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STRENGTH AND FRACTURE OF PAPER

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ABSTRACT

In spite of forty years of research, it is still unclear how the mechanical properties of paper, particularly strength, depend on the disordered geometry of the fibrous network. Most of our understanding of the fracture phenomena of paper is based on illustrative microscopic observations. Theoretical models have traditionally been focused on the behaviour of a typical element in the network. However, the failure process seldom starts from, or proceeds through, "typical" elements. Instead, the statistical distribution of local failures is crucial for the strength of paper. With the ever more powerful computers it is now possible to simulate numerically the behaviour of disordered systems such as paper. I believe that computer simulations, in combination with new measurements and effective data analysis will lead to a better understading and more accurate microscopic characterization of the strength and fracture properties of paper.

1. INTRODUCTION

I will discuss certain aspects of the current knowledge of the strength and failure properties of paper. Even though the basic empirical facts have been known for years, much remains unresolved. It is widely accepted that the mechanical properties of paper are related to the disordered or statistical geometry of the fibre network, nevertheless, a satisfactory mathematical description of that relationship is still lacking. There are reasons to believe that the situation may be changing. For one, computer simulations of theoretical models provide new insights that can be applied to paper. Also, sensitive measurement techniques coupled with powerful data processing make it possible to collect statistical information on the microscopic failure process of paper.

The proceedings of the previous Fundamental Research Symposia contain a lot of material on the structure and mechanical properties of paper and provide a good overview of the research trends. In the first symposium in 1957, "Fundamentals of papermaking fibres", beating was one of the main topics. Steenberg (1) reviewed research on beating effects and stressed the need to study the stress distribution in paper under different conditions in order to understand the properties of paper and the influence of its composition. Arlov (2) observed that the shape of the stress-strain curve for paper remained constant with increasing degree of beating, while changing the beater changed the shape. Nordman (3) showed how the irreversible elongation of paper is related to an increase in the reflectance (Fig. 1) and that the light scattering coefficient is directly proportional to the energy loss in a straining-destraining cycle.

In the next symposium, "Formation and structure of paper", in 1961, a large number of interesting papers were presented on the microscopic structure and mechanical properties of paper. Jayme and Hunger (4), and Page, Tydeman and Hunt (5) described the structure of fibre bonds as seen under the electron and light microscopes, respectively. The optical contacts between crossing fibres had irregular shapes and very different areas. Page *et al.*(6) also observed that most fibre-to-fibre bonds failed only partially when paper sheets were strained to failure. The fractional loss in bonding area was independent of beating but depended on the

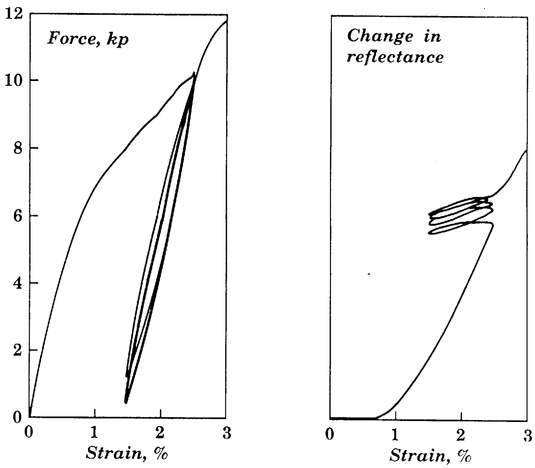

Fig. 1: Comparison between a stress-strain curve and the corresponding change in reflectance (arbitrary units), after Nordman (3).

drying tensions and direction of the test strips. Page and Tydeman (7) showed that the plastic elongation of the bonding sites was probably related to the "microcompressions", or wrinkles, induced by drying shrinkage. McIntosh and Leopold (8) measured the strength of individual fibres and fibre-to-fibre bonds, and Higgins and De Yong (9) discussed the effects of beating on the properties of pulp and paper.

At the same symposium, Corte and Kallmes presented their famous theory of the statistical geometry of two-dimensional random fibre networks, which agreed very well with the measured geometric properties of thin paper sheets (10). They also calculated the number of bond failures that should occur when a two-dimensional fibre network is strained to failure. The result was not in disagreement with that obtained by listening to the "clicks" that were emitted from thin paper sheets (11). Kallmes and Bernier (12) found that measured elastic properties of thin sheets were in fair agreement with those predicted from a model. Nissan (13) contended that it is necessary to consider the statistical distribution of hydrogen bonds in order to obtain accurate results for the mechanical properties of paper. Sternstein and Nissan (14) derived a mathematical model that gives the non-linear stress-strain curve of paper in terms of the non-linear behaviour of the hydrogen bonds. Van den Akker argued on theoretical grounds that the stresses acting on fibre-to-fibre bonds arise from the anisotropic drying shrinkage of the fibres and the stress transfer from one fibre to another through the bonds (15). According to Van den Akker, the latter mechanism causes the bonds to fail, which in turn leads to the non-linear tensile behaviour of paper. Ranger and Hopkins also explained the failure process of paper through the "peeling" rupture of the bonds, but under compressive forces (16).

At the third symposium in 1965, "Consolidation of the paper web", the mechanical properties of pulp fibres were discussed in several contributions. Van den Akker, Jentzen and Spiegelberg (17) found that the tensile strength and Young's modulus were high when the fibres were dried under tension. Very similar observations were reported by Kallmes and Perez (18). Duncker, Hartler and Samuelsson (19) followed, during drying and wetting, the elongation and shrinkage of fibres under load. Nordman, Aaltonen and Makkonen (20) found that the bonding strength of fibres was essentially constant in pulps of low hemicellulose content, while in hemicellulose-rich pulps it appeared to increase with beating and decrease with wet pressing. R. J. Norman (21) found that when forming stock concentration was increased, the tensile strength of paper decreased, as one would expect to happen with increasing structural inhomogeneity. Kallmes and Perez (22) presented a mathematical model for the non-elastic properties of paper. Algar (23) reviewed the literature on the structure of paper and its load-elongation behaviour.

At the fifth symposium in 1973, "The fundamental properties of paper related to its uses", B. Norman and D. Wahren (24) continued the discussion of the mass distribution of paper in relation to sheet properties, and found that the decrease in paper strength with increasing basis weight variability depends on the web formation process. Lyne and Hatzell used interference holograms to show that the variability of the local strain increases with that of the local basis weight (25). Moffatt, Beath and Mihelich (26) observed that tensile failure lines in uncalendered newsprint pass predominantly through areas of low basis weight, but after calendering they often pass through areas of high basis weight. Lyne, Jackson, Ranger and Trigg (27) showed that shives and other similar

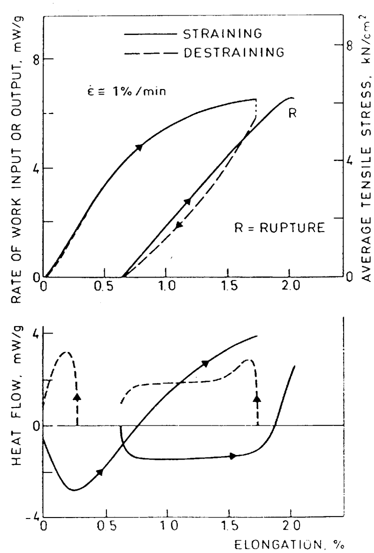

Fig. 2: Thermodynamic behaviour of 100 per cent rag paper MD, after Ebeling (32).

defects had no effect on the in-plane tear strength (the force required to start the tear), nor was the local pre-rupture strain field affected by their presence. Seth and Page argued that the runnability of paper webs could be characterized by a fracture resistance, which is equivalent to the in-plane tear strength with a 0° tearing angle (28).

Göttsching and Baumgarten measured the triaxial deformations of paper under tensile load (29). Radvan (30) concluded that the layered structure of paper has little effect on the in-plane mechanical properties. Dodson presented an extensive review of the published works on the mechanical properties of paper with particular emphasis on the fundamental parameters (31). He emphasized that the statistical network geometry governs the fracture process of paper. According to Ebeling (32), the small strain elongation of paper under tensile loads is consistent with Kelvin's thermoelastic principle, *i.e.* paper cools when strained. At larger strains, heat is released due to irreversible intrafibre deformation (Fig. 2).

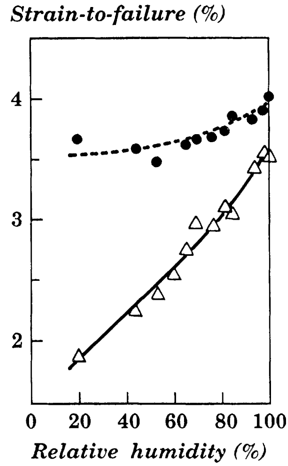

Fig. 3: Strain-to-failure against relative humidity under static (dots) and impact conditions (triangles) after Kimura *et al.* (35). Bleached softwood kraft pulp.

The seventh symposium in 1977, concentrated on the "Fibre-water interactions in papermaking". Htun and de Ruvo (33) showed that within reasonable error bounds, the internal stress of paper equals the final drying stress, and that the elastic modulus, tensile strength and compressive strength are proportional to it. Ionides, Jackson, Smith and Forgacs (34) argued that not only the solids content but also the conditions of dewatering affect the stress-strain behaviour of wet webs. Kimura, Usuda and Kadoya (35) measured the impact rupture properties of paper with a dynamic pendulum tester and found them to have a different moisture dependence than the static properties (Fig. 3).

At the 1981 symposium, "The role of fundamental research in paper-making", Perkins and Mark (36) discussed the effects of fibre orientation and drying shrinkage on the elastic modulus of paper using a micromechanical model. Seth and Page (37) presented a qualitative model for the stress-strain curve of paper in terms of the average (non-linear) properties of the fibres. Many different experiments were explained with this model. The authors argued that the stress-strain behaviour of paper does not depend on the network structure. Corte, Blinco and Hurst (38), on the other hand, claimed that the statistical network geometry determines the stress-strain behaviour of paper. Fellers, Westerlind and de Ruvo (39) determined the biaxial strength envelope of paper and found it to compare well with values measured for thick-walled paper tubes and corrugated containers. Htun and de Ruvo (40) argued that the effect of drying shrinkage on the strain to failure of paper can be explained using a linear superposition principle in which the drying process is divided into stages of constant drying strain.

At the eighth symposium in 1985, "Papermaking raw materials", Page, Seth and El-Hosseiny (41) reported how the strength of wood pulp fibres varies with the chemical composition. Page, Seth, Jordan and Barbe (42) also argued that various fibre defects like crimps, kinks and microcompression together with fibre curl have a strong influence on the mechanical properties of paper. Retulainen and Ebeling (43) reported measurements for the load-elongation behaviour of fibre-to-fibre bonds, and Steadman and Luner (44) results for the effect of wet fibre flexibility on the apparent density of paper. Jantunen (45) measured wet web properties under dynamic conditions and found that the rate of stress relaxation was roughly independent of the solids content, except above 80 %, where it decreased with increasing solids content. Back (46) showed that the compression strength of paper decreases with increasing moisture content much more rapidly than does the tensile strength. The effect of moisture on the compression strength was also reported by Fellers and Bränge (47). The effects of fillers, coatings and sizes on paper properties, including tensile strength, were discussed in several papers.

At the ninth symposium 1989, "Fundamentals of papermaking", Page reviewed the literature on beating and its effects on fibre properties (48). The effect of beating on wet fibre flexibility and sheet properties was discussed by Abitz and Luner (49). Nanko and Ohsawa (50) carefully analysed the structure of fibre-to-fibre bonds under an electron microscope and a confocal scanning laser microscope. The structure of the bond was found to be strongly dependent on the degree of beating. Ritala and Huiku (51) gave a few examples of the application of percolation theory and scaling arguments to paper, and described results of a computer simulation of fracture in a model system.

We can thus see that from the very beginning of these symposia, the focus has been on the basic mechanisms that relate the mechanical properties of paper to those of the constituent fibres and bonds and to the structure of the fibrous network. A lot of information describing the microscopic geometry has been obtained and the significance of the statistical network structure for the mechanical properties of paper has been demonstrated in many different experiments. The elongation behaviour of individual fibres and bonds is reasonably well understood, at least from a physicist's point of view. It is much less clear how the statistical distribution of these phenomena translates into the macroscopic properties of paper. This lack of understanding is clearly indicated by the on-going discussion of how beating affects the properties of paper.

On the other hand, it is also clear that the emphasis has spread from the fundamental tensile strength properties of paper to various other mechanical properties, such as the anisotropy of the elastic properties, viscoelastic behaviour and effects of moisture, and fracture properties. In practice, of course, many different requirements are imposed on the mechanical properties and strength, depending on the purposes the paper is to be used for. Two very obvious practical examples are the runnability of paper webs and the load endurance of packaging materials. The two cases illustrate how very different structural and functional aspects are relevant to the practical "strength" of paper.

In the first case, the paper web is run at very high speeds and a web failure is triggered by either a defect in the paper or a dynamic stress pulse in the machinery. Breaks in printing paper webs are very rare events, averaging only 2-3 breaks per 100 rolls. It is very difficult to characterize the runnability of paper by means of laboratory tests and perhaps even more difficult to quantify the structural factors of paper that govern runnability. The statistical nature of paper strength and the dynamics of crack propagation are critical for the runnability of a paper web.

In the case of load endurance, complicated static stresses may exist in the package because of its shape. The performance of the box is then governed by the elastic properties and failure envelopes of the paperboard. The statistical nature of these properties is of lesser importance. There are many excellent reviews dealing with these questions, including the compressive vs. tensile strength properties of paper (52-55). Interesting here is the observation that cyclic humidity changes greatly accelerate creep, which has to be taken properly into account when assessing the load endurance of paper and board (56, 57).

In the next sections I shall focus on the structure *vs.* strength relationship of paper under tensile loading conditions. In addition to the traditional paper physics literature, I will quote some more theoretical investigations on the general behaviour of disordered systems. These provide useful insights and, in some cases, can be directly applied to paper. Instead of trying to cover all the different aspects of paper strength, I will concentrate on the fundamental aspects, from which something rather more than mere speculations can be concluded. Although the strength of paper can often be improved most directly through the furnish properties and chemical additives, I shall consider such factors only in terms of their effect on paper structure. Numerous reports have been published on how the furnish and additives affect paper properties. For example, Seth (58) discussed in detail the effect of fibre characteristics and Retulainen *et al.* (59) studied the effect of fines and starch on the mechanical properties of paper.

Our understanding of why paper strength can be improved by a particular technical approach is usually based on three types of information: microscopic observations, indirect measurements and intuitive arguments. Perhaps cynically, I believe that the last two sources of information are at best unreliable if - or since - one does not know how the disordered structure of paper affects its macroscopic properties. In the following sections I will present my own view of how the situation might be improved. The first step will be to consider what is known experimentally and through models about the small-strain mechanical behaviour of paper (subsections 2.1 and 2.2). I shall then present a simulation-motivated stochastic model for the pre-rupture properties of paper (subsection 2.3). The phenomenological models developed for the tensile strength of paper will be analyzed in terms of their ability to predict the structural mechanisms of paper strength in subsection 3.1. Computer simulations of elastic models will then be discussed to demonstrate the generic effect of the disordered network geometry (subsection 3.2). Much less can be firmly said about the strength properties of disordered fibrous networks, though a relationship between paper strength and modulus may yet prove useful (subsection 3.3). The effects of formation will be considered in subsection 3.4. The literature on the mechanical runnability of paper will be reviewd in subsection 4.1, the linear elastic fracture mechanics in subsection 4.2 and the non-elastic fracture toughness of paper in subsection 4.3. Finally, paper toughness against structure will be considered in subsection 4.4.

2. PRE-RUPTURE PROCESSES

When the elongation of a paper sheet is increased starting from zero, microscopic or submicroscopic failures occur. These may involve the internal structure of the fibres, leading to a small permanent elongations, while only few fibre segments fail completely. The fibre-to-fibre bonds also rupture either in a brittle manner or gradually, through the cutting of fibrils and polymeric molecules. One may even view the entire failure process as corresponding to rupture of the hydrogen bonds (60-63). Whatever the nature of the failures and size of the related elements, the plastic, irreversible behaviour of paper is governed by microscopic failures. In order to understand what this means to paper properties it is not necessary to predict which elements fail. If desired, this can be determined by direct observation. More important is to be able to characterize the pre-rupture failure processes in paper in terms of measurable macroscopic quantities.

As a result of the microscopic failures, the local stresses in the network change during elongation. Stresses relax in some elements and increase in others. Each time a microscopic element in the paper sheet fails the local stress increases in the neighbouring elements in the lateral direction. Sooner or later most of the new failures occur sufficiently close to the old ones so that the entire sheet ruptures. The tensile strength and the macroscopic fracture of paper will be discussed Sections 3 and 4. Before that I shall concentrate on the initial "pre-rupture" behaviour of paper and discuss the related experimental observations and model calculations. I shall also describe how the stochastic nature of paper behaviour can be modeled with microscopic stress distributions.

2.1. Experimental observations

Working with very thin, really two-dimensional sheets, Corte *et al.* (11, 64, 65) were able to detect the failure of individual bonds when a sheet was strained. This was accomplished by mounting the sample on a slide projector so that the bond failures could be seen on the screen. Each bond failure was accompanied by a faint but audible "click". When the samples were strained at constant rate, a sudden drop was observed in the load-elongation curve each time a bond failed (Fig. 4). At higher, or normal, basis weights there are so many bond failures that they can no longer be distinguished in the load-elongation curve.

Page, Tydeman, and Hunt studied individual fibre bonds of a normal paper sheet under a light microscope (5, 6). They found that in contrast to the thin sheets studied by Corte, the bonds ruptured only partially when paper was strained to failure. In most cases the bonded areas were rather irregular. Beating made the bonds more regular but did not change the distribution of the bonding area loss (Fig. 5). The same qualitative observation was made when a sheet was strained cyclically, the fraction of partially opened bonds increasing steadily with increasing maximum strain while the number of completely broken bonds increased only little (66). The irregular shape of bonded areas obviously leads to stress concentrations within them. If the sheet strain is increased, the regions of highest stress fail first.

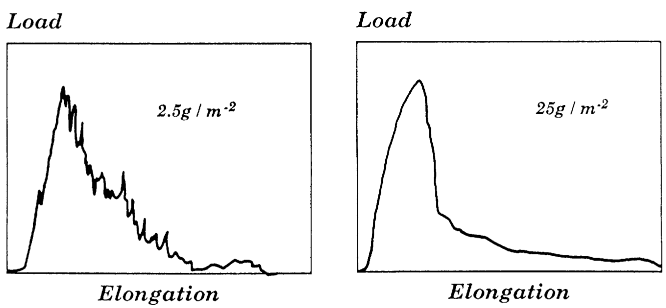

Fig. 4: Load-elongation curves of thin paper sheets, after Corte (65).

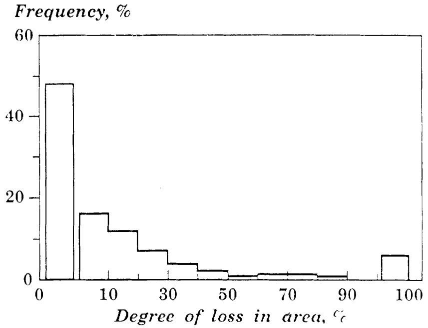

Fig. 5: Distribution of the degree of loss in bonded area at sheet rupture, after Page *et al.* (6).

Giertz and Roedland (67) measured the local strains as a function of the external elongation of a paper sheet. At first all fibre segments and bonds elongated fairly uniformly, but then some bonds started to open. Bonded fibre segments elogated generally more than free fibre segments (Fig. 6). In this case, too, it was observed that the bonds did not rupture completely. The bonds that yielded at small strain often did not deform more at higher strain. When the external load was removed, permanent elongation was found to have taken place primarily in the bonded areas of fibres. The free fibre segments relaxed elastically except in freely dried paper, in which the free fibre segments also underwent irreversible elongation. In summary, the bonded fibre segments seemed capable of governing the load-elongation behaviour of paper, which lead Giertz to propose the "bond strain theory of paper strength" (68).

The partial rupture of the bonds can be explained not only by the irregular shape of the bonded area but also by the stress distribution induced by drying (15). The anisotopic shrinkage of the fibres leads to shear stresses even when then bond is regular, as illustrated by the arrows in Fig. 7a. External load will cause a stress concentration on one edge of the bond (Fig. 7b), and the bond will first fail there. This may relax the shear stresses sufficiently so that the bond fracture stops, especially if the debonded fibre segment yields plastically (15). Even an axial stress along a fibre can lead to bond failure. Suppose that the axial strain is increased in fibre B, Fig. 7a. Then the shear stresses increase, too, and eventually exceed the failure threshold at the edges of fibre A.

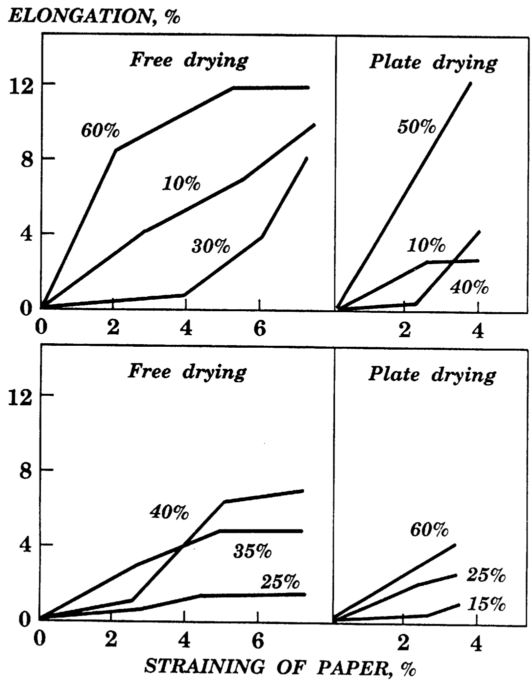

Fig. 6: The elongation of bonded and free segments (top and bottom) against the external strain of freely dried and plate dried sheets, after Giertz and Roedland (67). The segments were divided in three groups according to their behaviour.

From above, it is clear that bond failures must play an important role in the response of paper to external elongation. However, measurements on single fibres show that typical papermaking fibres, too, can elongate plastically, much like paper (37, 69) (Fig. 8). The irreversibility may be caused by fibre curl and defects such as crimps, kinks and microcompressions that are induced by the pulping and paper- making processes (42). In contrast, fibres with no defects are linearly elastic or nearly so (37, 70, 71). One can adopt the view that the irreversible elongation of bonded fibre segments, as discussed above, is also a manifestation of the non-linear behaviour of the fibres. For example, Seth

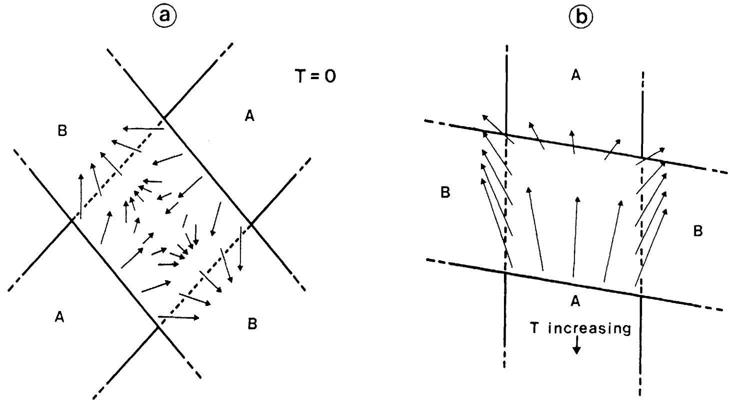

Fig. 7: Shear stresses on a interfibre bond as induced by the anisotropic shrinkage of the fibres, *a*, and by an external load, *b*, after Van den Akker (15).

and Page (37) argue that the load-elongation curve of paper can be explained entirely in terms of the average fibre properties with no reference to the statistical network geometry. This approach allows a plausible explanation to be given for very many experimental observations (37).

The experiments quoted above make it clear that when paper is stretched, fibre segments become permanently elongated and fibre-to-fibre bonds rupture, both in small steps. Complete failures are rare events. The failure process is stochastic, the occur in a seemingly random sequence throughout the sheet. The partial bond failures and the related yielding of fibre segments relax the local stresses, and the elastic modulus (or elastic energy at constant strain) of the network decreases. In some cases it happens that initially unloaded elements in paper become loaded when segments stretch and bonds fail around them. Then the elastic modulus of paper increases at small strains rather than decreases (67, 38). The local network geometry is crucial for this kind of activation effects. Although probably minor in well bonded papers (37), activation may be much more significant in loosely bonded mechanical papers (67, 72).

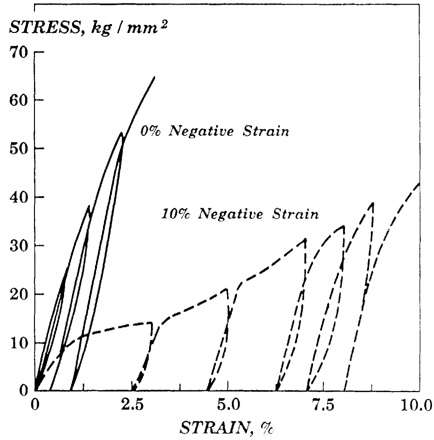

Fig. 8: Typical stress-strain curves measured by Dumbleton (69) for holocellulose fibres, dried under no drying strain and under 10% compressive strain.

In summary, the "pre-rupture" behaviour of paper is governed by the heterogeneous mechanical properties and random arrangement of the fibres and bonds. Still, it is quite possible that the statistical network geometry has no practical significance. I shall consider this question in the next two subsections.

2.2. Theoretical studies

Perkins and coworkers (73-76) have developed micromechanical models that may also be viewed as computer simulations of paper behaviour. The deformations of a typical fibre, treated as a mesoelement, are solved using the finite element method. This can be done by first integrating over the statistical distribution of the fibre segments to which the mesoelement is bonded. The model is thus essentially an effective medium approximation in which a uniform background or matrix is assumed. Combined with the finite element method, the micromechanical models allow very realistic fibre and bond dimensions and mechanical properties to be included and their effect on the elastic and plastic properties of paper to be described.

Because of their effective medium nature, micromechanical models cannot describe how the disordered network structure affects the fracture behaviour. They treat only the mean distribution of fibre orientations and the stress distribution along fibres. The non-linear mechanical behaviour of the fibres and bonds must be put in "by hand". The model output is a weighted average of the inputs, determined by the *mean* geometry of the network. In this respect the various closed-form models for the load-elongation curve of paper that have been proposed over the years (12, 22, 37, 77, 78), are very similar to the micromechanical models. The statistical nature of the microscopic failure process is ignored.

A completely different approach was taken by Nissan (14, 60), who expressed the small strain behaviour of paper as a sum over the reversible deformations of hydrogen bonds. Within a two-dimensional approximation this led to the following expression for the stress-strain curve of paper

$$\begin{aligned}\sigma &= const\ \left[\exp(-\varepsilon/\varepsilon_0) - \exp(-2\varepsilon/\varepsilon_0)\right] \\ &\approx const\ \left[\varepsilon/\varepsilon_0 - \tfrac{3}{2}(\varepsilon/\varepsilon_0)^2\right] \qquad (1)\end{aligned}$$

This expression is of course valid only when no bond failures occur. Also, the model should be derived for a three-dimensional assembly of hydrogen bonds, but then the mathematics becomes quite complicated (14). The point I want to make about Eq. (1) is that even the reversible behaviour of hydrogen bonds gives rise to a non-linear stress-strain behaviour closely resembling that of paper. Various mathematically convenient expressions are also often used to fit the elongation behaviour of paper (*e.g.*79, 80, 81). Such models have no microscopic interpretation but they may be useful in practice.

Dodson (82) was the first to derive a true statistical model for the pre-rupture failure process of paper. He assumed that, during elongation, the rate of energy dissipation is proportional to the stored energy, and that the dissipated energy per bond failure, *e*, is constant. From these two assumptions follows the cumulative number of bond failures, *B*:

$$\frac{eB}{F} = \frac{U}{F} - 1 + \exp(-\frac{U}{F}) \approx \frac{U}{F} - 1, \text{ when } \frac{U}{F} >> 1 \qquad (2)$$

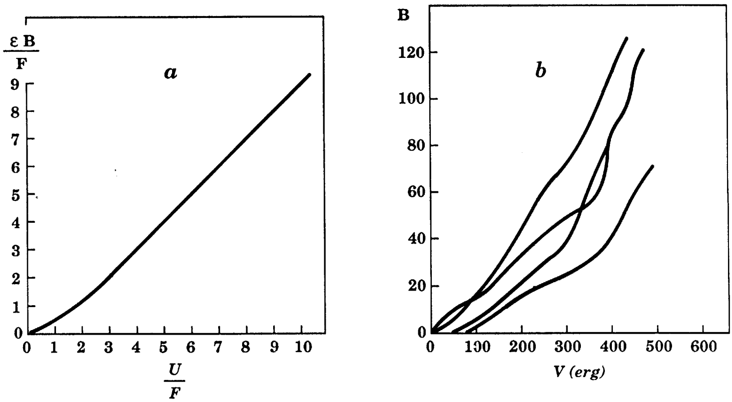

Fig. 9: Number of bond failures, B, against external work, as given by Eq. (2), a, and as measured for paper samples, b, after Dodson (82).

Where U is the external work done on the system and F is an adjustable scale parameter. Thus, in the end, all the work goes into the breaking of bonds (Fig. 9a). Dodson compared his model with experiments on thin paper sheets and found reasonable agreement (Fig. 9b) when B was taken to equal the number of discontinuous drops in the load-elongation curve. However, sample-to-sample variations are quite large with thin sheets. Also, Eq. (2) only applies to the small strain behaviour before the macroscopic fracture has begun.

In recent years, the elasticity and fracture of disordered systems have been studied extensively with simulation models defined on a lattice. It is possible to present only a small fraction of this material in this review; for those interested in learning more I recommend the book of Herrmann and Roux (83). In the computer simulations an underlying lattice greatly enhances the computation speed since the number of couplings per site is small. For this reason the simulations have mostly been limited to elastic lattice models with brittle failure of the bonds (links, segments) connecting nearest neighbours. On the other hand, lattice geometry is expected to have little significance. Disorder is implemented either as a random segment occupation or a random breaking threshold. To put it simply, in the first case (*i.e. dilution*) the local elastic modulus varies, while in the latter case the local breaking thresholds are random.

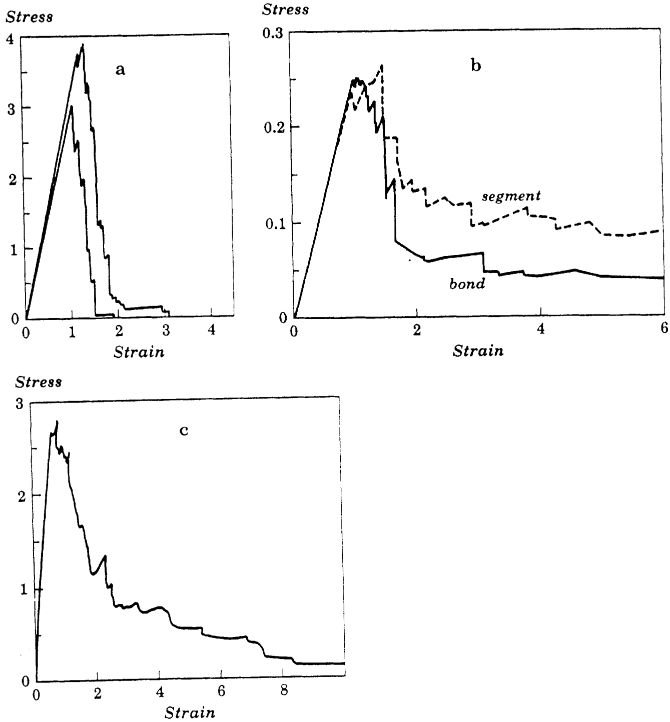

Fig. 10: Computer simulations of stress-strain curves (arbitrary units) for two-dimensional fibre networks: triangular lattice model (85), fibre length $l_f \in [1,5]$ and $l_f = 3$ (upper and lower curve, respectively), *a*; random network (86), *b*; "random vertex" model (89), *c*. Fibre segments failed in *a* and *c*, bonds and segments in *b*.

A very good example of the computer simulation approach is the study by Alava and Ritala (51, 84), who placed fibres of constant length l_f at random on a triangular lattice. The fibres carried axial and shear loads. Alava and Ritala (85) also studied the case where fibre length was taken from a uniform distribution $l_f \in [1,5]$. Except for low densities p, the length distribution appears to change only the elastic modulus and not the stress-strain curve (Fig. 10a). The density p is defined as the total fibre length per unit area. The authors concluded that it is the longer-than-average fibres that determine the mechanical properties. This is because the percolation threshold density p_c is inversely proportional to the r.m.s. average of fibre length and because the relative density p/p_c governs the mechanical properties (more on this in subsection 3.2).

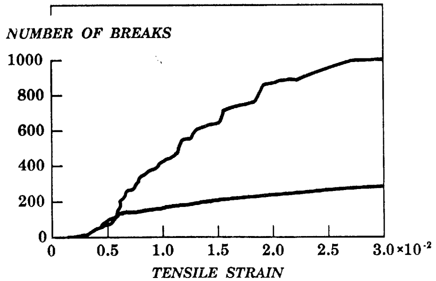

Fig. 11: Number of bond failures against external strain for the layered fibre network model of Hamlen (87), basis weights 1.0 and 2.5 g/m^2 (lower and upper curve).

Systems in which the geometric structure is disordered form another class of simulation models. Such models include two-dimensional random fibre networks (86), their layered generalizations (87) and "random vertex" models (88, 89). In a random fibre network the centre of mass and orientation of the fibres is random. A vertex model is composed of a random set of points that are each connected at random to a number of other points. The connection probability decreases with distance.

All the different models give qualitatively very similar results for the stress-strain curve, as illustrated in Fig. 10. The results also closely resemble those obtained for low basis weight paper sheets (Fig. 4), even though the models are linearly elastic and the non-linearity in the elongation behaviour arises from microscopic failures. The similarities also hold for other properties. For example, the simulated number of inter-fibre bond failures, *B*, against external strain is shown in Fig. 11 for the layered fibre network (87) and in Fig. 12 for the two-dimensional network (86). In both cases, the number of bond failures at a given strain is small when the fibre segments are flexible. In this respect an interesting observation was made by Hamlen (87), who found that when the fibre and bond properties of the model were taken from experiments, only a very small number of fibre segments failed. This agrees with the experimental observation that inter-fibre bond failure is the prevalent rupture mode in paper.

The examples quoted above clearly indicate that it would be difficult to discriminate between models for paper behaviour by means of the shape of the measured stress-strain curve. It is also clear that the stochastic

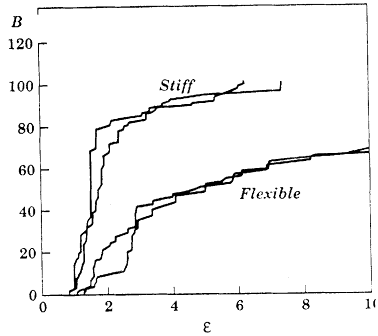

Fig. 12: Number of bond failures against external strain for two-dimensional network model (86), $p = 4p_c$, stiff and flexible fibres (fibre width-to-length ratio $w/l_f = 0.01$ and 0.06, respectively).

nature of the bond failures, even partial, must be crucial to the shape of the curve. However, if paper sheets of normal basis weight are considered, the stochastic rupture sequence leads to a smooth behaviour that may be fairly similar in different papers. Thus one may equally well argue that the statistical phenomena can be ignored and assume that all bonds and fibres are equivalent (37). One must know how the stochastic structure affects paper properties, otherwise the whole notion is of no practical significance.

The above applies also to the use of microscopic fracture sequence to determine the microscopic strength properties by measuring thin paper sheets. For example, since each inter-fibre bond rupture gives a drop in the load-elongation curve, one can try to determine the work of rupture. Fig. 13 shows the measured work against the number of breaks (90). It could then be assumed that Eq. (2) holds, in which case the asymptotic (large n) slope of the curve should be equal to the work of rupture per bond. There are, however, several problems with this approach. For example, sample-to-sample variations are large and it is not at all clear that the measured curve does in fact follow a straight line. From subsection 2.1 we also know that the failure process in thin sheets is different from that in ordinary paper (complete *vs.* partial rupture of inter-fibre bonds). A simulated curve for the work against the number of failures is shown in Fig. 14. It should correspond to the experimental ones in Fig. 13. It can be seen that there is a change in the slope at about B =

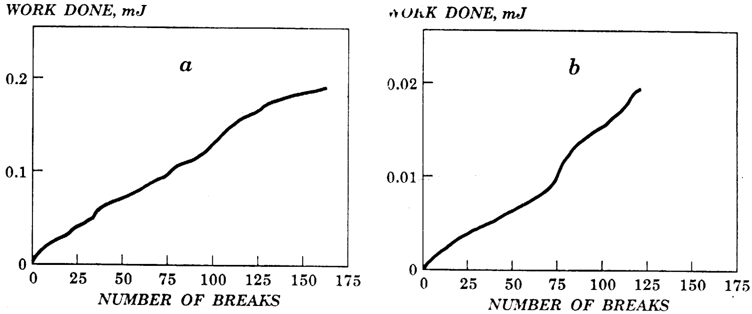

Fig. 13: External work against number of bond failures for handsheets made of beaten, *a*, and unbeaten, *b*, Southern softwood kraft pulp, after Smith and Graminski (90).

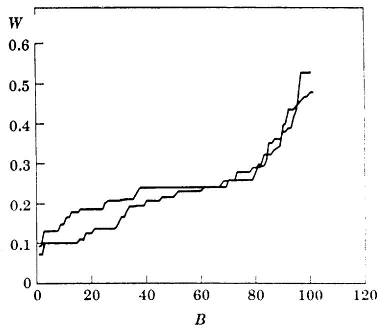

Fig. 14: External work against number of bond failures for two-dimensional network model (86), $p = 4p_c$, $w/l_f = 0.06$.

80 bond failures. This "kink" corresponds to the onset of ultimate, macroscopic failure of the network. When this happens the nature of the failure process changes and the macroscopic stress starts to decrease (83, 91). The same phenomenon occurs in the experiments (see Fig. 4) as a clear-cut or smooth cross-over in the load-elongation curves. In fact, the measurements of Smith and Graminski (90) (Fig. 13) were almost exclusively made in this region. Thus they do not characterize the small strain behaviour of paper at ordinary basis weights, nor does the model Eq. (2) apply in this case.

To determine the microscopic pre-rupture properties, it is necessary to be able to analyse paper of ordinary basis weights. In other words, one must be able to link mathematically the stochastic failure process to the measurable properties of paper. In the next subsection, I will describe one possible approach to this problem.

2.3. Stochastic model for the elongation of paper

The computer simulations of disordered systems that have been reported in the literature show conclusively that at small external elongations the microscopic failures are independent of each other (or "random") if the disorder is sufficiently "strong" (51, 83). The initial random sequence of microscopic failures is determined completely by the stress distribution of the microscopic elements at infinitesimal external strain, prior to any failures. The elements that will fail first with increasing external elongation are those for which the stress over the breaking threshold is largest. Even though the local stress increases close to each failure, in a large system there are other elements far away that have the next highest stress-over-threshold and will therefore fail next. Of course, as more and more failures occur, the mean distance between failed elements decreases and the failures gradually become more correlated. Eventually the system fails macroscopically. The extent of disorder in the local stresses or breaking thresholds determines how far the non-correlated failure process continues (92). In some cases, failure processes can also be classified by certain characteristics of the threshold distribution (93).

In paper, the validity of the above state of affairs has been demonstrated through direct observations (*cf.* subsection 2.1). In other words, no crack propagation occurs in paper at small strains. One explanation for this is the residual bonding: cracks do not propagate if the local failures are incomplete. This has been demonstrated explicitly with a lattice model in which the elastic modulus of the bonds was given a non-zero value even after the failure (94). This is obviously the case in paper as the inter-fibre bonds fail only partially. Even complete bond breaking leaves behind residual strength since there are other bonds along the fibre that have to be broken, too. In the case of segment breaking, the residual strength arises from the layered structure and the continuous distribution of fibre

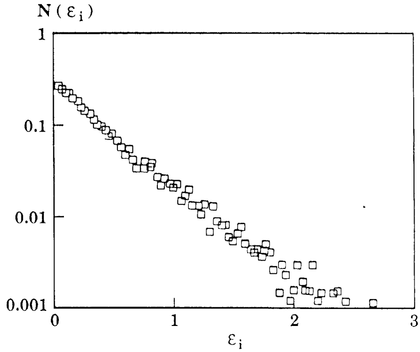

Fig. 15: Initial distribution of inter-fibre bond strains for two-dimensional network model (86, 96), $p = 4p_c$, $w/l_f = 0.06$. Strain values are in units of external strain.

orientations. Next to a broken segment there is usually another one which, because of its orientation, fails much later (91).

The *non-correlated* nature of the initial failure process is important, because it allows the mechanical behaviou of paperr to be calculated from the stress distribution P of the microscopic elements (95). Furthermore, P seems to be an exponential distribution, as will be discussed shortly.

As a concrete example, let us consider what happens in the random network of linearly elastic fibres (86). The simulated distribution of strain increments ε_s across the inter-fibre bonds is shown in Fig. 15 for a small external strain, at which no bonds have yet failed. The distribution is reasonable well reproduced by (96)

$$P(\varepsilon_s) \propto \frac{1}{\beta_s} \exp(-\beta_s \frac{\varepsilon_s}{\varepsilon_x}) \qquad (3)$$

The quota $x = \varepsilon_s/\varepsilon_x$ is independent of *the external strain* ε_x because of linear elasticity. The parameter β_s depends on the fibre properties and network density. The strain increment, "bond strain", ε_s is defined as the change in the axial strain of the fibre across the bond, *i.e.* $\varepsilon_s = \tau A_b / E_f A_f$, where τ and A_b are the shear stress and area of the bond, and A_f and E_f are the cross-sectional area and Young's modulus of the fibre, respectively.

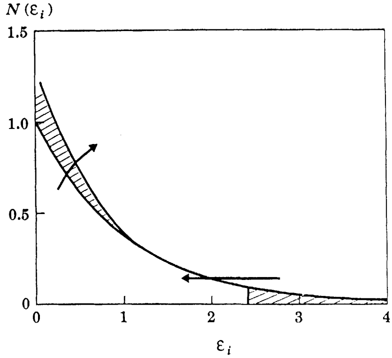

Fig. 16: Schematic illustration of how the microscopic strain distribution in shown Fig. 15 should change when external strain is increased.

The distribution of axial segment strain ε_a appears to be slightly more complicated (96):

$$P(\varepsilon_a) \propto \left(\frac{\varepsilon_a}{\varepsilon_x}\right)^{m-1} \exp\left(-\beta_a \frac{\varepsilon_a}{\varepsilon_x}\right) \tag{4}$$

However, the difference in the strain distribution function turns out to be insignificant for the small strain behaviour of the network (97). The exponential strain (or stress) distribution P, Eq. (3), appears to be rather general in systems of random geometry. It characterizes, for example, diluted resistor (spring) lattices (98) and real fibre networks made of rubber bands (99).

If the threshold for bond failures ε_c is constant, then at small external strain or stress (ε_x or σ_x) all the bonds are broken for which $\varepsilon_s > \varepsilon_c$, or

$$x > x_c = \varepsilon_c / \varepsilon_x . \tag{5}$$

As long as only a small fraction of the bonds have broken, the large strain (large x) end of the distribution $P(x)$ can be expected not to change, except that $P(x > x_c)$ becomes zero (95) (cf. Fig. 16). Of course, some elements relax and others activate, but, this should affect foremost the small stress end ($x<<1$) of the distribution and hence give only a small contribution to the total elastic energy. At large external elongations the distribution changes more dramatically as the failure process localizes (96).

The fraction B of broken inter-fibre bonds is (Fig. 17a)

$$B = \int_0^{x_c} dxP(x) = \exp(-\beta/\varepsilon_x) \tag{6}$$

Where $\beta = \beta_s \varepsilon_c$. The load-elongation (σ_x - ε_x) curve of the network can be calculated from the fact that each time a bond fails, the elastic energy of the system decreases by $\alpha\varepsilon_s^2$ where the constant α accounts for the stress relaxation around the failed bond (96):

$$\sigma_x = E\varepsilon_x\{1 - \alpha(\beta/\varepsilon_x)^2 \exp(-\beta/\varepsilon_x)\} \tag{7}$$

The calculated stress-strain curve (Fig. 17a) is surprisingly linear at small strains and then starts to deviate rather abruptly. After all, there is no pre-set "yield" strain in the model. This character of the stress-strain curve is a direct consequence of the exponential dependence on inverse strain.

Before making a comparison with experiments, it should be pointed out that the above equations ignore the plastic deformations of fibre segments and the gradual failure of bonds. However, for the sake of argument it can be assumed (97) that Eq. (7) is still valid when the macroscopic strain ε_x is taken as the *elastic* strain of the network. In addition, each time a bond fails in part, the length of the corresponding fibre segments can be assumed to increase by a small amount γ. The irreversible, plastic, strain of the network is then proportional to B and the total external strain of the network, $\varepsilon_{x,\,tot}$ is

$$\varepsilon_{x,\,tot} = \varepsilon_x + \gamma B \tag{8}$$

The stress and the number of broken bonds against the total strain are shown in Fig. 17b.

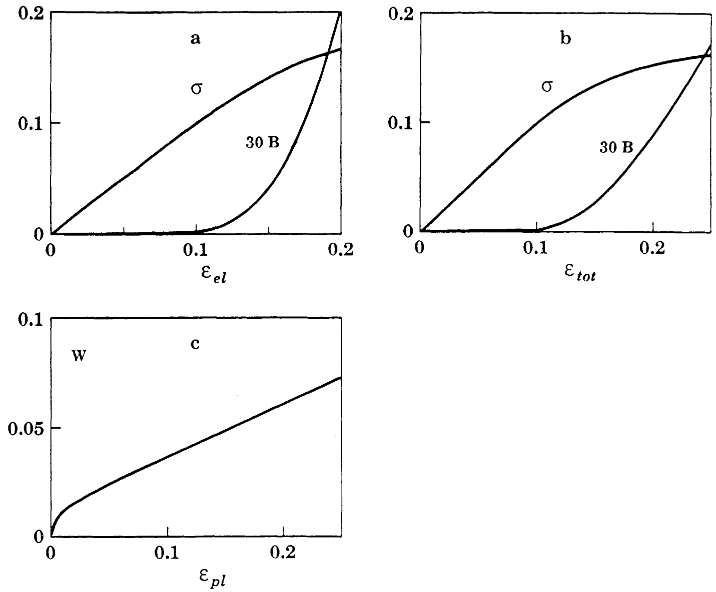

Fig. 17: Model results, Eqs. (6)-(8), for stress σ and number of broken segments B against elastic strain $\varepsilon_x = \varepsilon_{el}$, *a*, and total strain ε_{tot}, *b*; and for external work W against plastic strain ε_{tot}, *c*. Model parameters are $\alpha = \beta = 1$, $\gamma = 10$.

Detailed comparisons with experiments remain to be made, but the predicted behaviour is clearly very similar to that of real paper. To be honest, the same can be said about many other models, too. The virtue of the present one is that it offers a mathematically tractable description of the stochastic behaviour of paper under an external load. For the sake of illustration, Figs. 18 show examples of measured load-elongation curves for different industrial papers together with the plastic component of strain against the total elongation. The main point here is that the plastic strain (Fig. 18b) is very small at small external elongations and then very abruptly starts to increase *almost* linearly. This compares well with the model result for *B*, Fig. 17b. The plastic strain curve in Fig. 18b is very similar to the reflectance curve in Fig. 1. In fact, Sanborn (100) has shown that the light scattering coefficient increases linearly with the plastic component of external strain, in perfect agreement with Eq. (8). Also, Fig. 19 shows that if the external force is plotted against elastic strain, or work against plastic strain, then measurements with real paper yield curves that are almost linear. Comparisons with Figs. 17a and 17c show that the stochastic model is consistent with these observations, too.

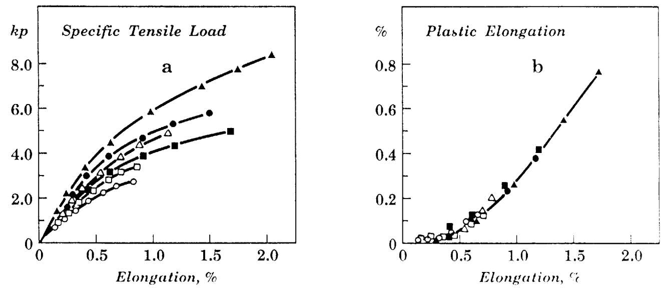

Fig. 18: Load, *a*, and plastic elongation, *b*, against total elongation (MD) for different industrial papers, each shown by its own symbol, after Göttsching and Baumgarten (29).

It must be kept in mind that, in spite of the agreement with experiments, the model ignores many aspects of real paper. The gradual failure of bonds and the related plastic elongation or even activation of fibre segments should be treated properly. Also, in real paper the failure thresholds have a distribution, while in the model they were all taken to be equal. Factors like density, fibre orientation and drying shrinkages have not been considered; presumably they would affect the model parameters α, β and γ. Because of all the simplifications, the above equations will probably have to modified later. It should also be remembered that the non-correlated failure process ceases to be valid before macroscopic failure commences. The model itself does not indicate when this happens and therefore does not predict the breaking stress or strain.

2.4. Conclusions

On the basis of the available experimental and computational evidence, the behaviour of paper at small strains can be characterized as a sequence of random failure events. The "random" events are not correlated, one failure does not imply the next. This state of affairs is independent of the nature of the microscopic failures, whether it is the fibres or bonds, or even the hydrogen bonds, that fail partially or completely. However, as the macroscopic elongation of paper is increased, the two-dimensional stochastic process changes over to the final localized rupture process. In the next chapter I will discuss the structural factors that govern the tensile strength of paper.

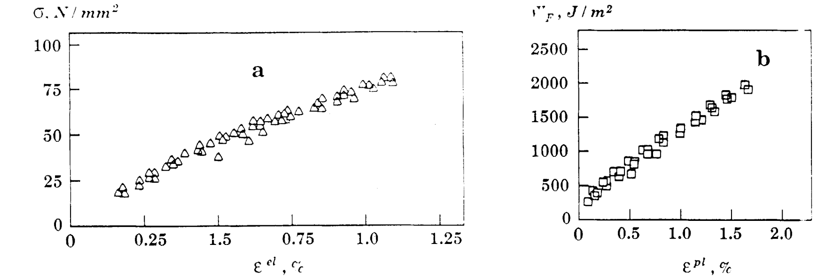

Fig. 19: Stress against elastic strain, *a*, and external work against plastic strain, *b*, for softwood kraft handsheets, after Paetow *et al.* (101).

The non-correlated fracture process at small strains is important since it means that the properties of paper can be understood in terms of a statistical distribution of the microscopic failures, *with no reference to their spatial arrangement.* Above I have shown of how this picture can be used to describe mathematically the load-elongation behaviour of paper.

The exponential strain (or stress) distribution, Eq. (3), follows from the disordered network geometry. Furthermore, in a linearly elastic system the local elastic strains are always linearly proportional to the macroscopic elastic strain. This means that if the irreversible elongation of such a system arises from failures at some structural level, then that behaviour must be given by a function of the inverse external strain. From this point of view it is generally rather questionable to model the load-elongation behaviour of paper with analytic[1] functions of the external elongation.

The statistical accumulation of microscopic failures can be analysed from experiments. The acoustic measurements of Corte (38, 64) and the microcalorimetric measurements of Ebeling were the first in that direction (32). More recently, the acoustic measurements have been developed by Yamauchi and Murakami (102-105). The amplitude distributions (Fig. 20) give direct information on the elastic energies that are released at bond or segment failure. The same authors have also used infra-red technique to measure the heat dissipation when paper is strained (104, 106). Both of these measurements could be compared with the stochastic model

[1] The function $\exp(-1/x)$ is not analytic, *i.e.* it cannot be expanded in powers of $x \ll 1$.

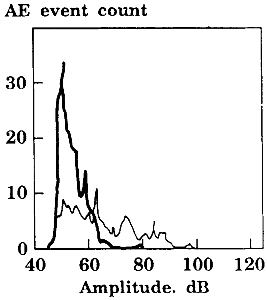

Fig. 20: Maximum amplitude distribution of acoustic emission during tensile straining of unbeaten and well-beaten softwood kraft handsheets (thick and thin line, respectively), after Yamauchi *et al.* (103).

described above or with a more refined theory. The same applies to the tensile testing of paper, *e.g.* changes in the reloading modulus and the plastic strain component against the total external elongation (59, 101) have direct interpretation in terms of the microscopic failures.

So, is the stochastic failure process relevant for the load-elongation behaviour of paper? My answer is yes. For example, the effect of network geometry gives a plausible explanation to the fact that the load-elongation curves of different papers are alike, much more so than the greatly variable curves of the constituent fibres and bonds. Because of this similarity or "universality" it is even possible to describe paper properties with models that completely ignore the microscopic failure phenomena.

3. TENSILE STRENGTH

The strength of paper is governed by two factors. First, paper fails through the partial breaking of fibre-to-fibre bonds and fibre segments. In simple terms, fibre breaking is the critical factor for paper strength only if the bonds are very strong. Second, the large-scale sheet structure (formation) governs the macroscopic fracture path and the small-scale structure the microscopic failures. We expect that uniform paper tolerates higher stresses than non-uniform paper. This section considers the effect of the disordered paper structure on tensile strength.

3.1. Strength phenomenology

The simplest estimate for tensile strength can be based on the assumption that fibres are solely responsible for the failure of paper. The tensile strength of paper would then be given by

$$T = \frac{1}{3}\rho\sigma_f = E\varepsilon_f \tag{9}$$

where σ_f is the tensile index and ε_f the breaking strain of the fibre, and ρ and E are the mass density and elastic modulus of paper, respectively. The factor 1/3 follows from the isotropic distribution of fibre orientations (107). Eq. (9) merely states that paper fails when the axial strain first exceeds the breaking threshold in those fibres that are parallel to the external strain. The inhomogeneity is ignored.

The failure of fibre-to-fibre bonds is usually at least as important as that of the fibres themselves, because the bonds are relatively weak. There are several reasons for this, such as mismatching between the bonding surfaces and the polymeric bonding molecules, and stress concentration on the bond periphery (15, 108, 109) (*cf.* Fig. 7).

Bond failure is usually explained by the "shear-lag" mechanism of Cox (107), according to which the stress is transmitted from one fibre to the next through the bonds. The axial stress in a fibre is zero at the ends and increases to a constant maximum value in the middle, while the shear stress on the bonds is largest at the fibre ends and decreases to zero in the middle (Fig. 21a). The bonds at the fibre ends should fail first. Signs of this have been observed in practice (43). However, because of the randomness in the network geometry, bonding and fibre curl, considerable shear stresses may even be induced on bonds that are far from fibre ends. In addition, as was explained in subsection 2.1, bonds may also fail if the axial stress along a fibre becomes too large. There seem to be several possible mechanisms for bond failure.

On the other hand, inter-fibre bond failure in paper is usually only partial and is associated with a plastic yielding of the bonded segments. One may then assume that the tensile strength of paper is governed by some kind of a yield limit of the bonds. In other words, paper fails when the

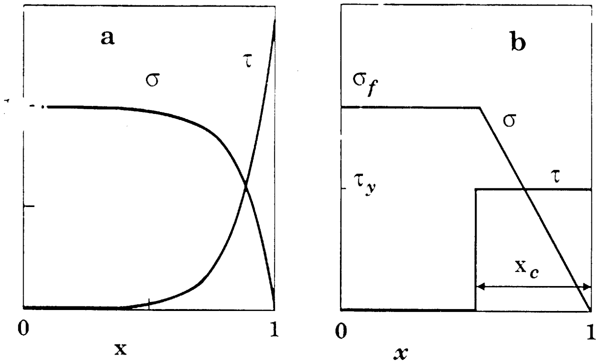

Fig. 21: Axial stress, σ, and the corresponding shear stress on the inter-fibre bonds, τ, from fibre centre (x=0) to one end (x=1). Cox model (107), *a*, and Shallhorn and Karnis model (110), *b*, where breaking thresholds for fibre and bond are σ_f and τ_y , respectively.

external stress equals the bond yielding threshold. This situation has been analyzed using various phenomenological models, in which the behaviour of typical fibres and bonds is considered. The description by Shallhorn and Karnis (110) is particularly pedagogical. They considered only the fibres parallel to the external strain and assumed them to be bonded to a homogeneous background. If they cannot break they must be pulled out of the sheet at the rupture line. In an ideal elastic-plastic system, all partially failed bonds carry a shear-lag stress τ that equals the yield limit τ_y of the bond. At macroscopic failure the shear stress along the entire fibre must be $\tau = \tau_y$. The axial tension σ increases linearly with increasing distance from the fibre ends with the slope $d\sigma/dx = 2\tau_y/r$ (Fig. 21b), and the average tension of the fibres is equal to the tensile strength T of paper[1]:

$$T = N\pi r\tau_y \, l_f/2 \,. \tag{10}$$

This applies if only the bonds fail. However, if the yield stress of the bonds, τ_y, is large, fibres break rather than pull out. Therefore, the axial stress σ is limited by the breaking threshold $\sigma = \sigma_f$, while at the fibre ends σ decreases linearly to zero over a length x_c that is determined by $\sigma_f = 2\tau_y x_c/r$. The tensile strength of paper is then given by

[1] Here r is the fibre radius and l_f the length, and τ the shear force per bond area.

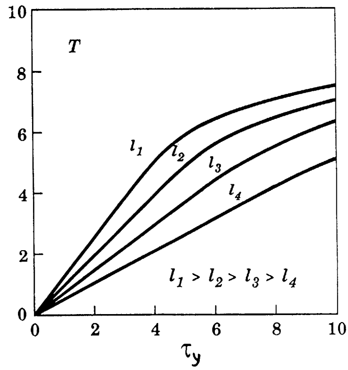

Fig. 22: Tensile strength of paper against the shear strength of bonds for different fibre lengths l according to Shallhorn and Karnis (110).

$$T = N\pi r^2\sigma_f\ (1 - \sigma_f r/2\dot{\tau}_y l_f) \quad . \tag{11}$$

The smaller of the two values given by Eqs. (10) and (11) defines the tensile strength of paper for any particular combination of fibre and bond properties. The result is illustrated in Fig. 22.

Page, on the other hand, derived his often quoted model starting with the assumption that tensile strength is proportional to the fraction of fibres that break along the rupture line (111). This assumption, motivated by experimental observations (112, 113), can be expressed as

$$T = \frac{n_f}{n_f + n_p} N\pi r^2 \sigma_f \tag{12}$$

where n_f and n_p are the numbers of fibres that respectively break and pull out intact. The second *ad hoc* assumption was

$$\frac{n_p}{n_f} = \frac{2\sigma_f r}{\tau_v l_f} \propto \frac{\sigma_f}{\tau_y} \tag{13}$$

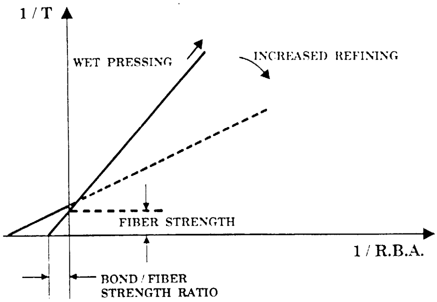

Fig. 23: Inverse tensile strength of paper against inverse relative bonded area according to Page (111), figure after (117).

Finally, Page linked the fibre strength to the zero-span tensile strength Z via $Z=(9/8)\cdot N\pi r^2\sigma_f$. Together, these three equations define the Page model, which is often written as (B is an effective bond strength)

$$\frac{1}{T}=\frac{9}{8Z}+\frac{1}{B} \quad , \tag{14}$$

In the limit of $n_p/n_f << 1$ the Page model yields $T=N\pi r^2\sigma_f\,(1\text{-}2\sigma_f\, r/\tau_y l_f)$ and thus equals Eq. (11) of Shallhorn and Karnis, except that the factor in front of the second term in parenthesis is 2 instead of 1/2. In the opposite limit, $n_p/n_f >> 1$, even the numerical factors are the same as in Eq. (10). The model of Kallmes, Bernier and Perez (78) also yields asymptotic limits very similar to those of the two other models. The differences between the models, arising from the considerations of geometry and statics, seem fairly insignificant in comparison to the unavoidable uncertainty in the microscopic parameters. If one is willing to accept the phenomenological approach, then especially Eq. (14) provides a simple means to characterize papermaking furnishes (Fig. 23).

For example, it is fairly easy to vary experimentally the relevant fibre properties, strength and length, in a qualitatively well governed manner. In this respect each of the phenomenological models has proven to be consistent with experiments (78, 110, 111, 114). On the other hand, Helle

(112) has argued that a nominal fibre strength has only limited significance for the tensile strength of paper, as one should really know the strength of fibres within the sheet. This property is often governed by statistically distributed local defects, induced in the pulping, beating and papermaking operations (42, 115). These defects greatly reduce the strength of individual fibres but not necessarily the strength of the paper. Van den Akker *et al.* (113) have pointed out that many fibres rupture only after macroscopic failure (crack propagation) has commenced, so that is possible that fibre strength has no effect on the tensile strength of paper. Indeed, one must view fibre strength as a phenomenological property whose microscopic interpretation is uncertain.

The fibre-to-fibre bonds in turn affect paper strength through two factors, the specific bond strength (strength per unit bonding area) and the number of bonds per fibre, or the relative bonded area, *RBA*. Stratton (116) has reported measurements on single bonds and has been able to show differences in the bonding ability of different fibres. On the other hand, Retulainen and Ebeling (43) compared different *indirect* measures of the bond strength but found poor correlation between them when the paper structure was varied. Since most bonds fail in several steps, the microscopic interpretation of bond strength is also unclear.

The lack of well-defined microscopic properties in the phenomenological strength models is reflected in the discussion of why paper strength increases with beating (55, 117). Basically, three different explanations have been given: beating increases either the effective strength of fibres, or the specific strength of the bonds (bond strength per unit area), or it increases the *RBA*. Some authors, see *e.g.* (118-121) favour the first explanation, claiming that beating removes cell-wall defects and fibre curl and increases fibril orientation. On the other hand, Helle and Van den Akker *et al.* (112, 113) have shown that the fraction of broken fibres crossing the rupture zone increases together with the tensile strength when beating is increased. Thus beating should make the bonds stronger in relation to the fibres. As to the specific bond strength, Stratton (116) and Mohlin (122) found that it does not change with beating. Shallhorn and Karnis, and Paavilainen (110, 123), among others, concluded that beating increases fibre flexibility and sheet densification (*i.e. RBA*) and therefore increases the tensile strength of paper.

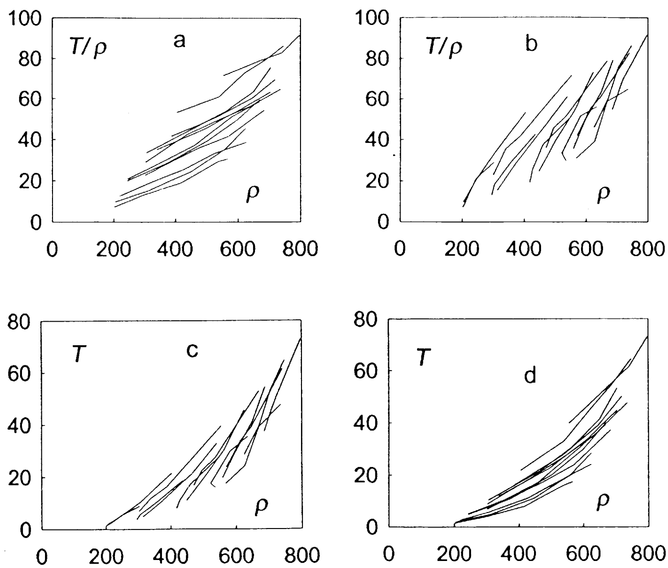

Fig. 24: Tensile index T/ρ (in units Nm/g) and tensile strength T (kPa) against apparent density ρ (kg/m^3) for handsheets of different high-yield pulps, data of Luner *et al* (129). For each pulp, the lines connect points of varying beating (ball mill), *a* and *c*, or wet pressing, *b* and *d*, when all other parameters were held constant.

Many other authors have sought to resolve the strength mechanisms of beating by analyzing changes in the *RBA*, see Waterhouse for a review (55). The specific surface area and light scattering coefficient of paper have been used as measures of *RBA*. However, this involves problems, since the normalization varies with beating and with fibre type (124, 125). *RBA* could also be related to the apparent density of the sheet, but even such a relationship depends on the furnish (55, 126). Thus the changes in bonding area and specific bonding strength induced by beating cannot, in general, be distinguished with certainty.

The apparent density of paper is the simplest measure of paper structure, but does not enter directly into the phenomenological strength models. An increase in density can raise the *RBA* but it can also make the bonds stronger. The tensile index of paper is often a linear function of the apparent density if the latter is varied through beating (127, 128) (Fig. 24). No generally accepted explanation is known for this. If wet pressing is the controlling variable, a different but still almost linear relationship is obtained. Even though the apparent density of paper is not a well defined quantity, it would be useful for practical applications to understand how density affects paper strength. In this respect phenomenological models perform no better than empirical regression analysis (130).

In summary, the tensile strength of paper has been modelled with various phenomenological approaches that give qualitatively very similar results. It seems irrelevant to argue that one particular version is better than all the others. Instead, it is important to remember that these models ignore the random geometry of the fibre network and the random properties of the fibres and the bonds. It is tacitly assumed that all bonds or segments (or those parallel to external strain) reach the failure threshold simultaneously.

3.2. Simulation of elastic modulus

The fundamental mechanical properties of two-dimensional random systems can be most readily studied by means of computer simulations. Before looking at the tensile strength of simulation models, I will discuss the elastic modulus because the effects of random structure are more easily seen in that case. In two-dimensional model systems the modulus E is defined as the force per unit displacement and per system width, and the density p as the total fibre length per unit area. One example is shown in Fig. 25 from the work of Alava and Ritala (51, 84), who placed fibres of constant length l_f at random on a triangular lattice. Qualitatively very similar results have been obtained in many other studies (131-134).

At low densities the elastic modulus obeys a percolation scaling law:

$$E = E_0(p - p_c)^t \tag{15}$$

Where the exponent t depends on the "universality class" of the model (51, 135). On the basis of literature (134), for paper the exponent should be $t \approx 4$ independent of the bending stiffness of the fibres. However, if the

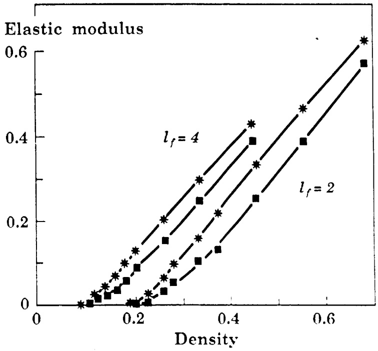

Fig. 25: Elastic modulus against density in the triangular lattice model of Alava and Ritala (51, 84) with two fibre lengths l_f. Stiff and flexible fibres correspond to model parameters β/α = 0.4 and 0.1 (stars and squares, respectively).

bending stiffness is precisely zero, then p_c is almost twice as high as otherwise (132). This higher critical point affects the elastic modulus even when the bending stiffness of the fibres is small but non-zero, the modulus of such systems being very low at all densities below $2p_c$.

At high densities, $p \geq 2p_c$, Eq. (15) does not apply. Instead, the elastic modulus is linearly proportional to density in many two-dimensional lattice models with geometric disorder (dilution) (131, 133, 134). However, in some cases the critical behaviour (Eq. 15) seems to extend to rather high densities and mask the linear behaviour. In paper, this might occur if the fibres were very flexible (*cf. e.g.* Fig. 3 in (51)).

In a two-dimensional random fibre network (no lattice) the linear high density behaviour if the shear-lag model of Cox (107) is written as

$$E = \tfrac{3}{8}E_f p(1 - Kp_c/p) = \tfrac{3}{8}E_f(p - Kp_c) \qquad (16)$$

Where E_f is an effective modulus of elasticity of the fibres and K is a shear-lag coefficient. A *small* K means *effective* inter-fibre stress transfer, *e.g.* when the fibre or bond stiffness is high. Eq. (16) is basically the well-known expression for the elastic modulus of paper (73, 75, 136) but usually there is a function of the segment width-to-length ratio w/l_s and bond shear stiffness (137) in place of Kp_c. The latter aspect has not been included in computer simulations, except for the study of Hamlen (87). All the other simulation models correspond to infinite bond stiffness.

The form of Eq. (16) is motivated by the fact that the random network structure enters through the critical density p_c. Dimensional arguments imply that p_c is inversely proportional to the fibre length l_f. It also depends on the symmetry of the lattice (131) or, in real paper, on factors like flocculation (138) and fibre orientation. For a two-dimensional random fibre network (*i.e.* no lattice) p_c is (139)

$$p_c = 5.71/l_f \tag{17}$$

Since the average segment length (= mean distance between bond centres) is $l_s = \pi/2p$ (10), at the percolation density there are an average of 2.6 bonds per fibre.

It is clear, of course, that Eq. (16) is not useful if K varies significantly with density. Such a density dependence could arise through the connection to the average segment length l_s. However, as pointed out above, in computer simulations the elastic modulus is generally a *linear* function of density. This is the case in Fig. 25 even though the mean segment length does vary with density. Thus K seems to be a function of the *fibre* width-to-length ratio, w/l_f, instead of the *segment* width-to-length ratio w/l_s. Since the elastic constants of the model are defined with respect to the underlying lattice, they characterize fibre properties rather than segment properties. Furthermore, as Uesaka has pointed out (137), if the bond shear stiffness is low, then it determines the elastic modulus of paper. This implies that the elastic modulus is linearly related to the number of bonds perfibre or to p/p_c.

The data in Fig. 25 yields, on linear extrapolation to zero modulus[1], K= 1.2 and 1.6 for the two fibre stiffnesses. We expect that $K > 1$ since the critical exponent $t > 1$. The shear-lag coefficient K can also be estimated from the results of the random fibre network simulations (86, 96), which gives K = 2-3 for fibre widths w/l_f = 0.06 - 0.01 (Fig. 26). Because of the shortage of data it is not possible to check that $K = K(w/l_f)$ independent of density.

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[1] The percolation density is approximately $p_c = (3l_f)^{-1}$.

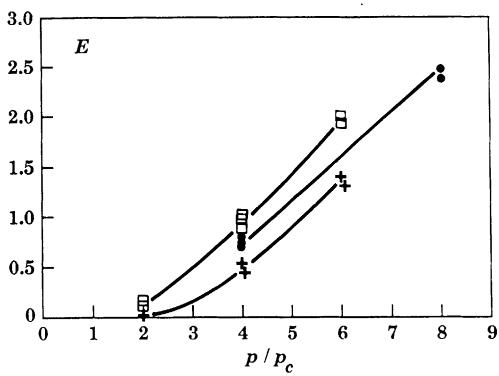

Fig. 26: Elastic modulus against density in two-dimensional network model (86), with w/l_f = 0.06, 0.03 and 0.01 (squares, bullets, and +'s, respectively).

These considerations are relevant to real paper only if the interwoven layer structure of paper (30) does not affect the in-plane mechanical properties. Indeed, Kallmes *et al.* (140) found that ordinary and "multi-planar" laboratory sheets have quite similar mechanical properties. The three-dimensional computer simulations carried out by Hamlen (87) also show that the out-of-plane deformations do not affect the in-plane mechanical behaviour of the fibre network.

Real paper and two-dimensional models could be compared at equal number of bonds per fibre (140). This number defines the network geometry of real paper, assuming that it is sufficiently close to random geometry. Of course, the number of bonds per fibre is difficult to measure. Equally well, the geometry is defined by the two-dimensional density p relative to the percolation threshold. However, no reliable connection between the somewhat ambiguous apparent density ρ of real paper and that of two-dimensional models, p, has yet been established. It is uncertain how this connection should depend on the number of fibre layers and the fibre flexibility (10, 140, 141). The new measurement technique for wet fibre flexibility will probably prove helpful in solving this problem (44, 142). Also, Mohlin (143) has shown that the mechanical properties of paper at low basis weights can be explained by assigning to the surface layers properties that are different from those of the inner

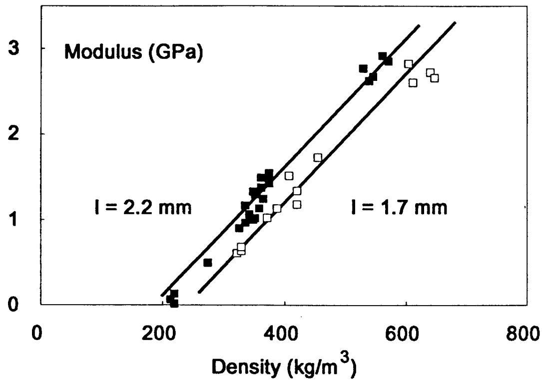

Fig. 27: Elastic modulus against apparent density for light-weight handsheets with two different mean fibre lengths l, after Hollmark *et al.* (145).

layers. Similarly, the apparent thickness of a paper sheet does not fall to zero with the basis weight because of the finite fibre thickness (144). There is, of course, a percolation threshold for the basis weight, too, which also explains the observations made by Mohlin.

In any case, it is reasonable to assume that at *normal* basis weights the mean thickness of one fibre layer equals fibre thickness. Then the apparent density ρ of real paper is related to the two-dimensional model density p (total fibre length per unit area) through

$$\rho = p\rho_f w \tag{18}$$

Where ρ_f is the mass density of the fibres and w their average width. The elastic modulus is in practice determined by measuring the force per sample width and dividing it with the apparent thickness of the sample. All this means in terms of applying Eq. (16) is that the ambiguity of density affects the values of K and ρ_c, but not the slope $dE/d\rho$.

Hollmark *et al.* (145) have measured the elastic modulus of very thin paper sheets for two average fibre lengths l_f = 2.2 and 1.7 mm (Fig. 27) for which Eqs. (17) and (18) yield the percolation densities[1] ρ_c = 101 and 131 kg/m^3. The measured modulus agrees with the linear model Eq. (16)

[1] With w = 0.03 mm and ρ_f = 1300 kg/m^3 (145).

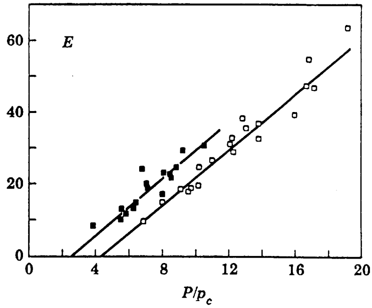

Fig. 28: Elastic modulus (arbitrary units) against density for ordinary and "multi-planar" handsheets. Data of Kallmes *et al.* (12, 140), unbeaten and beaten kraft (black and white squares, respectively). Density was calculated from the reported number of crossings per fibre (c_m): $p/p_c = c_m/2.6$, and E from the measured modulus per unit mass by multiplying it with p/p_c.

and on extrapolation intercepts the density axis at Kp_c = 185 and 240 kg/m^3 or K = 1.83 for *both* fibre lengths.

The data of Kallmes, Corte and Bernier (12, 140) for ordinary and mult-iplanar sheets can also be used, see Fig. 28. In this case, the two-dimensional density p was here calculated from the reported number of bonds per fibre, c_m. Again, the measured values are consistent with Eq. (16). The least-square fits shown in Fig. 28 correspond to K = 2.5 and 4.3 for the unbeaten and beaten kraft furnishes, respectively. The qualitative and quantitative match of these two experiments with Eq. (16) is surprisingly good, given the uncertainty of the fibre properties in Fig. 27 and the number of bonds per fibre, c_m, in Fig. 28.

To summarize, the computer simulations and experimental data suggest that the elastic modulus E of paper should obey the basically well-known Eq. (16) except at very low densities or basis weights where percolation scaling, Eq. (15), applies. The shear-lag constant K, ranging perhaps from 1 to 4, should be a weak function of the elastic properties of the fibres and bonds, but it should be independent of the network density (or mean segment length l_s). Any change that, at a given apparent density, affects only the network geometry can be expressed as a change in ρ_c. In particular, the equation should hold if the apparent density ρ is varied through wet pressing. If the density is varied through beating, then $E(\rho)$ may well become a non-linear function since the fibre or bond properties change and thereby K and E_f also change.

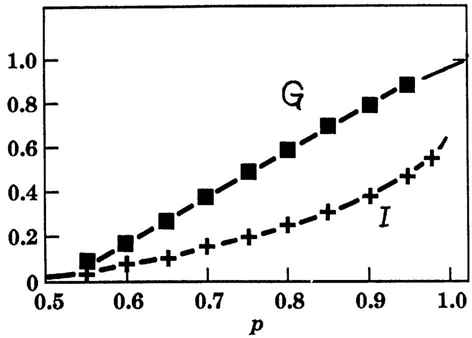

Fig. 29: Conductivity *G* (elastic modulus) and breakdown current *I* (tensile strength), squares and crosses respectively, against density for a diluted resistor lattice after Duxbury and Li (148). Density, conductivity and breakdown current are = 1 in the perfect lattice.

3.3. Tensile strength of paper as a random network

There are few computer simulations on breaking strength that could be applied to paper. It is very difficult to obtain statistically reliable values since the strength depends on the size of the model system. Most studies have focused on the microscopic fracture phenomena rather than macroscopic strength estimates. Also, perfect elasticity (brittle elements) is usually assumed to hold all the way to microscopic failure. In most studies, bond failure is not included and the inter-fibre bonds or their equivalents are taken to be rigid.

As Fig. 29 shows, the ideal, maximum, strength is not reached on a lattice until very close to the perfect system. Even small amounts of disorder reduce the tensile strength *T* much more than the elastic modulus *E*. This is quite easy to understand since *T* is determined by the highest values of the local stress, not the mean value. According to Fig. 29, the tensile strength of a two-dimensional system is generally *not* a linear function of density. The results of Alava and Ritala (84) for a triangular lattice with brittle fibres seem to imply a linear density dependence, but the data are so few that no firm conclusions can be drawn.

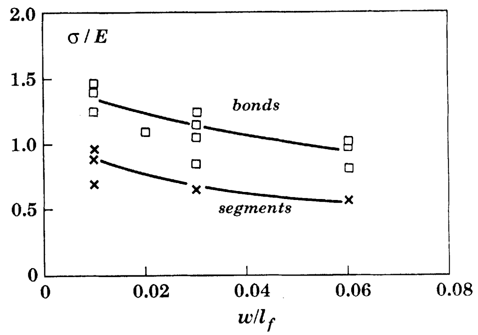

Fig. 30: Tensile strength over elastic modulus (σ/E) against the fibre width-to- length ratio w/l_f in two-dimensional random network (86) at $p = 4p_c$. Bond and segment failure, squares and crosses, respectively.

Close to the percolation threshold the tensile strength should be governed by a scaling law analogous to Eq. (15) with a new exponent $f = 2.5 \pm 0.4$ (134) in place of the elastic exponent $t = 4$. The percolation density p_c is the same for elastic modulus and tensile strength. Using the scaling concepts, Ritala (51, 146) showed that the reinforcing effect of kraft fibres is governed by a dimensionless combination, $\Gamma = pl_f = l_f m\omega$, where p is again the two-dimensional density (total length of fibres per layer and unit area), ω the mass-to-length ratio of the fibres, and m the basis weight of the fibres per layer. Using the percolation threshold $\Gamma_c = 5.71$ /PiSe74/, Ritala concluded that 3-12% (in mass) of reinforcement fibres are needed in order to produce a percolating network in all fibre layers. Below this concentration the reinforcement pulp should have little effect on the tensile energy adsorption and other strength properties, while above it the strength properties should improve dramatically.

The failure of fibre-to-fibre bonds in random fibre networks has been studied by Hamlen (87) and Åström and Niskanen (86). Hamlen showed that if realistic values are used for the fibre and bond properties, then only bonds fail, a finding that is in good agreement with experimental observations (subsection 2.2 and (108)). Åström and Niskanen, on the other hand, compared bond and segment failure at a relatively high density ($p = 4p_c$). The strength-to-modulus ratio T/E decreased with increasing fibre stiffness, or width, (Fig. 30). An analogous conclusion may

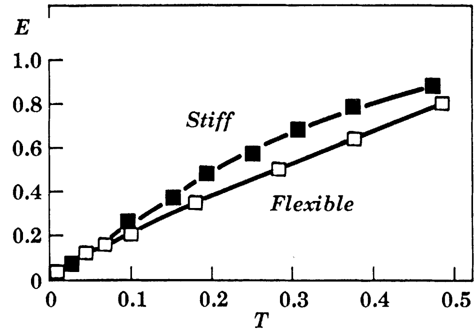

Fig. 31: Elastic modulus against tensile strength, for a diluted resistor and a spring lattice that are, respectively, equivalent to stiff and flexible fibres. Data quoted in (148).

be drawn from certain lattice simulations (Fig. 31). Thus flexible fibres (or segments) may give the paper higher strength than stiff fibres when compared at the same level of the elastic modulus of paper. A reduction in network density also seemed to lower the tensile strength more when segments failed than when bonds failed (86). This is quite natural since only a single segment failure but several bond failures are needed for a microcrack. This applies even more to real paper, as most real bonds fail in several steps. However, it should be remembered that these findings are rather uncertain because of the very limited amount of data.

Computer simulations thus give inconclusive results for the strength of paper, and more investigations are called for. It is clear that the effect of the network density on the tensile strength is not as simple as on the elastic modulus, Eq. (16). On the other hand, Fig. 31 suggests that plotting the tensile strength against elastic modulus may be a useful way to remove the effects of random geometry. More precisely, it seems possible that the elastic modulus and tensile index of paper at reasonably high densities could be given by

$$E = E_0 \cdot (\rho - K\rho_c) \text{ and}$$
$$T = T_0 \cdot f(\rho - K\rho_c) \ . \qquad (19)$$

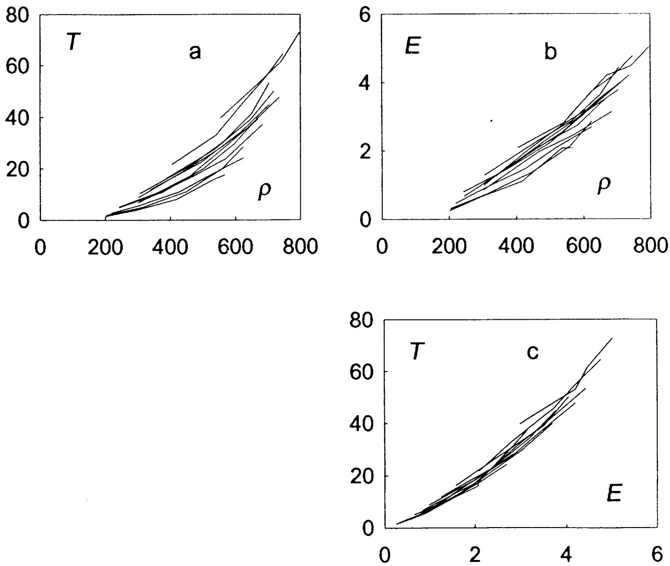

Fig. 32: Tensile strength T, a, in units kPa, and elastic modulus E, b, GPa, against apparent density ρ, kg/m^3; and strength against modulus, c, for handsheets of different high-yield pulps, data of Luner *et al.* (129). For each pulp and degree of beating (ball mill), the lines connect points of varying wet pressing.

Where E_0 and T_0 are constants, and f is an unknown *convex* function. The results of the previous section suggest that the shear-lag constant K should depend on the fibre and bond properties but *not* on the mean segment length l_s (or density ρ). If the hypothesis holds, then the function f can be determined experimentally by varying the geometric properties (density, fibre orientation, flocculation) of the network. Changes in such properties, particularly in density, should move the data along a single $T(E)$ curve, while changes in the mechanical properties of the fibres or bonds should change the curve.

I emphasize that Eq. (19) contains little intrinsically new, similar attempts to relate tensile strength to density have been made before (55, 147). One should realize, however, that quite generally the density dependence of the mechanical properties of paper should be different at low and high densities and thus the entire density dependence of, say, the tensile strength cannot be presented with one simple mathematical expression.

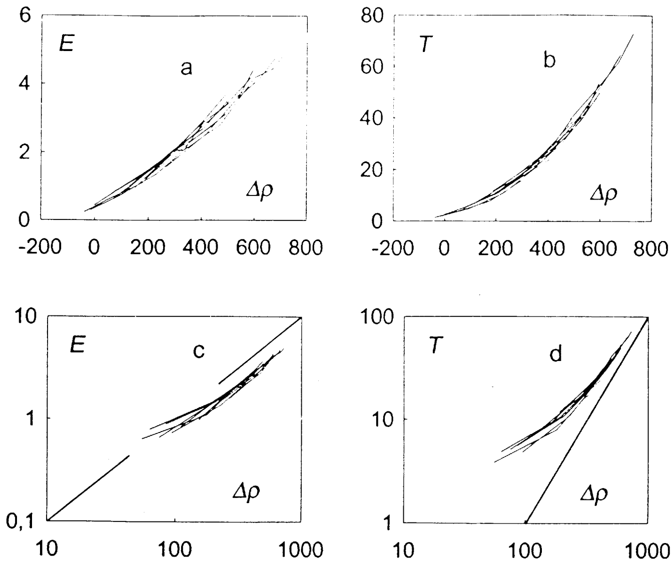

Fig. 33: Elastic modulus E, *a, c,* and tensile strength T, *b, d,* as in Fig. 32, but against a shifted apparent density $\Delta\rho = \rho - \rho_i$. Different constant ρ_i was chosen for each pulp and beating degree (see text for more details). Log-log plot in *c* and *d*, with the straight lines corresponding to a linear and quadratic behaviour, respectively.

As an example, consider the data of Luner *et al.* (129), already shown in Fig. 24. For the sake of convenience, I have redrawn the tensile strength against density in Fig. 32, where each pulp type and degree of beating are again connected with a line. Shown in Fig. 32b are the corresponding results for the elastic modulus. On the basis of these lines alone, little can be said about why beating increases strength. However, a single curve is obtained when tensile strength is plotted against elastic modulus (Fig. 32c). Furthermore, the lines for both $E(\rho)$ and $T(\rho)$ can be made to fall on a single curve for E and on another curve for T, when plotted against a shifted density $\Delta\rho = \rho - \rho_i$, as shown in Fig. 33. The required constants ρ_i grow monotonically with the beating, but otherwise the operation is quite arbitrary. At high density the elastic modulus is linearly related to $\Delta\rho$. The zero point of $\Delta\rho$ was chosen so that $E = \text{const}\cdot\Delta\rho$ at high density, and as a result the tensile strength is a quadratic function of density, $T = \text{const}'\cdot\Delta\rho^2$ in the same region (Fig. 33c and 33d). Furthermore, the curve $T(\Delta\rho)$ has a shape similar to that of $T(E)$ in Fig. 32c.

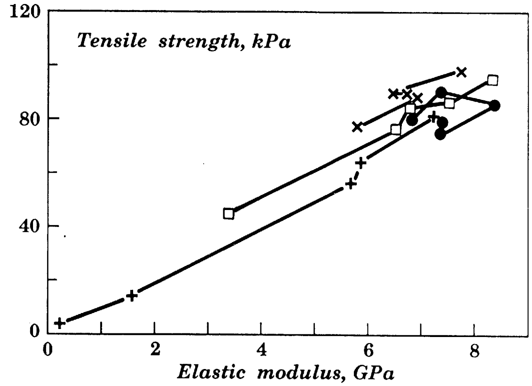

Fig. 34: Tensile strength against modulus for handsheets at different degrees of beating (PFI mill), CSF = 690, 510, 225 and 120 ml (+'s, squares, x's and bullets, respectively). Data of Alexander *et al.* (118), 20 mesh fractions from 48%-yield Norway Spruce springwood kraft pulp.

Considering Eq. (19), the collapse of the lines in Figs. 32 and 33 implies that the prefactors E_0 and T_0 must be independent of beating. The beating-induced increase in tensile strength at a given apparent density must have arisen from a decrease in the elastic property $K\rho_c$. Since beating makes the fibres more conformable, it easy to accept that either bonds become stiffer or ρ_c decreases. It is possible that the strength of the bonds or fibres may change in beating, but such effects are not necessary to explain the data. On the contrary, the analysis suggests that, in the ball mill beating, nothing happened to the microscopic strength properties.

The above is, of course, not a general case, as the effects depend on the type of beater. For example, it is quite possible that fibre length is reduced in beating, which would increase ρ_c. However, such a change would not affect the $T(E)$ relationship, provided the strength of the fibres and bonds did not change. On the other hand, changes in the microscopic strength properties would affect the tensile strength at a given elastic modulus. This seems to be the case with the data of Alexander *et al.* (118, 119), who employed a PFI mill (Fig. 34). In terms of Eq. (19), in this case beating increased the constant T_0 (change of slope in Fig. 34), except for the highest level of beating. Even in this case, however, most of the increase in modulus and strength seems to have come from a decrease in $K\rho_c$, *i.e.* from an increased elastic stiffness of the network.

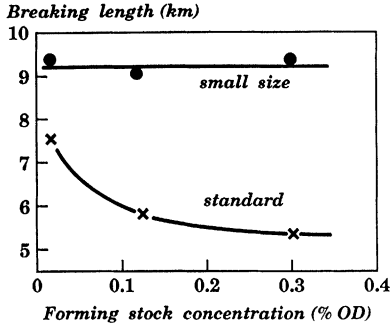

Fig. 35: Tensile index against forming stock concentration after Norman (21). Standard specimens (strip width 15 mm) and small size waisted strips (waist width 3 mm).

Suppose now that Eq. (19) is valid and that f is a quadratic function: $T = T_0 \cdot (\rho - K\rho_c)^2$. If the apparent density at a given wet pressing and the critical density ρ_c both depend in the same way on beating, then it follows that the tensile index, T/ρ, is linearly proportional to density. Could this explain why tensile index often increases linearly with beating?

The discussion should have shown that even changes in the network geometry (ρ_c) and the elastic properties affect the tensile strength of paper in a complicated manner. Thus it seems very doubtful that the tensile strength of paper could be predicted from the phenomenological models (subsection 3.1) even if one knew the the properties of fibres and bonds. On the other hand, Eq. (19) may not be generally valid, and in any case it does not describe explicitly the effects of the the elastic and strength properties of fibres and bonds. In my opinion, the tensile strength of paper is still very much an open problem.

3.4. Effect of formation

In the previous two subsections I have tried to illustrate how the disordered network structure affects the tensile strength of paper. In practice there is little one can do to change factors like density and fibre orientation that are dictated by other paper properties. On the other hand, the large-scale inhomogeneity of paper, or formation, is obviously a much more significant practical control parameter.

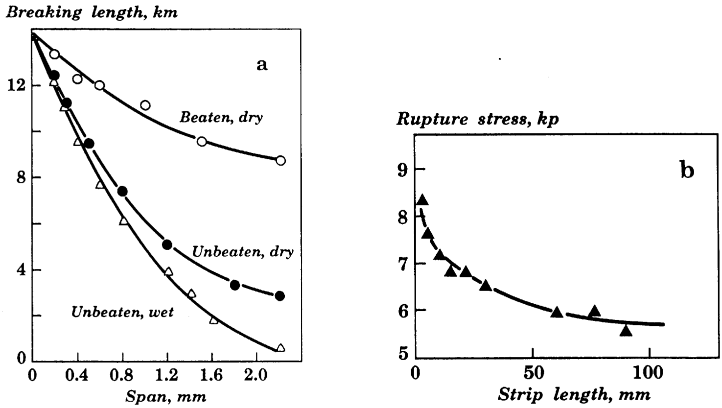

Fig. 36: Tensile strength against sample length with a short span tester (150), *a*, and ordinary tensile tester (154), *b*, softwood kraft handsheets and kraft wrapper, respectively.

It is well known and intuitively easily accepted that the strength of paper decreases as the basis weight of a sheet becomes more non-uniform. This was shown by R.J. Norman (21), who varied the forming stock concentration in a handsheet mould. However, when he measured the local tensile strength using strips with a narrow neck, Norman found that the mean value of the local strengths was independent of the forming concentration (Fig. 35). Thus the standard tensile strength is determined by the extreme low values of the local breaking stress. Similar results have been obtained by many others, as at length by B. Norman and D. Wahren (24). In particular, it has been demonstrated that the tensile failure line of paper generally goes through the thin spots of the specimen (26).

Because strength is determined by the extreme value statistics, tensile strength is lower in long paper strips than in short (148, 149) but the behaviour over short spans is determined by different statistics than over long spans (Fig. 36). In the first case, the variation of tensile strength with specimen length is determined by the average properties of the network. In other words, strength is dependent on the number of fibres that extend across the test span (150) and on the elastic stress distribution within the strip (151, 152, 153). In the limit of a very long strip, on the other hand, strength goes to zero very slowly, if at all. Whether it does or not is, of course, very difficult to determine experimentally. On the other hand, in practice the local strength of paper is never zero on the scale of a typical strip width squared (about 1 cm^2).

In considering the effect of formation, it is important to remember that it is not the local strength alone that causes the thin spots to break but rather the local stress (or strain) over local strength. Poor formation gives low strength because the variability of the local strain increases with increasing basis weight variability (20, 25, 35, 155, 156). Local strain is highest in the areas of low basis weight. The strain variations explain why tensile strength *decreased* when Norman (21) deposited stock spots on wet handsheets. Next to a heavy spot in the longitudinal direction the local strain should be higher than average and *vice-versa* if the heavy spot was in the lateral direction. This can be demonstrated with a simple lattice model (157) or with finite-element calculations (158). Thus the topology of the basis weight variations has a significant effect on the local strain distribution. If one also takes into account the fact that the local fibre orientation (159, 160) and density (20) may be connected to basis weight, it is easy to understand why the relationship between strength and formation may vary from one paper machine to the next.

A computer simulation study and related numerical analysis (157) indicate that for a given formation the local strain variations should be smallest when the inter-fibre stress transfer is effective (*eg.* short stiff fibre segments). The reason for this is that a tight coupling between adjacent local areas smooths out strain variations. Poor formation should thus be most detrimental for the strength of poorly bonded papers. On the other hand, if formation improves, paper may lose some of its ductility and may tolerate fewer local, or incipient, failures. I am not aware of clear experimental observations in this respect. It is also physically reasonable that the tensile strength of paper is more sensitive to poor formation when fibre strength is low (86). Many bonds per fibre have to be broken before the network fails locally, whereas breaking just one segment will cause the same qualitative effect. In other words, for segment failure the relevant length scale is smaller and hence the basis weight variability larger than for bond failure.

3.5. Summary

Paper strength is usually governed by the gradual failure of the inter-fibre bonds, though in some cases fibre failure may also play a role. The competition between these two failure modes can in principle be assessed using the well known phenomenological models of paper strength, but these models are rather difficult to apply if the network geometry changes. In particular, beating may affect fibre strength, specific bond strength and network geometry (degree of bonding). I have deliberately not considered in any detail the predicted relationships between paper strength and the various indirect measures of *RBA*. Even though changes or differences in the degree of bonding can be characterized by means of optical measurements, I find it difficult to believe that our understanding of paper strength can be increased in that way.

As one would expect, the disordered network structure has a strong effect on the tensile strength of paper. This dependence on the connectivity of the network is not properly described by the standard strength models of paper. Nor can it yet be specified on the basis of computer simulations, though certain clues in that direction are already available. In particular, changes in the elastic bonding can perhaps be separated from the other factors by studying tensile strength against elastic modulus. As a function of the apparent density, tensile strength exhibits two different regions: at low densities the strength is governed by the percolation threshold, while at high densities another asymptotic density dependence is expected. Poor formation generally leads to low strength, but the extent of strength reduction depends on the topology of basis weight variations, on the related variation in the local fibre orientation and density, and on the mean elastic properties of the network.

4. FRACTURE

Tensile strength characterizes the maximum load-carrying capacity of paper and its significance is easy to understand intuitively. However, tensile strength tells little about the "runnability" of paper, for example its performance in high-speed printing machines. The failure of a running web is triggered by a small flaw in the paper or by a transient peak in web tension. This section discusses the measurements and models that can be used to analyze the fracture resistance or toughness of paper.

Tear strength has been traditionally used to evaluate the paper's ability to resist crack propagation or fracture. However, it is very difficult rigorously to link tear strength to runnability, either experimentally or theoretically. In other fields of materials science, where one usually deals with three-dimensional bodies, a "tear strength" does not occur. Instead, what is measured is fracture resistance or fracture toughness. These concepts have also been applied to paper and many different measurement schemes have been devised.

4.1. Mechanisms of web failure

The mechanical runnability of paper webs is usually characterized by reporting the number of breaks per, say, 100 rolls. Typical values in pressrooms seem to range from 2 to 4 breaks per 100 rolls (161, 162) so that the breaks are very infrequent events. If the runnability is measured in terms of the mean distance between breaks then the typical result is perhaps 10^5 to 10^6 m /Roi90/. The low frequency of breaks makes the it difficult to evaluate the runnability of a particular paper. As Page and Seth (161) have shown, breaks occur at random intervals and the number of breaks for a given number of rolls obeys a Poisson distribution. Because of many disturbing random factors, under normal circumstances it is necessary to run 1000 to 10,000 rolls before it is possible to distinguish between papers from different manufacturers. It may take months to compile the necessary data, during which time climatic conditions and paper properties may drift significantly.

The off-line characterization of paper runnability is difficult. Pilot-scale test installations have been used to simulate the actual dynamic conditions (163, 164, 165). Such tests require heavy equipment and are rather elaborate. In some cases, experiments have also been conducted on full-size printing machines (166). Part of the problem naturally arises from the low frequency of flaws in paper. In order to circumvent this, high-speed elongation tests (in-plane tear) have been performed with relatively small flawed specimens, but apparently the results do not correlate with the actual runnability (167).

The runnability of paper is strongly dependent on the paper's moisture content. At low moisture contents paper is brittle and fails easily (35, 163,

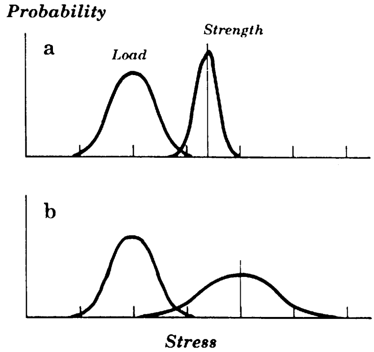

Fig. 37: Probability of web breaks is low if the overlap of load and strength distributions is small (top). Strength variations can increase break probability even if mean strength is reduced (bottom).

168). Thus runnability is generally poorer in the winter than in the summer (161). When the relative humidity approaches the saturation value, the impact rupture properties of paper become very similar to the static properties at the same RH (35) (*cf.* Fig. 3). In other words, at high moisture contents the speed of tension variations should not have any effect on paper runnability. Rapid stress relaxation improves runnability, or equivalently, paper can be stretched more under slow strain rates than high. The rate of stress relaxation increases with relative humidity (and temperature), but even more if humidity varies cyclically (56, 57). The moisture dependence of the relaxtion rate particularly affects runnability in the dryer section of the paper machine but has little effect at solids contens below about 80% (45).

A paper web fails if, somewhere in the web, either the local strength is too low or the momentary load too high. Even if the mean strength of paper is well below the mean load, fluctuations in either may cause the web to break. Therefore, it is very important that the variance in strength and load also remain sufficiently low (163, 164) (Fig. 37). It is clear that a low break frequency relies heavily on the proper operation of the paper machine and the printing press. Significant improvements have been made in this direction, flutter has been reduced in the dryer section of the paper machine and the web tension profiles have become more uniform because of better moisture profiles and winder governs (163, 166, 169).

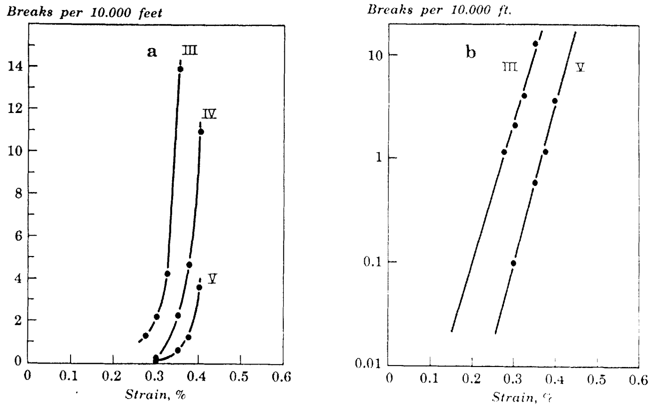

Fig. 38: Newsprint web break frequency against strain measured with a winding instrument at 1200 ft per min, Sears *et al.* (165). Linear, *a*, and log-linear scales, *b*; III, IV and V denote different commercial papers.

Many studies have indicated that web breaks are related to flaws in the paper (163, 166, 168). In newsprints, shives and calender scabs are common causes of breaks (164, 165, 170) and the runnability can be improved by effective screening of pulps and on-line monitoring of web defects. Larocque (168) has also shown that the number of breaks correlates with the number of "weak spots" in paper. He derived the latter number by measuring the burst strength of single plies and calculating the percentage of the measurements that fell below an arbitrarily chosen value. Thus paper formation may play a role in determining runnability.

It has been demonstrated that break frequency increases with increasing web strain or tension (164, 165, 166), perhaps exponentially (Fig. 38). The exponential dependence has been used to estimate the break frequency at low web tensions from measurements at high tensions (164) (Fig. 39). However, it is possible that some other functional form than exponential applies to the break frequency in which case the above estimates may be missleading. In subsection 2.4 it was pointed out that on general grounds failure events in an elastic system should be inversely related to the external strain. At the fibre level failure frequency seems to obey an exponential distribution, exp(-*const*/ε), in which ε is the external strain (subsection 2.3). Interesting enough, this behaviour, shown in Fig. 40, is consistent with the measured frequency of web breaks, Fig. 38.

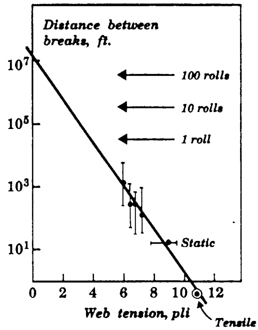

Fig. 39: Newsprint web break frequency against web tension measured with a pilot winder at 100 ft. per min. by Adams and Westlund (164). Standard tensile strength and static strength of 17 ft. long web span are also indicated. Straight line is exponential fit used to estimate runnability at low web tensions.

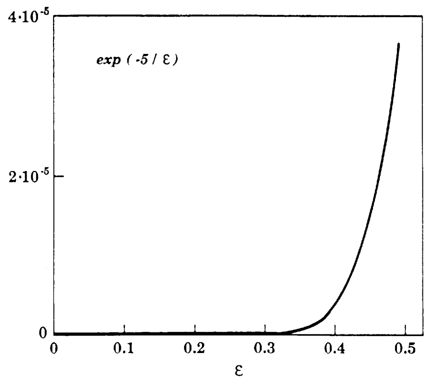

Fig. 40: Fracture probability against external strain (arbitrary units) calculated from an exponential strain distribution.

The tear resistance of paper, particularly the in-plane tear, has been conventionally employed as a measure of runnability (27, 171, 172, 173). It is often observed that on a particular paper machine a low tear strength is related to poor runnability (168). However, the general conclusion seems to be that there exists no universal connection between tear resistance (Elmendorf or in-plane) and runnability, nor can one predict runnability from other conventional paper properties (161, 163, 166, 168). As Roisum (163) points out, the correlations reported between runnability and paper properties are usually so weak that no reliable conclusions can be obtained from them. Of course, this is not too surprising given that web

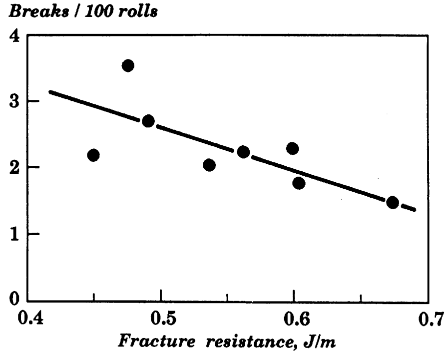

Fig. 41: Web break frequency of commercial newsprints against fracture resistance, after Page and Seth (161).

breaks are closely related to flaws or weak spots in paper. After all, in runnability tests paper fails at strains (or stresses) well below the ultimate values measured in a tensile test (Fig. 38).

From the above, it follows that in order to analyze and improve paper runnability one should have the means to characterize the ability of paper to resist fracture in the presence of small faults. In that respect it is clear that the Elmendorff and standard in-plane tear tests do not measure the right property since the sample is not loaded in the tensile "mode I" that presumably governs the failure of a paper web (28, 174, 175). Instead one should measure the fracture toughness or fracture resistance of paper. This corresponds to the in-plane tear resistance but with a 0° tearing angle. In fact, Page and Seth (161) showed that the fracture resistance of paper correlates with runnability when different paper mills are compared (Fig. 41). This appears to be the only case where a universal correlation has been demonstrated between paper runnability and a strength property. Fig. 41 is probably one of the reasons why many different measures of the fracture resistance or toughness of paper have been investigated recently. These developments will be discussed in the next two subsections.

4.2. Linear elastic fracture mechanics

Griffith (176) considered the fracture of an ideally brittle, linearly elastic material. His model forms the basis of linear elastic fracture mechanics (LEFM). Griffith postulated that during an increment of crack extension *da* there can be no change in the total energy of the system. In other words, the potential energy that is released when the new crack surface relaxes is equal to the surface energy required for the crack increment. The surface energy per unit crack area is denoted by γ. It describes the material's ability to resist crack propagation. As the crack grows, potential energy is released at a rate G that depends on the loading and geometry of the specimen.

The rate G increases with the specimen displacement even if the crack does not grow. For crack growth to be initiated, the displacement must be sufficiently high so that elastic energy can be released at the rate of $G=G_c$:

$$G_c = 2\gamma. \tag{20}$$

For a linearly elastic material, G against the displacement Δ can be determined experimentally. One just has to measure the area between the load-displacement (P-Δ) curves for specimens with slightly different crack lengths a and da (Fig. 42):

$$G = \frac{1}{2}P\left(\frac{\partial \Delta}{\partial a}\right)_P = -\frac{1}{2}\Delta\left(\frac{\partial P}{\partial a}\right)_\Delta \tag{21}$$

In this way, the measured G indicates how much energy is required to increase a crack length by *da*. If the measured value of $G(a,\Delta)$ is low, there is little potential energy available for crack growth at that particular displacement. On the other hand, the *fracture toughness* of the material is the critical value G_c that occurs at the onset of crack growth. It is of course necessary that the specimen is such that G_c is independent of its geometry. This is often not true, which has been one of the difficulties with the fracture toughness measurement of paper (below).

Note that $G(a,\Delta)$ is high for a given displacement Δ if the elastic modulus is high. However, this does not mean that the material is "tough". The

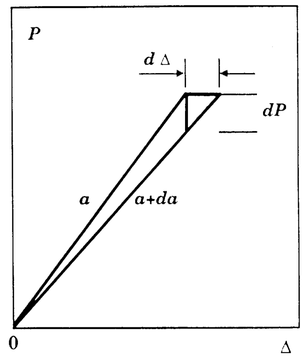

Fig. 42: Load P against elongation Δ for LEFM specimens with a crack of length a and $a+da$. The rate of energy release G corresponding to the crack growth is proportional to $Pd\Delta = \Delta dP$.

critical displacement may be small and toughness therefore low, as $G \propto E\Delta^2$. In comparison, the tensile strength of a high modulus material is usually high, since $P \propto E\Delta$.

Another criterion for crack growth in linearly elastic materials is obtained from the stress field σ in the vicinity of a crack tip. This is of the general form (see, *e.g.* chapter 3.1 in ref. 177)

$$\sigma(r,\theta) = \frac{Kf(\theta)}{\sqrt{2\pi r}} \quad \text{for } r \to 0 \tag{22}$$

where the distance r is measured from the crack tip and θ is the polar angle. The crack propagates in the direction θ = 0, where the form function f is largest. K is the *stress intensity factor*, which again depends on the dimensions of the cracked body and the external load P (or displacement Δ): $K = K(P,a)$. For example, for a strip of width $2b$, the stress intensity at the tip of a centrally located crack of length $2a$ is

$$K = \sigma\sqrt{\pi a}\,\sec^{1/2}\left(\frac{\pi a}{2b}\right) \tag{23}$$

where σ is the tensile stress far from the crack.

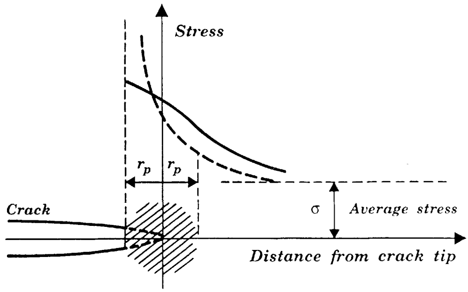

Fig. 43: Stress distribution at a crack tip, for an linearly elastic material (dashed line) and plastic material such as paper (solid line). Plastic flow occurs in the hatched region. Figure after Helle (175).

While the stress does not really diverge at the crack tip, the asymptotic amplitude K nevertheless characterizes the stress concentration in the linearly elastic medium. In order to trigger crack growth, the external stress must be increased, and the critical value of the stress defines the critical value K_c. If the LEFM is valid then K_c is related to the fracture toughness G through (178)

$$G_c = K_c^2/E \qquad (24)$$

where E is the elastic modulus.

The above LEFM equation (24) can be shown to hold for ideal brittle materials such as glass. However, if it is applied to ductile materials such as metals, inconsistent results are obtained (177). In reality, most of the energy G released does not go into surface energy but is dissipated as heat when plastic flow occurs around the crack tip (Fig. 43). The plastic deformation has a strong effect on crack propagation since it lowers the stress at the crack tip.

From this, one would predict that the fracture toughness of paper is affected by plastic deformations at the crack tip and therefore LEFM should generally not be applied to paper. For example, when a paper strip with an initial cut is strained, the opacity increases at the crack tip over a

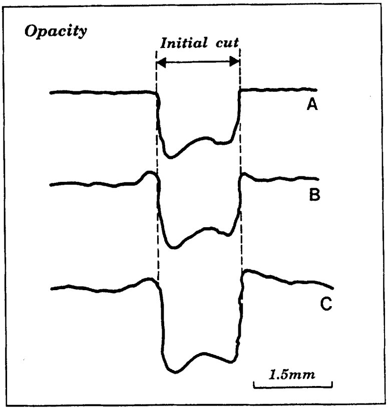

Fig. 44: Opacity at a transverse cut of a tensile strip at zero strain (A), at intermediate strain (B) and just before failure (C), after Andersson and Falk (182).

range that is comparable to the length of the cut (Fig. 44). This indicates that plastic yielding occurs far ahead of the crack tip. The same conclusion was drawn by Thorpe *et al.* (179, 180), who measured the local deformations at a crack tip using linear image strain analysis.

In the first fracture toughness investigations of paper, the LEFM parameters G_c and K were determined experimentally for samples with a small initial cut (181, 182). However, the results varied with the initial crack length and sample width. The question was then studied extensively by Seth and Page (174, 183, 184). When short specimens and slow strain rates were used, crack growth was quasi-static, *i.e.* the crack grew only with increasing displacement. In a typical case the load decreased linearly to zero with increasing displacement (Fig. 45). Assuming that LEFM can be applied to paper, the fracture toughness was then calculated as the work R done after the onset of fracture, divided by the total crack length. It was found (183) that the work of fracture R equals G_c.

The key in the work of Seth and Page was that the plastic deformations could be confined to a small zone at the crack tip by choosing a suitable initial crack length and specimen width. The plastic deformations at the crack tip are then insignificant and the fracture resistance R is

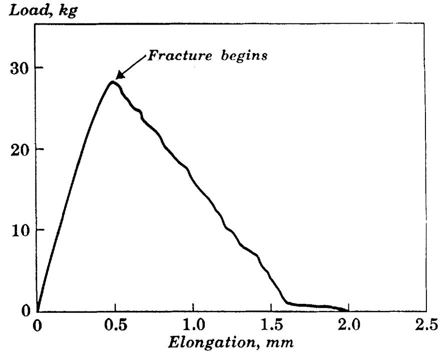

Fig. 45: Load against elongation for a fracture resistance specimen (length 50 mm, width 150 mm and length of initial edge cut 50 mm), after Seth (184).

independent of the specimen geometry, at least for *brittle papers* such as newsprint (183, 184). However, in the case of tough ductile papers (e.g. woodfree writing paper) very wide specimens would have been needed to measure R. Even then, the force T_n needed to initiate crack growth is equal to or larger than the yield stress T_Y of paper (cf. Fig. 46). Thus the applicability of LEFM seems questionable (185).

Similar conclusions concerning LEFM were reached by, among others, Heckers *et al.* 186), Uesaka (187) and Steadman and Fellers (207). By now it seems clear that linear elastic fracture mechanics cannot generally be applied to paper, although the fracture resistance (work of fracture) R (185) may give correct results for brittle papers.

4.3. Non-elastic fracture toughness

From the preceding it is clear that the fracture toughness of paper should in general be characterized in some other way than by LEFM. Rice (189) showed that the so called J-integral taken around the crack tip is independent of the integration path Γ. The integral is formally defined as

$$J = \int_{\Gamma} W dx_2 - T_i \frac{\partial u_i}{\partial x_1} ds \qquad (25)$$

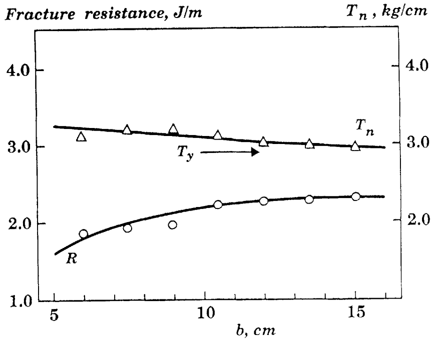

Fig. 46: Fracture resistance R (circles) and net load at failure, T_n (triangles) against specimen width for bleached softwood handsheets after Seth (184). Yield strain T_y of the paper indicated by the arrow.

Where W is the potential energy (*i.e.* $\partial W/\partial \varepsilon_{ij} = \sigma_{ij}$), T is the traction and u the displacement. Under certain assumptions J is equal to the energy release rate G:

$$J = -\frac{1}{t}\left(\frac{\partial U}{\partial a}\right)_{\Delta} = G \tag{26}$$

This also holds for the critical value, $J_c = G_c$. The point here is that plastic deformations at the crack tip have no effect on J when the integration path Γ is taken sufficiently far from the crack. This gives a justification for using the critical value of the energy release rate $J = G$ as a measure of fracture toughness. Following the work of Rice, the critical value J_c has become a standard measure of fracture toughness.

In plastic or ductile materials, the load-displacement ratio of a cracked specimen becomes non-linear before the initial crack starts to grow. To evaluate fracture toughness, the intrinsic non-linearity of the material must be separated from that caused by the crack growth. No general mathematical formula can be given for this purpose. The J-integral can be measured using multiple specimens with different crack lengths (BL method of Begley and Landes (190)). Several authors have applied this to paper with good results (*e.g.* 188, 191, 192, 193). Unfortunately, the BL method is rather tedious. Experimental work has recently been focused on finding ways to cope with just one crack length.

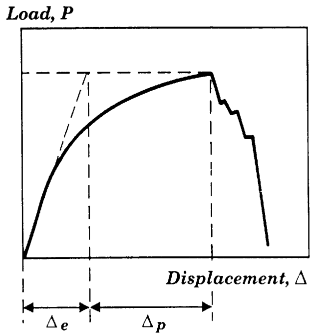

Fig. 47: Load against displacement for a notched specimen. Elastic and plastic elongation at failure are Δ_e and Δ_p (196).

One of the single specimen methods tested for paper is that of Rice, Paris and Merkel (194). It is assumed that the plastic displacement Δ_p (Fig. 47) of a specimen with a deep notch is a function only of the load per uncracked width b of the specimen:

$$\Delta_p = bh(P/b) \tag{27}$$

The unknown function h should be obtained from one set of measurements, after which the J-integral can be calculated from a single load-displacement curve for any sample. The RPM method has been shown to give the same results as the BL method for brittle filter paper (195) but not for tough papers (188), for which the RPM values depend on the specimen geometry(188, 192).

In order to overcome the dependence on geometry, Yuhara and Kortschot (196) generalized the above formula to read

$$\Delta_p = bh(P/b^m) \tag{28}$$

They found that a value m = 0.8 should be chosen. The J-integral values obtained from Eq. (28) were close to those obtained with the BL method.

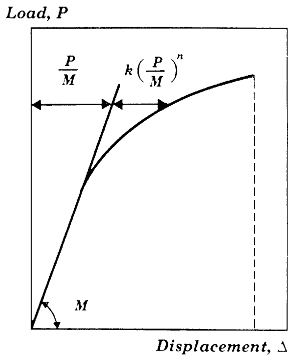

Fig. 48: Parametrisation of load-displacement curve in Liebowitz method (197).

In another single specimen method, proposed by Liebowitz *et al.* (197), the load-displacement P-Δ relationship of a notched specimen is approximated by (Fig. 48)

$$\Delta = \frac{1}{M}P + k\left(\frac{P}{M}\right)^n . \tag{29}$$

In the absence of a crack M is the Young's modulus of the specimen. Eq. (29) describes a strain-hardening material. With a few supplementary assumptions the critical value J_c can be calculated from the load-displacement curve for a single specimen. Westerlind *et al.* (192) found that the values were similar to the BL-values. Fellers *et al.* (193, 198) also obtained a reasonable correlation between BL and Liebowitz methods, though the latter values were somewhat lower (Fig. 49).

Seth *et al.* (199) pointed out that all the J-integral methods suffer from the fact that it is difficult to detect when the crack starts to grow. Sub-critical crack growth (180) may lead to errors in the crack length. To overcome these problems, Seth *et al.* (199) characterized the fracture toughness of paper using the concept of *essential work of fracture* (200). The work done on a notched specimen is divided into two parts, the essential work of fracture w_e and the non-essential work w_p. The first part w_e is consumed in a process zone where the actual fracture occurs (cf. Fig. 50),

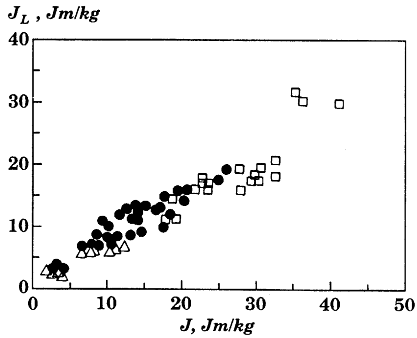

Fig. 49: Liebowitz fracture toughness, J_L, against J-integral of Begley and Landes for liner, sack paper and newsprint samples (bullets, squares and triangles, respectively), after Fellers *et al.* (193).

and under certain conditions $w_e = J_c$ (200). The latter part w_p is dissipated by plastic deformations in an outer screening region. The total work required to break a notched specimen is

$$W_f = Ltw_e + \beta L^2 tw_p \ . \tag{30}$$

Where L is the length of the central ligament of the specimen, t is paper thickness and β is a shape factor that depends on the specimen and notch geometry. Thus w_e and w_p can be separated when the work of fracture is measured for specimens with different ligament lengths /198/. The method thus requires multiple specimens.

Steadman & Sloane (201) have used a completely different approach in which fracture toughness is related to the plastic deformation at the crack tip just before crack propagation (see, *e.g.* (177), Ch. 5) Δ_c:

$$R = \Delta_c \, \sigma_y \, . \tag{31}$$

Where σ_y is the yield stress of paper. Both Δ_c and σ_y are difficult to determine for paper.

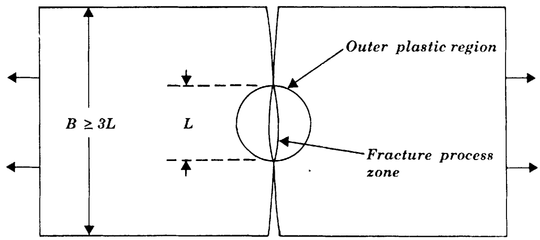

Fig. 50: Double edge notched specimen used by Seth *et al.* (199) to measure the essential work of fracture, consumed in the fracture process zone.

4.4. Paper toughness vs. structure

Different fracture toughness measurements have already yielded quite a lot of data. Although not all of these are comparable, it is perhaps still interesting briefly to consider the results. The LEFM fracture resistance R (indexed with the basis weight) is ca. 10 Jm/kg for newsprints and varies from 10 to 30 Jm/kg for chemical pulps (174). The essential work of fracture has been found to be (199) w_e = 20-25 Jm/kg for copying paper and 10-33 for kraft handsheets, while the Liebowitz method has resulted in somewhat lower values, J_c = 2-15 Jm/kg for kraft handsheets (198). Beating and free drying seem to increase the fracture toughness (199) as does an increase in the moisture content of paper (207). The effects of recycling (202) and filler (203) on fracture toughness have also been explored. Seth *et al.* (198) have pointed out that fracture toughness is closely related to the product of the breaking strain and stress of paper.

It is well known that fracture toughness is very sensitive to cohesive forces acting behind the crack tip (Fig. 43). Simple models for this have been introduced by Dugdale (204) and Barenblatt (205). In real paper there are usually fibres behind the crack tip that extend across the crack and hold the network together. Micrographs illustrating this can be found in ref. (52), p. 224. With respect to tear strength it has been shown that as the crack propagates across the sheet, a significant fraction of the fibres crossing the failure line break. Thus fibre strength is a much more important property for the tear strength or fracture toughness than it is for the tensile strength of paper (58, 113, 206).

Li and Duxbury (94), working with a lattice model, showed that if a post-fracture residual strength is assigned to all "broken" segments, crack arrest can occur and the work of fracture increases substantially. This means that a crack can first start growing but then stop until the external elongation is increased. This is precisely what happens in the fracture toughness measurements of paper: elongation must be increased for the crack to propagate. In the model studied by Li and Duxbury, crack arrest occurs if the residual strength was higher then 3% of the original (at $p = 0.7p_c$). It is clear the structure of the fibre network, particularly the degree of bonding, or density, has a great effect on the threshold of spontaneous crack propagation. This has not been investigated in any detail theoretically.

It is obvious that a better microscopic understanding of what happens at the crack tip would be needed. The linear image strain analysis employed by Thorpe *et al.* (179, 180) appears promising in this respect. According to (179), in tough papers such as kraft sack, linerboard, manila folder and photocopy paper, the local strain field deviates from the mean far away from the crack tip. Brittle papers, newsprint and tracing paper, have a relatively small zone of critical strain. Quantitative measures of the zone of influence of the crack tip might provide an alternative, intrinsic way to characterize paper toughness. In this connection it is also worth pointing out that measurements with an infra-red camera (104, 106) could yield similar information. In this case, the crack tip is seen as a hot spot that propagates ahead of the visible crack opening. Aside from information on the characteristic lengths, the IR measurements could reveal the amount of energy that is dissipated as the crack grows.

Considering the runnability of paper, one of the intuitively obvious facts is the rapid propagation of the crack. According to Andesson and Falk (182), the spontaneous crack growth in paper is slow at first but then approaches the velocity of sound ($v = \sqrt{E/\rho}$), whose order of magnitude is 1 km/s. This means that once at full speed, the crack should traverse an ordinary paper web within milliseconds. Kimura *et al.* (35) found that under impact rupture conditions the typical time to rupture of a narrow paper strip was of the same magnitude and therefore the initial acceleration takes a significant fraction of the failure time of a paper web.

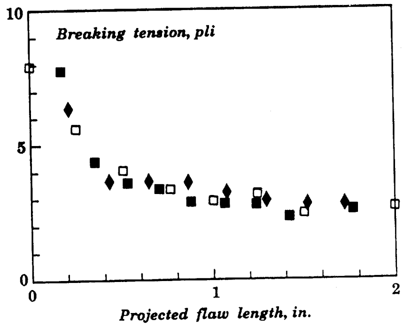

Fig. 51: Breaking load of newsprint specimens with edge cut against projected flaw length in CD, after Roisum (163). Orientation of the cut relative to CD was 0°, 30° or 45° (open squares, diamonds and filled squares, respectively).

It has also been found that the fracture toughness of paper is independent of the rate of elongation, at least up to 50%/min (182, 184, 193). Even though the situation may change at much higher strain rates (perhaps above 10^5 %/min, see Fig. 1 in (108)), it seems that the toughness values obtained with the relatively slow tests should characterize the runnability of paper in practice.

It has also been demonstrated that formation has no effect on the fracture toughness J_c (208). The same observation has been made with the in-plane tear resistance of paper (27). This is natural, since the initial cut or notch determines the starting point of the crack. Once the crack starts to propagate, local deviations from the "mean" structure only cause temporary disturbances in the energy consumption, but do not change its mean value. Indeed, the standard deviation of the fracture toughness was found to decrease with improved formation (208). It has also been shown with lattice simulations that formation-type disorder has no effect on crack propagation (92, 157). Once a crack starts to propagate in a system with weak disorder, it proceeds essentially straight across the sample.

Fracture toughness, on the other hand, measures the general ability of paper to resist crack growth. One assumes, of course, that the higher the toughness, the larger or more prominent are the defects that a paper web tolerates without breaking. Boadway *et al.* (209) measured the tension at rupture of newsprint strips with a cut. They found that the rupture tension was independent of the width of the strip. The relative loss in the rupture

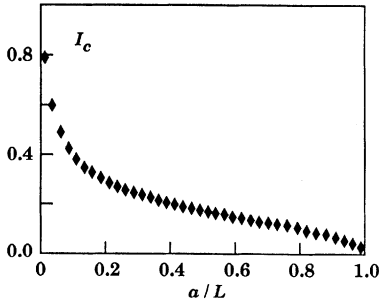

Fig. 52: Breakdown currect I_c (breaking load) for random resistor network of width L with crack of length a at one edge (148).

tension was constant for a cut of given size. Sears *et al.* (165, 170) used a special winder installation and found that shives longer than 3-4 mm trigger breaks in newsprint. However, many shive-induced small cracks were also found to have passed through the testing equipment without causing a break, so a paper web can indeed tolerate even small cracks without failing. The critical flaw size of a few millimetres was also found by Roisum (163). The curve he obtained for the breaking tension against flaw length (Fig. 51) agrees very well with an analogous result obtained from a lattice simulation study (Fig. 52). The rapid decrease at small flaw lengths corresponds to the rapid decrease in tensile strength when density is lowered from the saturation value (Fig. 29). At larger flaw sizes, boundary effects govern the breaking tension (148).

In tensile loading the failure of a paper web (or strip) usually starts from the edge (Fig. 53). To some extent this can be understood by the general fact that large stresses occur at the corners of a strip when it is stretched (151, 152, 153). In fact, however, in a non-homogeneous tensile specimen, the highest stress occurs with a high probability in a narrow region at the edges (Fig. 54). In a lattice simulation (98) the probability that the highest stress occurs at the edge was found to diverge as $L^{0.3}$ with the width L of the system. Notice that in contrast, the probability for the *lowest* stress value to occur at the edge is inversely proportional to the width L. Thus the paper structure at the edges of a wide web is crucial to runnability. It also seems that the elastic properties and their anistropy must affect in the stress concentration at the edges.

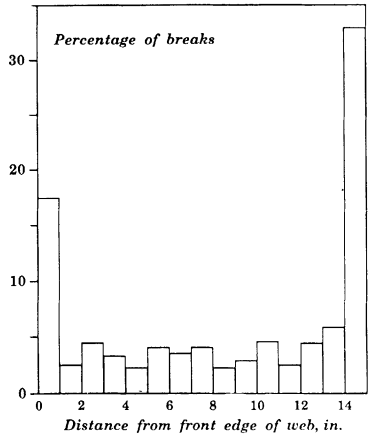

Fig. 53: Across-the-web distribution for longest shive in newsprint breaks, after Sears *et al.* (165).

Quite recently, Kertész (210) measured the fractal dimension of fracture lines in different papers. The root mean square displacement of the fracture line scaled as L^{ζ} with the size of the measurement window, where the exponent ζ = 0.63 - 0.72. It is not clear that these differences in the fractal exponent are significant enough to indicate that paper might exhibit different types of crack propagation. However, since the exponent $\zeta < 1$, fracture lines in paper are one-dimensional on large length scales.

4.5. Summary

Good runnability of paper is a product of two properties: web defects are few in number and small in size, and fracture toughness is high. Many fracture toughness measures have been developed and tested on paper. They may give numerically different results but probably rank papers of a given grade in the same order. The number of defects can also be controlled on-line. Thus it should be possible to characterize the potential runnability of a given paper as a material. It is not surprising that this may have little to do with the runnability observed in practice, the same occurs with many other functional properties of paper: they cannot be predicted precisely with laboratory measurements. Thus, fracture toughness should not be regarded as an irrelevant property for the papermaker. On the

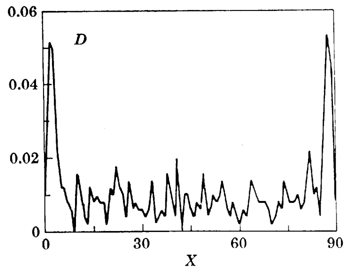

Fig. 54: Probability D that a bond at distance x from one edge of a random resistor network (size 90x90) carries the largest current (largest stress), after Li and Duxbury (98).

other hand, the runnability of a paper web probably also depends on the elastic properties since these control the stress distribution across and along the web in an open draw. These questions are outside the scope of this presentation.

5. FINAL REMARK

Systems where large numbers of microscopic variables interact are common problems in statistical physics. The macroscopic properties of such systems depend on the general form, or "symmetry", of the interactions. The macroscopic, observable phenomena can then be explained with simple microscopic models. It is not necessary nor possible to describe all the details but the essential incredients must be included. Above, I have done my best to show that such models can also be applied to paper physics. With regards to strength and fracture of paper, the crucial feature that a model must include is randomness in structure.

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Transcription of Discussion

STRENGTH AND FRACTURE OF PAPER

K Niskanen (Review Paper)

Prof Lyne kindly pointed out that the sentence below Figure 2 on p645 in my presentation should read as follows:
".... defects had a big effect on the in-plane tear strength (force required to start the tear), and that the local pre-rupture strain field was affected by their presence."

I apologise for the misrepresentations.

K Niskanen

Prof C T J Dodson, University of Toronto, Canada
Thank you very much. The last point you made was one of the most important ones, it has been made before by Heinz Corte I believe. I would like to comment on the situation of moving from random to non random. The case of a flocculated structure is qualitatively different and rather hard to cope with; there is no information as far as I know in the literature on the strain distributions simulated, or actual measurements, on flocculated structures. It is quite tricky but the point I would like to make is that from the work we've been doing it appears that where the negative exponential distribution crops up, we have only one parameter so you cannot change the variance if you are going to fix the mean. In the transformation to the flocculated situation, the gamma distribution appears to take over and there you do have two parameters and it contains the case of the negative exponential but if you want to have a variance change you may at the same mean. This appears to be the key to free fibre length distributions in the move to flocculated structures and it may well be the key to the strain distributions of others. I notice that

without saying so there is some unpublished work that appears to be using gamma distribution in your equation 4 so we look forward to seeing that.

(EDITOR'S NOTE: THE SPEAKER DECLINED TO RESPOND AND TOOK THE ABOVE AS A COMMENT)

Dr C Fellers, STFI, Sweden

You have read the literature carefully, so well I have a very provocative question for you. In your view why is the stress/strain behaviour of paper so interesting in the first place. Is it because it is so easy to measure? In the real world we want resistance for long term loading and that points to the creep behaviour or we want resistance of fracture in the presence of defects and so on for runnability. So why should we devote so much time to the stress/strain behaviour?

K Niskanen

The stress/strain curve in my opinion is something that you can hope to predict from the first principles so that if you measure the stress/strain curve and work hard and develop theoretical understanding then we can gain information about the microscopic strength properties like the effective strength of fibre bonds. The microscopic strength properties are something very difficult to measure. I am always thinking that when we build these models one of the motivations is to provide means so that by doing experiments we can learn about the microscopic properties of paper.

Dr D Page, PAPRICAN, Canada

I was on the committee which made the decision as to who should be giving these review papers and one of my comments was let's give the job to the younger chaps and then he'll have to read the literature. I am delighted to see that you have done so and done a very fine job of it. I truly hope that it has helped in your thinking. I have one comment to make and that is you have dealt with the tensile strength and there are three equations which you mentioned. There is one which is generally called the Page equation and there are the Kallmes, Bernier and Perez and the Shallhorn and Karnis equations. You make the comment that the difference between these models seems fairly insignificant in comparison to the unavoidable uncertainty in the microscopic parameters. The differences are not insignificant. They are actually quite substantial and they are very important in practice for the following reason. The Kallmes Bernier equation says that when the tensile strength of the sheet is less than 4/9ths of the zero span tensile strength, fibre strength does not matter at all so if you have an 18km zero span strength when the strength of the paper is below 8km fibre strength is of no importance. The Page equation says that it is of equal importance to bond strength at that level. This has enormous practical importance for the permanence of paper because most papers are made around 8km breaking length or so and these sheets do lose strength upon ageing because of loss of fibre strength. I think Dr Priest is going to be giving a paper on permanence later on in our programme and I don't think his results could be explained by either of the other two equations. I have been through a number of oscillations of believing and not believing in the Page equation and I am now going through a stage of believing in it again.

(Written note added after the Symposium)

K Niskanen

I was not allowed to answer Dr Page on the spot so perhaps I may do it afterwards. My point in making the cited statement was that as the models are uncertain (or actually incorrect), I see little point in arguing about which one is the best. I am not surprised to learn that the models vary in their ability to reproduce sets of experimental data. As Dr Page makes such a strong point in favour of his model, I may point out why I am sure his model cannot be correct. Suppose fibre strength is very large, in which case the Page model predicts that the tensile strength of paper is linearly proportional to RBA. That is wrong. Firstly, tensile strength must vanish at the percolation threshold, where RBA ~2-3 bonds per fibre (I grant this is not very relevant for most papers). Secondly, anyone who has studied models with disorder knows that strength is never a linear function of a density-type property such as RBA over the entire range. I refer to section 3.3 in my written contribution.

Dr S Loewen, Abitibi Price Inc, Canada

Do we in fact know that in printing presses, the failure mode is usually an in plane type fracture mode? Have there been studies that experimentally investigate the actual failure mode in a printing press?

K Niskanen

I don't know the answer to the first part of the question , whether we know it's in plane this failure. As to the latter part I would say that I don't know the answer either. I seem to remember that there isn't very much work on that. These things are difficult because of the nature of statistics and the amount of paper you need.