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THE INFLUENCE OF THROUGH-THICKNESS VARIATION ON THE MECHANICS OF PAPER DRYING

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ABSTRACT

A material model for drying paper is presented. Moisture-dependent material parameters, hygroscopic shrinkage, the elastic and the time-dependent responses of the material to load, and the effect of unloading at a higher stiffness than the load was applied at are modelled. The model is used to determine the effects of a varying moisture ratio through the paper during drying on free shrinkage development and stiffness development at free drying. Simulation results for the stress development during drying and the state of residual stress immediately after drying are also presented. The model predicts a variation of in-plane elastic moduli through the paper, a prediction that is studied by experiments.

INTRODUCTION

Paper is thin compared to its in-plane dimensions and is often treated in a simplified manner as a 2D (two-dimensional) structure. Variables thus may vary for different positions within the web, but not through the thickness. However, the effects of a possible through-thickness variation should be known in order to clarify when the 2D approximation is appropriate. In this paper, numerical simulation is used to study the influence of a z-dependence during drying, *z* being a coordinate in the thickness direction.

Simulation requires a model for the mechanics of paper drying. Such a model would be useful in many respects. The mechanisms involved during drying are difficult to separate and study independently, and seeing the whole picture would aid the understanding of the underlying physics. This would help in product development and design of drying sections for paper machines. A model could also enable tailoring of the paper properties to the requirements on end-use properties and on runnability in converting. The solidification of paper has some general similarities with phases of the manufacturing processes for other materials such as the injection moulding of polymers and the curing of wood. Many of these have been modelled through the years and some of the aims of the modelling are common with paper. The models are however very specific to the material and process in question. There are, to the knowledge of the authors, very few attempts made to model the mechanics of paper drying. In 2003, Mäkelä [1] presented a material model for drying paper with moisture-dependent mechanical properties. Finite elements simulations were performed assuming this material behaviour at every in-plane position of the web. In [2], a similar model was used.

The possible variation of state variables through the thickness of the web was neglected in these studies. It is known from experimentally determined moisture profiles of boards and pulp sheets that the moisture ratio varies significantly through the web during drying when the moisture ratio is below the fibre saturation point (FSP). The FSP is the highest moisture ratio at which all the water is contained within the fibres, which would typically be before the drying begins or soon after the drying starts. Given the moisture-dependence of the material properties, the moisture gradients imply a variation of the mechanical state variables as well. The presence of residual stresses in paperboard after drying – a distribution of in-plane stresses through the unloaded board – is interpreted as an effect of this z-dependence during drying [3].

In this paper a material model is presented that is based on [1] but takes the z-direction dependence into account. It should be quite possible to use the model also for paper webs and whole drying sections, though this requires numerical tools like FEM and this course is not pursued here. Instead, the model was used to study the mechanics of a paper subjected to a homogeneous loading situation during drying, i.e. with no inplane variation. These conditions would also correspond to a typical point on a paper web. The objective was, as mentioned above, to analyze the effects of the z-dependence of the moisture ratio on the mechanical properties and their development during drying. The purpose of modelling in this context is thus to increase the knowledge of the drying process, rather than to obtain quantitative results.

The papers of primary interest to study were 300 g/m^2 boards made from kraft pulp and dried in the biaxial drier at STFI-Packforsk [4], used in a previous study [5]. All experimental data in this work are for these boards when not stated otherwise. However, data from experiments on other boards have been used in some instances. In a few calibration experiments, machine-made 75 g/m^2 papers made from kraft pulp and dried in the same biaxial drier [4] were used. In two verification tests, a machine-made 300 g/m^2 board made from all chemical pulp, from another previous study [6], and a board made from six 86 g/m^2 papers formed and pressed separately and couched together before drying were used. That the data refers to these papers are stated explicitly in the respective cases.

MATERIAL MODEL

Drying paper is a complex material. Paper is hygroscopic and decreases its dimensions at desorption of water, a phenomenon that is analogous to thermal contraction at cooling. The mechanical properties of paper change during drying. A paper is loaded during drying, and will contract less when released than it expanded upon loading, due to the increase in stiffness. Furthermore, paper exhibits time-dependent material behaviour when loaded. These are considered the basic features of drying paper and are taken into account in the proposed model. The calculations are performed incrementally, since the phenomena depend on state variables that are typically not known beforehand.

The basics of paper drying is a decreasing moisture ratio, u. The variation of the moisture ratio through the board during drying is the source of the effects to be discussed in this work. The moisture gradients originate in the moisture transport from the paper to the ambient environment being quicker than the moisture transport within the paper.

In the equations below, greek indices α and β each stand for either MD or CD, $\alpha \neq \beta$. All the equations thus represent two different relations depending on the “value” of the α on the left side of the equality sign.

Hygroscopic shrinkage

The in-plane change of dimensions of the paper from de-swelling of the fibre material was modelled by hygroscopic strains ε_α^{sh}. The hygroscopic strains were assumed to be functions of the moisture ratio, see Equation (1). As a first approximation, the evolution of the hygroscopic strains was obtained from curves fit to data of free shrinkage against moisture ratio, as in Figure 1, from which values for the fitting parameters $C_{1\alpha}$, C_2 and C_3 are obtained. The moisture data for the experiments in Figure 1 were obtained through a calibration curve relating time to moisture ratio, a curve obtained by weighing of partly dried specimens.

$$\varepsilon_{\alpha}^{sh} = C_{1\alpha}\left(1 - e^{C_2(u_{FSP} - u)^{C_3}}\right) \tag{1}$$

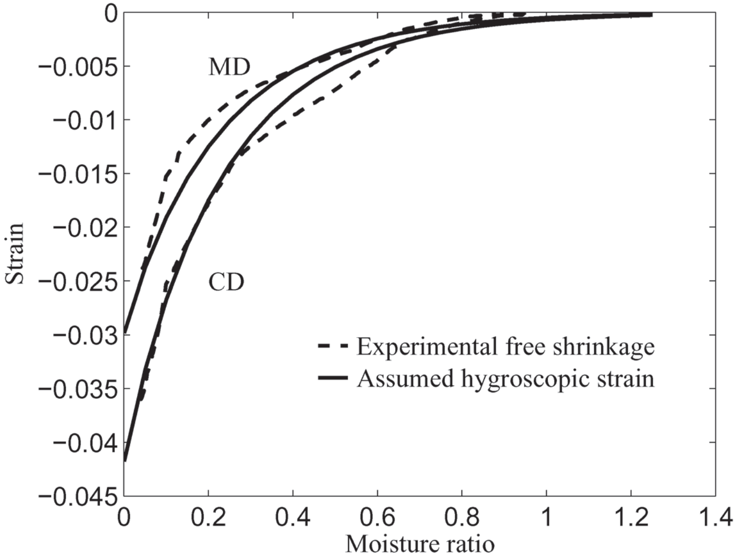

Figure 1 Free shrinkage data and fitted curves.

The expression in Equation (1) assumes that the in-plane shrinkage begins at the FSP (u_{FSP} being the corresponding moisture ratio), which agrees well with the literature. This formulation captures that increased beating, which increases the swelling of the wet pulp, causes free shrinkage to begin at a higher moisture ratio. Hornification might affect the reversibility of the process, but for drying this is not an issue. As an alternative to determining the FSP, the water retention value, WRV, can often be used. In this common centrifugation test, some water is expected to be pressed out of the fibres, while approximately the same amount of water is left in the pores [7]. The WRV would then be a good approximation of the FSP.

Elastic moduli development at free drying

The development of the elastic moduli E_α at free drying was modelled by Equation (2). This equation was fitted to the experimental data in [1], to determine the parameters $E_{\alpha,wet}$, $E_{\alpha,0}$, A_2 and A_3. The behaviour was then scaled by changing $E_{\alpha,0}$ (as well as reducing the anisotropy in $E_{\alpha,wet}$ slightly), for the model to accurately predict the stiffnesses in the dry state of the board considered here. Figure 2 shows the obtained stiffness development. The Poisson ratios $v_{\alpha\beta}$ were estimated from the Baum approximation [8] using the relation between the Poisson ratios that applies for orthotropy, as in Equation (3).

$$E_\alpha = E_{\alpha,wet} + E_{\alpha,0}\left(1 - e^{A_2(u_{FSP} - \mathrm{u})^{A_3}}\right) \tag{2}$$

$$v_{\alpha\beta} = 0.293\sqrt{\frac{E_\alpha}{E_\beta}} \tag{3}$$

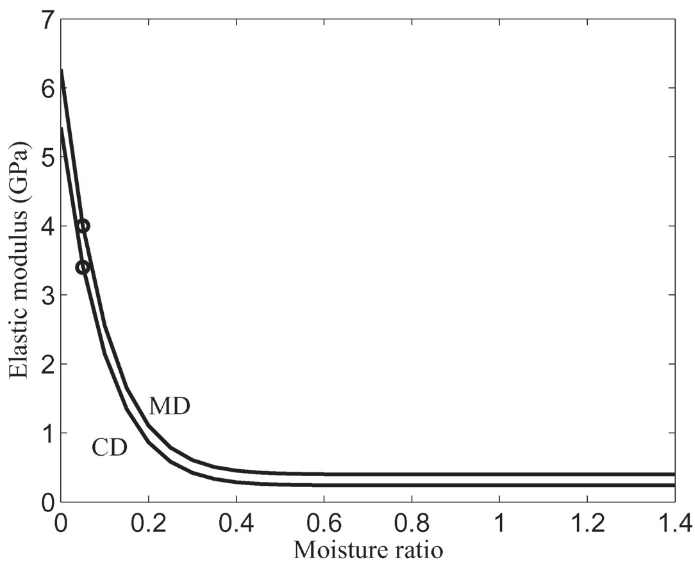

Figure 2 Stiffness development at free drying, with circles representing the experimental dry stiffness data of the studied freely dried board.

Stiffness development was thus just as the hygroscopic shrinkage assumed to start at the FSP, i.e. when the dewatering of the fibres starts. Equation (3) was assumed to hold throughout the drying, which finds some support in the literature [9].

Loss of elastic strain due to stiffness increase

Since the moisture ratio varies in the through-thickness direction of the sheet, so do the hygroscopic strains. As the layers of the paper are bonded to one another, it is a fair approximation that the total strains ε_α do not vary with position in the through-thickness direction for a flat board. The difference locally between total strain and hygroscopic strain would lead to the development of elastic strains ε_α^e. The presence of elastic strain brings in two other phenomena. Firstly, the stiffness increases during drying, which results in a loss of elastic strain as the paper would be unloaded at a higher stiffness than it had when the elastic strain developed, see Figure 3. Inelastic strains ε_α^r would then increase by the same amounts as the elastic strains decrease.

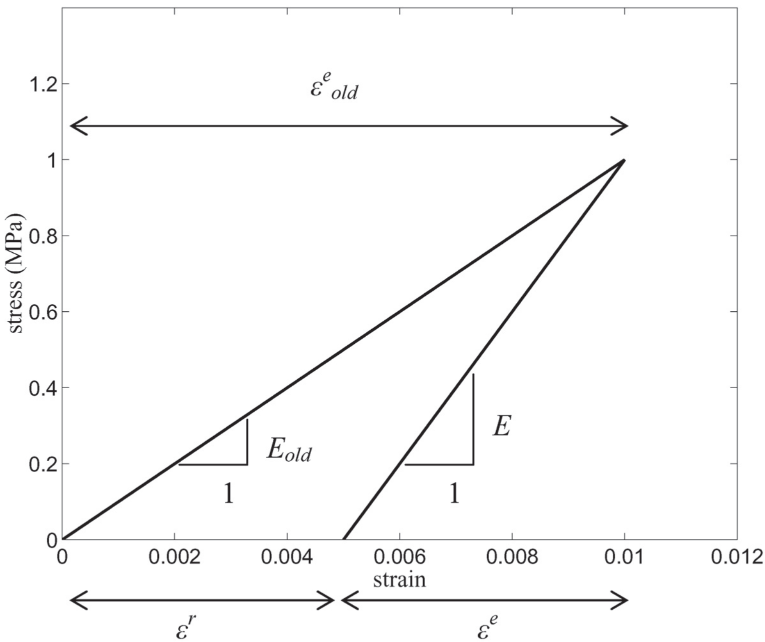

Figure 3 Illustration of the inelastic strain, which is the strain that remains when the stress (and the elastic strain) has returned to zero in the example.

Given that the current stiffness can be calculated, the developed inelastic strain in the increment is analytically known, see Equation (4). In Equation (4), $E_{\alpha,old}$ is the modulus before the increment and $\varepsilon^{e}_{\alpha,old}$, likewise, is the elastic strain before the increment.

$$\Delta\varepsilon^{r}_{\alpha} = \left(1 - \frac{E_{\alpha,old}}{E_{\alpha}}\right)\varepsilon^{e}_{\alpha,old} \tag{4}$$

Stress relaxation

The other phenomenon caused by the presence of elastic strain is stress relaxation, which can be treated as a time-dependent conversion of elastic strain to viscous strain ε^{v}_{α}. Results from stress relaxation tests of paper at room temperature and various moisture ratios are shown in Figure 4a. Machine-made 75 g/m^2 papers made from kraft pulp taken out of the machine after pressing were used. The various moisture ratios were obtained by partly drying sheets without restraints in the biaxial drier at STFI-Packforsk [4]. The relaxation tests were performed by straining standard tensile test specimens using a hydraulic tensile tester. To preserve the moisture ratio during a test, the test specimens were wrapped in thin plastic foil before the tests. For the data in Figure 4a, the applied strain was 1.0%, and the time zero was the last data point of the test with zero force.

The experimentally measured line loads in Figure 4a exhibit the classic linear dependence on the logarithm of time (c.f. [10]) for much of the tests. This behaviour is modelled most easily by parallel Maxwell elements with a set of time constants τ_i, as in Equations (5) and (6), where Δt is the time increment and i indicates the respective parallel element. Five elements (N in Equation (5)) were found to be sufficient for the relevant time spans during drying. The material behaviour in Figure 4a can be modelled with the same set of time constants for all the tests, indicating that stress relaxation is practically independent of moisture ratio. This, and that the relaxation is linear in applied strain, is used in the Equations (5) and (6). The time constants during drying should be lower than those used to model the experiments in Figure 4a because of I) high temperature during the drying and II) mechano-sorptive effects due to the change in humidity during the drying. The time constants in Equation (6) were set (using the fitting parameter B_3) so that the MD and CD drying stresses predicted by the model at restrained drying matched measured values. The relaxation behaviour used in the model and the behaviour observed in the experiments are shown together in Figure 4b, for comparison.

$$\Delta\varepsilon_{\alpha}^{v} = \frac{1}{N}\sum_{i}^{N} \Delta\varepsilon_{\alpha}^{v,i} = \frac{1}{N}\sum_{i}^{N}\frac{\Delta t}{\tau_i}\,\varepsilon_{\alpha}^{e,i} \tag{5}$$

$$\tau_i = 10^{-2 + B_3 i} \tag{6}$$

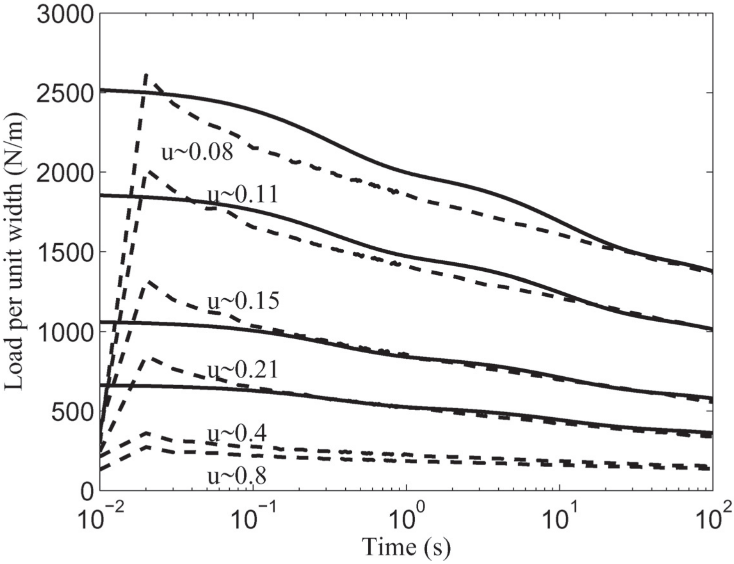

Figure 4a Stress relaxation data at various moisture ratios and 1.0% applied strain (dashed), and modelling of this relaxation (solid curves) for a few of the tests once the constant strain has been applied, using the same set of time constants for all the solid curves.

Figure 4a also shows that at very short times, relaxation was very fast although the strain was still increasing at that stage. However, since the elastic moduli in Equation (2) were determined from tensile tests, the relaxation at times shorter than about 0.1 s is captured by the moduli here and was not modelled as stress relaxation (i.e. by Equations (5) and (6)). Only at lower applied strain, there appeared to be slightly less relaxation of the dry paper, which is corroborated in the literature [11, 12] for the hygroscopic region. The strains typically become large during drying, so it is more important to model the behaviour at large strains, and relaxation tests at 2.0% and 3.0% applied strain showed the same trend as Figure 4a. The relaxation model will become

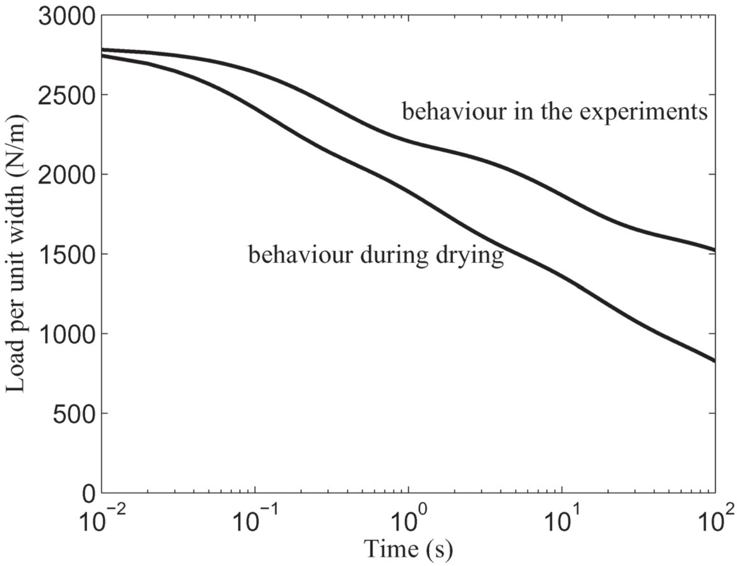

Figure 4b Simulations of relaxation tests. The upper curve is obtained with the time constants in Figure 4a, the lower curve shows the same test with the time constants used in the present drying simulations.

unstable for too large time increments, when more than 100% of the stress in the most viscous Maxwell element would be relaxed.

Constitutive equation

In order to use Equations (5) and (6) to model stress relaxation, the elastic strain in each parallel element of the viscoelastic model must be known. The elastic strain increments are calculated using Equation (7) assuming an additive decomposition of the total strain, i.e. that the different types of strain are independent. The inelastic strains were considered to be the same in each of the parallel elements when using Equation (7), since the inelastic strains are due to a change in elastic moduli and the viscoelastic formulation used is based on constant moduli. If the forces on the web are prescribed rather than the total strains, the latter have to be calculated from the known average stresses (force over total area) σ_{α}^{k} and the conditions of equilibrium, Equation (8). In Equation (8), n is the number of layers considered in the simulations. Remember that so that $\alpha \neq \beta$ each equation represents two different relations depending on whether α stands for MD or CD (β automatically being the other).

$$\Delta\varepsilon_{\alpha}^{e} = \Delta\varepsilon_{\alpha} - \Delta\varepsilon_{\alpha}^{sh} - \Delta\varepsilon_{\alpha}^{r} - \Delta\varepsilon_{\alpha}^{v} \tag{7}$$

$$n\sigma_{\alpha}^{k} = \sum_{j}^{n} \frac{E_{\alpha}}{1 - \nu_{\alpha\beta}\nu_{\beta\alpha}} (\varepsilon_{\alpha}^{e} + \Delta\varepsilon_{\alpha} - \Delta\varepsilon_{\alpha}^{sh} - \Delta\varepsilon_{\alpha}^{r} - \Delta\varepsilon_{\alpha}^{v} + \nu_{\beta\alpha}(\varepsilon_{\beta}^{e} + \Delta\varepsilon_{\beta} - \Delta\varepsilon_{\beta}^{sh} - \Delta\varepsilon_{\beta}^{r} - \Delta\varepsilon_{\beta}^{v})) \tag{8}$$

Once the strains have been updated (by adding the result of Equation (7) to their previous values), the stresses σ_{α} can be calculated with the orthotropic linear elastic constitutive law, Equation (9).

$$\sigma_{\alpha} = \frac{E_{\alpha}}{1 - \nu_{\alpha\beta}\nu_{\beta\alpha}} (\varepsilon_{\alpha}^{e} + \nu_{\beta\alpha}\varepsilon_{\beta}^{e}) \tag{9}$$

It is possible to express Equation (9) on an incremental form, which may be more suitable for implementation in a finite element (FE) code, and use an incremental form of (8) instead. The present form is preferred here because I) the reasons for the changes in stress are easier to see when the stresses depend only on the current elastic strains, and II) the accumulated round-off errors in the force equilibrium when the incremental form of (8) is used are avoided. Equation (9) applies to plane stress conditions. It assumes that loads are applied only in the principal directions of the material, and only such cases will be studied here. Also, Equation (9) is only valid for small elastic strains, which will be discussed further below.

Effect of tension during drying on elastic moduli

Finally, the stiffness is influenced by straining of the web during drying, relative to the shape the web would have had without external loads. This effect was modelled as stress-dependent, to be able to fulfil resulting stiffnesses both when different straining histories are used and when different drying conditions are used. Equation (2) is then replaced by Equation (10), where the effect of straining is included through the variables ΔE_{α}, which would have been zero for the case of free drying covered earlier. The ΔE_{α} were in the simulations updated after each increment according to the stresses the paper was subjected to during the increment, see Equation (11), to be used in the next increment in Equation (10). Drying experiments with different straining histories were used to study the effect of straining on the stiffness, as in Figure 5. The specimens were again machine-made 75 g/m^2 papers made from kraft pulp and the drying/straining took place in the STFI-Packforsk biaxial drier [4].

A linear stress-dependence in Equation (11) would result in that the stiffnesses of all the specimens strained at different parts of the drying were underpredicted compared to the papers strained for the entire drying history. One way of handling this deficiency is to instead model the effect of straining on the stiffness as a nonlinear stress-dependence as in Equation (11), which decreases the influence of the very high loads applied at the end of drying in all of the cases where the paper was strained for the entire drying history. This nonlinear stress-dependence instead causes underprediction of the stiffnesses of boards strained for the middle part of the drying. Together with a dependence on the moisture ratio at which the stress is active, this can be rectified. With a two-parameter (u_1,u_2) weight function f as in Equation (12) and Figure 6, the experimental results were predicted with satisfactory accuracy. This form (Figure 6) of moisture-dependence might be interpreted as a smaller effect of load at high moisture ratio because the load would prevent fibre bonds to form or develop fully, and a smaller effect at low moisture ratio because the fibres are less flexible when dry. It should be emphasized that Equation (12) is a simple function that exhibits the local maximum during drying. The values at either end of the curve may thus not be entirely realistic. It is quite possible to improve the predictions by using more parameters in Equation (12). The parameters $B_{1\alpha}$ in Equation (11) were set so that the increase in the stiffnesses when drying under restraint instead of freely was predicted correctly for the modelled board. Parameter B_2 was taken from the analysis of the experiments exemplified in Figure 5. The stress-dependence of the tensile stiffness would be linear up to a line load of about 500 N/m in Figure 5 with Equation (11) and the value of parameter B_2 used.

$$E_\alpha = E_{\alpha,wet} + (E_{\alpha,0} + \Delta E_\alpha)\left(1 - e^{A_2(u_{FSP} - \mathrm{u})^{A_3}}\right) \tag{10}$$

$$\Delta E_\alpha = \Delta E_{\alpha,old} + B_{1\alpha} f(u)\left(1 - e^{B_2(\sigma_\alpha - \nu_{\alpha\beta}\sigma_\beta}\right) \tag{11}$$

$$f(u) = \frac{u^3}{3} - (u_1 + u_2)\frac{u^2}{2} + u_1 u_2 u \tag{12}$$

Compressive stress was modelled to decrease the stiffness in the same way as tensile stress increases it (in Equation 11, this requires changing the sign of the stress and of the effect). Certainly, there is little experimental support for this. However, if compressive stress was to have no effect on the elastic modulus, the stiffness of a freely dried paper would be heavily influenced by

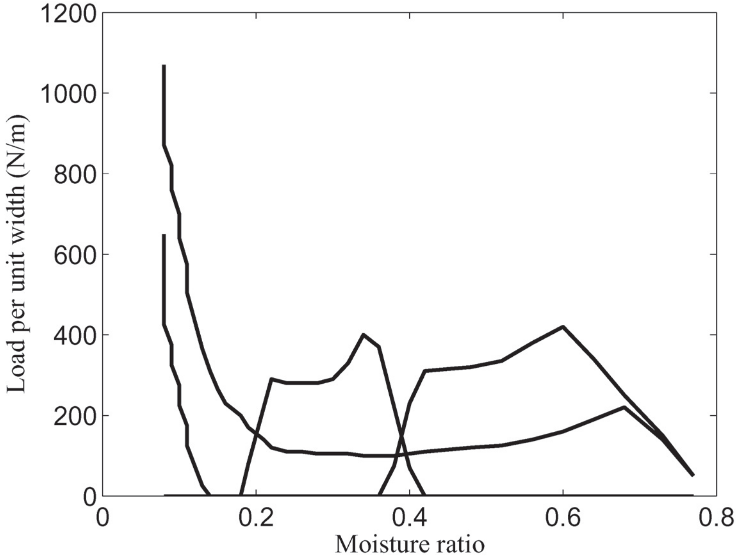

Figure 5 Uniaxial load histories during drying for four different tests, as support to the proposed model of the effect of drying restraints on stiffness (Equations (10–12)). The corresponding stiffnesses are given in Table 1.

Table 1 Measured tensile stiffness and corresponding model predictions for the tests in Figure 5.

Time interval of loading	Resulting tensile stiffness	Predicted tensile stiffness with Equations (11,12)
0–100 s	500 kN/m	520 kN/m
100–200 s	470 kN/m	455 kN/m
200–400 s	540 kN/m	490 kN/m
0–400 s	750 kN/m	750 kN/m

the tensile stresses present in individual layers during drying. For the 300 g/m^2 boards in this paper, the 'MD' elastic modulus for the hypothetical case of no stresses would then actually be lower than the corresponding 'CD' modulus. This is unlikely, since fibre orientation is expected to be the reason both for a higher stiffness after free drying and for a larger additional stiffness resulting from a given tensile load during drying.

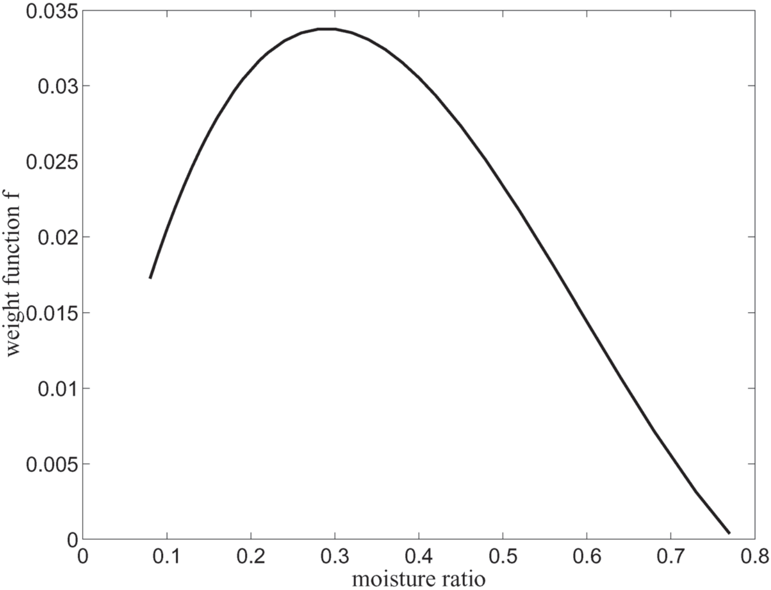

Figure 6 Weight function for the influence of stresses on elastic moduli (Equation (12)).

Equations (1, 3–12) form a mathematical material model for drying paper. Parameters such as A_2, A_3 and C_2, C_3, are probably not independent, but current knowledge is insufficient to state how they are related.

THE EFFECTS OF THE THROUGH-THICKNESS VARIATION

Simulations of the mechanics of paper drying were performed using the model developed in the previous chapter for a paper subjected to homogeneous loads. The values used for the parameters are given in the Appendix (Table 2). No boundary conditions are needed for points at the respective surfaces of the web, indeed the only coupling between layers is through the condition of equal total strain. The moisture history was prescribed using literature data. The input moisture history was based on the experimental data of Kirk and Jones [13], see Figure 7. With these data, the drying starts slightly above the FSP. Note that the thickness of the paper was not used in the model. It influences only through the prescribed moisture history. The

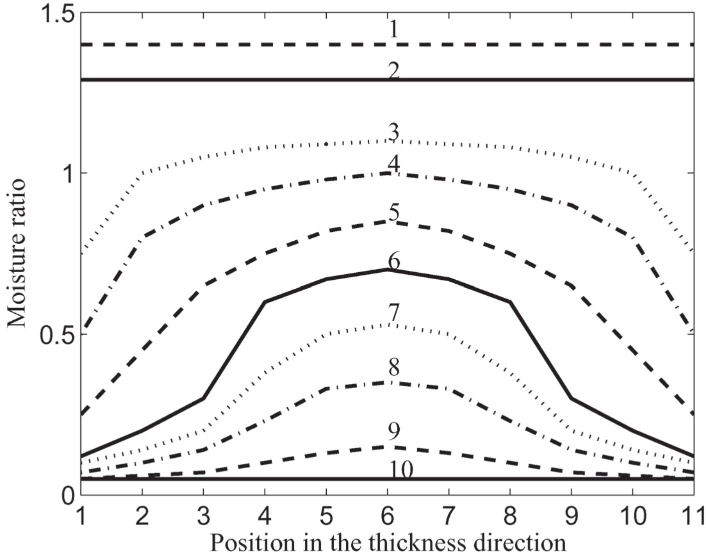

Figure 7 Moisture history used in the simulations, with 3 minutes between the states shown in the graph, the states being numbered to be able to reference them later.

moisture history in Figure 7 would correspond to quicker drying for a thin paper than it would for a thick paperboard.

First, the principal behaviour of the model will be shown. That different positions through the thickness experience different moisture histories is seen more clearly when the moisture data are plotted as in Figure 8a. The corresponding shrinkage is shown in Figure 8b for the different positions. These curves may be interpreted either as the hygroscopic strain in the different layers at free drying (from Equation (1)), or as the total shrinkage of unconnected layers subjected to this moisture history. Figure 8c shows that the stiffness resulting from this moisture history (by Equation (2)) would also vary through the thickness during drying. In Figure 8d, the effect of enforcing the same total strain in all layers is seen. Stress builds up in response to the constraint, even though no loads are acting on the paper. The total strain (through Equation (8)) and the stiffness (through Equations (10–12)) would also be affected by the constraint, compare Figures. 8b,c. The effects of the through-thickness variation is studied in more detail below.

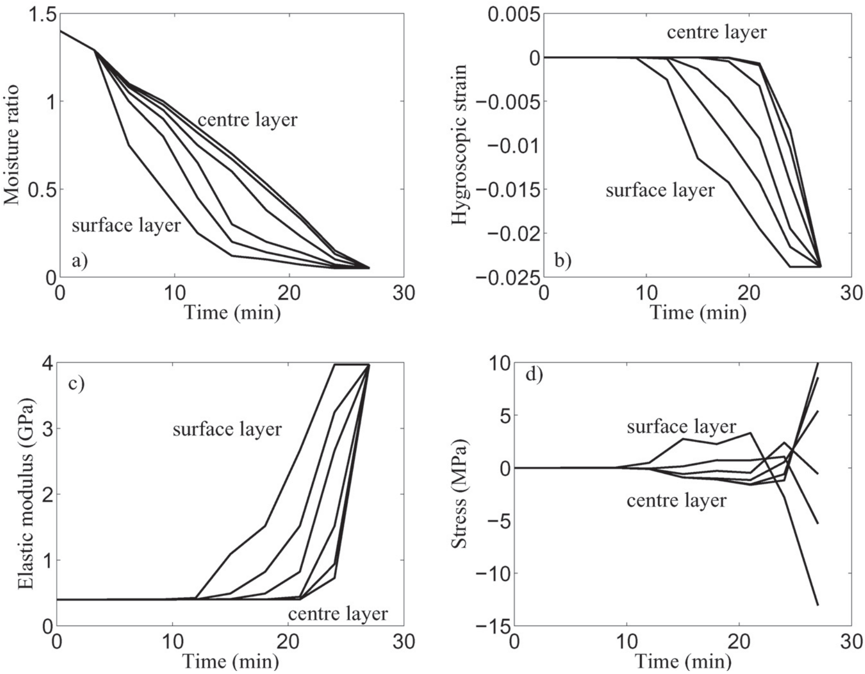

Figure 8a,b,c,d In a), moisture history for the different layers. In b), hygroscopic shrinkage history for the different layers. The surface layers dry and shrink first. In c), elastic stiffness development for the different layers. The surface layers stiffen first. In d), stress development at free drying for the different layers. The surface layers end up with the most compression.

The development of shrinkage and stiffness at free drying

The first effect to be discussed is seen when a simulation is run for conditions of free shrinkage (prescribed forces equal to zero). The total strain predicted by the model is plotted against average moisture ratio in Figure 9 together with the hygroscopic strain relation used in the model and assumed to hold locally for every point in the material. The simulation shows a higher rate of shrinkage at the first part of the drying than a gradient-free paper would have (all the layers and the board as a whole would follow the local curve then). Toward the end of drying, the rate of shrinkage of the board (with a gradient) is lower than that of the local curve, yielding roughly the same total shrinkage after the completed drying process. This effect of the z-dependence is caused by the accelerating behaviour of shrinkage against

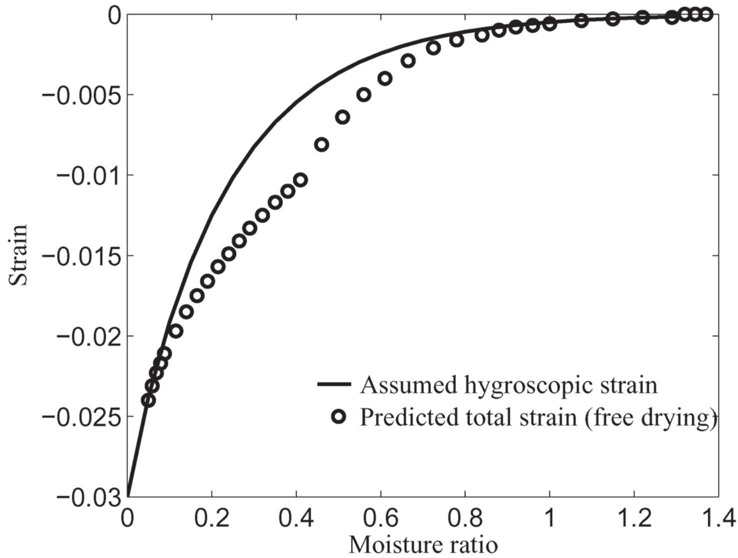

Figure 9 Free shrinkage in the MD as a function of moisture ratio.

moisture ratio (Equation (1)). The smaller shrinkage of the points with higher than average moisture ratio is not able to compensate for the higher shrinkage of the points with lower than average moisture ratio, due to the nonlinear relationship. The effect of the through-thickness variation increases and then decreases because the moisture gradients do. The total strain given in Figure 9 is not steadily accelerating. Instead, its second derivative changes sign twice. This is obviously not in agreement with the free shrinkage behaviour of paper.

A very similar result is obtained if tensile stiffness development during free drying is studied instead of free shrinkage development. This case may be easier to understand, as the elastic modulus of the board is simply the average of the moduli of its different layers. The importance of these results is that neither the free shrinkage development during drying nor the stiffness development at free shrinkage of paper as a material are known or can be obtained from experiments unless the papers are dried with a uniform moisture distribution. It is very difficult to dry paper without moisture gradients developing. The intrinsic material behaviour is of course only needed if variations within the paper are to be studied. (Preventing inplane variation of shrinkage to obtain such data is somewhat easier, using for example the biaxial drier of STFI-Packforsk [4].)

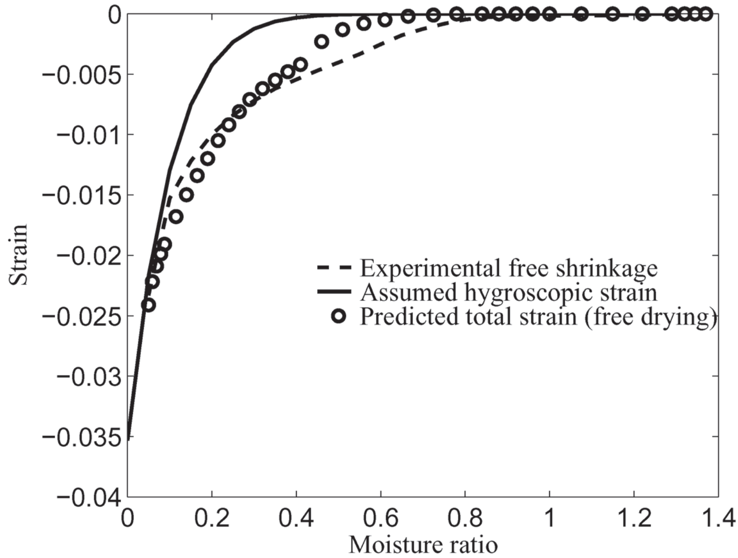

Figure 10 Free shrinkage in the MD as a function of moisture ratio.

The poor predictions of the model in Figure 9 can be remedied by adjusting the assumed local hygroscopic strain behaviour to yield results for the board in better agreement with the experiments, see Figure 10. The stiffness development would also need to be changed in the same way. These adjustments were done for the remaining simulations of this paper. The hygroscopic strain development in Figure 10 and assumed in the model was found by trial-and-error, which is not quite satisfactory. The local hygroscopic shrinkage behaviour is however reminiscent of data on fibre shrinkage (c.f. [14]). Perhaps it is not so surprising if the moisture dependence of the shrinkage of papers and of fibres is the same, as paper shrinkage would be driven by fibre shrinkage. Fibre data may thus constitute an alternative option for obtaining the shape of the hygroscopic shrinkage behaviour (though not the magnitude).

Residual stress

An effect of the through-thickness variation during drying that has been mentioned previously in the literature is the build-up of residual stress. Figures 11 and 12 show the predicted developments of MD stress as functions of

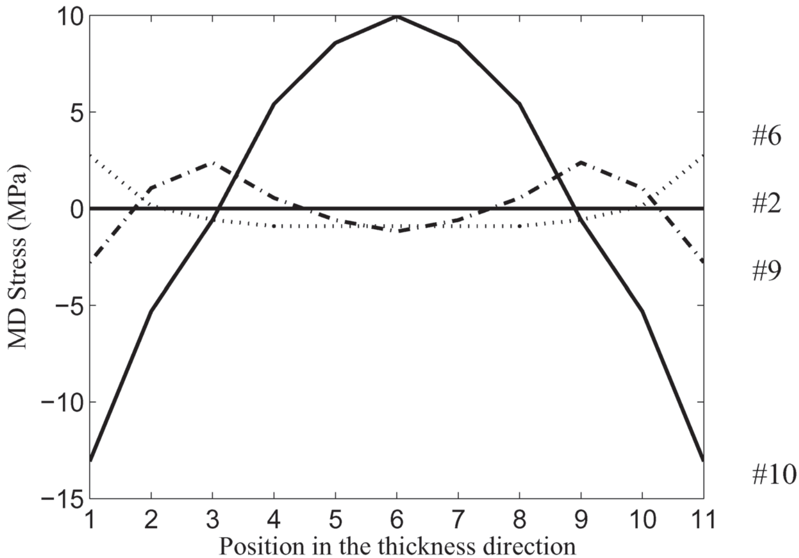

Figure 11 Distribution of MD stress through the thickness for different times during free drying. The number of the moisture profile in Figure 7 that the results correspond to is shown at the right end of the curves.

position in the thickness direction for free and restrained drying, respectively. Stress is built up even in the case of free drying because the shrinkage of the board, and of all its layers, is more than some of the layers would want and less than other layers would have shrunk without constraints, based on their respective moisture ratio. The results are well inline with the mechanism for residual stress build-up in paper suggested in the literature (c.f. [3] or [15]). Tensile stress is built up in a layer that would shrink (due to water removal) more than the board as a whole. Compressive stress is built up in the layers that do not want to shrink and that to some degree restrain the shrinkage of the board. This means that at the start of drying, tensile stress is built up near the surfaces as the moisture ratio decreases there (moisture being transported out through the board surfaces). The stiffness of the middle layers of the board is low and so are the stresses caused by this process. At the end of the drying, tensile stress is built up in the interior as moisture leaves while the middle layers are unable to shrink, and the surface layers are compressed trying to resist the shrinkage. These stresses are higher than those at the start of drying because the surface layers are now dry and have significant stiffness. In the case of the restraint-dried board in Figure 12, stresses built up in the

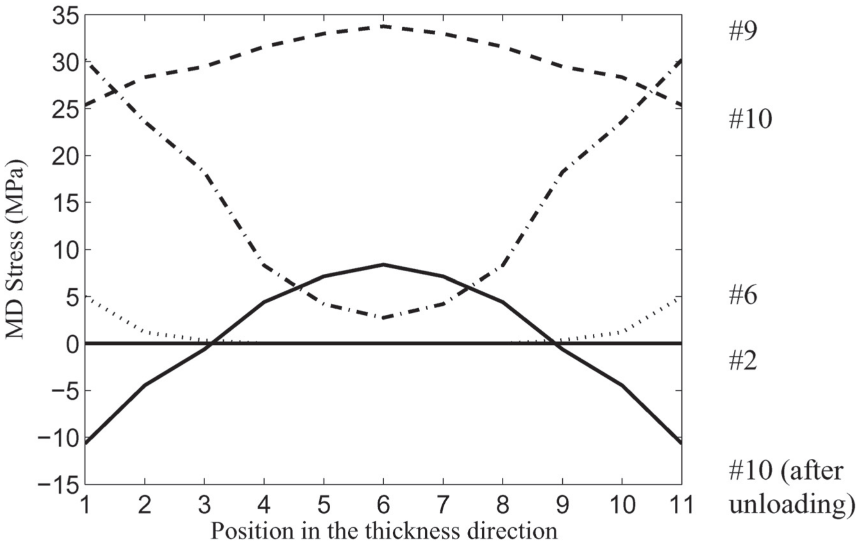

Figure 12 Distribution of MD stress through the thickness for different times during restrained drying. The number of the moisture profile in Figure 7 that the results correspond to is shown at the right end of the curves.

material as a response to the applied load (noticeable through the non-zero average stress) are superimposed on the self-equilibrating stresses caused by the moisture gradients. It may be noted that practically all the stress development takes place toward the very end of the drying, while the elastic strains that cause the stresses start developing already at the FSP.

Paper is a viscoelastic material, so the stresses should continue to change with time after the board is dry (i.e. the last state shown in Figures 11 and 12). The rate of change will however decrease steadily. The decreasing rate of change is partly because the stress in paper at constant strain depends linearly on the logarithm of time, and partly because the mechano-sorptive effects that accelerate the viscous processes during moisture changes will gradually decrease when moisture ratio is kept constant [16]. The latter may be due to fibre-level strains caused by the anisotropic shrinkage of fibres, acting as driving force for the additional relaxation at short times, with the understanding of mechano-sorption in [17]. In [16], it was interpreted as physical aging (increased free volume due to sorption).

The residual stresses in Figures 11 and 12 (the last states of these stress developments) are higher than the residual stresses found experimentally in the corresponding board (Figure 2 in [5]). It has been noted previously that the residual stresses determined from experiments do not differ significantly depending on when the test was performed (covering a time span of up to two years). This independence of the residual stresses on time is interpreted primarily as an effect of the linear dependence of stress on the logarithm of time. After the first few weeks, the rate of change of the stresses should be negligible. The stresses right after completed drying would then have been significantly higher (because of the high rate of change of the stresses at short times with a linear dependence on the logarithm of time) and may well have been in accordance with Figures 11 and 12, apart from the asymmetry of the experimental result. A smaller time-dependence of paper at small load [10] may also play a part in the independence of the residual stresses on time. The simulation results in Figures 11 and 12 thus confirm the experimental observation in [5] that the residual stresses after drying are practically independent of restraint during drying. Incidentally, all trends of the simulation results in this paper are the same in the MD and in the CD, since both originate from the moisture history.

A build-up of either inelastic or viscous strain would be sufficient in order for residual stresses to develop. The variation of elastic strain through the thickness that corresponds to the residual stress distribution must be balanced by another type of strain if total strain is to be kept constant. With plasticity not being included in the model, and with the assumption that the moisture ratio (and thus the hygroscopic strain) is constant in the dry paper, viscous and inelastic strains will together balance the elastic strains in the dry paper.

Through-thickness variation in mechanical properties after drying

The final important effect of the z-dependence in this work regards the variation of stiffness through the thickness of paperboard. The well-known increase of stiffness when paper is subjected to tension during drying was modelled as stress-dependent, and the stress varies through the thickness during drying. With this model, the varying stress histories in the different layers lead to variation of the elastic moduli through the thickness of the final board for all straining histories during drying. Studying the case of restrained drying (total strain prescribed equal to zero), the model predicts higher moduli at the surfaces compared to the middle of the board, see Figure 13. The simulation result can be interpreted as the drier surface layers having higher stiffness during drying, leading most of the load applied on the board to be taken up by these layers. A higher elastic modulus at the surfaces is positive regarding bending stiffness, which would be one reason to understand such an effect better.

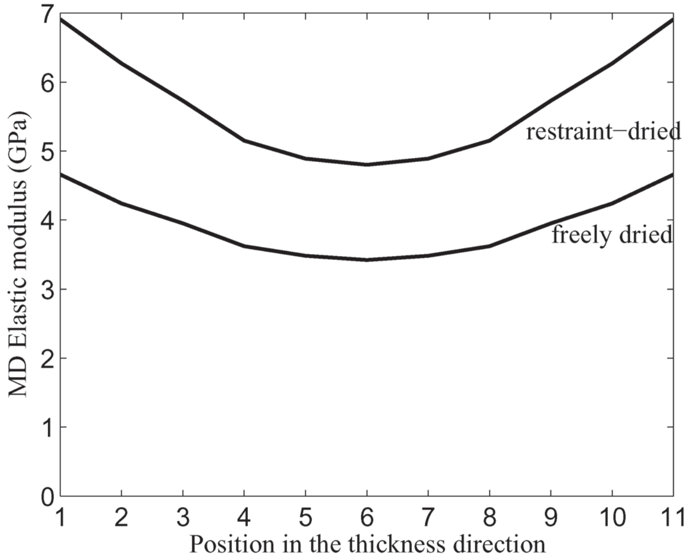

Figure 13 MD elastic modulus as a function of position through the thickness, simulation results for restrained and free drying.

Predictions of the elastic moduli for the case of free drying exhibit the same trend (see Figure 13) with this model. This result is however much more sensitive to the details of the model, in this case primarily the questionable effect of compressive stresses during drying on stiffness.

Certainly, experimental evidence of such an effect of the moisture gradients during drying is needed after seeing this model prediction. However, it is difficult to exclude from such experiments any variation in structural properties that may also influence the stiffness. To study experimentally the prediction of a varying stiffness, five different layers from a machine-made board made from all chemical pulp were isolated using surface grinding. This board was known from earlier tests ([6], Figure 4) to have residual stresses closely resembling the result in Figure 12. The residual stresses thus indicated not only the presence of moisture gradients such as in Figure 7 during drying, the stresses were also of a significant magnitude. Above all, however, the residual stress distributions were smooth, which means that the structural variation from ply to ply (it was formed in multiple plies) should be insignificant. A lower stiffness in a centre ply would cause lower stress to build up there during drying, which in turn would lead the average stress in the surface plies to be higher, because of equilibrium conditions. In some experiments, this has been observed to lead the maximum tensile stress of the board to be in the surface ply (at the interface), and would at any rate be observable as an anomaly in a stress plot. The stress distribution was practically parabolic in this case ([6], Figure 4).

The variation of the elastic moduli through this board, shown in Figure 14 as obtained from tensile tests on the isolated layers, corresponded very well

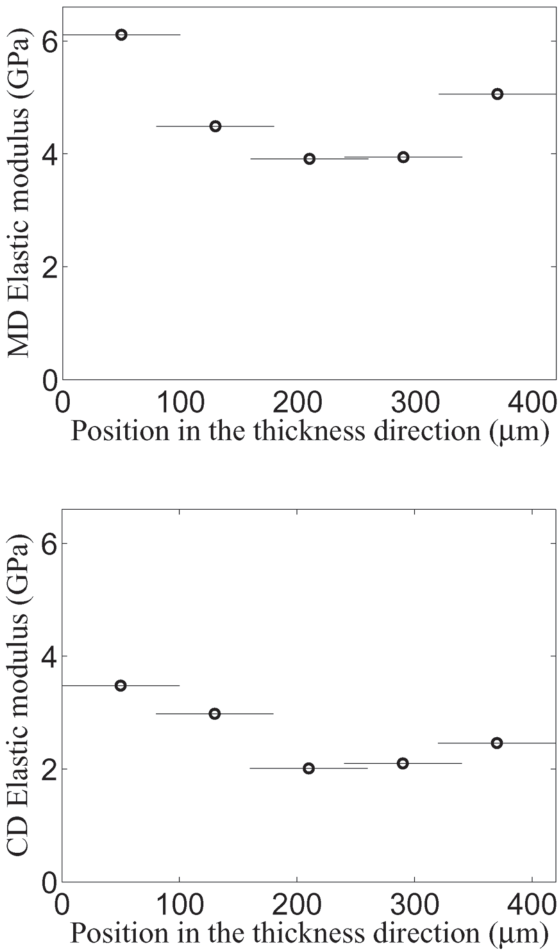

Figure 14a,b Experimental elastic moduli for layers of a board (circles), with horizontal lines showing the positions and thicknesses of the layers.

with the prediction in Figure 13. For the board in the experiment, there may still be other factors influencing the stiffness variation, such as a possible gradual variation in density. The varying opportunity for ZD shrinkage (larger near the boundary) would be one reason for density variation, similar to the more well-known CD shrinkage profiles. Still, the correlation between this result and the prediction indicates that moisture gradients during drying may be an important contributing factor regarding the through-thickness variation of stiffness after drying. Of course, many other mechanical properties are affected by tension during drying in a similar fashion to stiffness. CD results are also shown in Figure 14(b). That the trend is the same for MD and CD results would rule out a varying fibre anisotropy as a main reason for the experimental results.

However, to further study the prediction, the stiffness in the different plies was investigated also for another board, made by forming and pressing six 86 g/m^2 papers from kraft pulp. These were couched together on a drying cylinder with a drying wire tightened around the board to keep the drying restrained. The plies were after drying separated by hand, and tensile stiffness was determined in tensile tests. Figure 15 shows the results, with the cylinder side to the left in the graph. The stiffness variation is small if at all significant. If the draw-back of the first test (Figure 14) is that it relies on the residual

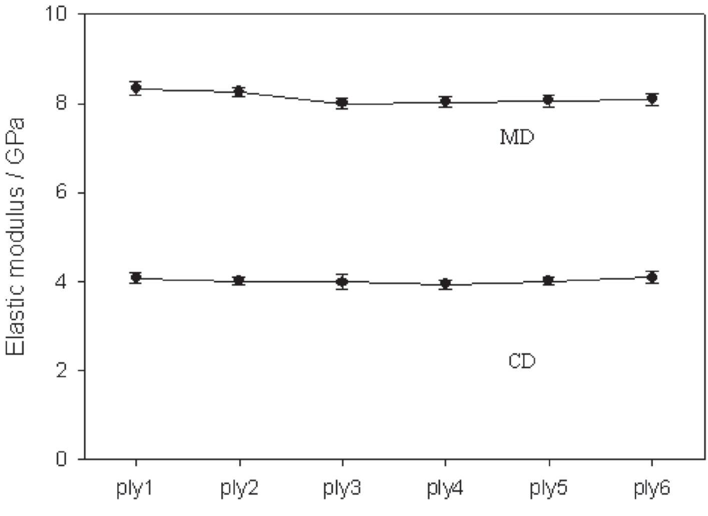

Figure 15 Experimental elastic moduli for plies of a board couched from papers formed individually and couched before drying, to avoid structural variation.

stress distribution to support both the supposed moisture history and its structural homogeneity, for this board the moisture history is completely unknown. More important perhaps is that plies that can be separated by hand may not transfer load to one another during drying, which would have eliminated the stiffness variation also in the simulations. The conclusion would be that the effect of moisture gradients during drying on variation in stiffness needs further investigation.

The results in Figures 13–15 should be put in the context of the result of [18] that the final stiffness is linear in total strain during drying. Total strain would be clearly dominated by inelastic strain (including viscous and perhaps plastic strain), which may vary through the thickness as opposed to total strain. As the boards in Figures 13 and 14 were known to have compressive residual stresses near the surfaces, the elastic strains would also have been compressive near the surfaces. The inelastic strains would then be higher near the surfaces than in the centre to achieve the constant total strain, and this correlates well with the higher stiffness near the surfaces in Figure 14 and

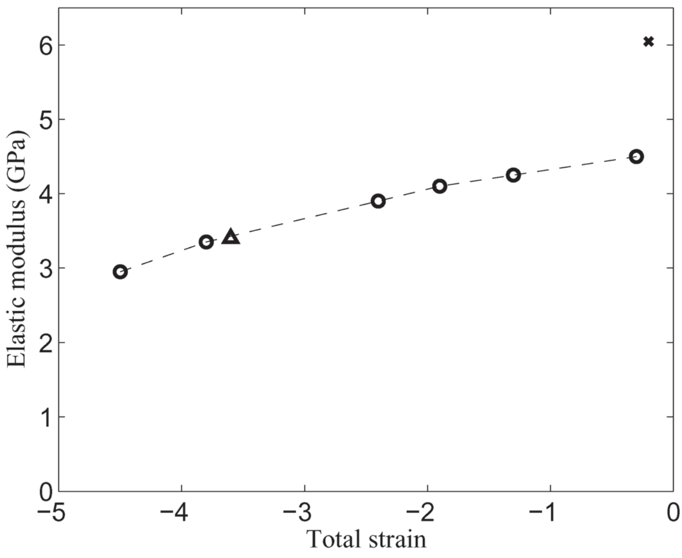

Figure 16 Elastic modulus simulation data as a function of total strain (%) after drying. The circles connected by a dashed line are the CD results for varying applied strain in that direction when the strain in the MD was restrained. The MD results for the same cases are shown by the crosses (all on top of each other). The triangle is the CD result for free drying in both directions and indeed falls practically on the dashed curve.

with the result of [18]. This is brought up as the inelastic strains indicate a possible structural reason for the higher stiffness of restraint-dried boards. Here, however, both inelastic strains and increased stiffness are seen as results of elastic strain present during drying. In [19], it was shown that stress relaxation during drying affects the final tensile stiffness negatively at restrained drying. The result of [19] may be captured by the phenomenological model used here, even if the reason for a lower stiffness with more relaxation is still unclear. The model presented here is able to capture the linear dependence of stiffness on total strain during drying (for conditions of equal drying time and temperature) even though it was not prescribed, which is exemplified by Figure 16. For the cases in Figure 16, load histories from restrained drying were scaled to obtain the different strains, in order to maintain the same distribution of the applied loads over the drying process.

DISCUSSION

A comment on the validity of the constitutive equation (9) and the possibility that plasticity would have to be modelled is in order. The surface layer of the simulated board reaches an MD elastic strain of 0.001 with a moisture ratio just below u=0.25. This is not plainly outside of the elastic region at that humidity. In the centre layer of the free-shrunk board, the MD elastic strain is predicted to be –0.0015 and still decreasing at a moisture ratio of u=0.7. Thus, the fibres probably buckle or yield. Much is however unclear regarding the compressive properties of paper and it could be that the fibres just straighten up when the elastic strain is reversed in a later stage of the drying process. After drying the values are –0.002 at the surface and 0.002 in the centre of the freely dried board according to the model. For the restraint-dried board, the elastic strains are positive in all of the board during drying and it is more a question of whether the elastic strain in the centre just prior to unloading (0.004) is below the proportional limit. It is not straightforward to compare these elastic strains with tensile tests, since residual stresses and the corresponding strains are typically unknown in such test specimens.

The model can simulate arbitrary loading histories. It enables for example the determination of the effect of a change in web tensions on mechanical properties such as tensile stiffness without actually changing the machine parameters and making the paper. If the development of strength properties of paper during drying were known, this could easily be used in the model and studied in the same way. Using Equations (1,3–12) as a material model in a finite elements model of a paper web would enable the investigation of variations in the CD of the web to find optimal drying strategies to avoid such problems.

Apart from the difficulty of calibrating the model, its biggest weakness would be the empirical descriptions of hygroscopic shrinkage and stiffness development, Equations (1) and (10), and to some extent (11). These phenomena should be modelled based on knowledge of the underlying physics, which is a demanding task. To achieve good resolution and accuracy of the results, the moisture data should be calculated from boundary conditions using a model for the moisture and heat transport and not prescribed from literature data.

CONCLUSIONS

A mathematical material model for drying paper with z-dependent state variables has been developed. It was used to investigate the effects of through-thickness variation on the mechanics of paper drying. The presence of moisture gradients through the thickness was shown to influence the free shrinkage behaviour and the stiffness development during free drying. These properties of paper as a material can therefore not be determined unless papers dried with uniform moisture distributions in the thickness direction are used. The model shows stress development in the board during drying that agrees well with experimentally determined stresses in paperboard after drying (residual stresses). Also, the model predicts higher tensile stiffness near the surfaces of paperboard than in its centre, which would lead to improved bending stiffness. Stiffness variation through the thickness was observed in one experiment, while a second experiment showed no stiffness variation.

ACKNOWLEDGEMENTS

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APPENDIX: VALUES OF THE PARAMETERS OF THE MODEL

Table 2 Values for the material parameters used in the model

u_{FSP}	1.3
C_{1MD}	–0.34
C_{1CD}	–0.493
C_2	$-4*10^{-3}$
C_3	13
$E_{MD,wet}$	0.4 GPa
$E_{CD,wet}$	0.24 GPa
$E_{MD,0}$	57 GPa
$E_{CD,0}$	50 GPa
A_2	$-1*10^{-4}$
A_3	29
B_{1MD}	7.65 TPa/(number of increments)
B_{1CD}	4.032 TPa/(number of increments)
B_2	$-0.2*10^{-6}$ Pa^{-1}
B_3	1.05
u_1	0.29
u_2	0.9

For the simulation results in Figure 9, the stiffness development was modelled using $E_{MD,0} = 51.5$ GPa, $E_{CD,0} = 45.5$ GPa, $A_2 = -0.004$ and $A_3 = 13$, which are the curves shown in Figure 2. Also in that simulation, the hygroscopic strains were modelled according to $\varepsilon_{MD}^{sh} = -0.03e^{-4.1u^{0.96}}$ and $\varepsilon_{CD}^{sh} = -0.042e^{-4.1u^{0.96}}$, which are the curves presented in Figure 1.

Transcription of Discussion

THE INFLUENCE OF THROUGH-THICKNESS VARIATION ON THE MECHANICS OF PAPER DRYING

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Øyvind Gregersen NTNU

In your results, you show that there was a reversal in the tension distribution through the thickness from 9% moisture content down to 5% moisture content. If I am correct, that must mean that, between these values, there is some moisture content corresponding to an almost flat tension distribution through the thickness. It should be possible to calculate that moisture content. Would that be of benefit? I guess that such sheets would be quite insensitive with respective to curl.

Magnus Östlund

Well, it could be quite a long answer for that question I think. That you would be able to obtain possibly fairly flat stress distribution at 0.07 does not mean that it is possible to produce board with no residual stresses. It would probably always have a fairly flat stress distribution at the moisture ratio slightly above the final moisture ratio. So, if you were drying to end moisture ratio of 0.09 perhaps you would have a flat stress distribution at 0.11 instead. So you should not interpret the results such that it is possible to produce board free from residual stresses.

Discussion

Øyvind Gregersen

So is the problem that you still have quite substantial moisture variation through the thickness of the sheet together with the flat tension distribution?

Magnus Östlund

Well, the reason for the stress reversal is that basically you have a situation where moisture is leaving only from the centre of the board. That is going to happen at the end of the every drying scenario.

Doug Coffin Miami University

I would just like you to clarify something on the last comparison shown between the commercial dried sheet and the sheet made in the lab. I think the difference in the modulus can be almost a direct function of how much shrinkage you have during drying. In a commercial sheet, you have some shrinkage variations through the sheet and you expect a modulus difference, and with your handsheets, you have dried them with restraint on both sides so that there is very little shrinkage through the sheets so that the modulus would not change much. That is my impression. I think you have to clarify this because what you said was contradictory.

Magnus Östlund

Well the simulation result indicated the stiffness variation for both restraint-dried and freely-dried so I would not expect any influence of shrinkage on the variation itself. Shrinkage would have an influence on the mean modulus, I would not expect an influence on the variation.

Doug Coffin

Could you say what is driving the change of modulus if it is not shrinkage?

Magnus Östlund

Well, there are sort of two ways of looking at it. Obviously at the start of the drying, there would be higher stresses in the surface layers and at the end of the drying, there would be higher stresses in the interior of the board and one interpretation is that the stresses at the start of the drying are more influential or that stresses at the surfaces are higher for a longer part of drying. Personally, I would like to see it more as an effect. The surfaces dry first and would be stiffer. Because they are stiffer, the surface layers take up more of the load and this is why we would obtain higher stiffness in the simulation. As I said we are not entirely sure yet what the model should predict. So, we need more research on that.

Torbjörn Wahlström The Packaging Greenhouse AB

It may be this is more of discussion than a question. You assume in your presentation and article that varying stresses in the different layers lead to a variation of elastic moduli through the thickness of the final board for restrained drying. It is shown in Figure 13, as discussed here.

I have never seen any experimental evidence showing this behaviour and I cannot say I am too surprised, since it is quite well known that, depending on the drying strategies used, you can get large differences in drying stress while the stiffness is still constant, for example Zhang has shown this. You present two sets of experiments: one from a multi-ply board production machine and one from one-sided lab-drying. Regarding the production machine results, the difference in stiffness in the top ply compared to the middle is quite obvious since it is the reason you have multiply boards. Therefore, this is not giving, at least to me, any new information about stress history and its influence on stiffness. However, regarding the one-sided lab drying, I strongly agree on the elegant experimental results in Figure 15 showing no effect from stress history on stiffness. You have given us two explanations here as to why to doubt those results. First you say that the moisture history is completely unknown – I do not agree with this. For the production machine simulations, you used literature data. There is also lot of data available for exactly the same lab trial as you performed showing the same type of moisture gradients, but done one-sided of course. Secondly, you say that the plies may not transfer load and therefore, conclude a need for further investigation in this area. I think everyone has realized by now that I do not think this needs any further investigation, but if you really want to pursue this trail, I propose you add some starch between the plies, use more refined pulp or maybe couch them in a more wet state to get better plybond.

Magnus Östlund

I think the only thing I need to comment is what you said about multiply boards intentionally being made to be stiffer at the surfaces of the board. This board was made entirely from chemical pulp indicating that obtaining a higher modulus near the surface was not a major concern at the machine that made this board and I showed residual stresses indicating that there would not be a major stiffness distribution through the board from the stock in the different plies.