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The *J*-integral as a Parameter for Characterizing the Fracture Toughness of Paper

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ABSTRACT

In this study, the methods used by both industry and paper physicists for evaluating paper toughness (resistance to breakage) are critically reviewed, and a new method for determining the critical value of the *J*-integral, J_c, is presented. Difficulties arising from the use of tensile strength and tensile energy absorption in the evaluation of paper toughness are highlighted. J_c values obtained with the Leibowitz non-linear technique were relatively close to those obtained with a new method developed by the authors when the latter was used in conjunction with key-curve analysis for determining the critical point. This result suggests that the Liebowitz non-linear technique may give a relatively accurate *J*-integral value at the crack initiation point with less experimental effort. Experimental results show that it may also be possible to use notched specimens in the standard tensile testing configuration for *J*-integral estimation, which would be an attractive method for industry.

INTRODUCTION

Although paper most often experiences in-plane fracture during converting operations and in service, the paper industry does not have a standard test method for determining the in-plane toughness of a paper sheet. The most common method for evaluating paper toughness has been the Elmendorf tear test. In this test however, specimens are subjected to out-of-plane tearing, and it has been recognized that there may be significant differences in the way in which a particular sheet will respond in the two different modes of fracture. In spite of this potential problem, the Elmendorf tear test is the most widely used method for measuring paper toughness, and it may be that a combination of traditional properties (including the Elmendorf tear test) is sufficient to predict the runnability of sheets from a single mill where the structural variations are minimal. Nevertheless, a more realistic measure of fracture toughness is needed if papers with a wide variety of fibres and structures are to be compared or if an attempt to design the structure of paper for maximum toughness is to be made.

In this paper, the limitations of conventional methods for evaluating paper toughness will be discussed, and several methods for estimating the J-integral, including the Liebowitz non-linear technique (1,2), the essential work of fracture (3), and the recent method of Yuhara and Kortschot (4), will be compared. A novel method for determining the J-integral, which can be performed with a conventional tensile tester without any modifications, will also be described.

THE IMPORTANCE OF THE IN-PLANE FRACTURE TOUGHNESS OF PAPER

Runnability is still a serious concern in the newsprint and the newspaper industry. Paper web breakage in the printing press is generally attributed to in-plane crack propagation from a flaw tip in the paper web (5). The factors related to runnability have been summarized by Smook (5) and Snider (6). According to these descriptions, the number of flaws in paper, the flaw carrying ability of paper and the press tension are the major factors controlling press room breaks. Although a high value of in-plane toughness is not in itself sufficient to ensure good runnability, good resistance to crack growth is required and the paper maker must therefore monitor this property.

In the near future, there will be a need to diminish the use of virgin fibre because of environmental pressures, and it may therefore be necessary to reduce the basis weight of paper (7). At present, transportation and storage costs have already led to some pressure to reduce basis weight. For example, in Japan, 43 g/m^2 newsprint is gaining in popularity. The trend to lighter papers will produce added pressures on the newsprint manufacturers, and there will be a need for better quality control and a more complete understanding of the fracture process.

In many other applications, including sack paper and linerboard, in-plane crack propagation also leads to service failure. Fellers and co-workers (8,9) introduced an in-plane fracture test for the evaluation of die cutting behaviour, and they showed a good correlation between the in-plane fracture toughness and "die-cutting toughness".

In spite of a great deal of research and interest in the area of in-plane fracture toughness measurement, the paper industry has yet to adopt an in-plane test.

PREVIOUS APPROACHES FOR EVALUATING PAPER TOUGHNESS

Elmendorf tear strength

The Elmendorf tear test is still the most common method for evaluating paper toughness. However, in terms of runnability, the Elmendorf tearing (out-of-plane) mode is not appropriate, because paper web breakages on the printing press typically result from in-plane tearing. In fact, paper is seldom exposed to out-of-plane tearing in service, although in some cases the local mode of crack propagation may be Mode III because of local buckling, even when the overall loading seems to be in-plane. Comparisons of in-plane and out-of-plane fracture resistance values have shown that they are not necessarily well correlated (10).

Tensile strength

Tensile strength is another common parameter used to evaluate paper toughness in combination with the Elmendorf tear test. For many materials, however, a high tensile strength does not always mean good in-plane tearing resistance. In Fig. 1, the unnotched tensile strength of fine paper (bleached hardwood kraft) is higher than that of bond paper (bleached softwood kraft). On the other hand, if the specimen contains flaws (notch length 2, 4, 6, 8 mm) fine paper has a lower tensile strength than bond paper. The use of tensile strength to rank papers in terms of toughness would not be appropriate in this case.

Tensile energy absorption (TEA)

TEA is very commonly used as a critical parameter for sack failure. However, if the failure is caused by crack propagation from a flaw tip, TEA has the same deficiencies as tensile strength as illustrated in Fig. 2. In Fig. 2, although the TEA of fine paper is higher than that of bond paper in the unnotched specimen, the TEA of fine paper containing flaws (notch length 2, 4, 6, 8 mm) is lower than that of bond paper.

Linear elastic fracture mechanics (LEFM)

In early studies, LEFM was applied directly to paper (11). However, the extensive plastic deformation at the notch tip in a typical paper specimen invalidates the underlying principles of LEFM unless the specimen is prohibitively large. Uesaka (12) showed that the plastic zone sizes defined by McClintock and Irwin (13) in double-edge-notched (DEN) paper specimens can be

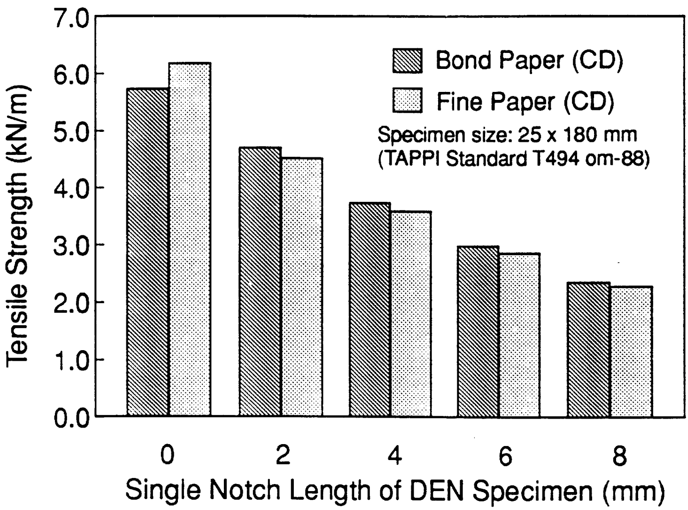


Fig. 1: Tensile strengths of notched and unnotched specimens.

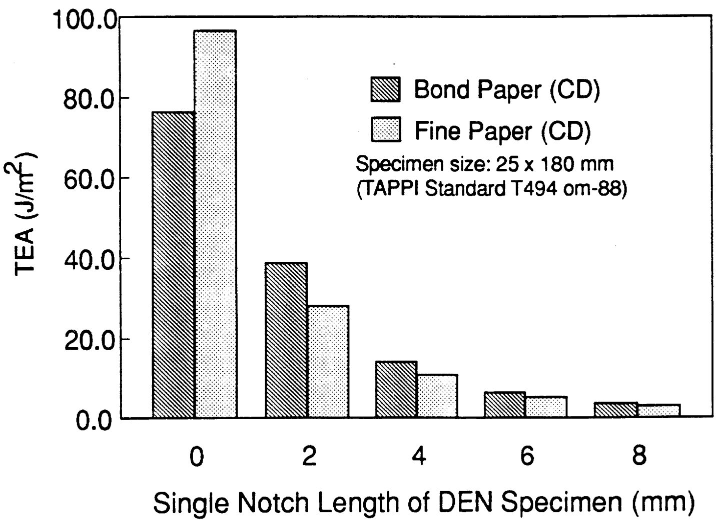


Fig. 2: Tensile energy absorption of notched and unnotched specimens.

relatively large when compared to the crack length. There is therefore a need to employ a fracture parameter developed specifically for dealing with plastic or viscoelastic materials.

J-INTEGRAL AND ITS DETERMINATION

Original concept and its interpretation

The *J*-integral was originally developed as a means of characterizing the stress-strain singularity around crack tip in non-linear elastic and small-scale yielding materials. For these materials, the *J*-integral can be interpreted as the energy available for the crack propagation. Although this interpretation does not hold for materials with extensive plasticity, the *J*-integral has nevertheless been shown to have value as a toughness parameter in these materials (14).

There are a number of techniques which can be used to evaluate the *J*-integral. A brief summary of the most commonly used methods is provided below.

Multiple-specimen method

The multiple-specimen method is directly derived from the energy interpretation of *J*-integral (Fig. 3). The *J*-integral can be calculated with the following equation:

$$J = -\frac{U}{a_2 - a_1} \tag{1}$$

where

J: J-integral (J/m^2),
U: dissipated energy divided by specimen thickness (J/m),
a_1, a_2: notch lengths (m).

In spite of the conceptual simplicity of the multiple-specimen method, it requires substantial experimental effort in practice.

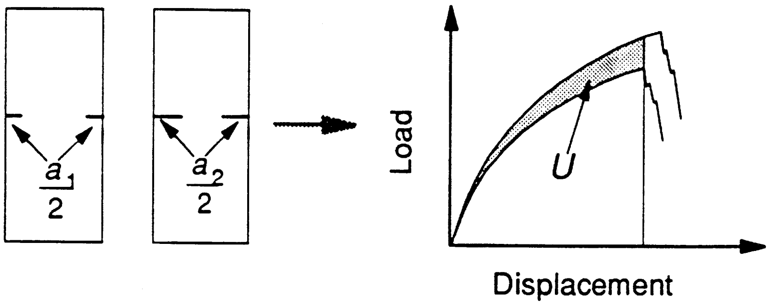


Fig. 3: Schematic of the multiple-specimen method.

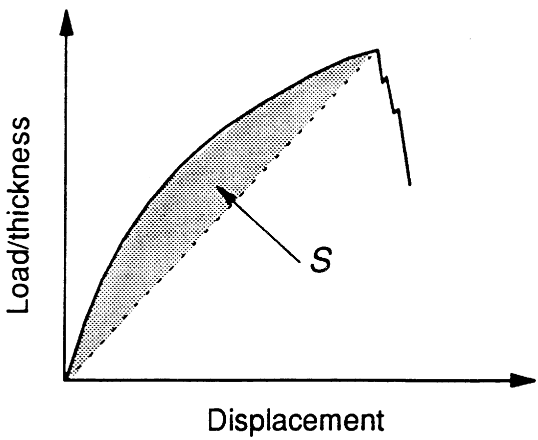


Fig. 4: Schematic of the single-specimen method.

Single-specimen method

The single-specimen method is also based on the energy interpretation. However, this method is much easier to implement and is used in many material testing standards. According to the original definition of the single-specimen method (15, 16), the J-integral is given by the following equation:

$$J = J_e + J_p = \frac{K^2}{E} + \frac{2S}{b} \tag{2}$$

where

J_e: elastic portion of J-integral (J/m^2),
J_p: plastic portion of J-integral (J/m^2),
K: stress intensity factor (N/m$^{1.5}$),
E: elasticity (N/m^2).
S: shaded area in Fig. 4 (J/m)
b: uncracked ligament length

Modified single-specimen method

As reported by many researchers, J-integral values estimated with the original single-specimen method do not always agree with the values obtained with the multiple-specimen method in paper and other ductile materials (17,18). Recently, Yuhara and Kortschot (4) proposed a simple and novel modification of the single-specimen method which yields values very similar to those obtained with the multiple-specimen method. More precise J-integral values are given by the following equation:

$$J = J_e + J_p = \frac{K^2}{E} + \frac{1}{b}\left[(1+m)S_1 + (1-m)S_2 - S_3\right] \tag{3}$$

where

S_1: area under the load-displacement curve up to the critical point (area OAB in Fig. 5, J/m)

S_2: area under the initial line up to the critical load point (area ODE in Fig. 5, J/m)

S_3: critical load times critical displacement (area OABC in Fig. 5, J/m)

m: non-dimensional parameter.

Thus far, the parameter m seems to depend only on the degree of anisotropy and/or the material properties. m can be determined using two specimens with neighbouring crack sizes.

Liebowitz non-linear technique

Liebowitz and co-workers (1,2) proposed a non-linear fracture toughness parameter, G_c, which is based on the same energy interpretation as the J-integral, and concluded that G_c is approximately equal to J_c. Recently, Westerlind and co-workers (19) introduced the Liebowitz non-linear technique as a less labour intensive and more accurate method of evaluating the toughness of paper. They showed that the values of fracture toughness obtained with this method were independent of crack length. The Liebowitz non-linear technique is based on a Ramberg-Osgood description of the non-linear load-displacement curve:

$$\delta = \frac{P}{M} + k\left(\frac{P}{M}\right)^n \tag{4}$$

where

δ: displacement (mm),

P: load (N),

M: initial stiffness of the specimen (N/m),

k, n: non-dimensional parameters.

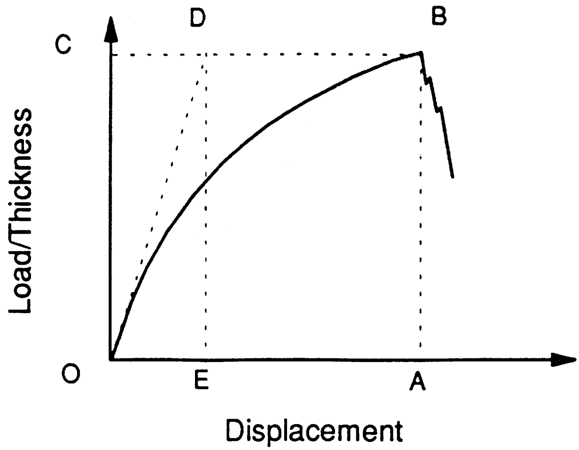


Fig. 5: Schematic of the modified single-specimen method.

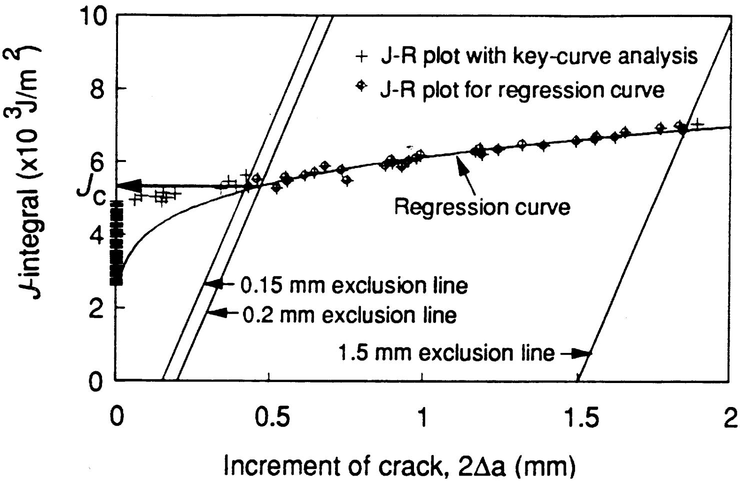


Fig. 6: J-R curve derived from the load displacement curve of bond paper (CD). Specimen width: 90 mm. Notch length: 2 x 20 mm.

The Leibowitz fracture toughness parameter is given by the following equation, and its value may be approximately equal to the J-integral (2):

$$\begin{aligned} \bar{G} &= (1+\beta)J_e \\ &\approx J \end{aligned} \tag{5}$$

where

$$\beta = \frac{2nk}{n+1}\left(\frac{P}{M}\right)^{n-1} \tag{6}$$

Essential work of fracture

Seth and co-workers recently introduced the measurement of the essential work of fracture to paper sheet (3), which was originally developed for characterizing the fracture behaviour of ductile materials (20). The total work of fracture per unit ligament area is written as:

$$w_f = w_e + \beta L w_p \tag{7}$$

where

w_f: total work of fracture (J/m^2),
w_e: essential work of fracture (J/m^2),
w_p: plastic work of fracture (J/m^2),
β: shape factor of plastic zone (m^{-1}),
L: ligament length (m).

In a graph of total work of fracture against ligament length, the intersection of the regression line with the y-axis gives the essential work of fracture.

Mai and Cottrell (21) demonstrated that ductile fracture in polymers can be characterized by w_e, which is obtained when L is extrapolated to zero. Moreover, Mai and Cottrell (21) suggested that the physical meaning of J_c and w_e was similar, and Paton and Hashemi (22) found this to be true for thin sheets of polycarbonate.

PROBLEMS WITH *J*-INTEGRAL ESTIMATION

Difficulty in the determination of crack initiation point

In order to use the *J*-integral value as a fracture criterion, the onset of crack growth must be identified experimentally. This determination is unusually difficult in paper. Except in a few types of very brittle paper, the onset of stable crack growth can not be identified either visually or on the load-displacement curve. Therefore, the maximum load point is commonly used as the critical point for J_c estimation. Although it may be possible to apply a high-speed video or acoustic emission technique to determine the point of crack initiation, a method based on the load-displacement curve alone is highly desirable in order to reduce the experimental burden.

Key Curve Method

Ernst and co-workers (23) developed a technique to evaluate the crack length increment Δa and the *J-R* curve (J_c-value as a function of crack growth), using an assumption of confined plasticity in the uncracked ligament region. The technique involves scaling the load-displacement curve to a common curve for each notch length, and the whole load-displacement curve including the region of stable crack growth is assumed to be expressed as a function of the common curve. Although this assumption is not strictly correct for paper, a rough approximation of crack extension can be made with this approach.

In the analysis of Ernst (23), the load-displacement curve (including the region of stable crack growth) is described with the function *F*:

$$\frac{Pw}{b^2} = F\left(\frac{\delta}{w}, \frac{a}{w}, \ldots\ldots\right) \tag{8}$$

where

P: load per thickness (N/m),
a: crack length (m),
b: uncracked ligament length (m),
w: specimen width (= $a + b$, m),
δ: displacement (m).

Eq. (8) implies that the normalized displacement at the applied normalized load for a certain crack growth, Δa, can be calculated using a scaled down specimen with an initial crack length proportional to $a+\Delta a$ and no crack growth.

Using the differential of Eq. (8) with δ/w and a/w as variables:

$$dP = \frac{\partial P}{\partial \delta} d\delta + \frac{\partial P}{\partial a} da \tag{9}$$

and estimating both coefficients with Eq. (8), the increment of crack length, da, can be described as:

$$da = \frac{\dfrac{b^2}{w^2} \dfrac{\partial F}{\partial(\delta/w)} d\delta - dP}{\dfrac{2b}{w} F - \dfrac{b^2}{w^2} \dfrac{\partial F}{\partial(a/w)}} \tag{10}$$

Yuhara and Kortschot (4) have elucidated the non-linear relationship between load, plastic displacement and ligament length, as:

$$\delta_p = bh\left(\frac{P}{b^m}\right) \tag{11}$$

The function F of a common (master) curve in Eq. (8) can be redefined using this relationship, as:

$$\frac{Pw^{m-1}}{b^m \sigma_y} = F(\frac{\delta_p}{b})$$

$$= h^{-1}(\frac{\delta_p}{b}) \frac{w^{m-1}}{\sigma_y} \quad (12)$$

where

h^{-1}: the inverse function of h,
σ_y: yield stress (N/m^2)

Using a power law approximation for the expression of h^{-1}, as:

$$h^{-1}(\frac{\delta_p}{b}) = w^{1-m} \sigma_y \alpha \left(\frac{\delta_p}{b} \right)^{\beta} \quad (13)$$

the differential of F in Eq. (10) can be obtained, so that the crack growth can be evaluated with Eq. (10).

It is, therefore, possible to obtain a *J-R* curve from a single load-displacement curve (Fig. 6), so that a *J*-integral value at the onset of crack growth can be obtained more accurately. This value of J_c is more physically meaningful than a value determined using the maximum load point.

Specimen width restrictions

In spite of the wide application of the *J*-integral to other materials, this method is not commonly used in the paper industry. One difficulty is the need for a relatively wide specimen and rigidly attached grips. Previous studies of paper have employed specimens, for instance, 50 mm (8) and 80 mm (16) wide, with wide grips and a rigid connection between a top grip and a load cell. Most industrial paper labs, however, are equipped only to deal with standard 15 or 25 mm specimens (TAPPI Standard T 220 or

T 494 om-88 (tensile testing standard)). In this study, a 25 mm wide specimen and a universal joint between a grip and a load cell have been used to evaluate the *J*-integral.

EXPERIMENTAL

Specimens were cut from the machine- and cross-machine directions of fine paper made from fully bleached hardwood kraft pulp, bond paper made from fully bleached softwood kraft pulp and newsprint made from TMP and kraft pulp.

Wide (90 mm wide) and narrow (25 mm wide according to TAPPI Standard T494 om-88) double-edge-notched (DEN) specimens were prepared with edge notches of length 10, 15, 20, 25, 30, 35 mm for the wide specimens and 2, 4, 6, 8 mm for the narrow specimens. For the measurement of the essential work of fracture, 90 mm wide DEN specimens with edge notches of length 30, 33, 36, 39 were used.

Anti-buckling guides made of acrylic were used to sandwich the specimen using spacers of approximately twice the specimen thickness. The guides were 110 mm wide and 190 mm long. Investigation with unnotched specimens showed that the effect of the guide on the load-displacement curve was negligible.

All tests were performed at 23 °C and 50% relative humidity. A Sintech 1 tensile tester equipped with 100 mm wide grips rigidly attached to the frame was used to test the wide specimens. For these tests, the initial distance between grips was 200 mm and the cross-head speed was 2 mm/min. For the measurement of the essential work of fracture, a cross-head speed of 2 mm/min was used. For testing the narrow specimens, a universal joint was used between the upper grip and the load cell, the initial distance between grips was 180 mm, and the cross-head speed was 25 mm/min, in accordance with TAPPI Standard T494 om-88.

In order to eliminate the influence of strain rate on the comparison between the narrow and wide notched specimens, the data in Fig. 8 were generated with a set of wide J_c specimens tested with a cross-head speed of 27.78 mm/min. This produces a strain rate identical to that found in the narrow (and slightly shorter) notched specimens tested with a cross-head speed of 25 mm/min.

The load-displacement curves were recorded digitally and analyzed using a personal computer. For each notch length, the load-displacement curves for eight specimens were recorded, and J-integral values were computed for each curve. Average load-displacement curves for each notch length were used for determining the J-integral with multiple-specimen method.

RESULTS AND DISCUSSION

Comparison of J_c values obtained with various methods

Fig. 7a shows J_c values (corrected for grammage and thickness variations) as a function of notch length for wide specimens of bond paper oriented in the cross-machine direction. The maximum load points were used as the critical point except where the method proposed by Yuhara and Kortschot was used in which case the critical point was estimated with the key-curve method. In this case, J_c was obtained by using a J-R curve, evaluated with the key curve method, according to ASTM E318-87, where the blunting line was obtained as $J = 2\Delta a\sigma_y$. As shown in Fig. 6, J_c is given by the intersection of the power law regression line (which is obtained with the data points between two offset lines (0.15 and 1.5 mm) each drawn parallel to the blunting line) with a line parallel to the blunting line drawn at an offset of 0.2 mm.

The authors' proposed method gives good agreement with the values obtained with the multiple-specimen method, which is directly derived from the energy interpretation of J-integral. The values obtained with the Leibowitz non-linear technique were relatively close to the values obtained with the authors' proposed method where key curve analysis was used to determine the crack

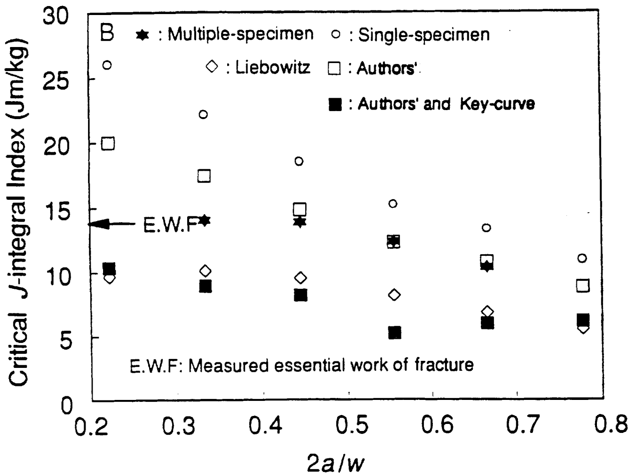
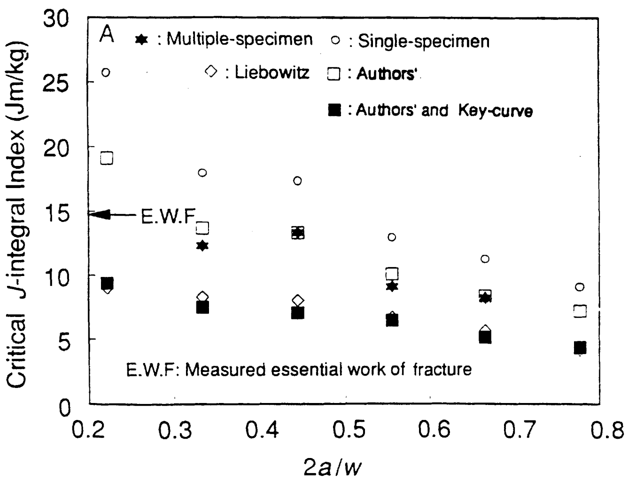


Fig. 7: The effect of notch length on J_c. Specimen size: 90 mm x 200 mm. a) Bond paper (CD). b) Fine paper (CD).



initiation point for the latter. Similar trends were observed with other samples of paper (Fig. 7b). In these experiments, J_c values evaluated with every method displayed some geometric dependency, as shown in both Fig. 7a and 7b.

The measured value of the essential work of fracture was the same or higher than the J_c values obtained with multiple-specimen method. The maximum load point was used as the critical point to determine J_c for the multiple-specimen method, and thus the J_c values obtained in this way may be slightly elevated since the point of crack initiation precedes the maximum point. In this case, the essential work of fracture values are substantially higher than the true value of J_c. This might possibly be attributed to the difference in crack velocity in the two tests. Similar results were obtained for bond and fine paper in both machine and cross-machine directions. For newsprint, in both machine and cross-machine direction, unstable rapid crack growth occurred during the essential work of fracture test, and the values obtained are therefore overestimates of the true work of fracture since they include some stored elastic energy. For the samples of fine and bond paper, the stable crack growth condition was also violated in some cases. Another frequently encountered problem was misalignment of the propagating cracks, which leads to a long tail in the load displacement curve.

J_c values from narrow specimens

Fig. 8 shows a comparison of the J_c values obtained with narrow specimens (width: 25 mm, single notch length: 6 mm) and the values obtained with wide specimens (width: 90 mm, single notch length: 20 mm). The J-value was estimated with the authors' proposed method and the maximum load point was used as the critical point. The values obtained using the narrow specimens were approximately 50% of those obtained with the wide specimens. This disagreement means that the value of J_c itself (measured with the maximum load point as the critical point) may depend on the specimen width. However, the correlation between the values was quite good, so that it may be possible to use the

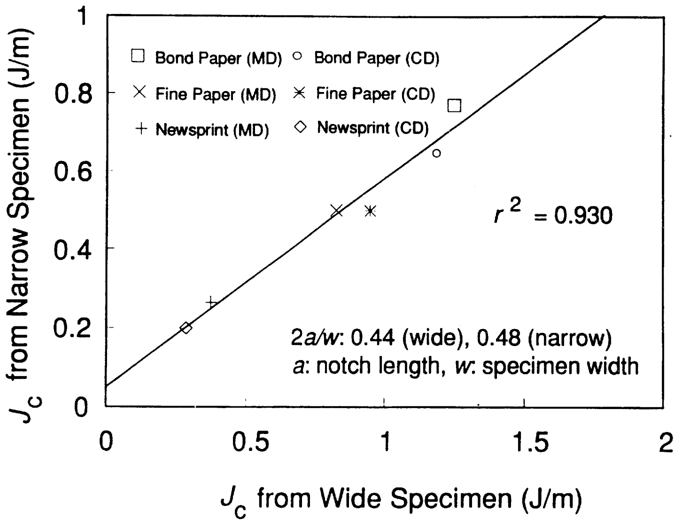


Fig. 8: Comparison of J_c for narrow and wide specimens.

standard tensile testing configuration to estimate a relative value of the critical *J*-integral.

In order to obtain a material property which is independent of specimen geometry, the ASTM Standard Test Method for *J*IC (E813-91) (24) recommends the following conditions:

$$a', B, w - a' \geq 25 \frac{J}{\sigma_y} \tag{14}$$

where

a': total crack length (= 2*a*)

For specimens with relatively large *J*-values such as MD bond paper (see Fig. 8), the right side of the Eq. (14) is 5.83 mm. According to Eq. (14), the thickness of samples was much smaller than required by the standard. For the wide specimens, the same results were also obtained. Clearly the plane-strain condition, which is required for J_c tests of other materials, can not be obtained for thin paper sheets. In any case, paper sheets with different thicknesses (or basis weights) may have quite different structures, and would not be expected to have a constant value of J_c even if plane strain conditions were satisfied for all thicknesses. The dependence of J_c on specimen thickness is therefore not a problem for paper, because comparisons should be limited to materials of uniform thickness.

FUTURE WORK

The difficulty in determining the onset of crack growth continues to be the main limitation of fracture toughness testing for paper. Although direct observation of the crack front with a video system might prove to be a useful tool in a laboratory setting, a much less labour intensive method is required before the technique can be adopted by industry.

Although the J-integral is widely recognized as a criterion of fracture toughness for elastic-plastic materials, it can not be interpreted in terms of the energy available for crack growth. Moreover, it is well known that the J_c is not independent of specimen width and geometry, as demonstrated in this study. Nevertheless, the J-integral is still the most reasonable parameter for characterizing the onset of unstable crack growth in materials such as paper. Further development of the J-integral and the development of alternative methodologies is clearly warranted.

CONCLUSIONS

1) The authors' proposed method for determining the J-integral gives values very close to those obtained with the multiple-specimen method, which is itself directly derived from the energy interpretation of J-integral.

2) The J_c values obtained with the Leibowitz non-linear technique using the maximum load point for the critical point were relatively close to those obtained with authors' proposed method where key-curve analysis for determining the critical point. This result suggests that the Liebowitz non-linear technique may give a relatively accurate J-integral value at the crack initiation point with less experimental effort.

3) It was not possible to measure the essential work of fracture for the papers tested here (fine, bond and newsprint) because of the difficulty in ensuring stable crack growth and connection of the propagating cracks.

4) It may be possible to use narrow notched specimens in the standard tensile testing configuration for J-integral estimation. This method would be very suitable for industry.

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APPENDIX

Simplified calculation for the key curve method

Applying the key curve concept directly to the non-linear stress strain behaviour of paper described by Yuhara and Kortschot (4), the calculation may be simplified. The non-dimensional function F', which is similar to F in Eq. (8), is assumed to be:

$$F'\left(\frac{\delta_p}{b'},\frac{a}{w}\right) = h^{-1}\left(\frac{\delta_p}{b'},\frac{a}{w}\right)\frac{w^{m-1}}{\sigma_y} \tag{15}$$

where

b': initial ligament length (m),

The load P_n on the measured load-displacement curve (OAB, Fig. 9) obtained for a specimen having an initial ligament length b_0, can be described as:

$$P_n = b_0^{\,m} w^{1-m} \sigma_y F' \tag{16}$$

On the other hand, the load P_n on the measured load-displacement curve obtained for a specimen having a stationary crack of length b_n, can also be described using Eq. (11), as:

$$P_n = b_n^{\,m} h^{-1}\left(\frac{\delta_{p,n}}{b_n}\right) \tag{17}$$

From Eq.s (16) and (17):

$$b_0^{\,m} w^{1-m} \sigma_y F = b_n^{\,m} h^{-1}\left(\frac{\delta_{p,n}}{b_n}\right) \tag{18}$$

where the inverse function h, h^{-1}, is defined by Eq. (13). Therefore, the crack growth, a, can be expressed as:

$$\begin{aligned} \Delta a &= b_n - b_0 \\ &= \left(\frac{F' b_0^{\,m}}{\alpha \delta_{p,n}^{\,\beta}}\right)^{\frac{1}{m-\beta}} - b_0 \end{aligned} \tag{19}$$

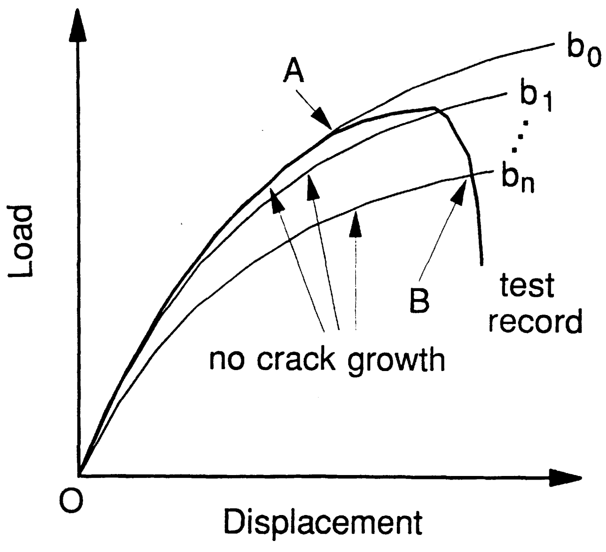

Fig. 9: Schematic load-displacement curve with actual test record and lines denoting the behavior of specimens without crack growth.

Transcription of Discussion

PLANE STRESS FRACTURE TOUGHNESS AND ITS MEASUREMENT FOR PAPER

Prepared contribution

R Seth, PAPRICAN:

THE J-INTEGRAL AS A FRACTURE TOUGHNESS PARAMETER OF PAPER

T Yuhara

FRACTURE TOUGHNESS AS A PULP CHARACTERISATION METHOD FOR RUNNABILITY ASSESSMENT OF PAPER

A Astrom (Paper presented by A Nordstrom):

(EDITOR'S NOTE: WHAT FOLLOWS IS THE PROCEEDINGS OF A DISCUSSION OF ALL THREE PAPERS)

Dr F El Hosseiny, Weyerhaeuser Paper Co

This question is addressed to whoever can answer it. For the past two decades fracture mechanics of paper was introduced hoping to be able to characterise the behaviour of paper in the converting process and especially press room newsprint runnability. Seth and Page have shown that newsprint runnability is governed only by the rare event phenomena irrespective of its fracture toughness. If fracture toughness or fracture mechanics failed to do what it was supposed to be doing, that is to predict the flow carrying ability of a sheet, what hope do we have for extending this study. We could do it for another 20 years and come to the same conclusion. Why are we continuing doing work in the fracture mechanics of paper?

R Seth

Yes, I was involved in that study. You are quite right. The reason for my getting back into fracture was for the softwood kraft market pulps and tear. Tear is a millstone around the necks of market kraft pulp producers. Tear goes down when you refine the pulp implying you shouldn't refine. Fracture toughness tells us now that you can refine without fear of losing tear because what matters in the end use processes is the stresses in the plane of the sheet, therefore don't be afraid to refine, and that was the purpose. If you wish I can draw curves of fracture toughness against refining and tear against refining; fracture toughness goes up and tear factor goes down. This convinces papermakers that there is a merit in refining and thus using the full potential of the fibre. We now have tear resistance also in plane stress, like tensile strength and elastic modulus, and they all increase as we refine, unlike the out-of-plane tear. That was the only reason I got into this and I think the industry does appreciate that.

Dr D Page, PAPRICAN

I guess my comment is similar to F El-Hosseiny's. There has been a lot of work on fracture mechanics and debate and how to measure fracture mechanics. But what surprises me is that fracture toughness has become a religion. When you actually look for the data to find out if it is important to runnability you will find one plot that R Seth and I published many years ago when we collected 1½ years runnability data. We measured fracture toughness of newsprints and obtained runnability data for a large press room and got a correlation which was significant but very poor between fracture toughness and runnability. It seems to me that the difficult step is to prove that in general it is worthwhile using fracture toughness as a means of evaluating pulps or sheets. That's the difficult step. The easiest step is what you three people are doing now, namely trying to devise a method for measuring it. I think the important step though has to be to show a relationship between end use and a test. I can visualise for example that you can take sheets or reels of paper which have 1% stretch to break and rereel them using 0.5% stretch they won't break, but use 1.5% stretch and they will. This will have nothing whatsoever to do with fracture toughness. What is the proper criterion for failure? I believe we don't know and I believe that's where the next step has to take place, not in a laboratory but in the workplace where the real data exist. In the absence of evidence we are at the mercy of evangelists and their beliefs.

Yuhara *(in response to El Hosseiny question)*
If we want to talk about runnability we have to cover all three factors discussed by Niskanen in the first presentation and also mentioned in my presentation. The three factors are the number of flaws present, uneven tension and finally the paper toughness. For example, Japanese newsprint does not always have high fracture toughness so the industry focuses on decreasing the number of flaws and controlling the calliper profile, especially in the cross machine direction. Not only calliper but also moisture content and of course the basis weight profile. These things influence the tension on the printing press, and the fracture toughness may not be the dominant factor. We must consider other things for good runnability.

R Seth
I agree with Dr Yuhara. We do measure tensile strength and other stress-strain properties, and write review papers on these properties. Why are we doing this; do these properties matter at

all. If we go back to basic materials science, it tells us that there are certain mechanical properties, and if they are OK, we have a hope that the material will survive and fracture toughness is one of them.

Dr K Ebeling, Kymmene Corp, Finland

Thank you for your answer to the first question Dr Seth. One comment I would like to make - it is not the papermakers, it's the pulp merchants that promote the importance of tear strength. I think they are so short minded that tear strength is the only thing they understand.

R Seth

It is unfortunate. We have to educate them.

Dr F-J Chen, Kimberly Clark Corp, USA

One of the key reasons for breaks is web non-uniformity which may be particularly sensitive to your notch method. What happens to the coefficient of variance in your measurement? Is the average value more important or is the coefficient of variance more important? Would the two numbers reported together give us better indication as to whether breaks may occur?

R Seth

I will pass this question because I am not measuring fracture toughness at present to relate to breaks. I am measuring only to evaluate pulps. So those who are doing it to predict breaks should answer this question.

Yuhara

Maybe we have to use some sort of safety assessment method for achieving the fracture toughness for high runnabilitv.

R Seth: *(Question for Yuhara)*
Let's go back to your figure 7 in the text on page 799. In each figure there are about 25-30 points. Which one of them can I call the fracture toughness value?

Yuhara: Actually as I presented in my conclusions, there is no way, so far, to evaluate the energy for crack extension. From that point of view none of those values have a physical meaning which relates to the energy available to drive the crack. Your method, the essential work of fracture cannot give the energy available for crack growth in a machine made paper.

R Seth
Because of the experimental difficulties? I'll come to that point later. So am I correct in assuming that the J methods that you have used, (and you are using 5 of them), none of them gave you an unambiguous result. Am I right in saying this?

Yuhara
Yes, absolutely right.

R Seth
As far as difficulties with my method are concerned, first of all my method may not work for brittle papers such as newsprint because of unstable crack propagation. It was not meant for newsprint, but for ductile sheets, and I have been using it for tough handsheets, sack kraft etc; it worked very well for copying paper. Further, if you have difficulties in having the two edge cracks not connect on propagation, there is nothing in that method that stops you from using a single edge notched sample. There will be experimental difficulties of keeping the clamps from turning. There are guided

clamps which we use and I am happy to give the drawings of them to anyone who wants them. These difficulties can be met, but you have to recognise that the essential work of fracture method is for ductile materials and that is what my February 1993 paper in Tappi said.

Dr J R Parker, Messmer Instruments, UK

I seem to remember from very old data that one of the few properties of paper that had any relationship to runnability was moisture content. I wonder if this gives any pointer to the sort of toughness measurement that might be appropriate.

R Seth

A small increase in moisture content can lead to a higher fracture toughness. It's a piece of work which we did 15-16 years ago, and that was the indication. We measured fracture toughness of newsprint between 40-60% humidity at that time and 60% humidity results were higher.

J Waterhouse, IPST, USA

We need to distinguish between rare events and fracture mechanics but I think it is clear that there are a number of different areas where fracture mechanics can be applied, and obviously rare events will sometimes occur. I always remember Christer Fellers showing the beer bottles on the floor because of failed packaging in one of his presentations on fracture mechanics. There are examples of scoring, durability, perforations, and more recently die cutting, so I think as we can analyse the end use application of paper more correctly, we can see quite a number of applications of fracture mechanics. Obviously fracture toughness is key to this and ultimately what we want to know is: how the processes at the micro mechanics level relate to fracture toughness, ie what do we have to do to improve the fracture toughness of paper? Also, does fracture mechanics give us any clues as to the ultimate strength properties of paper again going back to K Niskanen's excellent review paper this morning.

Prof M Kortschot, University of Toronto, Canada

Let me defend the use of fracture toughness. I think it is still true that in spite of the rarity of breaks in a printing press and in spite of the influence of moisture and other factors, some paper does run better than others. When papermakers have problems with runnability they have to respond by adjusting the furnish. We therefore need some method of addressing the relationship between furnish parameters and the eventual runnability. Intuitively it seems that fracture toughness is the most likely means of characterising this relationship, but I agree with Dr Page that we have to explore this in much more detail.

R Seth

There was an earlier question from Dr Chen that I tried to evade and that was regarding whether I was measuring an average value and what was the spread. If we look at the test, we are measuring an average in the same way as for example tear strength; an average value along a certain path. So, what I am doing is giving an average for the material that I am testing. If you want to see what is the spread or error in that value I would suggest that you look up the Tappi paper (February 1993) which explains the method and you will see that the measurements are fairly good, and the error is comparable to the error in tensile strength etc.

Prof H Kropholler, UMIST, UK

We seem to have a very powerful technique, lots of mathematics and different ways of doing it and there was one very interesting problem which I don't think has been mentioned. There is folklore in the paper industry that some grades of strong papers are best made with a wild formation, corrugating medium is one of these and Prof Göttsching showed some 15 years ago that this was folklore and not true. Another interesting one is sack kraft where it is suggested that the fracture strength is better if you have a wild formation. I don't really believe this but surely you could prove this with fracture mechanics.

R Seth

I will think about it.

Dr K Ebeling, Kymmene Corp, Finland

I think we have to keep the testing methodology on two levels in our minds. The process engineer would like to have a simple method to follow for example if the raw material he is using has a constant quality. As scientists we should have methods that allow us to understand what really is going on and allow us to predict how a better product should be made. The fracture toughness is very important for wood containing printing paper manufacturer because when you have a machine producing 250,000 tonnes a year and if you can save 1% - unit a year in your expensive chemical pulp by using pulp that provides a better fracture toughness to your web, you have earned your salary many times.

R Seth

I agree with you.