Preferred citation: D. Radoslavova, J.C. Roux and J. Silvy. Hydrodynamic modelling of the behaviour of the pulp suspensions during beating and its application to optimising the refining process. In **The Fundametals of Papermaking Materials**, *Trans. of the XIth Fund. Res. Symp. Cambridge, 1997*, (C.F. Baker, ed.), pp 607–639, FRC, Manchester, 2018. DOI: 10.15376/frc.1997.1.607.

HYDRODYNAMIC MODELLING OF THE BEHAVIOUR OF THE PULP SUSPENSIONS DURING BEATING AND ITS APPLICATION TO OPTIMISING THE REFINING PROCESS

Authors: D Radoslavova, J C Roux, J Silvy*

UMR Génie des Procédés Papeties 5518 CNRS

EFPG-INPG

Domaine Universitaire, BP 65, 461 rue de la Papeterie

38402 Saint-Martin d'Héres, Cedex

FRANCE

* Present address: Universidade da Beira Interior

Rua Marques d'Avila e Bolama

6200 Covilhà

PORTUGAL

INTRODUCTION

The theoretical approach proposed in this research paper aims to relate the energy efficiency coefficient of the refiner to the rheological pulp properties. The theory is then used to predict the observed energy consumption as a function of the desired paper properties. Various aspects of the beating process have been previously discussed in review articles [1],[2] and this discussion will not be repeated in this paper.
Even in the sixties, the major characteristics of the beating as a dynamic process, were pointed out very clearly. Halme [3], in an interesting survey , mentioned that the pressure in the beating zone must be combined with the relative motion of the surfaces to produce acceptable beating effects on the fibres. It was also clear that the beating effect of the turbulence alone was negligible. As a matter of conclusion, for future works on the beating process, Halme [3] recommended a better characterisation of the behaviour of stock in the refining zone. 25 years latter, Page [2] pointed out the necessary knowledge of the stresses that fibres can experience in the beating zone and their answer to these stresses, this last remark implies 'the development of a mechanistic theory of the beating'.
Nowadays, we have not yet directly developed this approach but an other general one. The hydrodynamic aspect of the beating process is considered with a pulp apparent viscosity of the suspension in the beating zone.
Various studies of flow in the refiners have been reported in the literature. Frazier [4] tried to use the hydrodynamic theory to analyse the process in high-consistency refining. The fact that the pulp apparent viscosity is unknown, was a major drawback and led to a limited success. Rance [5] and Steenberg [6] said earlier that we can develop the modelling of the beating through the lubrication theory but the beating process, in itself, modifies the pulp lubricant film - that is not the case in tribology.
The previous authors have drawn the difficulties of such a research but have not solved the problem. How to do that is the purpose of the following paragraphs.
We shall prove in this paper that it can be a fruitful approach. But, several conditions must be required before:

1. The flow of the pulp suspension in the beating zone is laminar,
2. The value of the pulp apparent viscosity is known, at any time of the beating process.

APPLICATION OF LUBRICATION THEORY TO THE REFINING ANALYSIS

By considering the refining machine as a bearing in which the pulp acts as a lubricant, it is possible to establish several relationships between the key refining parameters. The development starts with the equation of motion: the momentum transfer balance, written for the refining zone and for an incompressible fluid flow [7]:

$$\rho \frac{\partial}{\partial t} \vec{v} = -(\vec{v}.\vec{\nabla})\rho\vec{v} - \vec{\nabla}p - \vec{\nabla}.\overline{\overline{\tau}} + \rho\vec{g} \tag{1}$$

This equation contains different forces expressed per unit volume:

- the inertial forces are expressed from the derivatives of the velocity of the flow and regroup an accumulation term and a convective term: $\rho \frac{\partial}{\partial t} \vec{v} + (\vec{v}.\vec{\nabla})\rho\vec{v}$
- the pressure forces can be deduced from the gradient $\vec{\nabla}p$
- the viscous forces are included in the divergence of the stress tensor $\overline{\overline{\tau}}$ (spatial derivatives of the shear stresses according to the chosen co-ordinates)
- the gravity forces.

The shear stress tensor implies the knowledge of the rheological pulp behaviour in the refining zone. But, even for a non-Newtonian behaviour, it is always possible to consider a pulp apparent viscosity [7]. This parameter seems difficult to obtain owing to the complexity of the flow and to the pronounced modifications during the beating process. We obviously know that it affects the fibre length distribution, the hydration and the fibrillation and brings to change in the fibre flexibility. The rheological equations for particulate suspensions, already published in the literature [8] indicate that changes in the fibre characteristics (length, diameter) can lead to modifying the pulp apparent viscosity.
In other hand, the presence of inertial terms introduces a set of supplementary parameters: the derivatives of the speed of the flow.

Time sampling of the problem

The pulp properties change continuously during refining but if we consider a small period Δt, a mean value of the suspension apparent viscosity can be assumed. Then, if we sample the refining process, during each elementary period, a continuous stationary flow is supposed to establish in the refining confined zone. The initial equation can be written under the following form:

$$-(\vec{v}.\vec{\nabla})\rho\vec{v} - \vec{\nabla}p - \vec{\nabla}.\overline{\overline{\tau}} + \rho\vec{g} = \vec{0} \tag{2}$$

Rheological pulp behaviour during an elementary period

The apparent viscosity is, by definition, the non-Newtonian effective viscosity [7]. In an elementary period of time $(t_i, t_i + \Delta t)$, the pulp apparent viscosity is coincident with the dynamic viscosity of the equivalent Newtonian fluid μ_{a_i} but μ_a is a function of time during the whole refining process. This point is illustrated on the figure 1 in order to reconstruct the pulp apparent viscosity function from the different discrete numerical values μ_{a_i}.

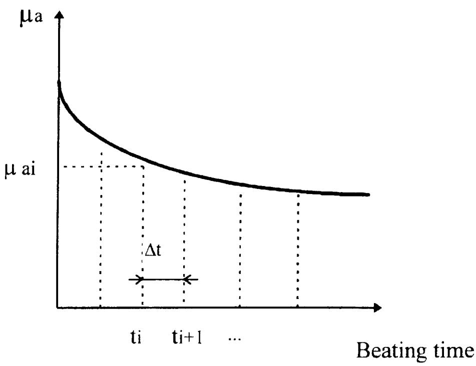


Figure 1. Reconstruction of the pulp apparent viscosity function vs. time during the refining process.

This last statement is very important in the proposed analysis. It simply means that the equation (2) is a set of Navier-Stockes equations for each period of time during the whole refining process.

$$-(\vec{v}.\vec{\nabla})\rho\vec{v} - \vec{\nabla}p - \mu_a \nabla^2 \vec{v} + \rho\vec{g} = \vec{0}$$
$$\mu_a = \mu_{a_i} \quad \text{for } (t_i, t_i + \Delta t) \tag{3}$$

Estimation of the Reynolds number

In order to determine the nature of the flow in the refining confined zone, the Reynolds number must be calculated. In other hand, this number characterises the momentum flux transferred by convection versus those transferred by viscous diffusion, taking into account the viscosity μ_{a_i}.

The Reynolds number is defined by the following equation:

$$\mathrm{Re} = \frac{\rho \Phi \overline{U}}{\mu_a} \tag{4}$$

where

ρ is the pulp density	$kg.m^{-3}$
$\overline{U}$ is the mean velocity of the flow	$m.s^{-1}$
μ_a is the pulp apparent viscosity	$Pa.s$
Φ is the hydraulic equivalent diameter of the gap clearance	m

For example, if we consider a refiner gap equal to 300 μm, then $\Phi = 6.10^{-4} m$, ρ can be chosen as the density of the dry cellulose $\rho = 1{,}54.10^3 kg.m^{-3}$. A mean value of the velocity is $\overline{U} = 12 m.s^{-1}$. Then, according to Batchelor equation [9], an apparent viscosity of 20 *mPa.s* is retained for the calculation.

The Reynolds number can be calculated as:

$$\mathrm{Re} = \frac{1{,}54.10^3.12.6.10^{-4}}{20.10^{-3}} = 554 < 1000$$

(5)

Even if the numerical values above are slightly different, we must conclude that the nature of the flow is laminar in the refining zone. The inertial term is equal to 0 owing to the orthogonality of the flow ($(\vec{v}.\vec{\nabla})\vec{v} = 0$). Thus, the equation (3) can be written in the following form if we neglect the gravity forces :

$$-\vec{\nabla}p + \mu_a \nabla^2 \vec{v} + \rho \vec{g} = \vec{0} \tag{6}$$

Considering the above assumptions, the gravity forces will be neglected then the modelling of the refining process will be based upon this last equation:

$$-\vec{\nabla}p + \mu_a \nabla^2 \vec{v} = \vec{0} \tag{7}$$

It is a single form of the Navier-Stockes equation but the resolution requires first the selection of the most appropriate co-ordinate system [7]. In other terms, the resolution depends on the refining geometry. This knowledge then allows supplementary simplifications of the model to be performed. After the model resolution, the laminar nature of the flow will be validated a-posteriori.

THE MODEL RESOLUTION

The following developments are performed for a laboratory Voith Hollander beater chosen for its simple geometry, described on figure 2. In order to obtain the input parameters of the model , the refiner is equipped with sensors. These allow measurements of the vertical displacement of the cylinder, the loading force, the shaft torque, the rotation speed and the temperature of the pulp suspension. An automatic control system is developed to run in different beating modes:

- maintaining a constant vertical displacement of the cylinder (i.e. constant set of gap clearance)
- maintaining a constant loading force (normal beating mode for this refiner)
- maintaining a constant net power
- maintaining a constant torque (if the rotation speed is also constant, the running mode keeps the total power constant).

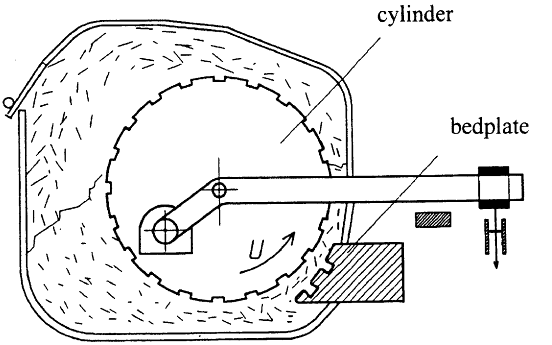


Figure 2. Scheme of the Voith Hollander beater.

Owing to the refining geometry, the cylindrical co-ordinate system is chosen according to the figure 3. First of all, the two bars of the cylinder in the converging zone are considered (see figure 6) but we assume that the bedplate is smooth with no bars inside. This last assumption will be validated afterwards. The flow according to the z-direction is not taken into account in this analysis. The problem to be solved remains bidimensionnal $(0, \vec{i}_r, \vec{i}_\theta)$.

In the lubrication theory, the thickness of the wedge zone is very small compared to the other characteristic dimensions (radius of the cylinder, length of the bedplate,...). The complementary following assumptions can be reasonably considered:

no radial flow $v_r = 0$

no radial pressure gradient $\dfrac{\partial p}{\partial r} = 0$

The vectorial equation (7) finally leads to only one scalar equation :

$$\frac{1}{r}\frac{dp(\theta)}{d\theta} = \mu_a \left(\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r v_\theta) \right) \right) \tag{8}$$

r radial co-ordinate in the gap clearance m

v angular co-ordinate of the speed of the flow $m.s^{-1}$

It is easier to manipulate dimensionless formulations so several reference parameters must be chosen accordingly.

p	- the reference pressure	Pa
U	- the tangential speed of the cylinder	$m.s^{-1}$
R	- the radius of curvature of the bed plate	m
$R\Delta\Theta$	- nearly the width of a bar in the cylinder	m

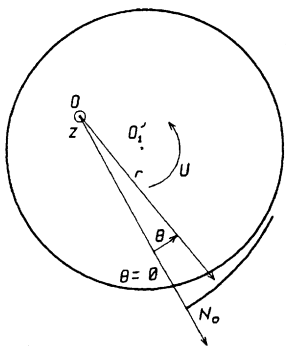


Figure 3. Cylindrical co-ordinate system retained.

Considering the dimensionless quantities defined below :

$$X = \frac{r}{R} \quad \Theta = \frac{\theta}{\Delta\theta} \quad P = \frac{p}{p_0} \quad V_\Theta = \frac{v_\theta}{U} \tag{9}$$

the dimensionless form of the equation (8) is obtained :

$$\left(\frac{p_0 R^2}{\mu_a U R \Delta\theta}\right)\frac{1}{X}\frac{dP}{d\Theta} = \frac{\partial}{\partial X}\left(\frac{1}{X}\frac{\partial}{\partial X}(X V_\Theta)\right) \tag{10}$$

Thus, this equation can be integrated analytically taking into account the following boundary conditions :

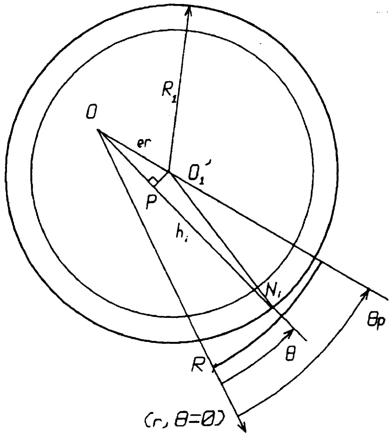
$$\text{for: } X = 1 \qquad V_\Theta = 0$$
$$\text{for: } X = \alpha_i = \frac{h_i}{R} \qquad V_\Theta = 1 \tag{11}$$

The dimensionless radial co-ordinate α_i refers to the points N_i located on the surface of the cylinder as seen on the figure 4. This yields to the analytical expression of the dimensionless angular velocity of the flow :

$$V_\Theta = \frac{1}{2}\left(\frac{p_0 R^2}{\mu_a U R \Delta\Theta}\right)\frac{dP}{d\Theta}\left(X \ln X - \left(\frac{X^2 - 1}{X}\right)\left(\frac{\alpha_i}{\alpha_i^2 - 1}\right)\alpha_i \ln \alpha_i\right) + \left(\frac{\alpha_i}{\alpha_i^2 - 1}\right)\left(\frac{X^2 - 1}{X}\right) \tag{12}$$

But this expression cannot be used for determining of the numerical values of the dimensionless angular velocity of the flow in the refining zone since $\frac{dP}{d\Theta}$ is unknown.

The calculation of the pressure distribution $P(\Theta)$ implies to use, as second equation in the model, the continuity equation written in a simple form:



$$OP = er.\cos(\theta_p - \theta)$$
$$O_1'P = er.\sin(\theta_p - \theta)$$

$$PN_i = \sqrt{R_1^2 - O_1'P^2}$$

$$h_i = OP + PN_i$$
$$h_i = er\cos(\theta_p - \theta) + \sqrt{R_1^2 - er^2.\sin^2(\theta_p - \theta)}$$

Figure 4. Details of the boundary conditions.

$$Q_\Theta = const \tag{13}$$

where Q_Θ is the dimensionless flow in the refining zone, defined as follows :

$$Q_\Theta = \frac{q_\theta}{RUL} = \int_{\alpha_i}^{1} V_\Theta dX \tag{14}$$

where q_θ is the volumetric flow and L is the length of a bar in the z-direction.
Giving the analytical form of the dimensionless angular velocity , the dimensionless flow in the refining zone is obtained :

$$Q_{\Theta} = \frac{1}{2}\left(\frac{p_0 R^2}{\mu_a U R \Delta\Theta}\right)\frac{dP}{d\Theta}\left(\frac{1}{4}\left(\alpha_i^2 - 1\right) - \left(\frac{\alpha_i^2}{\alpha_i^2 - 1}\right)\ln^2\alpha_i\right) + \left(\frac{\alpha_i \ln\alpha_i}{\alpha_i^2 - 1} - \frac{1}{2}\alpha_i\right) \tag{15}$$

In order to determine the dimensionless pressure profile $P(\Theta)$, the derivation over the variable Θ is undertaken . Finally, the following expression is :

$$\xi_1(\Theta)\frac{d^2P}{d\Theta^2} + \xi_2(\Theta)\frac{dP}{d\Theta} + \xi_3(\Theta) = 0 \tag{16}$$

where ξ_1, ξ_2 and ξ_3 are functions of the dimensionless angular co-ordinate Θ, defined below:

$$\xi_1 = \frac{1}{2}\left(\frac{p_0 R^2}{\mu_a U R \Delta\Theta}\right)\frac{dP}{d\Theta}\left(\frac{1}{4}\left(\alpha_i^2 - 1\right) - \left(\frac{\alpha_i^2}{\alpha_i^2 - 1}\right)\ln^2\alpha_i\right) \tag{17}$$

$$\xi_2 = \frac{d\xi_1}{d\Theta} = \frac{d\xi_1}{d\alpha_i}\frac{d\alpha_i}{d\Theta} \tag{18}$$

$$\xi_3 = \frac{d}{d\alpha_i}\left(\left(\frac{\alpha_i}{\alpha_i^2 - 1}\right)\ln\alpha_i - \frac{1}{2}\alpha_i\right)\frac{d\alpha_i}{d\Theta} \tag{19}$$

where $\alpha_i(\Theta)$ is described on the previous figure 4 :

$$\alpha_i(\Theta) = \frac{er}{R}\cos\left(\theta_p - \Theta\Delta\theta\right) + \frac{1}{R}\sqrt{R_1^2 - er^2 \sin^2\left(\theta_p - \Theta\Delta\theta\right)} \tag{20}$$

and

$$\frac{d\alpha_i}{d\Theta}=\frac{er}{R}\Delta\theta\sin\left(\theta_p-\Theta\Delta\theta\right)\left(1+\frac{er\cos\left(\theta_p-\Theta\Delta\theta\right)}{\sqrt{R_1^2-er^2\sin^2\left(\theta_p-\Theta\Delta\theta\right)}}\right) \tag{21}$$

The second order differential equation (16) can be numerically integrated using Choleski's method . After this calculation, the pressure distribution can be derived . As an example, the figure 5 gives the shape of the curve $P(\Theta)$.

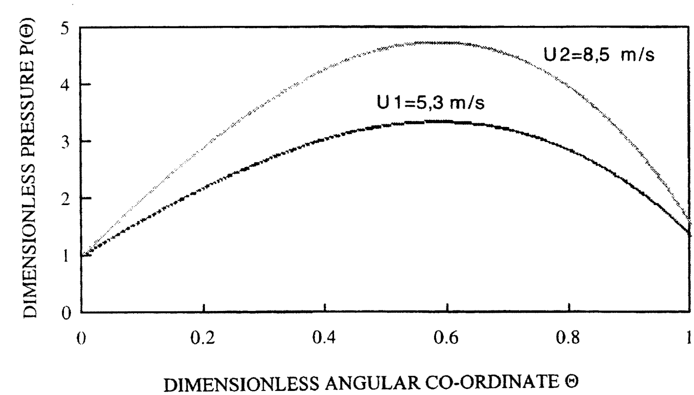


Figure 5. An example of a dimensionless pressure profile in the refining zone for two different tangential speeds of the cylinder

Determination of the pulp apparent viscosity.

This iterative and numerical procedure consists in assuming an initial value of the pulp apparent viscosity . Using this value in the model, the normal and tangential forces exerted in the gap clearance are determined taking into account the two converging zones concerned in this analysis.

Normal forces.

The normal forces or loading forces are calculated owing to the pressure distribution $P(\Theta)$ previously determined. For one bar, the bearing load exerted by the pulp on the cylinder is as follows :

$$r_p = p_0 LR\Delta\theta \,.\int_0^1 P(\Theta)\alpha_i d\Theta \tag{22}$$

where p_0 is the reference pressure, L is the length of a bar and $R\Delta\theta$ the curvilinear length of a converging zone (approximately the width of a bar).

The dimensionless bearing load appears clearly from the previous equation :

$$R_p = \frac{r_p}{p_0 LR\Delta\theta} \tag{23}$$

In the elementary period of time Δt, both wedges contribute to the resultant bearing load, as described on the figure 6 .

Tangential forces.

On other hand, the shear force, exerted by the pulp suspension in the confined zone, is defined in the general case [7] by the following expression :

$$\tau_{r\theta} = -\mu_a r \frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) \tag{24}$$

Taking into account the dimensionless quantities (see equations (9)), it follows:

$$T_{X,\Theta} = \frac{\tau_{r\theta}}{\left(\mu_a \dfrac{U}{R}\right)} = -X\frac{\partial}{\partial X}\left(\frac{V_\Theta}{X}\right) \tag{25}$$

In order to determine the resulting friction force exerted on a moving bar, the appropriate co-ordinate must be chosen ($X = \alpha_i$). For one bar, the modulus of the previous friction force can be calculated as follows :

$$f_t = \mu_a U L \Delta\theta \, . \int_0^1 T_{\alpha_i \Theta} \alpha_i \, d\Theta \tag{26}$$

From this equation, a dimensionless form of the friction force is defined :

$$F_t = \frac{f_t}{\mu_a U L \Delta\theta} \tag{27}$$

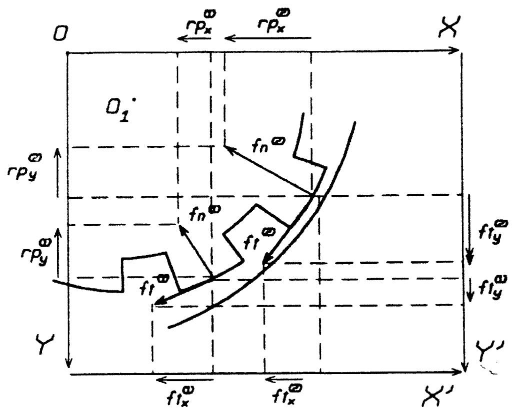


Figure 6. Determination of the resultant forces (normal and tangential) exerted on the cylinder

As described on the figure 6, both wedges contribute to the whole friction force. In an elementary period of time Δt, during the rotation of the cylinder, an infinite set of positions is occupied by the 2 moving bars. Numerically, the consideration of only 20 positions is sufficient to obtain stable numerical values of physical quantities. Then, if we take the mean of these 20 calculated normal and tangential forces respectively, it allows macroscopic values to be determined. Sensors can detect these values at our scale.

Comparison between the measured and calculated forces.

The measured loading force is the vertical component of the resultant and must be compared to the corresponding mean calculated force.

The measured friction force results from the total power consumed; for example, if the speed of rotation is kept constant :

$$f_t^m = \frac{pw_{total}}{U} \tag{28}$$

The idling loss must be taken into account. If we follow in the same condition, then :

$$\left(f_t^m\right)^i = \frac{pw_{total} - pw_{idling}}{U} \tag{29}$$

When the measured and calculated forces are numerically different, a new value of μ_a is forced and the iterative procedure restarts from the equation referenced (8) until a new comparison. If both the calculated and measured forces are identical (less than 5% of relative error), the value obtained for the pulp apparent viscosity is determined and considered constant over a period of time equal 1 min (refer to figure 1).

In order to characterise the rheological behaviour of the pulp suspension in the refining zone, the mean shear rate is also calculated for each period of time Δt. In a refining trial of 30 minutes, quite 30 discrete numerical values are obtained for the pulp apparent viscosity and the mean shear rate.

Dimensionless analysis.

Using the model and the analogy of bearings for the refining machines, considering the following set of values for the :
-vertical displacement dh (i.e. gap clearance),
-tangential velocity U of the cylinder,
-pulp apparent viscosity μ_a,

$$\begin{array}{lllll} 10 & < & dh & < & 350\ \mu m \\ 4{,}2 & < & U & < & 8{,}6\ m/s \\ 50 & < & \mu_a & < & 500\ mPa.s \end{array} \tag{30}$$

a dimensionless analysis is performed.
For the mean resultant bearing load $\overline{R_p}$, one dimensionless number Smp, very close to a Sommerfeld number, is obtained:

$$Smp = \frac{\mu_a U}{p_0 R}\left(\frac{R}{dh}\right)^2 \tag{31}$$

$\overline{R_p}$ can be determined when the number Smp is known accordingly [8]. The relationship is expressed on the figure 7. On other hand, the mean resultant friction force $\overline{F_t}$ is fully determined, when a dimensionless number Smf is known:

$$Smf = \left(\frac{dh}{R}\right)^{-2/3} \tag{32}$$

A simple relationship can be obtained, as described on the figure 8.
So, from this dimensionless analysis, the forces exerted by the pulp suspension in the refining gaps can be foreseen if the input parameters are known; among these: the vertical displacement of the cylinder, the tangential velocity of the cylinder and the pulp apparent viscosity.
These previous relationships will be used for the energetic interpretation of the refining process.

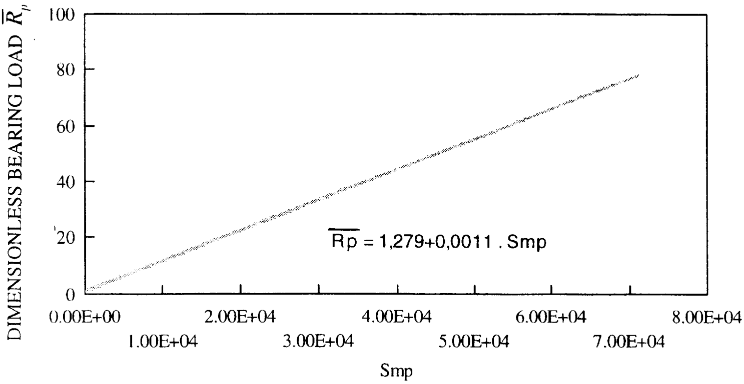


Figure 7. Dimensionless analysis of the bearing load $\overline{R_p}$.

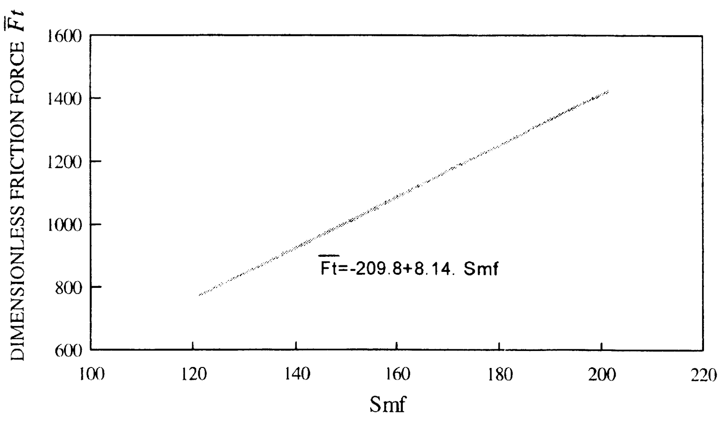


Figure 8. Dimensionless analysis of the friction force $\overline{Ft}$.

THE MODEL VALIDATION

The proposed hydrodynamic model is validated using Newtonian and non-Newtonian fluids with known rheological behaviour. In this case, the Hollander beater runs as a viscometer in high shear rate conditions $(10^4 - 10^5 s^{-1})$.

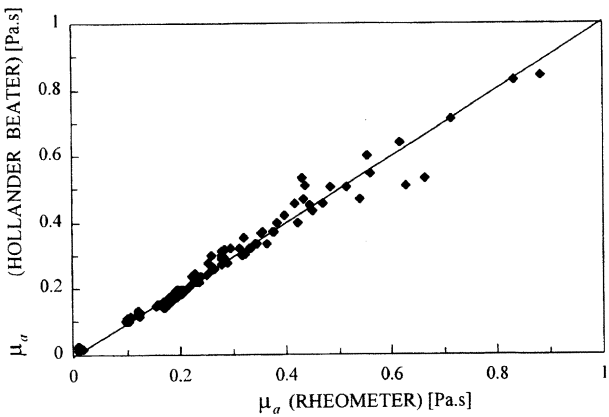


Figure 9. Validation of the model with glycerol-water mixtures.

Figure 9 shows the comparison between the dynamic viscosity of glycerol-water mixtures (known to be Newtonian), determined by the procedure and the corresponding values obtained on standard laboratory rheometer. The quantitative agreement is as good as expected.

For non-Newtonian fluids, a CMC was used and the results are described on the figure 10.

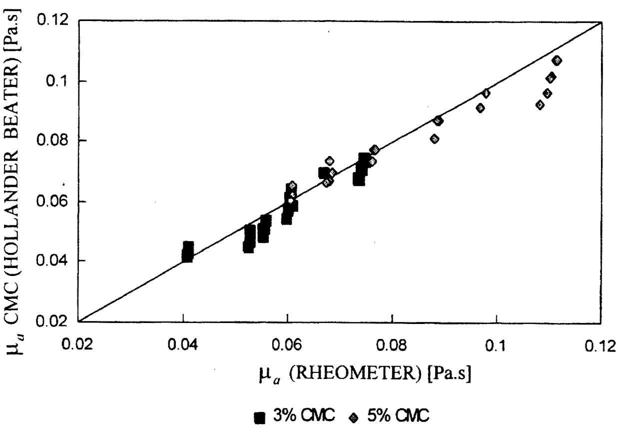


Figure 10. Validation of the model with Non-Newtonian fluid.

A quantitative agreement is also obtained and validates a-posteriori all the assumptions done and the procedure previously developed.

A RHEOLOGICAL EQUATION FOR PULP SUSPENSIONS.

Since we are able to determine a pulp apparent viscosity over each period of time during a refining trial, it is of a great interest to try to develop a rheological equation for the

behaviour of pulp in the refining zone. The pulp used is a mixture of bleached softwood and bleached hardwood kraft pulps. Different compositions are studied.
This is experimentally done under the following conditions for the consistency C, the mean shear rate $\dot{\gamma}$, the fibre aspect ratio l/d, the hydration index WRV% (water retention value) and the drainage index °SR:

$$
\begin{array}{ccccc}
2{,}5\% & < & C & < & 5{,}0\% \\
10^4 s^{-1} & < & \dot{\gamma} & < & 5.10^4 s^{-1} \\
15 & < & {}^\circ SR & < & 90 \\
90 & < & WRV & < & 200 \\
14 & < & \frac{l}{d} & < & 75
\end{array}
\tag{33}
$$

In order to minimise the number of refining trials, a factorial analysis is undertaken. From the experimental results obtained in the constant gap clearance beating mode, a polynomial equation is found:

$$
\begin{aligned}
\ln\mu_a = {} & 242{,}5 - 1{,}03C - 0{,}25WRV - 0{,}49.{}^\circ SR - 1{,}14.10^{-3}\dot{\gamma} - 2{,}23\ln\frac{l}{d} + 0{,}50.10^{-4}\dot{\gamma}.{}^\circ SR + \\
& 0{,}25.10^{-5}\dot{\gamma}\, WRV + 6{,}6.10^{-5}\dot{\gamma}\ln\frac{l}{d} + 0{,}16.10^{-2}WRV{}^\circ SR + 0{,}19.C^2 - 0{,}74.10^{-5}\dot{\gamma}^2 + 0{,}93)SR^2 \\
& + 3{,}6.10^{-4}WRV^2 + 0{,}3\left(\ln\frac{l}{d}\right)^2
\end{aligned}
\tag{34}
$$

This empirical equation can be considered as a rheological equation for the pulp suspension in the refining zone if it is independent of the running mode. This last statement is clearly demonstrated on the figure 11 where an agreement is found between the values of the pulp apparent viscosity obtained either by the iterative procedure or by the rheological equation (34) using the known values of the parameters C, $\dot{\gamma}$, °SR, WRV and l/d.

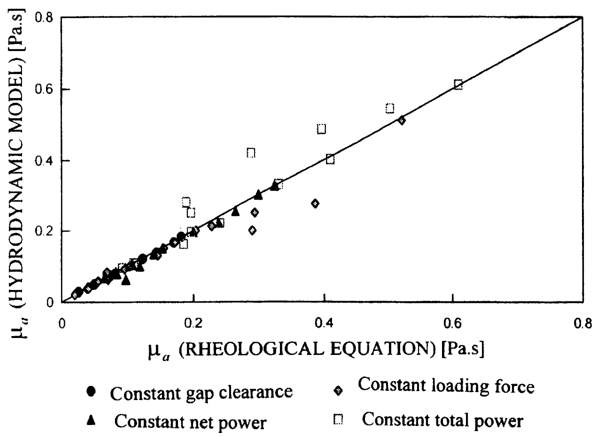


Figure 11. Validation of the model with pulp suspension for different beating modes.

According to this rheological equation, the pulp apparent viscosity is related to the fibre characteristics, to the pulp consistency and to the mean shear rate. An increasing in the drainage index °SR and/or the hydration index WRV leads to a decreasing in the pulp apparent viscosity. The same effect is obtained owing to a reduction in the fibre aspect ratio. From the results, it is shown that the pulp apparent viscosity decreases with an increasing in the mean shear rate. A shear-thinning behaviour is found for the pulp suspension in the refining zone of a beating machine (see Figure 12).

According to this rheological equation, the pulp apparent viscosity is related to the fibre characteristics, to the pulp consistency and to the mean shear rate. An increasing in the drainage index °*SR* and/or the hydration index *WRV* leads to a decreasing in the pulp apparent viscosity. The same effect is obtained owing to a reduction in the fibre aspect ratio. From the results, it is shown that the pulp apparent viscosity decreases with an increasing in the mean shear rate. A shear-thinning behaviour is found for the pulp suspension in the refining zone of a beating machine (see Figure 12).

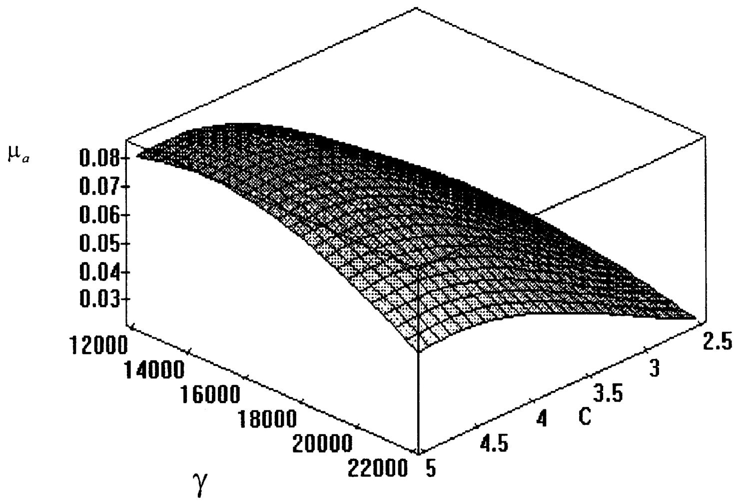


Figure 12. The apparent pulp viscosity as a function of the shear rate and the pulp consistency

ENERGETIC ANALYSIS OF THE REFINING PROCESS

Using the hydrodynamic model, the energy efficiency coefficient can be calculated if the input parameters are known.
For example, the net specific energy consumption is determined as follows, during one refining trial :

$$E_{net}(N\Delta t) = \frac{1}{m_s}\sum_{i=1}^{N} \overline{f_t}(i)\, U.\Delta t \tag{35}$$

where m_s is the dry mass of pulp in the beater and $\overline{f_t}(i)$ is the mean resultant friction force developed by the pulp in the refining zone during the period i of time.
Substituting the friction force by its dimensionless form (see equation (27)), we obtain:

$$E_{net}(N\Delta t) = \frac{1}{m_s}\sum_{i=1}^{N} L\Delta\theta\left(\mu_a U^2 \overline{F_t}\right)_i \Delta t \tag{36}$$

Taking into account the idling power loss pw_{idling}, the energy efficiency coefficient is determined accordingly :

$$\eta(N\Delta t) = \frac{\sum_{i=1}^{N} L\Delta\theta\left(\mu_a U^2 \overline{F_t}\right)_i \Delta t}{\sum_{i=1}^{N} L\Delta\theta\left(\mu_a U^2 \overline{F_t}\right)_i \Delta t + \sum_{i=1}^{N} pw_{idling}(i)\Delta t} \tag{37}$$

Replacing the pulp by water in the refiner, the idling power loss was obtained. It was shown that it only depends on the vertical displacement of the cylinder (i.e. gap clearance) and the tangential velocity of the cylinder either:

$$pw_{idling} = aU + b\frac{U^2}{dh^{1/3}} \tag{38}$$

If U is expressed in $m.s^{-1}$, dh in mm and pw_{idling} in W, the numerical values obtained for a and b are the followings:

$$a = 148$$
$$b = 0{,}170 \qquad (39)$$

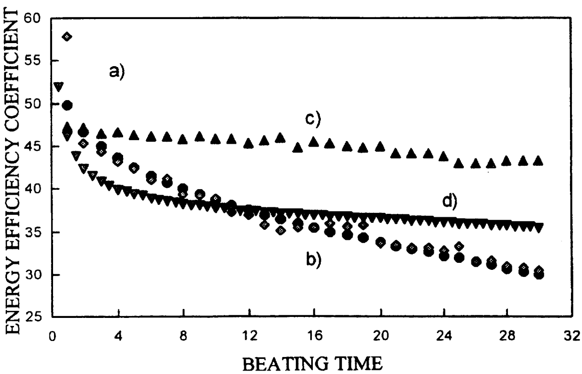
The simplification by Δt in equation (37) and the substitution of the idling power accordingly lead finally to the formulation of the energy efficiency coefficient:

$$\eta(N\Delta t) = \frac{\sum_{i=1}^{N} L\Delta\theta\left(\mu_a U^2 \overline{F_t}\right)_i}{\sum_{i=1}^{N} L\Delta\theta\left(\mu_a U^2 \overline{F_t}\right)_i + \sum_{i=1}^{N}\left(aU + b\frac{U^2}{dh^{1/3}}\right)_i} \qquad (40)$$

The energy efficiency coefficient depends on the evolution of 3 input parameters during a refining trial:

-the vertical displacement *dh* of the cylinder (i.e. gap clearance),
-the tangential velocity *U* of the cylinder,
-the pulp apparent viscosity μ_a,

but the third parameter is strongly related to the others owing to the rheological equation previously obtained. For example, decreasing the gap clearance keeping constant the tangential velocity gives an increasing in the mean shear rate and, consequently, a decreasing in the pulp apparent viscosity. On other hand, during a refining trial in a constant gap clearance mode, an increasing in the rotation of the cylinder leads to a decreasing in the pulp apparent viscosity. Remember that both refining effects and running conditions modify the pulp apparent viscosity.
On the figure 13, the evolution of the energy efficiency coefficient is given for refining trials specified in the 4 different beating modes implemented on the Hollander machine.



a) Constant gap clearance beating mode
b) Constant loading force beating mode
c) Constant net power beating mode
d) Constant total power beating mode

Figure 13. Evolution of the energy efficiency coefficient for different beating modes.

HOW TO OPTIMISE THE REFINING PROCESS?

General methodology

The refining process optimisation consists in determining of the conditions for which the desired paper properties are obtained with a minimum total energy consumption and a maximum energy efficiency. For instance, for the Voith Hollander beater, these input conditions are the following:

- the pulp consistency
- the pulp composition
- the tangential speed
- the set point of the control variable according to the chosen beating mode
- the time of beating

During the trials, the temperature of the pulp is maintained constant. As the beating mode is a qualitative variable, it is clear that different analysises must be performed depending on the chosen beating mode.
The optimisation needs to include the relationships between the refining conditions, the total specific energy consumption, the energy efficiency coefficients and the value of the pulp quality. Thus, the hydrodynamic model described in the previous paragraphs is useful to perform the refining optimisation.
In other hand, the optimisation results will depend on the nature of the desired paper properties. An example of optimisation for a packaging paper will be discussed.

Paper properties modelling

To predict the paper properties as function of the beating conditions previously described, mathematical quantitative models must be developed. Not only the beating conditions reflect the paper properties; other sub-processes as the formation, the pressing, drying... can influence to a great extent these paper properties. We will assume that the conditions during these sub-processes are constant.
A factorial analysis must be conducted in order to reduce the number of refining trials.
As output variables the following pulp and paper properties are considered:

- °SR -
- WRV %
- bulk $cm^3.g^{-1}$
- burst index $kPa.m^2g^{-1}$
- tear index $N.m^2.kg^{-1}$
- tensile index $N.m.g^{-1}$
- opacity -
- brightness -
- light absorption coefficient K' $cm^2.g^{-1}$
- light scattering coefficients S' $cm^2.g^{-1}$

The model form is a quadratic polynomial expression of second degree according to the formula:

$$Y_i = a_0 + \sum_i a_i X_i + \sum_{i \neq j} a_{ij} X_i X_j + \sum_i a_i X_i^2 \quad (41)$$

a_i the determined coefficients from a statistic method
Y_i the modelled paper properties
X_i the considered beating conditions

Thus, the refining optimisation can be presented as a typical optimisation problem with two blocs of equations:

1. Based on the factorial analysis empirical equations - the paper properties predictions

2. Based on the hydrodynamic model equations predicting the total specific energy consumption and the energy efficiency coefficient.

In this paper, the results obtained in constant loading force beating mode will be discussed. Other beating modes have been studied in [8], [10]. In the case of constant loading force beating mode, the limit conditions for the 5 input variables X_i are presented in table 1.

variable	**Index**	**minimal value**	**maximal value**
Pulp consistency [%]	1	2,5	5,0
Tangential speed [m/s]	2	4,21	8,52
[%] of softwood fibres	3	0	100
Beater loading force set point [kN/m]	4	5,30	6,43
Beating time [min]	5	3	30

Table 1. Limit values of the input variables X_i

As it was shown in the previous paragraph, the net and the total energy consumptions increase during refining. On the contrary, the energy efficiency coefficient decreases with the increasing of the beating time.

In other hand, the paper properties evolutions are specific for each property during refining. For instance, the tensile and the burst indexes increase continuously, the tear index can reach a maximum and decreases, the opacity and the brightness fall down. Thus, the optimisation results depend on the desired paper properties and if the methodology developed in this paper is general, the results are dependent on the target. Usually, a compromise must be made between the paper properties development and the refining efficiency decreasing.
The refining process must be interrupted after the target reaching.

Example of optimisation for a packaging paper

The desired paper properties target is determined for a packaging kraft paper. The properties measured and the retained targets for the studied commercial paper are presented in table 2.

Property	**Measurements**	**Target**
Mean tensile index	43,5 Nm/g	=or > 58,0 Nm/g
Tear index	11,4 Nm^2/kg	= or > 14,3 Nm^2/kg
Burst index	2,9 $kPa.m^2/g$	= or > 3,8 $kPa.m^2/g$

Table 2. Measured properties and target determined for the studied packaging paper

Using the target in table 2, two different sub-targets are determined. The first consists to amplify the burst index and to chose a value located in the interval (3,8 - 4,2). The second target is less strong - in this case, the burst index can be equal to 3,8. The optimisation results obtained for the corresponding targets are presented in the tables 3. and 4.
For instance, if a burst index equal to 3,8 $kPa.m^2/g$ is considered sufficient, the refining conditions in table 4. allows:
1728 - 1331 = 397 MJ/t
to be saved with respect to target 1 optimisation.
If not, the refining conditions presented in table 3. must be used.
It can be noticed in tables 3 and 4 that the values of the apparent pulp viscosity remains high, independently of the chosen pulp consistency.

target 1.

Values chosen for the tensile, tear and burst indexes:

32	<	*Energy eff. coefficient %*	<	*42*
58	<	*Tensile index Nm/g*	<	*61*
14,3	<	*Tear index Nm^2/kg*	<	*15,0*
3,8	<	*Burst index $kPa.m^2/g$*	<	*4,2*

Paper property		**Result**
Tensile index Yt	[Nm/g]	61
Tear index Yd	[Nm^2/kg]	15,0
Burst index Ye	[$kPa.m^2/g$]	3,9
Apparent pulp viscosity μ	[Pa.s]	0,233
Mean shear rate in the gap clearance γ	[s^{-1}]	$1{,}12.10^4$
Refining parameter		
Pulp consistency	[%]	3,9
Tangential speed	[m/s]	4,1
Pulp composition	[% softwood]	50
Loading force per unit length	[kN/m]	6,0
Beating time	[min]	20
Specific edge load	[J/m]	3,1
Total specific energy consumption	[MJ/t]	1728
Energy efficiency coefficient η [%]	[%]	39

Table 3. Optimisation results obtained considering target 1.

target 2.

Values chosen for the tensile, tear and burst indexes:

32	<	*Energy eff. coefficient %*	<	*42*
58	<	*Tensile index Nm/g*	<	*61*
14,3	<	*Tear index Nm²/kg*	<	*15,0*
3,8	≤	*Burst index kPa.m²/g*		

Paper property		**Result**
Tensile index Yt	[Nm/g]	60
Tear index Yd	[Nm²/kg]	15,0
Burst index Ye	[kPa.m²/g]	3,8
Apparent pulp viscosity μ	[Pa.s]	0,198
Mean shear rate in the gap clearance γ	[s^{-1}]	1,10. 10^4
Refining parameter		
Pulp consistency	[%]	2,4
Tangential speed	[m/s]	4,0
Pulp composition	[% softwood]	69
Loading force per unit length	[kN/m]	6,0
Beating time	[min]	11
Specific edge load	[J/m]	2,4
Total specific energy consumption	[MJ/t]	1331
Energy efficiency coefficient η [%]	[%]	32

Table 4. Optimisation results obtained considering target 2.

With the second target, the pulp consistency is very low: 2,4%, but the pulp apparent viscosity after 11 minutes of beating remains high: 0,198 Pa.s. This result is obtained owing to the low shear rate and the increasing rate of softwood fibres in the pulp composition.
The refining optimisation implementation is easy to realise owing to the performances of the software used. It is only necessary to define the target of desired paper properties and the refining conditions, optimising the total specific energy consumption and the energy efficiency coefficient are obtained accordingly.

CONCLUSION

A hydrodynamic modelling of the running of the beating machine is developed mainly based on the momentum transfer balance and the continuity equation in steady-state conditions. The model assumes that, in the beating confined zone, the pulp suspension behaves as an equivalent fluid with a given apparent viscosity. Then, the model resolution is performed for a Voith Hollander beater owing to supplementary assumptions: simple geometry and flow orthogonality leading to a bidimensionnal problem.
In order to compare the calculations of tangential and normal forces that the pulp exerts in the refining zones and their measurements, a specific metrology is developed. It allows the gap clearance, tangential velocity, bearing load, friction force and temperature to be measured. Four different beating modes can be studied in this refining machine. An iterative procedure is developed in order to determine, during each elementary period of time of a refining trial, the pulp apparent viscosity.
All assumptions made in this modelling are well validated a-posteriori. To give a general characterisation of the refining process, a dimensionless analysis is developed. The main previous forces can be foreseen if the input parameters: gap clearance, tangential velocity and pulp apparent viscosity are known. It is also shown that the energy transfer depends on the evolution of these 3 variables.
Then, the study of the rheological behaviour of the pulp, carried out in the Hollander Voith beater, yields an important result - it is a shear thinning fluid. A rheological equation is found, whose numerical coefficients are independent of the beating mode. The pulp apparent viscosity depends on the pulp consistency, the mean shear rate in the gap clearance, the fibre swelling, the fibre aspect-ratio and the drainage index. It is both affected by the state of the pulp and by the beating conditions. The model contributes to a

better understanding of the refining process both for the fundamental explanation of the pulp suspension behaviour and for the establishment of the running conditions in the beating process.
To select the best refining conditions for a given refiner, an optimisation method must be used. Several factors must be taken into account: the pulp consistency, the pulp composition, the tangential velocity, the beating time and the set-point of the control variable depending on the chosen beating mode. Also, determination of the best refining conditions depends on the desired paper properties. A factorial analysis is used to obtain the relationships between the paper properties and the selected refining conditions.
An example of optimisation of the properties for a packaging paper is demonstrated. Then, the hydrodynamic model is used to interpret the refining results in order to calculate the total specific energy consumption and the energy efficiency coefficient. Important energy savings can be realised by selecting the optimal refining conditions according to the model.

REFERENCES

1. EBELING K., « A critical review of current theories for the refining of chemical pulps », International Symposium of fundamental concept of refining. Preprint. Appleton p. 1-36, 16-18 September 1980

2. PAGE D. H., « The beating of chemical pulps - the action and the effects », Fundamentals of Papermaking, Mechanical Engineering Publications Limited, London, Vol 1., p.1-38, 1989.

3. HALME M., « How to use formulae to study refining equipment », Paper Trade Journal, N°9, p.32-35, 1964

4. FRAZIER W. C. « Applying Hydrodynamic Lubrication Theory to Predict Refiner Behaviour », Journal of Pulp and Paper Science, Vol. 14, N°1, January 1988

5. RANCE H. F., Proc. Tech. Sect. Br. Paper and Board Makers'Assoc., 32 (2) p. 360, 1951

6. STEENBERG B., Proc. Tech. Sect. Br. Paper and Board Makers'Assoc., 32 (2) p. 388, 1951

7. BRODKEY S. and HERSHEY H.C., Basic concepts in transport phenomena, Transport Phenomena - a unified approach, MacGraw Hill, 1988

8. RADOSLAVOVA D., « Modélisation hydrodynamique du processus de raffinage des pâtes à papier », PhD. thesis, Institut National Polytechnique de Grenoble, FRANCE, juin 1996

9. DINH S. M., AMSTRONG R. C., The Soc. Rheo. Inc. J. Rheo., 28(3), pp.207-227, 1984

10. RADOSLAVOVA D, SILVY J., ROUX J.-C., « The concept of the apparent viscosity of pulp for beating analysis and the development of the paper properties », Conf. Proc., TAPPI, March 1996, Philadelphia, USA

11. STEENBERG B., Review of the Effect of Mechanical Treatment of Fibres Paperstindning N°22, pp.933-939, 30 November 1963

Transcription of Discussion

Hydrodynamic Modelling of the Behaviour of the Pulp Suspensions during Beating and its application to optimising the Refining Process

Professor Jean-Claude Roux, EFPG, France

Theo G M van de Ven, Director, Paprican/McGill, Canada

Since all the action is between the cylinder and the bed plate - how sensitive is the model to the shape or curvature of the bed plate?

Jean-Claude Roux

That is an excellent question and we were aware of this problem. As you have seen gap clearance is about 50 microns. We used calibrated pieces of metal and measured step by step in order to be sure that the dimension was accurate - so we have this validation also and high shear rate can be calculated if we take into account gap clearance. If a mistake is made on the gap clearance this can have consequences on the results so we have taken this into account.

Steve Eichhorn, Student, UMIST, UK

You talked about the pressure variation with an angle co-ordinate dimensional analysis - is this a theoretical prediction?

Jean-Claude Roux

Yes.

Steve Eichhorn

How reasonable do you think that is?

Jean-Claude Roux

It is a theoretical prediction but as you have seen we have obtained results step by step in order to validate all the procedures, measurements and data - so it is a result in this strategy but we have not validated the pressure distribution. If you study the lubrication bearings or hydrodynamic bearings you can see curves with this shape - it is not very unusual in fluid mechanics books.

Ilkka Kartovaara, R&D Vice President, Enso Group, Finland

You have stated here that the energy efficiency coefficient depends mainly on the gap clearance and the tangential velocity and the apparent viscosity and you give one case where the combination of lower apparent viscosity and lower gap clearance gave higher energy efficiency. Is this a general rule or a single case? Can you say something general about the effects of these three parameters.

Jean-Claude Roux

You have to be aware that we have worked on the Voith Hollander beater and in this kind of equipment we have idling loss or power loss which are high. We do not place too much importance on the energy efficiency coefficient of this batch refiner. With this kind of equipment the better parameter seems to be the total energy consumption.

Ilkka Kartovaara

I was asking about the total energy efficiency. It is certain that the total energy consumption correlates with the strength but how is the energy efficiency related to the three parameters?

Jean-Claude Roux

We did this but it is not shown in the Paper. We can talk about this later. You take the evolution of the paper properties divided by the energy consumption in order to see the results.

Dr Gary Baum, Vice President - Research, IPST, USA

You selected a co-ordinate system ideally suited for the Voith Hollander beater. Will you need to re-do the mathematics or be able to extend the concepts when you apply this to commercial systems with different refining geometries?

Jean-Claude Roux

On disc or conical refiners the problem is three dimensional. We were lucky to choose the Voith Hollander beater because it can be solved. Some results in the literature can be explained by this hydrodynamic formulation. It is possible to relate the dimensionless parameters to other parameters - one related to the Sommerfeld number - one to the mean gap clearance - but the work stopped due to lack of funding.

Theo G M van de Ven

You used your beater as a rheometer to measure rheological properties of pulp fibres and I notice in your results for the apparent viscosity against shear rate, then there is a maximum in this curve, showing shear thickening - is that a real effect?

Jean-Claude Roux

No - you are perfectly right. You point out a problem with numerical simulations you have to be certain of the results and in the presentation you have inter-related effects. It is not possible if you take a SR30 or 35 to get a WRV of 200 we have to be very careful in the analysis of these kind of results. As you have seen in the main they are interesting and can be used to explain many results.

Kenneth J Zwick, Research Scientist, Union Camp Corp, USA

You seem to use a continuum model for the pulp - can you comment on how you think flocculation of the pulp might affect your results. Secondly how did your apparent viscosities compare with earlier work like that of Guthrie?

Jean-Claude Roux

We have chosen an engineering point of view. We can obtain interesting results so you raise a question on the physical description of the phenomena in the gap clearance. The flocculation is very important but perhaps we need to take into account some parameters - the crowding factor or some others on the flocculation state of the pulp. We have these ideas but no funding.

Professor Borje Steenberg, KTH PMT, Sweden

I first suggested the Sommerfeld equation in 1952 in a Paper with my good friend Toby Rance. After my presentation a certain Dr Alfred Nissan said that the fitting of the data to the Sommerfeld equation does not necessarily mean that this is a hydrodynamic lubrication case because it is dimensional analysis and has very little physical interpretation and he was correct. If Alfred Nissan is sitting here today what would he say about the Sommerfeld equation? I have one more point before Alfred gets the last word. If you look at the diagram 1 in this paper you will find that this paper is applicable to a small interval from T to T1. That means that the friction coefficient is changing all the time. I showed in 1952 that if you added well beaten fibres or polymers suddenly the gap size changed and he showed if you added unbeaten fibres to beaten pulps, the gap increased. So the size of the gap is changing all the time. My experiment was conducted with rayon pulp which you cannot beat . You can work it for hours and hours in the beater and the SR is always constant. That meant that there was no change in the pulp. Consequently I could change the speed, pressure of the refiner and the consistency. That was how I demonstrated the Sommerfeld equation. Now it is time for Alfred to prove why we are all wrong today.

Alfred H Nissan, Retired

I think we can claim the record in the Guinness Book for the length of time on a discussion (45 years) except there is one that is longer and that is the discussion between Einstein and Bohr on quantum mechanics. Now that I am on my feet, I wish to make a comment: we have excellent papers on studying basic dimensions of fibres, on the contribution of these dimensions to properties and on the fundamentals of beating and high energy efficiency. I would like to address a question to my generation "What on earth have we been doing for the last 40 years that these questions are raised today?" Have we been engaged in utilitarian research in which we have to justify as we have today the utility of every idea put forward? Have we been studying too much little bits here and there and forgetting the basics? I am saying this so that the young generation can take note and decide to discard the notion that every piece of research has to be short-term utilitarian research; let us do more basic research.

Jean-Claude Roux

Obviously you faced this problem many years ago - you laid the foundations - we use very quick and efficient computer systems . There is also increasing improvement in metrology so now it is possible to study easily very difficult problems with these kinds of tools. If you had these tools in your time you probably would have done the same things.