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A model for the retention of particles in evolving fibre networks

C.T.J. Dodson and W.W. Sampson

Department of Paper Science,

UMIST, PO Box 88,

Manchester, M60 1QD, UK.

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Abstract

A model is presented for the entrapment of a distribution of particles by a distribution of pores. The fraction of particles retained and their distribution of sizes can be calculated for evolving and static porous structures. For evolving structures the change in distribution and fractional retention through the structure can be calculated. Also, the variance in these parameters between zones in the plane of the sheet can be calculated. The agreement between the theory and experimental data is good. The theory has relevance to the retention of fillers and fines in the papermaking process and to more general problems of stochastic porous media.

INTRODUCTION

The pore radius distribution in paper is known to be approximately lognormal with standard deviation proportional to the mean [1, 2, 3, 4]. This is in agreement with the theory for pore radius distribution in random networks [2]. More recently the pore radius distribution has been derived for non-random fibrous networks [5] and has been shown to be well approximated by the gamma distribution which has a probability density function given by equation (1),

$$f(R) = \frac{b^k}{\Gamma(k)} R^{k-1} e^{-b\,R}, \tag{1}$$

with mean, $\bar{R} = \frac{k}{b}$ and variance $Var(R) = \frac{k}{b^2}$.

It was also shown in [5] that distribution characterising parameters, k and b are linearly dependent on each other for changes in grammage and flocculation for a given furnish and single ply forming. Also, presumably as a consequence of the Central Limit Theorem in statistics, fibre length distributions and particle size distributions for pigments are often described by the lognormal distribution (equation (2)):

$$g(r) = \frac{1}{\sqrt{2\,\pi}\,r\,\sigma}\, e^{\frac{-(\log(r)-\mu)^2}{2\,\sigma^2}}, \tag{2}$$

where r has mean, $\bar{r} = e^{\mu+\frac{\sigma^2}{2}}$ and variance $Var(r) = e^{2\,\mu+\sigma^2}\,(e^{\sigma^2} - 1)$.

The z-directional distribution of filler is found to be concentrated towards the wireside in handsheets and towards the topside in machine made papers [6, 7]. Radvan [8] states that in the laboratory former the sheet acts as a filter whereas, on the forming table of a paper machine drainage is faster and discontinuous, causing filler in the initially formed layers to be washed out. Here we model only the entrapment of particles by the sheet acting as a filter; washout of particles will be addressed in a future article. Studies of the source and retained particle size distribution have been made by Williams [9] who found that the retention of filler on paper machines increased with particle size resulting in fractionation of the pigment in the forming process. The effect is illustrated by comparing the retention of mineral with the same mean particle size, though with different distributions.

THEORY

Mechanical entrapment

The probability that a randomly chosen particle meets a pore of radius not greater than r is:

$$F(r) = \frac{\epsilon\,\int_0^r R^2\,f(R)\,.dR}{\int_0^\infty R^2\,f(R)\,.dR} \tag{3}$$

since ϵ is the fractional area of all pores and the likelihood of hitting a particular pore is proportional to its area.

Now, suppose that a total mass, M of particles arrive with mass-weighted probability density function, $g(r)$, independently and randomly scattered over the network. The arriving mass in the radius range r to $r + dr$ is $M\, g(r)\, dr$ and the retained mass in this range is $F(r)\, M\, g(r)\, dr$. We observe that the net mass fraction retained is:

$$\rho_{mech} = \int_0^\infty F(r)\, g(r)\, .dr \tag{4}$$

and of order ϵ in size.

The probability density function of retained particles is

$$G(r) = \frac{F(r)\, g(r)}{\rho_{mech}} \tag{5}$$

and hence, the frequency distribution of radii of the fractional mass retained is given by $\rho_{mech}\, G(r)$.

We use the variables $k(t)$ and $b(t)$ to characterise the evolution of the inter-fibre pore radius distribution on the forming table and calculate the distribution of radii of retained particles and the change of this distribution through the z-direction of the sheet. From the data of Ng *et al.* [10] and that of Bancroft [11] the empirical relationship between k and b for changes in mean grammage, $\bar{\beta}$ has been taken as:

$$b = \begin{cases} -0.33 + 0.40\,k & \text{for machine made papers with } 30\; g\,m^{-2} \le \bar{\beta} \le 50\; g\,m^{-2} \\ -0.13 + 0.12\,k & \text{for handsheets with } 5\; g\,m^{-2} \le \bar{\beta} \le 25\; g\,m^{-2} \end{cases} \tag{6}$$

This experimental relationship between k and b is significant since it seems to infer that possible paper structures are more limited than might be expected, there being infinitely many gamma distributions with a given mean. For the purpose of this study it is sufficient to state that the relationships given above have been found to give good agreement with experimental measures of pore size distribution in handsheets and machine-made papers.

The expected retention by mechanical entrapment of lognormally distributed particles with mean radius, $\bar{r} = 5\,\mu m$ and variance, $Var(r) = 5\,\mu m^2$ is shown in Figure 1 for increasing grammage as described by equation (6) for machine made paper. The

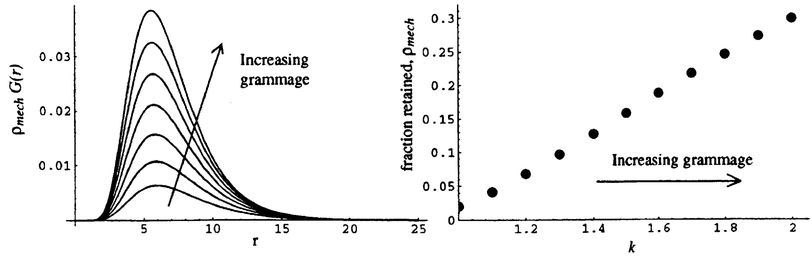

Figure 1: **Predicted mechanical retention on paper machine of particles with $\bar{r} = 5\,\mu m$ and $Var(r) = 5\,\mu m^2$.** *Left: retained particle radius distribution, $\rho_{mech}\, G(r)$; right: fractional retention of particles, ρ_{mech}.*

same figure also shows the change in fractional retention as fibre is deposited. The range of k used represents a grammage range of 30 $g\,m^{-2}$ to 60 $g\,m^{-2}$.

Effect of surface charge

Equation (3) holds for entrapment of particles by pores of radius less than or equal to the particle radius. Clearly, some physical chemical effects, such as pH, zeta-potential, surface chemistry, for example, will influence the way small particles aggregate and how they interact with the matrix of fibrous cellulose. No molecular model is available to represent such influences but presumably the net effect is that either the small particles are more easily trapped in the network or not. Crudely, this means that either they have a larger effective radius than their geometric radius, or not. Modelling particles as spheres, the available surface either to carry a charge or for chemical reaction is proportional to the square of its radius. Hence, a particle may be captured by a pore of radius $r + a\,r^2$ where a is a parameter accounting for the surface charge density and has units of reciprocal length. The probability that a randomly chosen particle meets a pore of radius not greater than $r + a\,r^2$ is:

$$H(r) = \frac{\epsilon \int_0^{r+a\,r^2} R^2 f(R)\,.dR}{\int_0^\infty R^2 f(R)\,.dR}\,. \tag{7}$$

The net mass fraction retained is

$$\rho = \int_0^\infty H(r)\, g(r)\, .dr, \tag{8}$$

and the frequency distribution of the radii of the fractional mass retained is $\rho\, H(r)$. Thus when $a = 0$, $H(r) = F(r)$ and $\rho = \rho_{mech}$.

The fractional retention of particles with radii in a given range, $r_1 \leq r \leq r_2$ is given by:

$$\rho_r = \frac{\int_{r_1}^{r_2} H(r)\, g(r)\, .dr}{\int_{r_1}^{r_2} g(r)\, .dr} \tag{9}$$

The absolute value of a is of little intrinsic importance; what is significant is whether, under a change of a physical chemical parameter x, the relative rate of change of a, $\frac{1}{a}\frac{da}{dx}$ is detectable. If it is, then it indicates the scale of the effect and points the way to a physical model.

VALIDATION OF THE THEORY

Data is presented by Bown [12] for the entrapment of minerals by laboratory handsheets. Full experimental details are not provided though sufficient information on sample preparation and data is presented to allow testing of the model. The pulp used was a bleached sulphite pulp beaten to 300 *ml* CSF. Electrochemical forces were eliminated by treating the pulp and fillers with a dispersant; retention in a dynamic drainage jar was negligible. Particle size data is reported in terms of the parameter, d_{50}, where 50 % of particles have diameter above or below this value.

The experimentally observed effect of grammage on the retention of clay is shown in Figure 2. The filler was close cut by multiple centrifuging to give steep particle size distributions about d_{50}; pore size distributions for the filled and unfilled papers were not measured. The prediction of the model has been calculated by numerically integrating equation (9) for 0.5 μm intervals of particle radius, r; values of k and b were calculated from the data of Bancroft [11] for machine made papers from 100 % bleached Kraft Pine furnish at grammages of 40 $g\,m^{-2}$, 50 $g\,m^{-2}$ and 60 $g\,m^{-2}$. The prediction of the model is shown in Figure 3 for $a = 0.1$; agreement with experimental data is good. The pore size distribution for the experimental sheet was unknown and whilst different absolute values would be obtained for a different pore size distribution, the observed trend would be the same; additionally, choice of parameter a is entirely

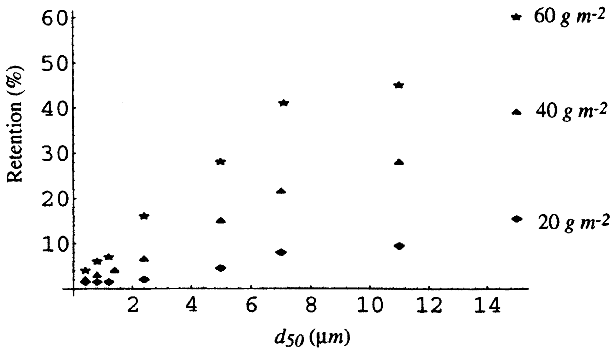

Figure 2: **Effect of grammage on retention of close cut clay samples by handsheets.** *Data is shown for 20$g\,m^{-2}$, 40$g\,m^{-2}$and 60$g\,m^{-2}$. Mineral samples were close cut using multiple centrifuging.*

arbitrary. The modelled response is typical of a cumulative frequency distribution and, though experimental data is unavailable above 15 μm to show the predicted plateau, the initial low gradient at low r is consistent with a cumulative distribution.

The fractional retention of clay, talc and calcium carbonate by 60 $g\,m^{-2}$ handsheets is shown in Figure 4. The modelled response for increasing surface charge is shown in Figure 5. Values of ρ_r were calculated using k and b values from the data of Bancroft [11] for 60 $g\,m^{-2}$ sheets. The source particle radii were lognormally distributed with mean, $\bar{r} = 2.5\,\mu m$ and variance, $Var(r) = 1\,\mu m^2$, though for a given pore size distribution, the value of ρ_r is extremely insensitive to the source particle radius distribution.

The effect of fibre grammage and retention aids on the retention of clay is shown in Figure 6. The clays used were Superfill ($d_{50} = 6\,\mu m$) and Grade C ($d_{50} = 2\,\mu m$); retention aid was added at 0.015 mass % with 1 mass % alum on fibre. The modelled effect of increasing a from 0.5 to 1 is represented by the dashed lines in Figure 6 for lognormally distributed particle radii with $\bar{d} = 6\,\mu m$ and $Var(d) = 1\,\mu m^2$. The values of k and b used to describe the pore size distribution were taken from the data of Bliesner [4] for unbeaten pulps over a similar grammage range to that used by Bown [12].

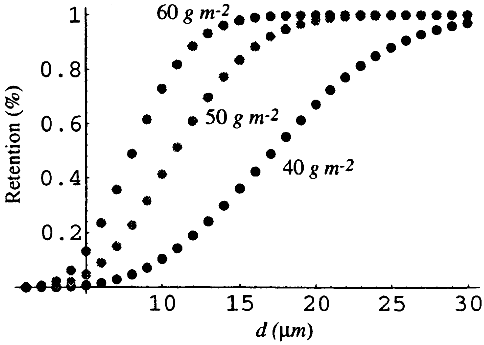

Figure 3: **Modelled retention of small particles.** *Data is shown for* $40 g\, m^{-2}$, $50 g\, m^{-2}$ *and* $60 g\, m^{-2}$*; Particle radii lognormally distributed with* $\bar{r} = 2.5\, \mu m$ *and* $Var(r) = 1 \mu m$*; parameter* $a = 0.1$

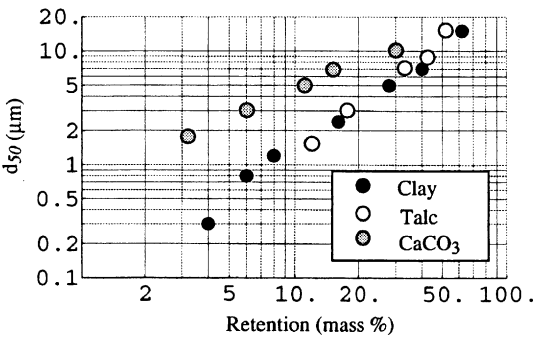

Figure 4: **Percentage retention of close cut mineral samples by 60** $g m^{-2}$ **handsheets.** *Data is shown for clay, talc and calcium carbonate.*

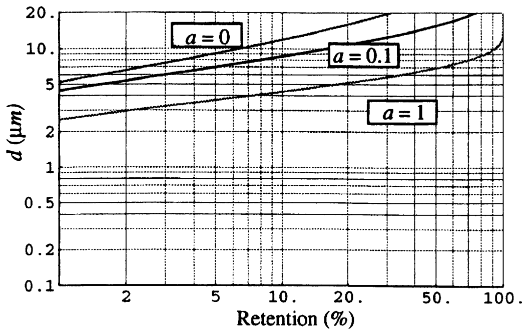

Figure 5: **Modelled percentage retention of particles by 60 $g\,m^{-2}$ handsheets.** *Data is shown for increasing surface charge denoted by parameter a.*

However, as the retention data is for beaten pulps we would expect a narrower pore size distribution with smaller mean and hence higher retention. Calculations of retention for lognormally distributed particle radii with $\bar{d} = 2\,\mu m$ and $Var(d) = 1\,\mu m^2$ gave retentions below 2 % for $a \leq 1$, though the shape of the curves and the change with increasing a was in agreement with the experimental data.

The predicted distribution of retained and transferred particle radii is shown in Figure 7 for retention of lognormally distributed particle radii with mean, $\bar{r} = 5\,\mu m$ and variance, $Var(r) = 5\,\mu m^2$ by a 60 $g\,m^{-2}$ sheet using values of k and b from the data of Bancroft [11]. The figure shows the particle size distribution introduced to the system and the split of retained and transferred particles for $a = 0$ and $a = 0.1$. The fractional retention of particles is 0.162 for $a = 0$ and 0.331 for $a = 0.1$. Work is continuing to investigate the effect of white water recirculation by including some or all of the transferred particles in the source distribution. This work will provide an insight into the evolution of a steady-state particle size distribution in the feedstock, white water and the sheet.

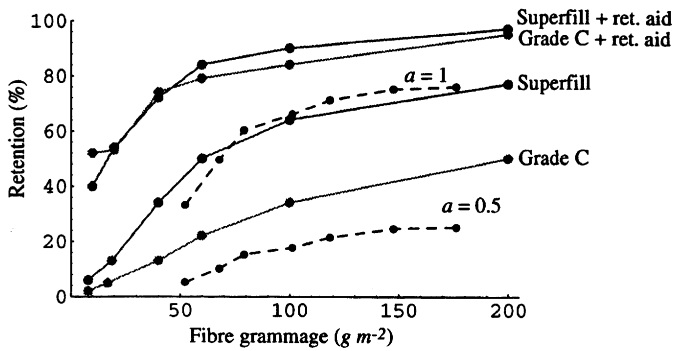

Figure 6: **Effect of retention aids and fibre grammage on retention.** *Data is shown for Superfill ($d_{50} = 6\mu m$) and Grade C ($d_{50} = 2\mu m$); predicted retentions are represented by dashed lines.*

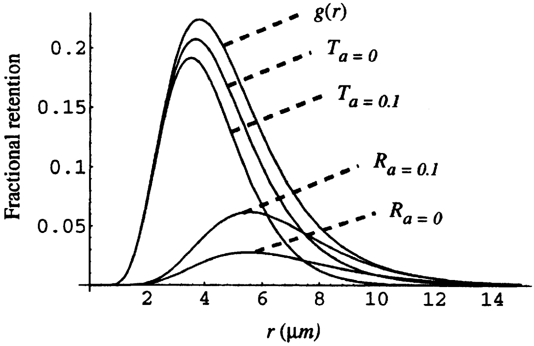

Figure 7: **Retention and transfer of particles.** *The effect of a on the particle size distribution of retained (R) and transferred (T) particles is shown. The source distribution of particles is denoted by $g(r)$.*

DISTRIBUTION OF FILLER IN THE PLANE

The effect of grammage on the retention of particles has been demonstrated above (see Figures 2,3,4,5). The expected distribution of mass in a sheet is, by the Central Limit Theorem, Gaussian [13]. Also, for Bancroft's data, the relationship $k = 0.164+0.021\bar{\beta}$ is found to hold with $r^2 = 0.812$ for grammages in the range 40 $g\,m^{-2}$ to 60 $g\,m^{-2}$. The distribution of fractional retained mass by zones of mean grammage, 50 $g\,m^{-2}$ and coefficient of variation of local grammage, 7 % was calculated by randomly selecting 1000 values of grammage from a normal distribution with this mean and variance; each grammage having associated values of k and b. Retention was calculated for these zones for $0 \leq a \leq 1$. The retention of particles with lognormally distributed radii with mean, $\bar{r} = 2.5\,\mu m$ and variance, $Var(r) = 1\,\mu m$ is shown in Figure 8 and for particles with mean, $\bar{r} = 5\,\mu m$ and variance, $Var(r) = 2\,\mu m$ in Figure 9. As the variance and hence coefficient of variation of local grammage are dependent on zone size, different papers may exhibit the same variance at different scales. Definition of the size and shape of zones is therefore not required in the model. A coefficient of variation of local grammage of 7 % is typical of commercial papers at the 1 mm scale. Interestingly the histograms in Figures 8 and 9 exhibit a negative skew despite the base structure of fibres having a normal distribution; also, the extent of skew increases with increasing a. The histogram for $a = 0$ is not shown as the fractional retention was below 4 % for all zones.

The mean, variance and coefficient of variation of fractional retention are show in Figure 10 for the two particle size distributions. As expected, the variance exhibits a maximum as fractional retention increases and the coefficient of variation decreases monotonically for $\rho\, H(r) > 0$. It should be noted that if all or no particles are retained by the sheet, then the variance and hence the coefficient of variation are zero.

DISCUSSION AND CONCLUSIONS

A model has been presented for the entrapment of particles by an evolving network of fibres. The theory gives good agreement with experimental data. The between zones variance of particle retention has been calculated and has relevance to the uniformity of optical properties and of ink transfer in printing processes. Also, a distribution of filler is associated with a distribution of bonding and will affect mechanical properties of the sheet. Work is continuing to characterise the effect of particle washout in regions of high shear close to the forming fabric. Also, the effect of recirculation of transferred

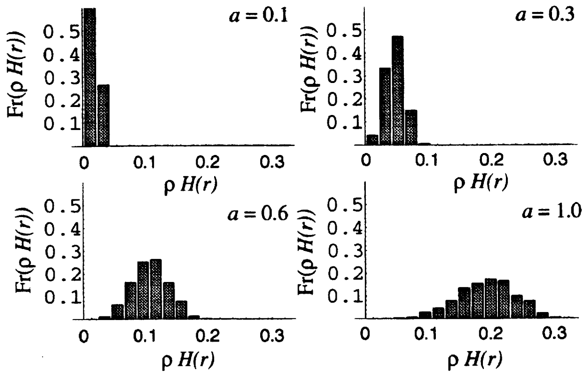

Figure 8: **Effect of retention aids on distribution of fractional retention for** $\bar{r} = 2.5\,\mu m$, $Var(r) = 1\,\mu m^2$. *Increase in charge modelled by increase in parameter a results in increased skew of the distribution.*

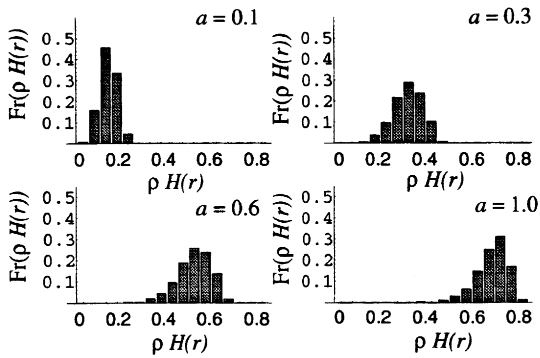

Figure 9: **Effect of retention aids on distribution of fractional retention for** $\bar{r} = 5\,\mu m$, $Var(r) = 2\,\mu m^2$. *Increase in charge modelled by increase in parameter a results in increased skew of the distribution.*

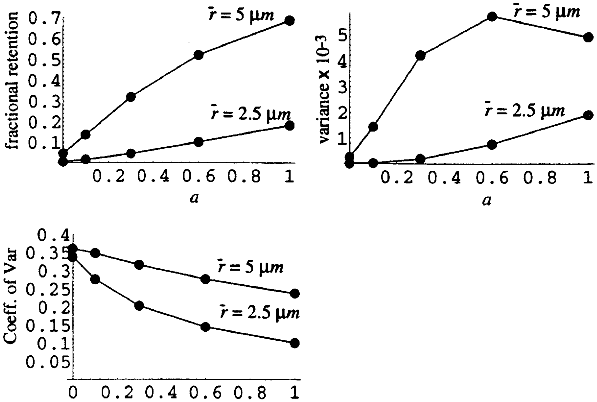

Figure 10: **Effect of retention aids on mean, variance and coefficient of variation of fractional retention.** *Top left: mean; top right: variance; bottom; coefficient of variation.*

particles is under investigation; this work has relevance to evolution of steady state conditions on the machine and hence in the product. Experimental work is also underway to obtain pore size distributions and associated retained and transferred particle size distributions for machine made papers under a range of forming conditions and chemistries. This will allow determination of the effects of quantifiable physical chemical characteristics of the system on the charge parameter a providing more rigorous testing of the model.

NOMENCLATURE

a	charge parameter	μm^{-1}
b	variable characterising gamma distribution	μm^{-1}
β	local grammage	$g\,m^{-2}$
d	particle diameter	μm^{-1}
ϵ	porosity	{ }
k	variable characterising gamma distribution	{ }
M	total mass of particles in system	g

μ	mean of $\log r$	{ }
r	particle radius	μm
R	pore radius	μm
ρ	retained net particle mass fraction	{ }
ρ_{mech}	mechanically retained net mass fraction	{ }
ρ_r	fractional particle retention in a given range	{ }
σ	standard deviation of $\log r$	{ }

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Transcription of Discussion

A model for the retention of particles in evolving fibre networks

Professor C T J (Kit) Dodson, UMIST, UK

Bruce Lyne, Senior Manager, International Paper, USA

As I understand it your model is essentially two dimensional. It's a fishing net. If you're looking at retention the tortuosity in the three dimensional structure would be important as it would increase the interaction of the fibre network with the particles that are going to be retained. Have you thought about the third dimension and how that would influence the result?

Kit Dodson

Yes. The problem is solved. It was already solved in 1965. The multiplaner model provides the layering effect; it means that what you have to do is to sum over the available structures vertically. In the random case it was trivial because they were independent. In the flocculated case we could say they were independent and each one is like the others, then the total variability would be the sum of the individual variabilities. That we can do. It doesn't alter qualitatively the results. What would be very neat would be to feed in the vertical interaction which clearly must be there. If you have a piece of a floc near the top it's unlikely you're going to have a piece of a hole right underneath it. It could be another piece of the floc. As soon as one makes a choice here one is open to criticism. So, at the moment, qualitatively I would claim that the results would be the same. Experimentally we were able to represent the behaviour of 40 and 50 gsm sheets which clearly are not mono fibre layers.

Bruce Lyne

If you were to put in some realistic values for retention and size of fibres would you expect your model to grossly under-estimate the size of the pores to compensate for the tortuousity?

Kit Dodson

That's any interesting question. This is something we will probably have to investigate experimentally. I like the idea of using these really close cut distributions as probes and then wash them out or perhaps add others successively as probes. Averaging on highly non linear things is very difficult to make guesses about. One's intuition is insecure here.

Steve Keller, Associate Professor, Syracuse University, USA

You indicated you were trying to model retention in your net. I wondered if the charge that you modelled was a repulsive charge which would increase the apparent diameter of the particle or an attractive charge which would increase the retention on the net as we normally characterise retention aids. Could you clarify that for me please?

Kit Dodson

Yes. The parameter that we switch on with increasing values of 'a' (charge parameter) increases the effective size of the particle It appears to the net bigger than geometrically it is.

Steve Keller

It strikes me that it might be opposite to the way we normally model retention aids. They are normally modelled through attraction of particles to the fibres. Am I correct in that?
Kit Dodson
So what do you suggest, that we alter the distribution, the net, and leave the particles? That would be an interesting thing.

Robert Pelton

I would like to thank you for reducing all our chemical problems to a single parameter.

Kit Dodson
At least we can pronounce 'a'.

Robert Pelton

A question of clarification. When you capture particles in a bed you change the permeability of the bed, in other words you start plugging up the holes. Does your model account for that?

Kit Dodson

Indeed not at the present. What we have is a changing structure. It's like the old fracture problem - as soon as the first bond is gone you have a new structure. You are absolutely right and it may be remarkable that we get such a good fit, so the model is insensitive. Certainly it would be very nice to have a way to model the evolving structure as the pores become clogged. The total mass of the particles being collected here is not great. We feel we are secure.

Mukhtar Hussain, Senior Consultant, Omni Continental, Canada

What happens if you have different consistencies of the fibre mat? I think the question is related to what Bruce Lyne asked earlier as to the tortuosity. You are talking about what is really a solid mat. The other point I have is that we are using equivalent spherical diameter, of course a particle shape is going to have an effect as well. Any comments on that.

Kit Dodson

To deal with the second one first, shape is on the list of things to do. Can I come back to the first point. In fact that is addressed because the main aim of using a gamma distribution is to be able to represent well the effects of flocculation on the free fibre length distribution. That is exactly what is occurring, at least in one layer. We have a representative layer for the flocculated structure and the effect of flocculation was beautifully represented by the k-b plot for differently formed papers.

Dr Richard Bown, Research & Technology Director, ECC, UK

Just a point of clarification. In the experimental data that Kit showed, the calcium carbonate has blocky particles and the kaolin and the talc particles are platey. This is a significant difference in shape and leads to a difference in the actual size of the particles at an equivalent e.s.d., as I discussed earlier in the week. You can see the difference in the data. The talc and kaolin particles have a greater major dimension than the calcium carbonate particles and show superior retention.

Dr Kari Ebeling, Director, UPM Kymmene Group, Finland

I would like to put some boundary conditions on the vertical interaction you mentioned. We all know that wet fibres are flexible and drainage phenomena is extremely short, just a few milliseconds, the compressive forces are enormous so you have a 'z' directional distribution of pore forms and that means that the water tries to go faster and faster and so the washing efficiency increases. The second point is that most of the drainage systems have pulses and then you have a pulse which opens up and that's where the retention becomes extremely critical. If you can include that also then you have a good model. Point three is that there is an enormous interaction that nobody has tried to tackle as the fibres which are against the wire try to go into the openings. This means that because of the fibre/wire interaction in the openings, the pore structure on that first layer is very difficult to characterise.

Kit Dodson

These are extremely valuable comments and I read them as future support from UPM Kymmene for our students.

Dr Pekka Mauranen, Research Director, KCL, Finland

This discussion about the flows is very interesting. I think that when we are dewatering fibrous suspensions by filtration, then the fibres follow the flow and the flow goes to the pores. We then get fibres to the thinner places on the web structure and that's the way to get a better structure or distribution of the fibres than a random one. I think that if you add such a parameter to the model which includes the hydrodynamics, you will have an even more accurate response. Of course we then have the problems of retention and flocculation so the situation gets extremely complicated, but in your flow considerations you have probably thought about this possibility.

Kit Dodson

Yes. In fact the representation of structures more uniform than random is possible within the gamma family of distribution, so that is possible. The smoothing effect arising from hydrodynamics yielding very small scale greater uniformity than randomness, this we have actually measured and discussed in a paper in JPPS recently. To incorporate it in the model is possible. What we would like to do first is to look at extremely flocculated structures and study their pore size distribution and the flow of fluids in those kind of networks and to look also at extremely uniform ones. What we would like to know is if the model is capable of straddling this wide range of structures. If it is, and the hydrodynamic effects are sufficiently great, then it is worth modelling; in cases of practical interest, flocculation seems always to win over smoothing.

Professor Tom Lindström, Royal Institute of Technology (KTH), Sweden

Nice modelling work. I'm not sure how relevant your modelling is for papermaking and I will give you a little example on this. Usually shear forces are much higher than you anticipate and pulses during the dewatering affect the structure in a different way. If we consider a mechanical filtration the retention goes up with particle sizes - it's obvious. In practical papermaking in the presence of retention aids it's usually the other way round. Large particles are very difficult to retain because they have very high inertia/viscous drag ratios, so actually smaller particles are much easier to retain so retention often goes the reverse way with particle size, so we are far away from the practice.

Kit Dodson

Indeed. Once one spends one's life doing geometry, even stochastic geometry, it can be very far from reality.

Ben Radvan, Retired, UK

If I understand your model correctly it would apply if you were draining a suspension through an already formed web of fibres. That is not what is happening even in a handsheet machine. The first lot of stock drains through a very thin web and then gets thicker and thicker so it seems to me as if it is a complication which can be included in your model but hasn't been.

Kit Dodson

We have included it because we have experimental data from the students' work on increasing the grammage and measuring retention and pore size distribution. We have done it step by step as it were, and the model can cope with this and you can add the effects.

Ben Radvan

So in effect when you have a 60 gsm sheet you should calculate the first 10 grammes separately from the next 20g, and that is included in your model?

Kit Dodson

It's not included in what is presented here but yes the ingredient that is needed is the dependence of the k and b on grammage and that remarkably is very stable. Once you have that, then you can have an evolving gamma distribution which is feeding the stochastic network.

Ben Radvan

It is an evolving distribution?

Kit Dodson

Yes, fortunately we don't have to do the calculus by hand for the integrals in here, and it's amazing that they come out in closed form. The other co-worker was Mathematica.

Anthony Scallan, Principal Scientist, Paprican, Canada

I would like to add to what several people have already said. If you're simulating retention by a paper machine or even a handsheet machine, the consistency of the pulp suspension is about 0.5% and thus there is a very considerable separation of the fibres in the z direction. Most of the filler will have left the sheet before the fibres come together as in your 2-D structure. I would also like to say that flocculation is sometimes undesirable and, very often, it is desirable to have the filler adsorbed on the surface of the fibres. As a result of these considerations, perhaps you shouldn't be using the term basis weight but rather consistency. An effect of higher consistencies is to have more fibre surface area in the system and this which will capture more filler.

Kit Dodson

Yes, it is another way. It actually relates to the point Ben Radvan made which is to handle the process not as stratified stages but as a full suspension. Indeed one could do this; in the suspension state, concentration is the appearance of grammage in the future.

Tony Scallan

In some of the work we've done looking at retention, we had a suspension at about 1% consistency and we never drained it at all. We just took samples of the supernatent liquor and found it depleted in filler because of adsorption onto the fibre surfaces. Thus adsorption can occur before the formulation of any tiny pores.

Kit Dodson

Indeed this is undeniable. There are many micrographs around which show particles stuck on the side of fibres. There are others which show particles trapped in small pores.

Richard Bown

The data Kit used was taken from work where we were initially deliberately not trying to generate retention by flocculating the filler onto the fibres. So there was no real interaction between the fibre and fillers. Then we slowly increased the level of interaction to demonstrate the move from mechanical entrapment, through collection, to having the particles firmly fixed on the fibres before the sheet is formed. I think you should be aware of the types of data that Kit has been fitting into his models.