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Is fibre failure Griffithian? - Prepared Contribution

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The properties of paper depend ultimately on the properties of the fibres therein. In particular the fracture of reasonably well-bonded papers is a strong function of fibre strength. It follows that the mechanism of fibre failure should be important for the modelling of paper fracture. The underlying assumption in both phenomenological and stochastic models to date is that fibre failure is originated in traction free surfaces, ie Griffith cracks. A macroscopic crack in a paper specimen, such as a notch in fracture testing, is obviously Griffithian, but the assumption is not warranted in the microscopic realm of the fibre cell wall. Indeed in many similar cases fracture is controlled by Zener-Stroh cracks, coalescing from dislocations and without traction free surfaces [1]. As shown in Figure 1, the two crack types are complementary.

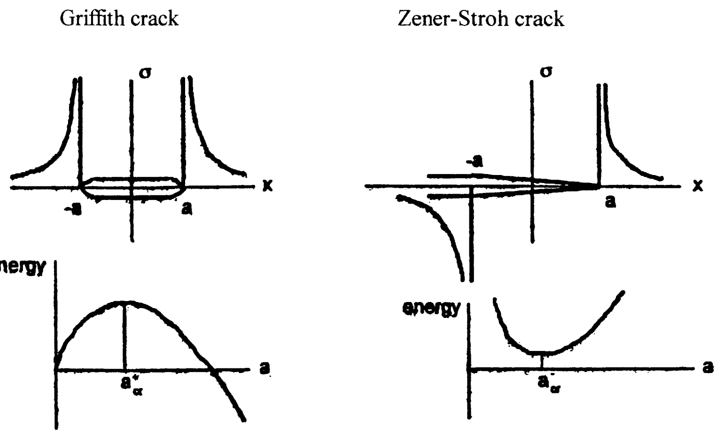

Figure 1: Comparison between the stress fields and energy functions of Griffith and Zener-Stroh cracks. Adapted from Weertman [2].

Whereas the Griffith crack has symmetrical stress fields, the Zener-Stroh crack stress fields are anti-symmetrical. While the energy function of a Griffith crack presents an unstable maximum at $a_{cr}+$, a Zener-Stroh crack has a stable minimum at $a_{cr}-$. Simplified equations for these critical crack half-lengths are:

$$a+ci = \frac{K2gc}{\pi.\sigma 2A} \quad \text{Where: } K2_{gc} = \frac{4.\gamma.G}{1-v}$$

$$a-cr = \frac{1}{\pi}\left[\frac{bT\,G}{2(1-v)K_{gc}}\right]2 \quad \text{Where: } b_T = n\,b$$

$\gamma = 500$ (J/m^2)
G = 10 (Gpa)
v = 0.2
Kgc = 5 (Mnm-$^{3/2}$)
σA = 1 (Gpa)
b = 3 (nm)

These equations can be used to estimate the size of each of the critical cracks between two materials or dissimilar layers [8]. Figure 2 shows that this is the case in micro-compressed wood fibres, where the cracks nucleate at the interface between the S1 and S2 layers.

Figure 2: Radial longitudinal section of wood in the region of a microscopic compression failure. From Keith & Côté [9].

A Griffith-Zener-Stroh crack formulation helps to explain the known importance of fibre singularities or 'defects' in fibre strength. It may clarify some of the discrepancies in paper fracture models and could also provide a theoretical basis for a unified theory of pulp refining and recycling.

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Comment by:

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You have nicely demonstrated that we know almost nothing about the fracture mechanics of a fibre. In fracture mechanics one is trying to predict failure load as a function of geometry, size and material properties. You showed us some approximations of material properties, as well as some principles on how to approach the problem of geometry. Then we have the size. All these have to be coupled together to be able say something. In order to say something about the behaviour of paper, one should also study the fracture mechanics of a bond between fibres, whatever the bond is. We ought to say something about the relationship between failure load or nominal stress and geometry, size and material properties. That sounds like a very challenging task; certainly challenging enough to be tried.

Gary Baum

In closing this session I would like to comment that a number of the papers have focused on the structure of the fibre cell wall. I am pleased with this because, like many of you, I believe the basic building blocks of paper are the fibrils, not fibres.