 8 times or larger than the nip width, and hence Hertz's equation applies. Dr. Peel and Dr. Baumgarten were quite correct in using Hertz's equation in their theses since these theses dealt with cotton-filled rolls and not with the polymer covers now commonly used in soft-nip calendering.

Now, if there is a discrepancy between the values of nip width and experimental measurements, it is often the case that this is due to the material properties used as input for these equations, rather than a breakdown of the usual assumptions (linear elasticity, small displacement, etc.). In effect, there is a well known adage of computer software: "**garbage in, garbage out.**" The results are only going to be as good as the input (and often they are worse). It is a fact that the **material properties of the materials involved are poorly known.** One needs accurate measurements of the modulus of elasticity and Poisson's ratio of these covers in the range of interest. By the range of interest I mean these properties **measured at the operating temperatures, strains, and strain rates.** The present situation is that these properties have not been properly measured and users are left to speculate as to what their values are. Another important variable is the paper between the two rollers, that may have an important effect on the pressure distribution, for example. None of the solutions previously mentioned include the effect of the paper.

I am not aware of any closed-form solution that gives the distribution of strains for contact problems of **layered** solids. Therefore, to obtain strain distributions for problems for which the nip width is the same or greater than the cover thickness one has to resort to numerical methods like the finite element method, for which computers are a must.

Viscoelastic effects have often preoccupied the mind of researchers in calendering. However, it becomes an important variable only with respect to the recovery of the paper after it leaves the nip. (viscoelastic recovery). With respect to what happens at the nip, what matters is the relaxation time of the materials (polymers have a **distribution of relaxation times!**). It is known that relaxation effects (at the nip) are important only when the contact time roughly coincides with one of the relaxation times of the materials (either the paper and/or the cover). It is under this particular condition that the contact problem becomes significantly asymmetric. At high speeds (those for which the contact time is shorter than the relaxation time of the material) the pressure distributions are approximately equal to the results obtained by applying elastic theory (and therefore, neglecting viscoelasticity), but with a "dynamic" elastic modulus. This "dynamic" elastic modulus is higher than the "static" modulus, due to the strain-rate effect.

Even under these conditions, this effect may not be as large as the actual reduction in elastic modulus that occurs due to temperature, since the properties of the materials involved (the paper and the polymer cover) are very dependent on the operating temperature.

The paper itself may have a very important effect on what happens at the nip. We do not have time to go in to it, but the audience may have noticed that the distribution of pressure became semi-elliptical (that is semi-Hertzian) for a Poisson's ratio of 0.48: but at a Poisson's ratio close to 0.0, there was quite a non-Hertzian distribution. There were two small lobes on the pressure distribution, one at each end. This is related to the paper and the cover properties.

With respect to the pressure and nip width measurements, one way to do this is by having load cells in the mating metal roller. The advantage being that one measures the pressure distribution (including nip width) at actual operating conditions, and one can include paper in the nip as well. However, I should warn you that one has to be extremely careful about having the load cell tangential to the circumference. If the loadcell is slightly higher ir slightly lower, or if it does not match the curvature of the roller, one will obtain very spurious results. If it is microns off, one will have completely wrong data.