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DAMAGE WIDTH: A MEASURE OF THE SIZE OF FRACTURE PROCESS ZONE

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ABSTRACT

The measurement of "damage width" from silicone-impregnated specimens reveals the area in which bond failures and other microscopic fractures take place. We demonstrate that damage width is a reasonable measure of the size of the fracture process zone in the sense of fracture mechanics. Firstly, the decay of cohesive stress against crack widening scales with damage width. Secondly, we can calculate the tensile strength of paper from fracture mechanics using damage width as the size of the fracture process zone. Armed with this interpretation, one can use damage width to evaluate, for example, the effective length and strength of fibers in paper.

INTRODUCTION

The tensile strength of paper has been traditionally explained with various fiber network models. These models consider the fracture of a small *representative* part of the network [1]. The microscopic parameters, such as bond

strength and fiber strength, are usually not measurable. The microscopic parameters are fundamentally problematic because they are sensitive to the sheet structure and the papermaking process. Furthermore, fracture is a macroscopic phenomenon where the "weakest point" fails. Fiber network models cannot describe the localization of fracture.

An alternative to explain paper strength is to use fracture mechanics. This approach separates the localization of fracture (the problem of defining "the weakest point") from the material properties. One can analyze the "weakest point" effect by studying the non-uniform formation and defects in paper. The material properties in turn are average properties of paper. Therefore one can relate them to the average properties of the fibers and to the average structure of the network, even using fiber network models.

The simplest version of fracture mechanics is linear elastic fracture mechanics (LEFM). It explains tensile strength using elastic modulus and fracture energy as the material properties. Because of its ductility, paper does not obey LEFM. This means that the microscopic fracture process spreads around a crack tip. However, one can include a correction in the LEFM equations that accounts for the nonzero size of the fracture process zone (FPZ). Also, the size of FPZ is a material property. Donner demonstrated [2] that if the size of FPZ is available, the tensile strength of paper agrees with the modified LEFM. Calculating back from tensile strength, Donner concluded that the size of FPZ is in the millimeter range, similar to fiber length.

In this paper we show that a quantity that we call "damage width" [3] gives a reasonable measure for the size of FPZ. We find agreement between measured tensile strength values and predictions of the modified LEFM. The variation of damage width with, for example, the fiber furnish gives a graphical demonstration of the intuitive differences between brittle and tough papers.

VISUALIZATION OF PAPER FRACTURE

Silicone impregnation allows the in-situ visualization of the microscopic fracture process of paper [4]. When cured, the silicone that we use forms a low modulus rubber. It does not alter the mechanical properties of paper, except when porosity is very high. Paper becomes translucent in the treatment because the transparent silicone gel fills all the voids in the sheet and covers free fiber surfaces.

Figure 1 shows a crack propagating through a silicone impregnated sheet. The images are taken in reflected light against a black background that makes

the paper seem gray. The fracture process zone (FPZ) shows up as a white area because light reflects from broken inter-fiber bonds and fiber ruptures. The change in the optical properties is irreversible.

The example in Figure 1 contains PGW and 10% softwood kraft. The fracture process zone is otherwise fairly uniform but extends far out along some long and presumably stiff PGW fibers. Comparison with Figure 2 demonstrates that in pure kraft handsheets the crack tip is more homogeneous and wider than in sheets containing mechanical pulp. These observations illustrate how silicone impregnation reveals details of the microscopic fracture process of paper.

When the paper specimen in Figure 2 breaks, the circular fracture process zone moves through the specimen, leaving behind it a zone where either inter-fiber bonds have broken or the fibers themselves have ruptured, as illustrated in Figure 1. We have developed a method to measure the microscopic fracture area after the in-plane tear test [3]. We call the result damage width, w_d. Damage width is equal to the combined width of the white zones in Figure 1 and to the diameter of the white circle in Figure 2.

The fracture process zones illustrated in Figures 1 and 2 do not have sharp boundaries. Thus a criterion must be selected to characterize the width of the zone. We have chosen to set the boundary of FPZ at the gray level that is 10% above the level seen in the undamaged parts of each specimen [3]. Different criteria would change the numerical values of damage width but would generally not alter the ranking of paper samples.

It is not self-evident that our measurement of damage width gives a

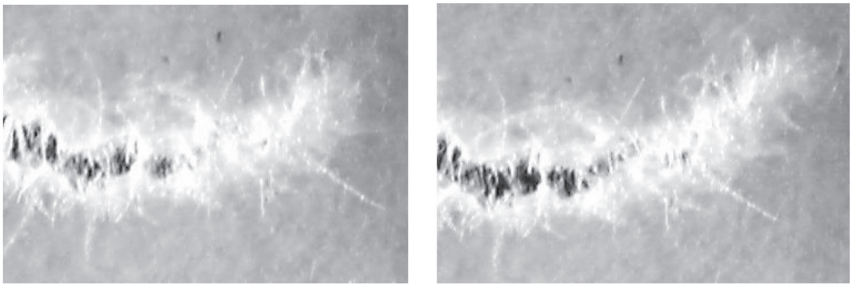

Figure 1 Two successive images of the fracture process zone (FPZ) in a silicone impregnated handsheet of PGW plus 10% softwood kraft. The crack propagates from left to right. The images are taken against a black background that shows through the crack opening. The white color arises from broken inter-fiber bonds and fiber fractures that reflect light. The white fracture process zone is roughly 1 mm wide.

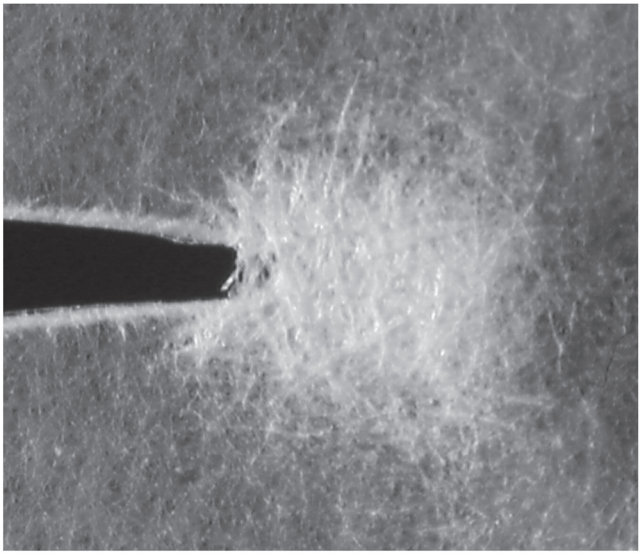

Figure 2 Fracture process zone (FPZ, the white area) in a handsheet of pure softwood kraft. The crack is just about to start propagating to the right. The FPZ diameter is 3–4 mm.

reasonable measure for the true fracture process zone (FPZ). The area detected in the in-plane tear test could include plastic deformation and therefore overestimate FPZ or it could not reveal all the relevant microscopic fractures. Damage width is a valuable evaluation tool of paper and fiber properties only if one can trust that its values are valid measures of the true fracture process zone. We present next two pieces of evidence corroborating this interpretation of damage width, from the analysis of the cohesive stress and tensile strength of paper. Other proof comes from thermographic images where energy dissipation is seen in the same area that damage width measures [5].

RANGE OF THE COHESIVE STRESS

If damage width measures the size of the fracture process zone, then it must correlate with the cohesive behavior of paper. In a stable fracture process, cohesive stress decreases as the displacement between the crack faces (crack

widening) increases. The cohesive stress is seen in a tensile test when the span length is sufficiently short [6]. The short span is necessary because otherwise the stored elastic energy leads to an unstable and very rapid fracture and the cohesive behavior cannot be detected. One assumes that when the fracture process starts, the local elongation, or widening of the FPZ grows. Elsewhere in the specimen, local elongation decreases in concert with the decreasing stress. The crack widening w can be calculated from the measured stress– elongation curve as follows:

$$w = (u - u_f) + \frac{\sigma_f - \sigma}{E} L \tag{1}$$

where u is the external elongation of the specimen at stress σ, u_f the elongation at the maximum stress σ_f, E the elastic modulus of paper, and L the span length.

An example of the cohesive stress–crack widening curve is in Figure 3. The difference between brittle and ductile papers should show up in the range of the crack widening values that give nonzero cohesive stress. We expect that a brittle paper has a short range cohesive stress so that even a small crack

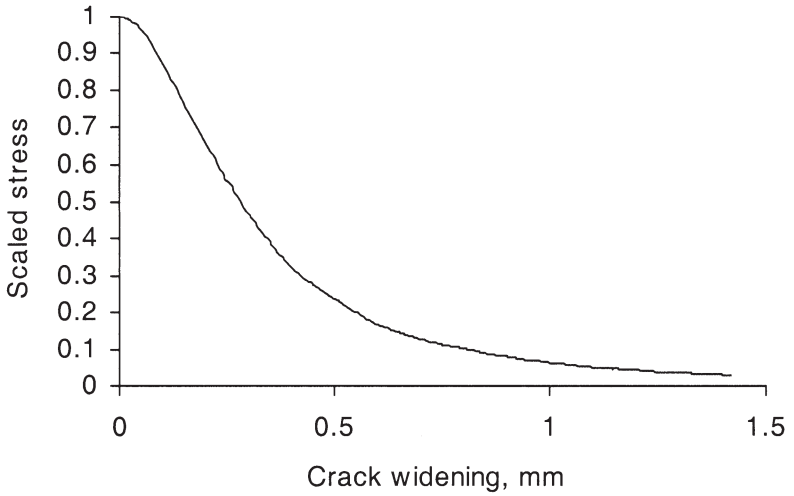

Figure 3 A cohesive stress–crack widening curve. The stress values are renormalized by the maximum value.

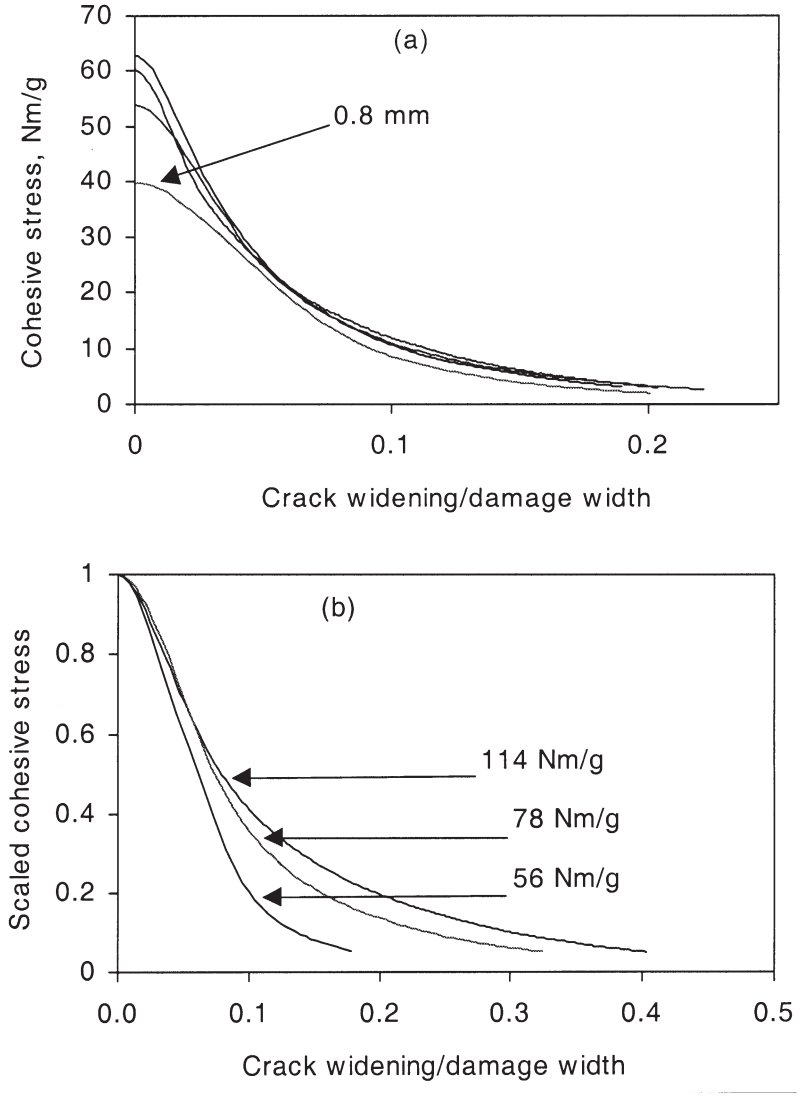

Figure 4 Cohesive stress vs. crack widening values over damage width for samples with different fiber length (a) and different fiber strength (b). Fiber length was varied by cutting and reslushing handsheets and fiber strength was varied by HCl vapor exposure of handsheets. The original curves [7] were rescaled. Of different fiber lengths in (a), only the 0.8 mm case differs from the rest. In (b) the numbers indicate the zero span strength.

widening would give complete failure. A ductile paper, in turn, should have a long range cohesive stress.

The maximum value of the cohesive stress is practically equal to the ordinary tensile strength of paper [7]. It has no connection to range of the cohesive stress. In ideal conditions the area under the curve is equal to the fracture energy. Fracture energy is, in many cases, linearly proportional to damage width [8]. Thus even internal consistency requires that damage width should correlate with the range of the cohesive stress.

Figure 4 demonstrates that when only fiber length or only fiber strength is varied, the range of cohesive stress seems to be proportional to damage width. The cohesive stress measurements come from an earlier study by one of us [7] but we have now rescaled the crack widening values with damage width. The scaled curves for mean fiber lengths 3.1 mm, 2.2 mm and 1.4 mm collapse quite well as Figure 4a shows. Only the curve for the 0.8-mm long fibers deviates from the others.

In the case of variable fiber strength (Figure 4b), we divided the cohesive stress values with the maximum value of each curve. Of the original four samples [7], the one with the strongest acid treatment was excluded because the sample was so brittle that stable crack widening could not be seen at all. Even part of the curve for the second weakest sample (of zero span strength 56 Nm/g) is unstable, as revealed by the linear section in the curve. The other two curves (zero span strength = 114 Nm/g and 78 Nm/g) fall together, as expected.

Figure 5 displays the cohesive stress–crack widening curves for mixtures of PGW and spruce kraft pulp. Again, we have divided the measured cohesive stress values with the maximum value of each curve, and the crack widening values with damage width. In this case the curves fall almost completely on one another. The small deviations do not depend systematically on the mixing ratio of the two pulps.

There is even a reasonable match between the *shape* of the cohesive stress curve and the spatial distribution function of damage. Thus the damage seen in the silicone-impregnated specimens can be directly related to the gradual fracture process of the fiber network. Details will be reported elsewhere [9].

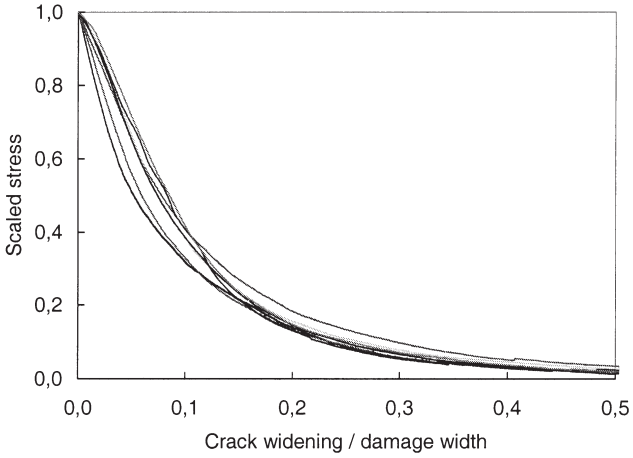

Figure 5 Cohesive stress curves for mixtures of PGW and coarse spruce kraft. The mixing ratio has no systematic effect on the curves.

PAPER STRENGTH VS. DAMAGE WIDTH

As the second demonstration of the validity of damage width as a measure of the size of the fracture process zone, we consider the tensile strength of paper. We have found that the following equation of modified linear elastic fracture mechanics approximates quite well the apparent strength and the ordinary tensile strength of paper:

$$\sigma_{\text{app}} = \frac{\sqrt{G_c E}}{\beta\sqrt{2\pi \cdot (a + w_d)}} \tag{2}$$

Here G_c is the fracture energy and E the tensile stiffness of paper. The factor β depends on the defect size, a, and the specimen width. Equation 2 is equivalent to the Irwin model of fracture mechanics [2], except that we cannot explain the empirical factor 2 in the denominator. It could arise from formation effects. The new feature of Equation 2 is the use of damage width in place of the theoretical correction term of the Irwin model.

In the ordinary tensile test, σ_{app} is tensile strength and $a = 0$ since there is no cut in the specimen. The failure usually starts from the specimen edge. This corresponds to a single edge notch (SENT) specimen. We used the following expression [10] for β.

$$\beta = \sqrt{\frac{2W}{\pi w_d}\tan\left(\frac{\pi w_d}{2W}\right)} \cdot \frac{1}{\cos\left(\frac{\pi w_d}{2W}\right)} \cdot \left[0.752 + 2.02\,\frac{w_d}{W} + 0.37 \cdot \left(1 - \sin\left(\frac{\pi w_d}{2W}\right)\right)^3\right] \quad (3)$$

where $W = 15$ mm is the specimen width. Typical values in this case are $\beta = 1.2 - 1.4$.

For the apparent tensile strength we used

$$\beta = \left[0.998 + 0.043\frac{a + w_d}{W} - 0.311\left(\frac{a + w_d}{W}\right)^2 + 1.87\left(\frac{a + w_d}{W}\right)^3\right] \quad (4)$$

where $2W = 50$ mm is the width of the specimen that had a $2a = 20$ mm-wide cut in the middle. Equation 4 comes from a numerical fit to the curve in Reference 12 for a specimen with a cut in the middle. We used the numerical expression because the ratios $(a + w_d)/W$ varied a lot in our sample material. Typical values for β in this case are $\beta = 1.1 - 1.2$.

Figures 6 and 7 compare Equation 2 with measured values of the apparent

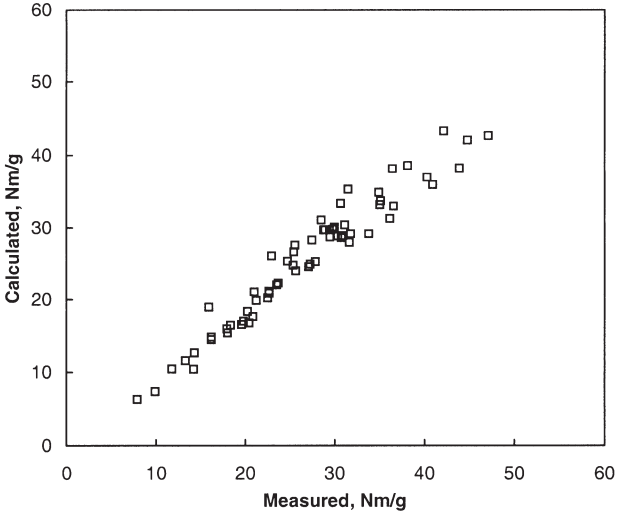

Figure 6 Calculated (Equations 2 and 4) vs. measured values of the apparent tensile index of handsheets containing various chemical pulps and their mixtures with mechanical pulps.

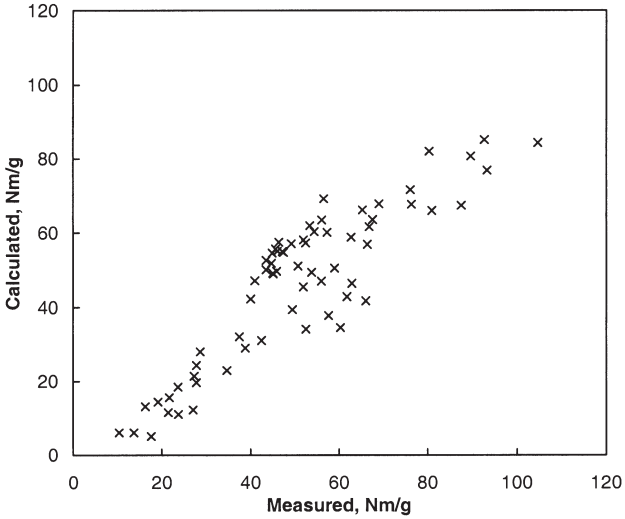

Figure 7 Calculated (Equations 2 and 3) vs. measured values of the ordinary tensile index of handsheets containing various chemical pulps and their mixtures with mechanical pulps.

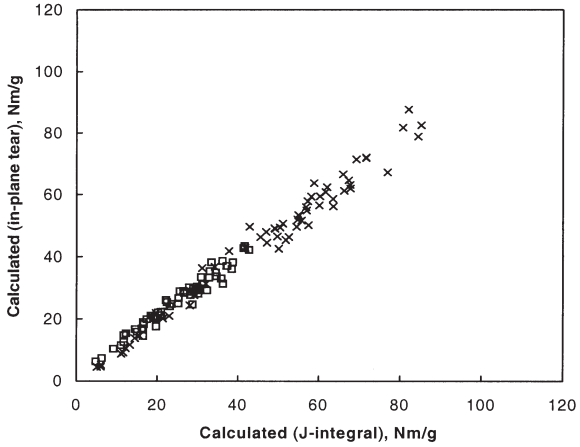

Figure 8 Calculated values of the apparent (squares) and ordinary tensile index (crosses) of handsheets containing various chemical pulps and their mixtures with mechanical pulps. Comparison of in-plane tear test and J-integral method for measuring fracture energy.

tensile strength and the ordinary tensile strength. The data are all for ISO 5269-1 handsheets (i.e., plate-dried), containing a variety of chemical pulps and their mixtures with either TMP or PGW. The agreement is reasonable in both cases. The deviations between Equation 2 and the measured tensile strength values in Figure 7 can come from at least two sources. Firstly, Equation 2 is not always theoretically valid for the relatively narrow (15 mm) tensile specimen. Secondly, Equation 2 assumes a completely uniform paper structure. The formation of paper reduces tensile strength from the prediction of Equation 2. We have made no attempt to estimate this effect.

The fracture energy value G_c used in Figures 6 and 7 came from the in-plane tear test [11]. We also had data from the Lorentzen and Wettres J-integral test. The two methods give essentially the same prediction for tensile strength (Figure 8). This suggests that eventual non-ideal features in these two fracture energy measurements are insignificant.

The quantitative link from damage width, fracture energy and elastic modulus to the tensile strength of paper in Equation 2 is still fairly rudimentary. The effects of formation and test dynamics can be included as discussed by Donner [2]. It is also possible that details of the evaluation method for damage width can be refined for better accuracy in the modeling of tensile strength. The magnitude of damage width depends on the choice that we made regarding the edge of the fracture process zone (the 10% gray level threshold). The simple addition of damage width and defect size in Equation 2 would change if a different choice were made in the evaluation of damage width.

DAMAGE WIDTH VS. FIBER PROPERTIES

The experiments that we have done with various handsheets demonstrate how damage width depends on fiber properties such as fiber length, strength, inter-fiber bonding and fiber segment activation [7,8,12,13,14]. For example, when fibers were made shorter by cutting and fractionating [7], damage width w_d decreased with the mean fiber length (Figure 9). Damage width was approximately twice the mean fiber length [12]. The fraction of broken fibers was low, 5–12%, and could not influence damage width.

Figure 10 shows data for mixtures of chemical and mechanical pulp fibers. Again, one can see how damage width is almost linearly proportional to the mean fiber length. The fraction of broken fibers was low also in these samples. Thus, when fibers do not break in the fracture process, damage width seems to be simply proportional to mean length of the fibers that cross the fracture line. The proper fiber length is the mass-weighted mean fiber length

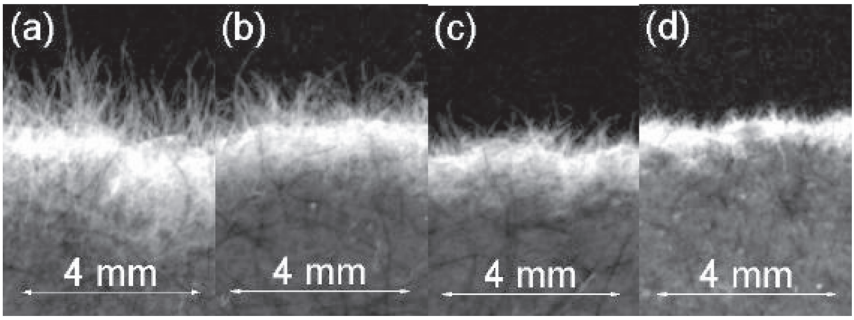

Figure 9 Fracture lines in handsheets of bleached softwood kraft pulp. Fiber length, changed by cutting, increases from (a) to (d). Black area is scanner background (at top), white damage zone (middle) and dark gray undamaged paper (at bottom). The arithmetic mean fiber lengths (measured using Kajaani FS-100) were 0.7, 1.2, 1.9 and 2.8 mm and the length-weighted mean fiber lengths 0.8, 1.4, 2.2 and 3.1 mm, respectively [12].

(see Appendix). It cannot be measured. The closest approximation is the length-weighted mean fiber length.

In the case of Figure 9 there is little difference between the arithmetic and length-weighted mean fiber length because the fractionated pulps had narrow length distributions. Figure 10 uses the length-weighted mean fiber length. Here the measured damage width is equal to about 1.2 time the mean fiber length. If the arithmetic mean fiber length were used instead, the ratio would be about 2.2 [12]. The value of the ratio has no fundamental significance since the value of damage width is fixed by the arbitrary choice of the 10% gray level threshold.

In addition to fiber length, there are many factors that can affect damage width. These include fiber strength and inter-fiber bonding. Reduction in fiber strength or enhancement in inter-fiber bonding often cause a reduction in damage width. For example, very highly beaten chemical pulps give very narrow damage width [14]. In contrast, low inter-fiber bonding or high drying shrinkage give a large damage width compared to the mean fiber length [13].

When considering the effect of lower fiber strength, higher inter-fiber bonding, or lower drying shrinkage, many of us think intuitively that these changes make paper more “brittle” or less “ductile”. The corresponding changes in damage width are consistent with this interpretation. Brittle papers are glass-like and have a narrow fracture process zone or a narrow damage width. Ductile papers, on the other hand, have a large fracture process zone or a large damage width.

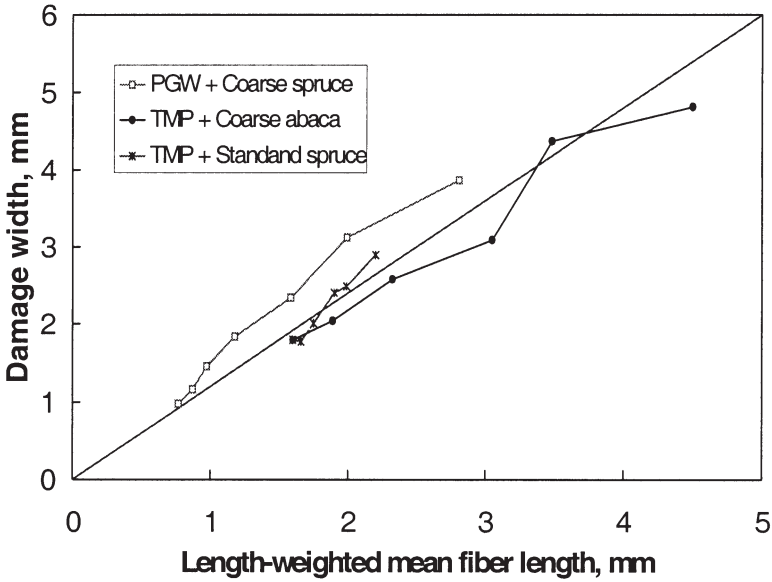

Figure 10 Damage width against the length-weighted mean fiber length in mixtures of chemical and mechanical pulps. The fraction of chemical pulp ranges from 0 to 100% (from bottom left to top right). The straight line is only for illustration.

SUMMARY

We have demonstrated how the visual observation of the microscopic fracture process zone in paper relates to the cohesive stress and strength properties of paper. The measurement of damage width gives a measure for size of the fracture process zone. Armed with this interpretation, one can use damage width to evaluate fiber properties such as the effective length and strength of fibers in paper. The results discussed here and elsewhere describe how fiber properties, sheet structure and drying stresses influence damage width and thus the strength of paper.

ACKNOWLEDGEMENTS

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APPENDIX: THE RELATIONSHIP BETWEEN FIBER LENGTH AND DAMAGE WIDTH

We calculate damage width w_d for an arbitrary pulp mixture, assuming that fiber length is the only relevant factor. Thus ignored are fiber fractures, fiber activation, etc. The principle of the calculation is to determine first the mass

of fibers that cross a unit length L of the crack line, and then set this mass proportional to paper grammage times L times w_d. This assumption is motivated because although the intensity of damage decreases continuously away from the crack line, the shape of the distribution does not vary a lot.

We use the following notation:

A	sheet area (m^2)
L	length of the crack line (m)
N	number of fibers in the sheet of area A
b	sheet grammage
l	fiber length (m)
ω	fiber coarseness (m/g)
x	mass fraction of a pulp fiber type (%)

The grammage of each fiber type in the sheet is

$$m = xb = \frac{N}{A}\omega l, \tag{A.1}$$

and the corresponding mass of fibers that cross the crack line is

$$M = \frac{2NL\omega l^2}{\pi A}. \tag{A.2}$$

For a mixture of different types of fibers, damage width is then proportional to

$$w_d \propto \frac{1}{bL}\sum_i M_i = \frac{2}{\pi}\sum_i x_i l_i. \tag{A.3}$$

where the sum is over the fiber types in the paper.

Thus should the damage width of a pulp mixture be proportional to the mean fiber length, weighted by the mass fractions of the pulps. The proper fiber length of each pulp type is, in principle, the mass-weighted mean fiber length but since this cannot be measured, the closest approximation is given by the length-weighted mean fiber length.

Figure 10 in the text shows that the measured damage width is equal to roughly 1.2 times the length-weighted mean fiber length. Considering the typical shape of the distribution of damage at a crack line [8], we can estimate

that the mass of the fibers contribution to damage width is equal to roughly half of paper grammage times L times w_d. This suggests that

$$w_d \approx \frac{4}{3} \sum_i x_i l_i \tag{A.4}$$

which is in reasonable *quantitative* agreement with Figure 10.

Transcription of Discussion

DAMAGE WIDTH: A MEASURE OF THE SIZE OF FRACTURE PROCESS ZONE

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Kit Dodson Department of Mathematics, UMIST

In the case of the presence of long kraft fibres for example then the decay from the very white centre of the fracture zone back to black, or whichever way around it was, was actually very slow, and it seemed proportionately longer than simply the length of the fibre. You have very sparse threads and in the fitting the model underestimated the strength. So you overestimated the fracture zone width in some sense. The other point that occurred to me is that the word strain is perhaps not the best thing for the 25%, this is a relative displacement, and it bears little relation to strain in the sense that strain is defined as response to a force, which is actually a movement. It isn't a change in state of a fixed entity.

Kaarlo Niskanen

Yes, I agree with what you are saying.

Eero Hiltunen Helsinki University of Technology

I wanted to ask the following: you mentioned that there is a very large calculated strain in the fracture process zone, in some cases it could be 25%. Has this any relationship to the old measurements by Giertz and Retulainen where they had microscopic evidence that the local strain in some fibres can be almost ten times higher that the total breaking strain of the paper.

Discussion

Kaarlo Niskanen

Yes I think this is good evidence. We ourselves measured the local strain at crack using the video image correlation technique and found something like 10% perhaps even more. So I do think that this (the local strain) is not complete rubbish. It is a real physical number, but at this point the evidence is somewhat scattered, and so I did not want to pursue the issue any further.

Kit Dodson

Can I just ask about the visibility of the white zone – the grammage, what size grammage can you go to for it still to be visible and representative.

Kaarlo Niskanen

Certainly not board samples. I think that we would at least for kraft hand sheets, we have gone up to 100 grammes per square meter. Somewhere there you start to lose the contrast.

Tetsu Uesaka Paprican

The reason why I am interested in this parameter, is that I introduced in the previous presentation that the characteristic length scale is exactly in the same range as the so called damage width that you defined. In your equation suppose you reduce the crack lengths into the level of wd – damage width, you still see quite a sharp dependence on the crack lengths, that is wd = 1 mm, introducing .0.5 mm cracks, you still see some strong dependence on the crack width according to the equation. Did you see such phenomenona in your test?

Kaarlo Niskanen

That is still undergoing work, but definitely if the crack size becomes small it no longer has any effect. According to the preliminary results the stress level for the smallest crack sizes it is not only related to the damage width, but also something like the formation amplitude come in. I think that it must, when we learn more we can combine the formation type of process you were describing with this defect induced factor.

Tetsu Uesaka

That is the point, when the crack size is reduced toward the order of wd then obviously no geno-elastic, or whatever the fracture may characterise type description can not be applied. So it is very difficult to do, the point is what to do in this area – just a comment.

Kaarlo Niskanen

Since we are not in a terrible hurry, I would like to point out that if we measure damage widths that are of the order of several millimeters, that is already the same order of magnitude as the width of the tensile specimen, and theoretically the expression we have given for tensile strength is invalid. The fracture process zone or defect is too large compared to the specimen size. It basically says that if you want to measure tensile strength then the standard specimen width is somewhat problematic. The specimen width should be larger. That is a comment on your comment.

Al Button Buttonwood Consulting

Have you seen impact of non-uniformities that Tetsu has talked about before in your testing and how do you accommodate that reality? I think that it is well established that this Weibull distribution is a big factor in what we see in performance, and wondering how you handle that non-uniformity in that type of approach?

Kaarlo Niskanen

This is the 'fudge factor' 2 I had in the equation. I was talking about this number 2 which seems to do a reasonable job in taking account also formation effects. This is all hand sheet data, machine made paper might require a different value for the constant. Basically the deviations between predicted and measured are at least partly due to variations in the formation of the specimens.