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PREDICTIONS OF ANISOTROPIC MULTIPLY BOARD PROPERTIES BASED ON ISOTROPIC PLY PROPERTIES AND DRYING RESTRAINTS

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ABSTRACT

There is a link missing between pulp properties and machine-made paper properties. The aim of this paper is to close a part of this gap by proposing an engineering model which, based on pulp or stock properties, makes it possible to predict the resulting anisotropic material behaviour of a multiply paper or board based on any given fibre anisotropy and drying restraint.

An anisotropic model for the shrinkage and stiffness interaction between the individual plies in a multiply structure is formulated. The input data to the model is the isotropic restrained dried and free dried stiffness, the free shrinkage strain and the density of each ply in the multiply structure. The basis weight, fibre anisotropy and total strain after drying are variables in the model. This means that besides the standard handsheet procedure only measurement of shrinkage and stiffness for one extra free dried handsheet is needed for the calibration of the model.

The proposed model is validated with a series of anisotropic handsheet trials. Various combinations of single ply anisotropic handsheets were couched together into seven different multiply boards, which were dried freely and restrained. The isotropic input data of the individual plies were used to predict the free and restrained dried tensile stiffness index and bending stiffness of the multiply boards. The agreement between the experimental and predicted results showed good agreement. The model constitutes a useful tool in engineering predictions and parametric investigations of the mechanical behaviour of multiply boards.

As a demonstration of the use of the model, the relation for tensile and bending stiffness versus total strain accumulated during drying was predicted for different board compositions. The basis weight and fibre anisotropy were varied in one of its plies. As another example the MD and CD stiffness profiles of a multiply board were simulated based on a given MD stretch and CD shrinkage profile.

INTRODUCTION

Strain during drying is known to have a large effect on most paper properties. Laboratory studies have for example shown that an 8% strain increase, from –4% (shrinkage) to +4% (stretch), can give a relative increase in tensile stiffness index of as much as 500%! The strain at break is reduced to the same extent and the tensile strength can be doubled [Setterholm and Kuenzi 1970]. On a paper machine, the paper web can be stretched to different extents in the machine direction (MD) by varying the speed, for example between the press section and the first dryer group. Over the dryer cylinders, the pressure from the dryer fabric prevents the paper from shrinking in the cross machine direction (CD). But in the free draw between each pair of dryer cylinders, there are no CD forces acting on the edges of the web and thereby the paper is allowed to shrink in the cross machine direction. The conditions in the free draw lead to a greater cross directional shrinkage at the edges than in the middle of the paper web [Viitaharju and Niskanen 1993]. Thereby a profile in paper properties in the cross machine direction is created. The shrinkage difference between the edge and middle of the web is often referred to as the shrinkage profile and is a well-known problem for papermakers. Producers of liner board and other grades that have demands on high stiffness, want to reduce the shrinkage at the edges in order to increase the stiffness. On the other hand, producers of sack paper wish to increase the shrinkage in the middle of the web in order to increase the strain at break and thereby the tensile energy absorption (TEA). A common desire of all producers is to even out the shrinkage profile.

The fibre anisotropy is indisputable important for the properties of paper, considering the large number of articles dealing with different aspects of the subject. A rather recent review of the factors affecting fibre anisotropy and its effect on paper properties is written by Odell (2001). The reported studies on fibre anisotropy and paper properties have often in common that fibre anisotropy is not measured directly. Instead it seems to be common practice to use the stiffness anisotropy for restrained dried paper as a measure of fibre anisotropy, as used for example by Htun and Fellers (1982). Some studies made with direct measurements of fibre anisotropy have been reported, for example by Setterholm and Kuenzi (1970) and Knotzer (2003). An established link between fibre anisotropy and tensile stiffness index is however still missing.

Anisotropic paper properties can be predicted from isotropic properties based on the finding that the geometric mean of MD and CD stiffness is constant with varying anisotropy [Schrier and Verseput 1967]. This observation was later refined by Htun and Fellers (1982). By separating the influences of drying restraint and fibre anisotropy on the in-plane stiffness of paper, they showed that it is only valid under equal drying conditions. Wahlström (2004) further showed that also the geometric mean of free shrinkage strain in MD and CD is constant when the fibre anisotropy is altered. These observations, together with assumed relations between the fibre-, stiffness- and shrinkage anisotropies, enable predictions of the anisotropic material properties of single-ply paper for any total strain after drying. Another approach was taken by Htun et al (1984) who used a network model formulated by Perkins and Mark (1981) to investigate the influence of drying restraints and fibre anisotropy on the tensile stiffness. The model predictions showed good agreement with experimental data for low fibre anisotropy. Rigdahl et al (1983) applied the same model to a series of low density paper sheets with different basis weight and anisotropies. Good agreement between predictions and experimental results was achieved with drying restraints disregarded.

There has been surprisingly few reported investigations on the properties of multiply paper and board in the literature. Most studies on the behaviour of paper materials have been conducted on single ply paper. The interaction between the different plies during the papermaking process has consequently been given little attention. One exception is the bending stiffness of a multiply board that can be calculated with laminate theory for the uniaxial situation based on the stiffness, density and thickness for each ply [Carlsson and

Fellers 1968]. Laminate theory can for example be utilised to optimise the amount of low density middle ply pulp for optimal bending stiffness.

Obviously the multiply interaction, fibre anisotropy and strain during drying are important when considering the end-use properties of paper and board. Still there are no established methods for how to account for these effects in standard pulp testing. A bridge between pulp properties (standard handsheets) and machine made paper properties is still missing. The aim of this study was to build this bridge by proposing an engineering model which, based on pulp properties, makes it possible to predict anisotropic behaviour for a multiply paper or board at any given fibre anisotropy and strain history.

Finally it should be stressed that the engineering prediction aspect of the work has been of great priority. Constitutive modelling and finite element methods are great tools but not so easy to use for an average engineer. The purpose here was to propose a readily available and intuitively understandable model, without compromises on the results.

MATERIALS AND METHODS

Handsheet making

Pulp was taken from the machine chests for the middle and bottom ply of a board mill. The middle ply pulp (22,5 °SR) contained CTMP, low yield sulphate, broke and a small amount highly refined sulphate. The bottom ply pulp (25,0 °SR) was an unbleached low yield sulphate. Anisotropic handsheets were made in a Formette dynamic sheet former with a conditioned basis weight of 110 g/m^2 for restrained dried sheets. The speed of the drum was 1100 rpm and a nozzle pressure of either 2,0; 2,5 or 3,0 bars were used to produce paper sheets with different fibre anisotropy. The handsheets were couched using only two blotters to keep the sheets wet. Pressing was made in a static press with 400 kPa during 5 minutes. Either of two different drying strategies regarding external boundary conditions was adopted, restrained or free drying.

The freely dried handsheets were dried for 25 minutes in an oven at 105°C and were thereafter conditioned over night in 50% RH and 23°C. The free shrinkage strain was determined by measuring the distance between two marks in the paper before drying and after conditioning [Wahlström and Goldszer 2004]. Material testing was made after conditioning. The restrained dried handsheets were dried in an STFI plate dryer. In the plate dryer, described by Htun and Fellers (1982), the sheet is clamped between two rigid and heated drying frames. The heated clamps are made of sintered (porous) metal in order to allow for water evaporation, which facilitates the drying of the paper between the clamps and thereby reduces the probability for breakage of the paper in the clamped region. The mean density for the restrained dried sheets was 347 kg/m^3 for the middle ply pulp and 530 kg/m^3 for the bottom ply pulp.

The multiply boards, or laminates, were made by couching together single ply anisotropic handsheets made of different pulp and with different fibre anisotropy. A more detailed description of the laminates is given in the trial procedure part below. The multiply boards were pressed after couching, dried either freely or restrained and thereafter tested in the same way as the individual plies.

A second set of Formette sheets were made to produce more fibre anisotropy data. A CTMP pulp (freeness 411 CSF) and a softwood bleached sulphate pulp (24,0 °SR) were taken from a board mill. The speed of the drum was 1100 rpm and the nozzle pressure 2,0; 2,5; 3,0; 3,5 and 4,0 bars for each pulp. The conditioned basis weight was 80 g/m^2. Pressing was made in a roll press, first pressing at 250 kPa and secondly at 450 kPa. The samples were dried restrained in a STFI plate dryer.

Testing methods

The fibre anisotropy was evaluated by Stora Enso Karlstad Research Centre using an image analysing method. A transparent adhesive tape was applied to both sides of the sample and then the tapes were pulled apart, leaving a layer of fibres on each of the two tapes. A new tape was applied to the delaminated surface and the tapes were pulled apart again etc. The samples in this study were separated into about 25 layers. A reflectance image against black background was produced on each layer using a scanner. The images were subsequently analysed to determine the fibre segment angle distribution of each layer. Thereafter a von Mises distribution function was fitted to the experimental data. The analysis and parameter definitions are described in detail by Rigdahl and Hollmark (1986). The number of fibre segments oriented in MD divided by the number of fibre segments oriented in CD was defined as the fibre anisotropy (short for fibre segment angle distribution anisotropy). The average of the evaluated fibre anisotropy of each layer was used as the fibre anisotropy of the sample.

The thickness of the samples was measured with an L&W micrometer according to EN 20534, tensile stiffness index according to SCAN P 67:93 and bending stiffness according to DIN 53121 (5°) with an L&W bending tester (two-point method).

Experiments

Isotropic input data to the predictions

Handsheets were made of a middle ply pulp and a bottom ply pulp from a board mill. The fibre anisotropy was 1,05 for the bottom ply handsheets and 1,06 for the middle ply handsheets. The freely dried and restrained dried tensile stiffness index, free shrinkage strain, density and basis weight of the handsheets were evaluated. The geometric mean values ($\sqrt{MD \times CD}$) of the stiffness and free shrinkage data for the handsheets (Table 2) were used as isotropic input data to the model when predicting the behaviour of anisotropic laminates.

Relations between anisotropies

Single ply anisotropic handsheets with three different fibre anisotropies, low, medium and high, were made of the middle- and bottom ply pulp. The fibre anisotropy, the freely and restrained dried MD and CD tensile stiffness indices and the MD and CD free shrinkage strains were measured. The fibre anisotropies of the handsheets are presented in Table 3. The relations between the anisotropies (ratio of MD and CD value) for these properties were evaluated and compared with the assumptions in the model (Equations (11) to (13)).

Anisotropic laminates for validation of the model

Single ply anisotropic handsheets were made of the middle ply pulp (MP) and bottom ply pulp (BP) in exactly the same way as the ones used for the validation of the relations between anisotropies. Three different fibre anisotropies were made; Low, Medium and High. Different combinations of the single ply anisotropic handsheets were couched together forming seven different multiply boards according to Table 1. Freely and restrained dried MD and CD

Table 1 Composition of simulated and laboratory made multiply boards (laminates).

Laminate	1	2	3	4	5	6	7
Ply 1	MP Low	MP Low	MP Low	MP Medium	MP high	BP Low	BP High
Ply 2	MP Low	MP High	BP Low	BP Medium	BP High	BP High	BP High

tensile stiffness indices, MD and CD free shrinkage strains, freely and restrained dried thickness and restrained dried MD and CD bending stiffness were experimentally measured and compared to model predictions. The input data to the predictions of the seven laminates in Table 1 were the single ply isotropic input in Table 2, and the fibre anisotropies in Table 3.

MODEL

Hygroscopic and mechanical deformation

The uniaxial deformation of a paper sheet can be characterised by the relative elongation or strain, ε, according to Equation (1) where δ denotes the elongation and L_0 is the original length of the sample.

$$\varepsilon = \frac{\delta}{L_0} \tag{1}$$

Paper materials exhibit hygroscopic deformation, i.e. the dimensions of a paper sheet are reduced when the moisture content of the paper is decreased. In analogy, a paper sheet expands when its moisture content is increased. The hygroscopic behaviour of a paper material can be characterised by measuring the dimensional changes of a paper sheet during drying, without any external forces acting on the paper sheet during the drying process. The lack of external forces has named this drying strategy to free drying. The free shrinkage strain, ε^{fs}, is here defined as the strain that is occurring during free drying of a paper from press dryness to an equilibrium final moisture content. A paper sheet subjected to externally applied loading will exhibit mechanical strain. Tensile loading of a paper sheet during drying will counteract the shrinkage of the paper material. A negative total strain during drying is often referred to as shrinkage, a positive total strain as stretch and zero total strain as restrained drying. Note that standard handsheets are dried restrained, which means that a counteracting mechanical strain, which is equally large as the free shrinkage strain, is induced in the paper during the drying process, resulting in zero total strain accumulated during the whole drying process. Equation (2) shows the decomposition of total strain accumulated during drying, ε, into free shrinkage strain, ε^{fs}, and mechanical strain, ε^{m} [Wahlström and Fellers 1999].

$$\varepsilon = \varepsilon^{fs} + \varepsilon^{m} \tag{2}$$

Relation between stiffness and total strain

If a paper is deformed by externally applied forces during drying, either stretched or restrained from shrinkage, it will have a great effect on most paper properties. Wahlström and Fellers (1998) showed that a linear relation between tensile stiffness index and the total strain accumulated during drying prevails for isotropic paper. This linear relation, which is expressed by Equation (3), can easily be determined for a certain pulp by measurements on two isotropic handsheets. First, the tensile stiffness index (E^r) is evaluated for a restrained dried handsheet and then the tensile stiffness index (E^{fs}) and the free shrinkage strain (ε^{fs}) are evaluated for a freely dried handsheet. Figure 1 shows data from such measurements for the bottom ply pulp used in this study ($E^r = 9.64$ MNm/kg, $\varepsilon^r = 0$ %, $E^{fs} = 3.16$ MNm/kg and $\varepsilon^{fs} = -6.23$ %, see Table 2). The behaviour of the model in Equation (3) is represented by the solid line in Figure 1.

The linear relation given by Equation (3) is valid also for tensile stiffness index versus total strain in MD and CD, respectively, for anisotropic paper, as shown for example by Wahlström and Fellers (1999) and Mäkelä (2003). These observations on anisotropic papers are obviously important for the understanding of the mechanics of paper materials. However, as will be shown hereafter, engineering predictions of the behaviour of anisotropic

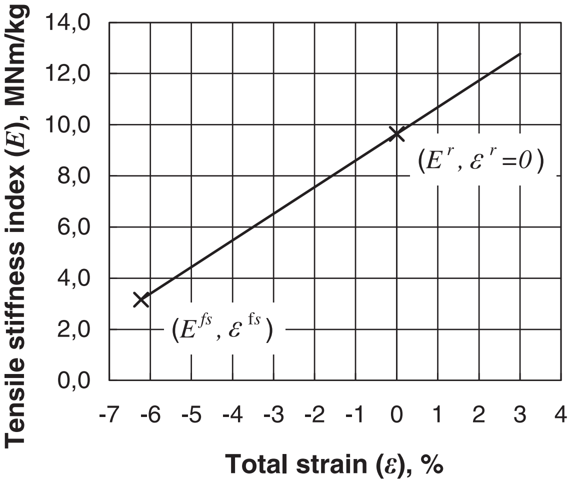

Figure 1 Linear relation between tensile stiffness index and total strain accumulated during drying for an isotropic handsheet.

papers can be performed solely based on the characterization of the pulp by measurements on isotropic handsheets.

$$E = \frac{E^{r} - E^{fs}}{\varepsilon^{r} - \varepsilon^{fs}}\,\varepsilon + E^{r} \tag{3}$$

Predicting anisotropic paper properties based on isotropic properties

The basis for the possibility to predict anisotropic paper properties based on isotropic pulp properties is the finding that the geometric mean of MD and CD stiffness is constant with varying anisotropy [Schrier and Verseput 1967]. Later this observation, which is only valid under equal drying conditions, was refined by Htun and Fellers (1982), who showed that the influences of drying restraint and fibre anisotropy on stiffness must be separated. Equations (4) and (5) state that the isotropic tensile stiffness index is equal to the geometric mean value of the tensile stiffness indicies in MD and CD for restrained and free drying, respectively. Wahlström (2004) further showed that a corresponding relation is valid also for the free shrinkage strain (Equation (6)), which makes it possible to predict anisotropic properties for any total strain after drying, as will be shown hereafter.

$$E^{r}_{Iso} = \sqrt{E^{r}_{MD}E^{r}_{CD}} \tag{4}$$

$$E^{fs}_{Iso} = \sqrt{E^{fs}_{MD}E^{fs}_{CD}} \tag{5}$$

$$\varepsilon^{fs}_{Iso} = \sqrt{\varepsilon^{fs}_{MD}\varepsilon^{fs}_{CD}} \tag{6}$$

In the proposed model, the fibre anisotropy was chosen as the controlling variable for prediction of the other anisotropic properties. Anisotropy is here defined as the ratio of some property between the two principal in-plane material directions of paper. These directions commonly coincide with the machine or manufacturing direction, MD, and the cross machine direction, CD. The fibre anisotropy, A_{Fibre}, is defined in Equation (7) where n is the number of fibre segments oriented in the respective direction. To realise the use of fibre anisotropy as a variable we need to formulate anisotropies in hygroscopic and mechanical properties in terms of the fibre anisotropy. The tensile stiffness index anisotropy for restrained dried paper, $A_{E^{r}}$, is defined in Equation (8), the stiffness anisotropy for freely dried paper, $A_{E^{fs}}$, in Equation (9) and the anisotropy in free shrinkage strain, $A_{\varepsilon^{fs}}$, in Equation (10).

$$A_{Fibre} = \frac{n_{MD}}{n_{CD}} \tag{7}$$

$$A_{E^r} = \frac{E^r_{MD}}{E^r_{CD}} \tag{8}$$

$$A_{E^{fs}} = \frac{E^{fs}_{MD}}{E^{fs}_{CD}} \tag{9}$$

$$A_{\varepsilon^{fs}} = \frac{\varepsilon^{fs}_{MD}}{\varepsilon^{fs}_{CD}} \tag{10}$$

Wahlström (2004) proposed Equations (11) to (13) to formulate the anisotropies in hygroscopic, $A_{\varepsilon^{fs}}$, and mechanical properties, A_{E^r} and $A_{E^{fs}}$, in terms of the fibre anisotropy, A_{Fibre}. Results from an investigation of the validity of these equations are presented in the experimental part of this work.

$$A_{E^r} = A_{Fibre} \tag{11}$$

$$A_{E^{fs}} = 2A_{Fibre} - 1 \tag{12}$$

$$A_{\varepsilon^{fs}} = 1/A_{Fibre} \tag{13}$$

The made assumptions in this section (see Equations (4) to (13)) makes it possible to predict anisotropic paper properties based on the isotropic pulp properties. The expressions for the anisotropic properties are given in Equations (14) to (19). Furthermore, the linear relation between tensile stiffness index and total strain accumulated during drying (Equation (3)) can be used to evaluated the influence of drying restraints on paper properties. Thereby it is possible to predict anisotropic paper properties based on isotropic pulp properties for any fiber anisotropy and drying restraints (any total strain accumulated during drying in MD and CD).

$$E^r_{MD} = E^r_{Iso}\sqrt{A_{Fibre}} \tag{14}$$

$$E^r_{CD} = E^r_{Iso}\Big/\sqrt{A_{Fibre}} \tag{15}$$

$$E^{fs}_{MD} = E^{fs}_{Iso}\sqrt{2A_{Fibre} - 1} \tag{16}$$

$$E^{fs}_{CD} = E^{fs}_{Iso}\Big/\sqrt{2A_{Fibre} - 1} \tag{17}$$

$$\varepsilon^{fs}_{MD} = \varepsilon^{fs}_{Iso} \Big/ \sqrt{A_{Fibre}} \tag{18}$$

$$\varepsilon^{fs}_{CD} = \varepsilon^{fs}_{Iso} \sqrt{A_{Fibre}} \tag{19}$$

Predicting multiply board properties based on ply properties

So far, single-ply papers have been treated. In this section, similar expressions for the prediction of multiply paper properties will be presented. The properties of a multiply board is dependent on the properties of the individual plies that form the board, the basis weight of the plies and the interactions between the individual plies. The restrained dried basis weight of the multiply board, w^{r}_{Lam}, is determined as the sum of the restrained dried basis weights of the constituent plies (Equation (20)). The restrained dried tensile stiffness index of the multiply board, E^{r}_{Lam}, is evaluated as a weighted average value of the restrained dried tensile stiffness indices (Equation (21)).

$$w^{r}_{Lam} = \sum_{i=1}^{n} w^{r}_{i} \tag{20}$$

$$E^{r}_{Lam} = \frac{\sum_{i=1}^{n} w^{r}_{i} E^{r}_{i}}{w^{r}_{Lam}} \tag{21}$$

The evaluation of the free shrinkage strain and the freely dried tensile stiffness index of the multiply board are less trivial. The plies constituting the multiply board are assumed to exhibit equal in-plane strains during drying, in other words no sliding between the plies are assumed to occur and the board is assumed to be flat. When plies with different free shrinkage strain are forming a laminate, the plies will interact with each other during free drying of the multiply board. This interaction will result in a free shrinkage strain of the laminate that is assumed to be controlled by the basis weights, free shrinkage strains and tensile stiffness indices of the individual plies. Appendix 1 treats the determination of the free shrinkage strain of a multiply board, ε^{fs}_{Lam}, based on the properties of the plies of the board, and concludes that it can be evaluated by using the approximate expression in Equation (22).

$$\varepsilon_{Lam}^{fs} = \frac{\sum_{i=1}^{n} w_i^r E_i^r \varepsilon_i^{fs}}{w_{Lam}^r E_{Lam}^r} \tag{22}$$

The tensile stiffness index of the individual plies after free drying of the multiply board, $E_i^{\varepsilon_{Lam}^{fs}}$, is dependent on the free shrinkage strain of the multiply board (Equation (23)). The freely dried tensile stiffness index of the multiply board, E_{Lam}^{fs}, may finally be evaluated as a weighted average value of the tensile stiffness indices of the individual plies (Equation (24)). The restrained dried basis weight, instead of the real basis weight, has been used in Equation (24), which simplifies the expression. This simplification can be introduced since the plies in the multiply board are constrained to exhibit equal in-plane strains during drying, which implies that all ratios between basis weights in different plies remain unchanged during drying.

$$E_i^{\varepsilon_{Lam}^{fs}} = \frac{E_i^r - E_i^{fs}}{\varepsilon_i^r - \varepsilon_i^{fs}} \varepsilon_{Lam}^{fs} + E_i^r \tag{23}$$

$$E_{Lam}^{fs} = \frac{\sum_{i=1}^{n} w_i^r E_i^{\varepsilon_{Lam}^{fs}}}{w_{Lam}^r} \tag{24}$$

Multiply board stiffness for any drying restraints

The tensile stiffness index of a multiply board can be calculated for any total strain accumulated during drying in MD and CD. Applying Equation (3) on a laminate gives a relation (Equation (25)) between the tensile stiffness index of the laminate, E_{Lam}, and the total strain accumulated during drying, ε, for a uniaxial situation.

$$E_{Lam} = \frac{E_{Lam}^r - E_{Lam}^{fs}}{\varepsilon_{Lam}^r - \varepsilon_{Lam}^{fs}} \varepsilon + E_{Lam}^r \tag{25}$$

The bending stiffness, S^b, of a laminate or multiply board with n plies can be calculated for the uniaxial situation using Equations (26)–(32) [Carlsson and Fellers 1968]. Calculations using these equations are easy to perform since no reference to a neutral surface is necessary. The input data needed for each ply is the tensile stiffness index (E) in each principal in-plane material direction,

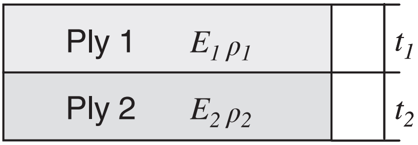

Figure 2 A laminate with two plies (n=2).

the density (ρ) and the thickness (t). The thickness was assumed to change proportionally with total strain according to Equation (33). The density is assumed to be constant with varying strain and can thereby be calculated with Equation (34). Figure 2 illustrates a two ply laminate and the relevant data for enabling the calculation of its bending stiffness. Equation (35) is given by Equations (33) and (34) but is included to show clearly how the basis weight changes with total strain according to the thickness and density assumptions.

$$S^b = D - \frac{B^2}{A} \tag{26}$$

$$A = \sum_{i=1}^{n} E_i \rho_i (z_i - z_{i-1}) \tag{27}$$

$$B = \frac{1}{2} \sum_{i=1}^{n} E_i \rho_i (z_i^2 - z_{i-1}^2) \tag{28}$$

$$D = \frac{1}{3} \sum_{i=1}^{n} E_i \rho_i (z_i^3 - z_{i-1}^3) \tag{29}$$

$$z_i = z_{i-1} + t_i \tag{30}$$

$$z_0 = -\frac{t_{Lam}}{2} \tag{31}$$

$$t_{Lam} = \sum_{i=1}^{n} t_i \tag{32}$$

$$t_i = \frac{t_i^r}{(1 + \varepsilon_{MD})(1 + \varepsilon_{CD})} \tag{33}$$

$$\rho_i = \frac{w_i^r}{t_i^r} \tag{34}$$

$$w_i = \frac{w_i^r}{(1 + \varepsilon_{MD})(1 + \varepsilon_{CD})} \tag{35}$$

Predictions using the model

MD and CD stiffness for a multiply board with a given total strain

As a demonstration of the modelling procedure, the bending stiffness in MD and CD for laminate 5 in Table 1 was predicted for 2% stretch in MD and 5% shrinkage (–5% total strain) in CD. The input data besides the given total strain in MD and CD were the single ply isotropic input data in Table 2, and the fibre anisotropies for laminate 5 according to Table 3.

Table 2 Single ply isotropic input data.

Pulp	Tensile stiffness index, Restr, MNm/kg	Tensile stiffness index, Free, MNm/kg	Free shrinkage strain, %	Basis weight, Restrained, g/m^2	Density, Restrained, kg/m^3
Middle Ply (MP)	5,88	2,83	–2,70	109,1	347
Bottom Ply (BP)	9,64	3,16	–6,23	109,1	530

Table 3 Fibre anisotropies of single ply handsheets.

Pulp	Low	Medium	High
Middle Ply (MP)	1,06	1,68	2,92
Bottom Ply (BP)	1,05	1,62	1,86

Varying total strain, basis weight and fibre anisotropy in one ply of a multiply board

As a demonstration of the use of the model, the relations between tensile stiffness index- and bending stiffness, respectively, and total strain accumulated during drying were simulated for different board compositions. The model was used to study the effects of varying basis weight (–20% to + 20%) and fibre anisotropy (low, medium and high) in the middle ply of Laminate 5 in Table 1. Furthermore, different total strain in the two in-plane principal material directions of the board was simulated. The input data besides the changed basis weight and fibre anisotropy were the single ply isotropic input data in Table 2, and the fibre anisotropies for laminate 5 according to Table 3.

MD and CD stiffness profiles based on a given shrinkage profile

As a demonstration of the potential use of the model, the tensile stiffness index profile and the bending stiffness profile in MD and CD of a full-scale paper web were simulated based on a given shrinkage profile in CD and a given stretch in MD.

RESULTS

Experiments

Isotropic input data to the predictions

Handsheets were made of a middle ply pulp and a bottom ply pulp from a board mill. Table 2 gives the isotropic input data used for predictions of properties of anisotropic laminates.

Relations between Anisotropies

Single ply anisotropic handsheets with three different fibre anisotropies were made of the middle- and bottom ply pulp. The fibre anisotropy, the tensile stiffness anisotropy for restrained drying, the tensile stiffness anisotropy for free drying and the free shrinkage strain anisotropy were measured. The fibre anisotropies of the handsheets are presented in Table 3.

The fibre anisotropy for each single ply anisotropic handsheet was determined as the mean value of the fibre anisotropy and the tensile stiffness anisotropy for restrained dried sheets, with the exception for the results in Figure 3, where plain fibre anisotropy data are presented. The mean value was adopted in order to even out experimental scatter. The use of the mean value was motivated by the results in Figure 3 and by the assumed equality in Equation (11).

The relations between the anisotropies given in Equations (11) to (13) are plotted as solid lines in Figures 3 to 5. In Figure 3 the open squares represents measurements on the middle ply pulp, open circles on the bottom ply pulp,

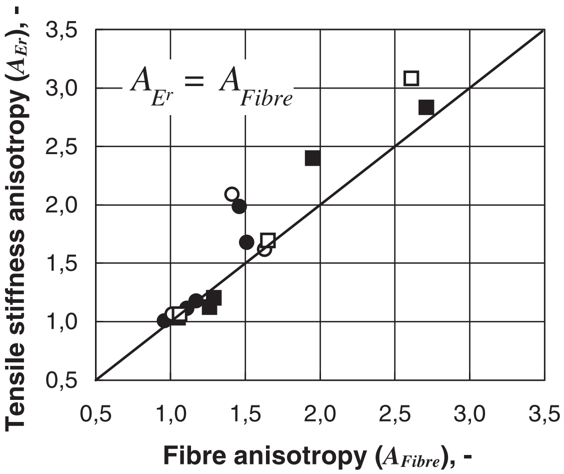

Figure 3 Relation between tensile stiffness anisotropy and fibre anisotropy for restrained dried single ply sheets.

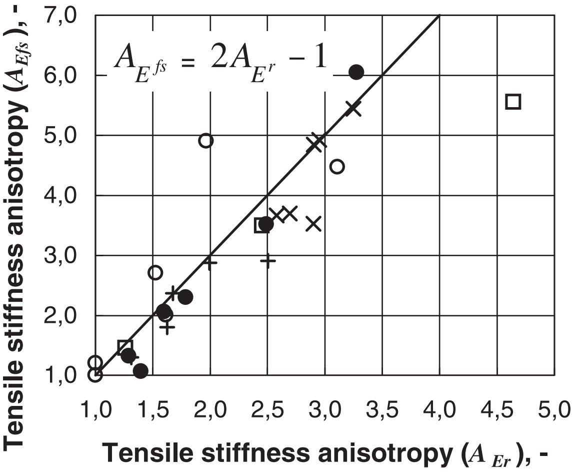

Figure 4 Relation between tensile stiffness anisotropy for freely dried single ply sheets and tensile stiffness anisotropy for restrained dried single ply sheets.

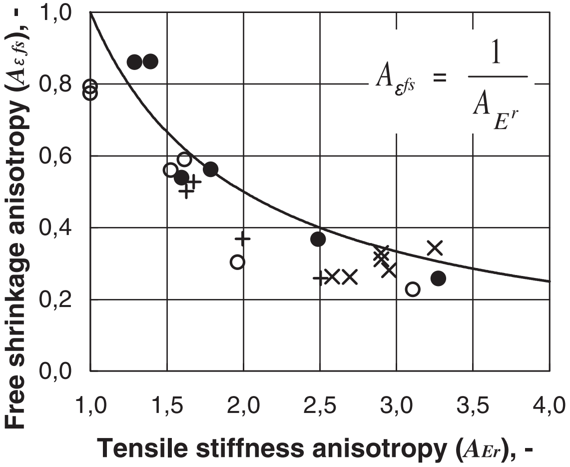

Figure 5 Relation between free shrinkage strain anisotropy and tensile stiffness anisotropy for restrained dried single ply sheets.

closed squares on the CTMP pulp and the closed circles on the bleached sulphate pulp. In Figures 4 and 5, the open circles represent experimental data produced in this study (bottom and middle ply pulp), while the filled circles [Wahlström 2004], x-marks [Westerlind et al 2004], squares [Htun and Fellers 1982] and crosses [non published data by Wahlström] comes from other studies. The data from the present study and the literature data support the assumed relations between the anisotropies given in Equations (11) to (13).

Anisotropic laminates for validation of the model

Single ply anisotropic handsheets with three different fibre anisotropies were made of the middle- and bottom ply pulp. Various combinations of the single ply anisotropic handsheets were couched together into seven different multi-ply boards according to Table 1. A number of properties were experimentally determined and predicted with the model based on isotropic input data, in order to validate the model.

Figures 6 and 7 show good agreement between predicted and experimental results for freely and restrained dried tensile stiffness index in MD and CD as well as for free shrinkage strain in MD and CD. The solid lines mark a one to one relation between predictions and experiments. The prediction of freely and restrained dried thickness and restrained dried MD and CD bending stiffness are overestimated for all samples as shown in Figures 8 and 9.

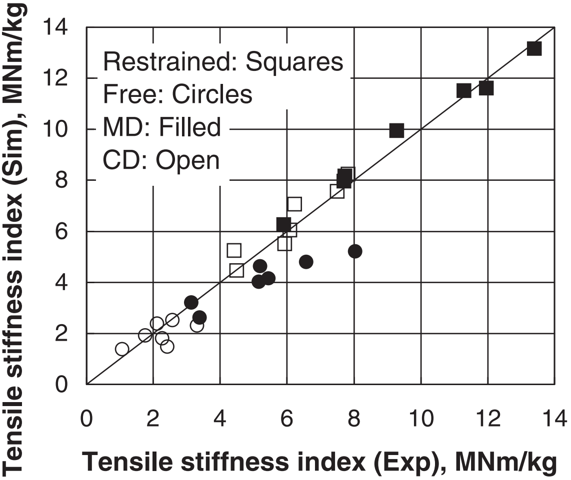

Figure 6 Predicted and experimentally determined tensile stiffness index in MD and CD for freely and restrained dried samples of the seven laminates in Table 1.

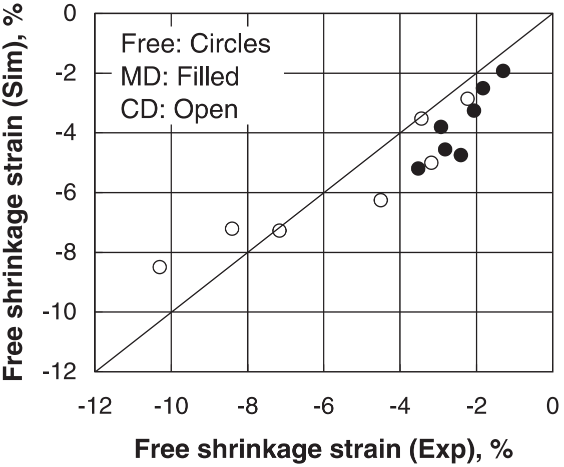

Figure 7 Predicted and experimentally determined free shrinkage strain in MD and CD for freely dried samples of the seven laminates according to Table 1.

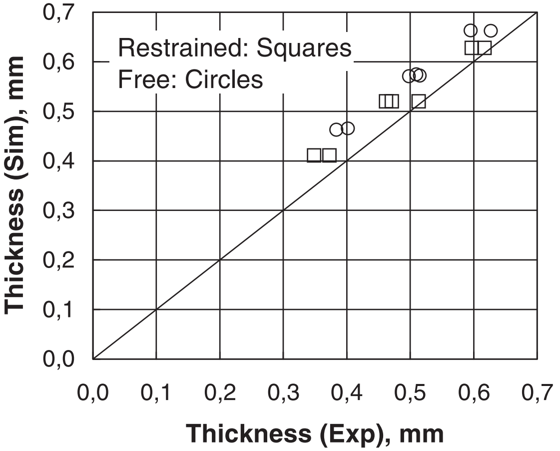

Figure 8 Predicted and experimentally determined thickness for freely and restrained dried samples of the seven laminates according to Table 1.

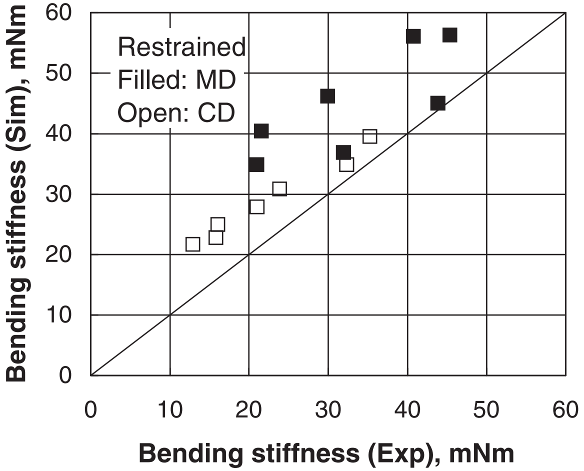

Figure 9 Predicted and experimentally determined bending stiffness in MD and CD for restrained dried samples of the seven laminates, according to Table 1.

Predictions using the model

MD and CD stiffness for a multiply board with a given total strain

As a demonstration of the modelling procedure, the bending stiffness in MD and CD for laminate 5 in Table 1 is simulated for 2% stretch (2% total strain) in MD and 5% shrinkage (–5% total strain) in CD. The input data besides the given total strain in MD and CD were the single ply isotropic data in Table 2, and the fibre anisotropies for laminate 5 according to Table 3 (BP=1,86, MP=2,92).

The solid lines in Figure 10 represents how the free shrinkage strain for the bottom ply pulp (BP) and Middle ply pulp (MP) develops in MD and CD with increasing fibre anisotropy. Equations (18) to (19) were used to evaluate these relations based on the isotropic input data. The anisotropic tensile stiffness index for freely and restrained dried papers is calculated in the same way with Equations (14) to (17). The circles in Figures 10 to 12 mark the values of the free shrinkage strain and the tensile stiffness index (free and restrained) with the fibre anisotropies given as input data (BP closed, MP open).

With the anisotropic data at the given anisotropies from Figures 10 to 12 (the circles), the linear relations in Figures 13 and 14 between tensile stiffness index and total strain accumulated during drying in MD and CD can be obtained for each ply in the laminate using Equation (3). The free shrinkage strain for the laminate, ε^{fs}_{Lam}, which is calculated using Equation (22), is required before the properties of the laminate can be predicted. The restrained dried and freely dried tensile stiffness indices of the laminate, E^{r}_{Lam} and E^{fs}_{Lam}, are also needed and are calculated by using Equations (21) and (24), respectively. The relations in MD and CD for the laminate can now be predicted by using Equation (25). The tensile stiffness index for laminate 5 at the given total strain is predicted to 15,8 MNm/kg in MD and 2,6 MNm/kg in CD, as illustrated with the closed circles in Figure 15.

Given the relations between tensile stiffness index and total strain in MD and CD for each individual ply, the bending stiffness of the laminate can be calculated for any total strain by utilising Equations (26) to (34). When the total strain is 2% in MD and –5% in CD, the bending stiffness is predicted to 85,1 mNm in MD and 9,8 mNm in CD.

Varying total strain, basis weight and fibre anisotropy in one ply of a multiply board

As a demonstration of the use of the model, the relation between tensile stiffness index- and bending stiffness, respectively, and total strain accumulated during drying was simulated for different drying restraints and board

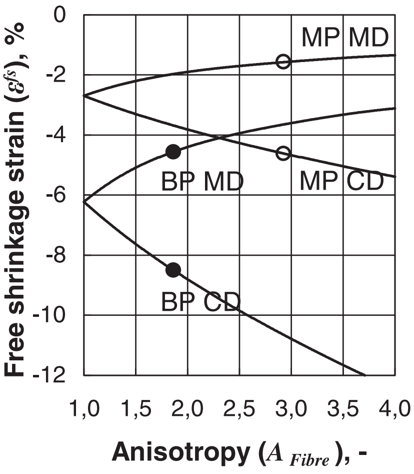

Figure 10 Relations between free shrinkage strain and fibre anisotropy for each of the plies in laminate 5.

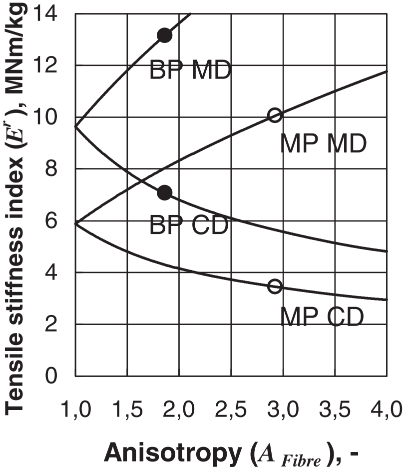

Figure 11 Relations between restrained dried tensile stiffness index and fibre anisotropy for each of the plies in laminate 5.

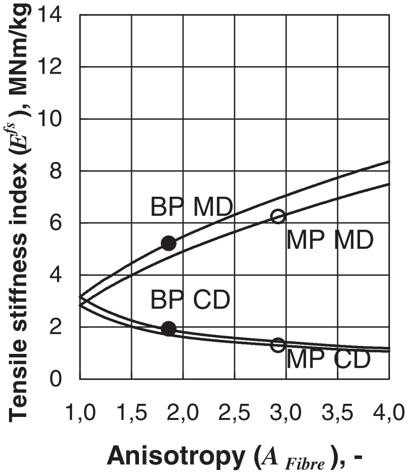

Figure 12 Relations between freely dried tensile stiffness index and fibre anisotropy for each of the plies in laminate 5.

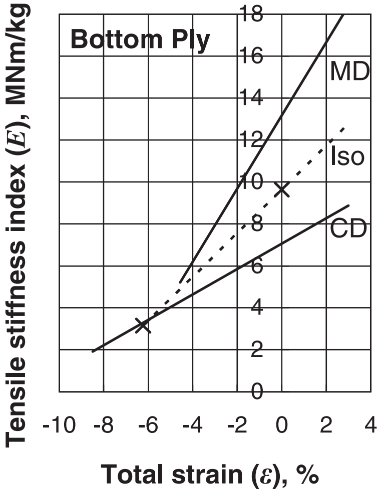

Figure 13 Relations between MD and CD tensile stiffness index and total strain accumulated during drying for the bottom ply in laminate 5.

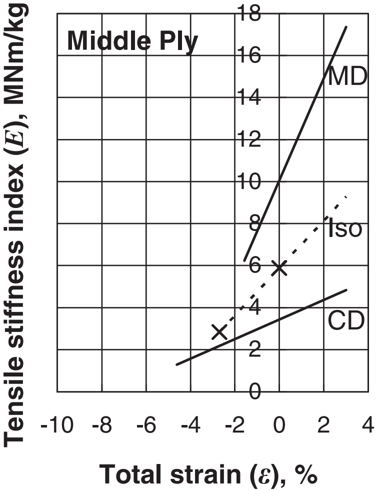

Figure 14 Relations between MD and CD tensile stiffness index and total strain accumulated during drying for the middle ply in laminate 5.

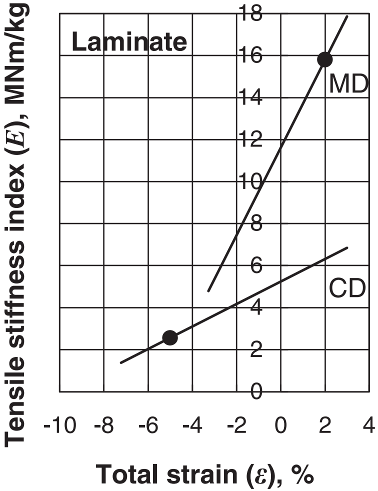

Figure 15 Relations between MD and CD tensile stiffness index and total strain accumulated during drying for laminate 5.

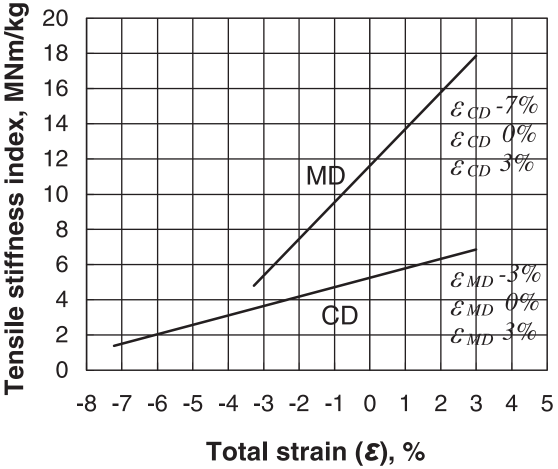

Figure 16 Relation between MD and CD tensile stiffness index and total strain accumulated during drying for varying strain histories for laminate 5.

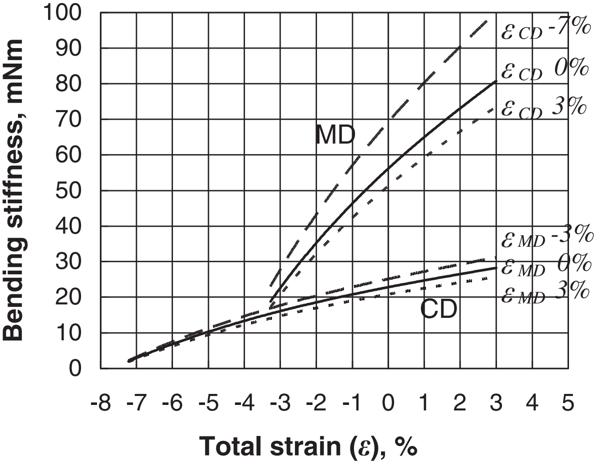

Figure 17 Relation between MD and CD bending stiffness and total strain accumulated during drying for varying strain histories for laminate 5.

compositions. Laminate 5 in Table 1 was simulated with the input data in Table 2, besides the changed drying restraints, basis weight and fibre anisotropy.

Figure 16 show the predicted relations between tensile stiffness index in MD and CD, respectively, and the total strain accumulated during drying, when different prescribed strain histories has been applied in the transversal in-plane direction. The relations are the same as presented in Figure 15 and remains the same for varying strain histories since the basis weight changes proportionally to the strain in the transversal in-plane direction in both plies. Figure 17 shows the same type of relations as in Figure 16 but for bending stiffness. The bending stiffness in one principal material direction increases when the total strain in the perpendicular direction decreases. The increase is explained by the increase in thickness caused by the negative strain (shrinkage).

Figures 18 and 19 show predicted relations between tensile stiffness index and bending stiffness, respectively, and the total strain accumulated during drying, for different basis weights (–20% to + 20%) in the middle ply. The tensile stiffness index is altered by changes in basis weight since the relative amount of the two plies changes. The bending stiffness is altered by the same reason and also due to the changed thickness.

Figures 20 and 21 show predicted relations between tensile stiffness index and bending stiffness, respectively, and the total strain accumulated during drying, for different choices of fibre anisotropies (low, medium and high according to Table 3) in the middle ply.

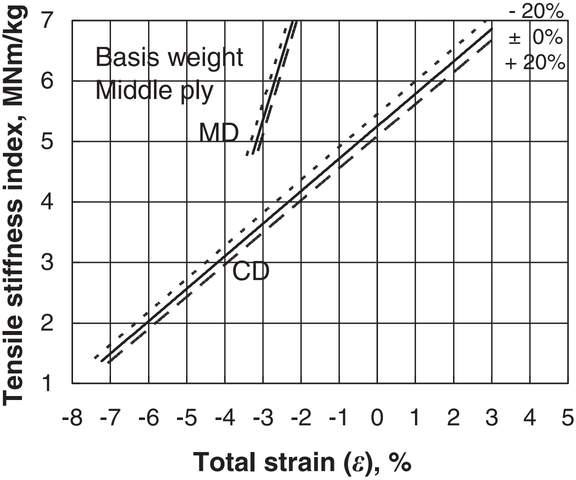

Figure 18 Relation between MD and CD tensile stiffness index and total strain accumulated during drying for varying basis weight in the middle ply for laminate 5.

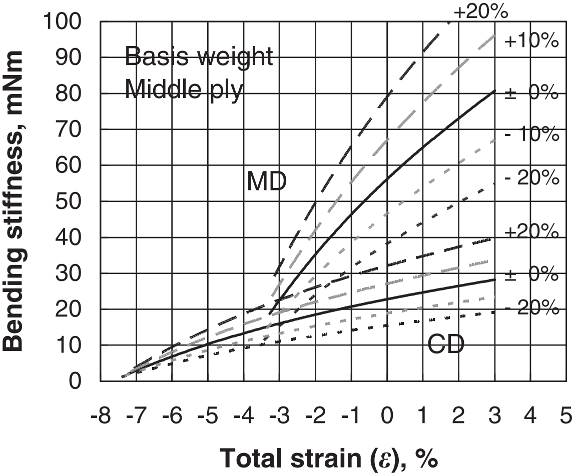

Figure 19 Relation between MD and CD bending stiffness and total strain accumulated during drying for varying basis weight in the middle ply for laminate 5.

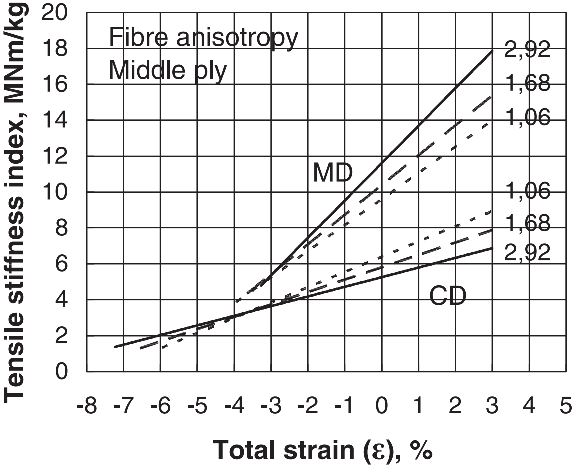

Figure 20 Relation between MD and CD tensile stiffness index and total strain accumulated during drying for varying fibre anisotropy in the middle ply for laminate 5.

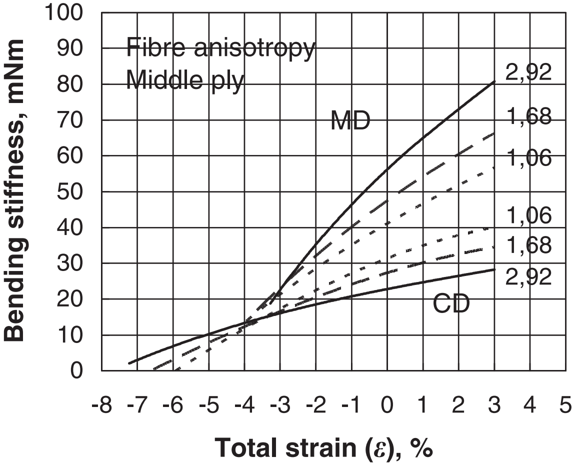

Figure 21 Relation between MD and CD bending stiffness and total strain accumulated during drying for varying fibre anisotropy in the middle ply for laminate 5.

MD and CD stiffness profiles based on a given shrinkage profile

As a demonstration of the potential of the model, the tensile stiffness index profile and bending stiffness profile in MD and CD was predicted based on a given shrinkage profile in CD and a given stretch in MD. Laminate 5 in Table 1 was simulated with the total strain in Figure 22 as input data. The predicted tensile stiffness index profiles in MD and CD according to Figure 23 can be constructed directly from the relations in Figure 16, whereas the predicted bending stiffness in Figure 24 must be calculated for each point on the web width with the given strain in MD and CD, as was evident from the example in Figure 17. The high negative CD total strain at the edges of the web gives correspondingly low stiffness at the edges. The increase in MD bending stiffness at the edges is due to the increased thickness at the edges that follows from the relatively higher shrinkage at the edges.

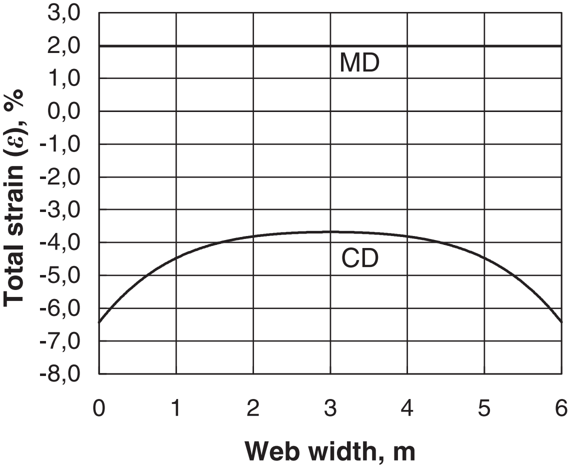

Figure 22 Shrinkage profiles used as input data to the simulated result in Figures 23 and 24.

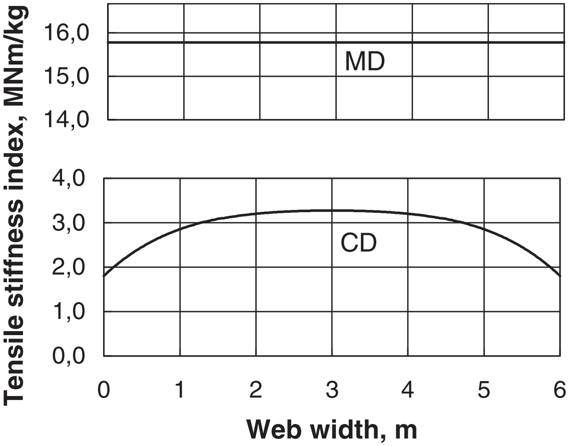

Figure 23 Tensile stiffness index profiles for laminate 5 in Table 1 and the shrinkage profiles in Figure 22.

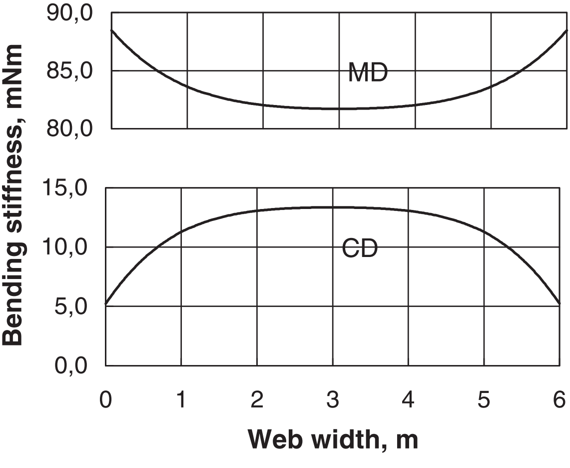

Figure 24 Bending stiffness profiles for laminate 5 in Table 1 and the shrinkage profiles in Figure 22.

DISCUSSION

The uniaxial linear relation between tensile stiffness index and total strain (Equation (3)) is only valid if there is an independence of MD and CD regarding properties and total strain. Wahlström and Fellers (1998) showed that the total strain accumulated during drying in one principal material direction (MD or CD) of the paper, does not have an effect on the relation between tensile stiffness index and total strain in the other principal material direction. Htun and Fellers (1980) also showed that different total strain during drying in one principal material direction does not influence the mechanical properties in the transverse direction. However, when a paper is stretched in one principal material direction it will contract, if allowed, in the transverse in plane direction. This mechanical deformation will be a part of the total strain accumulated during drying and will also influence the tensile stiffness according to Equation (3). Such a contraction with a corresponding decrease in tensile stiffness index has been shown by Baum et al (1984). These observations indicate that the linear relation is independently valid, although the total strain in one in-plane principal material direction can give contributions to the drying history in the transverse direction.

Another requirement for Equation (3) to be valid is that the relation between stiffness and total strain is independent of when the straining takes place during the drying process. Wahlström and Fellers (1999) showed that the linear relation between stiffness and total strain is independent of if the shrinkage takes place in the beginning or in the end of the drying process – the effect of the total strain on the paper properties is still the same. However, the stretching during drying has a reduced effect on paper properties in the late part of the drying process [Htun 1986]. Consider for example a stretch in the dry state, which is expected to have a negligible effect on the tensile stiffness, even though it might cause an increase in total strain (for example due to plasticity). The reduced effect of stretching at higher solids contents has not been considered a problem for the engineering predictions, since the major part of the stretching takes place in the draw between the press and dryer section. At such solids content, the linear relation holds as shown by Wahlström and Fellers (1998 and 1999). In commercial paper machines, the magnitude of the stretch or draw taking place between dryer groups at higher solids contents can commonly be neglected compared to the press draw.

Generally it can be said about Equation (3) that there is a benefit from using strain, rather than stress, as a controlling parameter for tensile properties. Wahlström and Fellers (1999) showed that the tensile properties will be the same for a certain level of total strain accumulated during drying, independently of the strain history. Such a relation does not apply between tensile properties and stress during drying. However, the linear relation may not be valid for large positive total strains. The stiffness has been observed to reach a maximum value at large values of stretching and may decline with further increased straining during drying [Setterholm and Kuenzi 1970]. The linear relation may furthermore not be accurate when the paper is subjected to compression during drying. This issue was briefly addressed in this work, during the development of the engineering expression for the determination of free shrinkage strain of multiply boards (see Appendix A). The tensile stiffness index of an individual ply is predicted by Equation (3) to decrease linearly with the total accumulated strain, even in situations when the accumulated total strain becomes lower than the free shrinkage strain of the ply. Consequently, the tensile stiffness index of the ply is predicted to be lower than the free drying tensile stiffness index when dried under compression. This might cause that a ply with small potential to shrink is predicted to have a very low or even negative tensile stiffness index as a consequence of its interaction with other plies exhibiting large potential for free shrinkage during free dying. This non-physical effect was considered by an alternative expression of the relation between tensile stiffness index and accumulated total strain (see Equation (A6)).

Since the aim of the model is to enable predictions of paper properties, the model should obviously be applicable on the paper making process. Therefore it is important to know whether the tensile stiffness index is dependent on drying time and temperature. Htun and de Ruvo (1980) showed a rather large effect of drying time and temperature on the tensile stiffness index, while Persson et al (2005) showed in a series of experiments that the tensile stiffness index is independent of drying time and temperature. The major difference between these studies were that Htun and de Ruvo separated the effects of drying time and drying temperature in their experiments, whereas Persson et al did not. In the dryer section of a paper machine it is not possible to independently control the relation between these two parameters. It therefore seems like the influence of drying time and temperature can be neglected in engineering predictions of paper properties after drying.

The prediction of anisotropic properties with the fibre anisotropy as a controlling variable according to Equations (14) to (19), requires that the fibre anisotropy is constant throughout the papermaking process. Danielsen and Steenberg (1947) showed for a grease proof paper that even as high stretch as 20% did not change the fibre anisotropy. Setterholm and Kuenzi (1970) measured fibre anisotropy during drying and, using different restraints, they could not confirm any changes. Hess and Brodeur (1996) showed that wet straining and drying shrinkage have no significant effect on fibre orientation angle. It seems to be common practice to use the stiffness anisotropy for restrained dried paper as a measure of fibre anisotropy (Equation (11)). It is therefore surprising that very few data supporting this assumed equality, and also some disagreeing data, has been found in the literature. Setterholm and Kuenzi (1970) present data supporting this equality but only for two anisotropies. Data from Hess and Brodeur (1996) and Chang (1983) show a different relation. The relation between stiffness anisotropy and fibre anisotropy according to Equation (11) was studied more in detail in the experimental results presented in Figure 3. These results show that for engineering predictions, the fibre anisotropy can be treated as equal to the tensile stiffness anisotropy for restrained dried sheets. One could argue that the fibre anisotropy could be replaced by tensile stiffness anisotropy for restrained dried sheets and that the relation between the two anisotropies is not important for engineering modelling applications. In this context, it has to be understood that the anisotropy of a paper is decided by both the fibre anisotropy and the drying history (shrinkage and stretch in MD and CD). Additionally, since the fibre anisotropy does not change after the forming section [Danielsson and Steenberg 1947] it can be measured in the dry paper. With knowledge of a papers strain history, the effects on paper properties from fibre anisotropy and drying history can be separated.

The geometric mean as an invariant quantity (Equations (4) to (6)) has been used in earlier studies [Schrier and Verseput 1967, Berg and de Ruvo 1972] and has furthermore been observed to be constant for different stiffness anisotropies for freely and restrained dried paper, respectively, [Htun and Fellers 1982]. Later also the isotropic free shrinkage strain has been found to be equal to the geometric mean value of the free shrinkage strain in MD and CD [Wahlström 2004]. Building the proposed model on a physical basis has been given great priority in the present work. But still the geometric mean as an invariant quantity can not, to our knowledge, be given a physical background.

The relations between the anisotropies presented in Figure 4 and Figure 5 were proposed by Wahlström (2004) but were revisited and re-examined in the present study. The experimental results and the data from the literature support the assumed relations in Equations (11) to (13). The relations between these anisotropies were further shown to be highly useful in engineering modelling applications.

The proposed model was validated with freely and restrained dried laminates made of single ply anisotropic paper with known properties. The predictions were based on isotropic input data for each ply and showed good agreement for tensile stiffness and free shrinkage strain. The predictions of thickness were overestimated with around 10% and the bending stiffness with around 30%. Based on the overestimation of thickness the result for bending stiffness is an expected succession since the bending stiffness is proportional to the thickness raised to the power of three ($S^b = Et^3/12$). The overestimations of the bending stiffness may therefore have its origin in the adopted method for measurement of the thickness. Also the addition of surface roughness to the thickness was present for both sides of each ply in the measurements of the single ply sheets used as input data, but only for the top and bottom sides of the laminate.

Wahlström et al (2000) showed that basis weight changes proportionally with total strain. Regarding thickness the situation is more unclear. Wahlström et al (2000) found that the thickness is proportional to the shrinkage during drying but the proportionality ceased to be valid for stretch. In the proposed model, the density is assumed to be constant for both positive and negative strains. According to the authors, a constant density constitutes a good assumption until more knowledge is gained in this area.

The model together with the described experimental procedure can be used as a simple and fast method for evaluation of changes of furnish, board composition or process parameters. Changes of furnish can be for example pulp raw material, pulp mixture, pulp treatment or additions to the pulp such as chemicals or fillers. Different board compositions that can be evaluated are for example the basis weight of the plies and the number of plies. The process parameters that can be studied, and for example compared to changes of the raw material, is mainly the fibre anisotropy in the different plies, the influence of shrinkage and stretch, but also pressing to different densities by performing this experimentally.

The proposed model constitutes one level of hierarchical modelling. Several ongoing studies are approaching mechanical properties of a paper based on fibre properties. If such studies results in the possibility to simulate restrained and freely dried stiffness, free shrinkage and density for isotropic handsheets, then the proposed model could be applied on those results. Thereby the coupling between fibre and paper properties could be extended to anisotropic multiply paper and also incorporate the influence of different drying restraints (strain histories).

In this paper the MD and CD profiles in Figure 23 and Figure 24 were simulated based on prescribed MD and CD total strain profiles (Figure 22). A logical expansion of the model would be to incorporate simulation of the CD shrinkage profile based on the properties of the multiply board and for example the geometries of the free draws in the dryer section. The effect of the free draws has been studied on a paper machine by splitting the paper web before the dryer section. Brecht and Wanka (1967) demonstrated the effect on dimensional stability, Malmelin (1993) on roughness, Hoole et al (1999) on shrinkage profile and Kniivilä (2002) on shrinkage and MD tension profile. Some knowledge has also been gained with finite element methods (Wahlström et al 1999, Mäkelä 2003). Additionally, Wahlström and Lif (2003) studied the effect of the free draw geometry in a static laboratory device. Next step would be to quantify the findings above in an engineering approach.

CONCLUSIONS

- The tensile stiffness index and bending stiffness of an anisotropic multiply board can be predicted for any fibre anistropy and any total strain accumulated during drying based on isotropic input data measured on handsheets made of the pulps of the individual plies.
- The agreement between experimental results and predictions obtained by using the proposed model showed good agreement. According to the authors the model can be used for engineering applications.
- The fibre anisotropy is equal to the tensile stiffness anisotropy for restrained dried sheets.

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APPENDIX A

Determination of the free shrinkage strain of a multiply board

This appendix treats the evaluation of the free shrinkage strain of a multiply board based on the knowledge of the basis weight, w, freely dried tensile stiffness index, E^{fs}, restrained dried tensile stiffness index, E^{r}, and the free shrinkage strain, ε^{fs}, of the individual plies of the board.

A plane stress orthotropic elastic material model was adopted for the description of mechanical material behaviour of each of the individual plies after drying. This model, when inserted into Equation (2), results in a hygro-mechanical model for the individual plies given by Equation (A1), where ε_{MD} and ε_{CD} are the accumulated total strains during drying in the in-plane principal material directions (MD and CD), respectively, and the specific stresses corresponding to these strains are denoted by σ_{MD} and σ_{CD}. The material parameters E_{MD} and E_{CD} are the in-plane tensile stiffness indices, v_{MDCD} and v_{CDMD} denote the in-plane Poisson’s ratios, and ε^{fs}_{MD} and ε^{fs}_{CD} are the in-plane free shrinkage strains.

$$\begin{bmatrix} \varepsilon_{MD} \\ \varepsilon_{CD} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{E_{MD}} & \dfrac{-v_{MDCD}}{E_{MD}} \\ \dfrac{-v_{CDMD}}{E_{CD}} & \dfrac{1}{E_{CD}} \end{bmatrix} \begin{bmatrix} \sigma_{MD} \\ \sigma_{CD} \end{bmatrix} + \begin{bmatrix} \varepsilon^{fs}_{MD} \\ \varepsilon^{fs}_{CD} \end{bmatrix} \tag{A1}$$

An approximate method for the determination of the Poisson’s ratios was used. The symmetry requirement on the stiffness matrix (Equation (A2)) combined with the use of the so-called Baum’s approximation [Baum et al, 1981] according to Equation (A3), was utilized to determine the Poisson’s ratios.

$$\frac{v_{MDCD}}{E_{MD}} = \frac{v_{CDMD}}{E_{CD}} \tag{A2}$$

$$v_{MDCD} v_{CDMD} = 0{,}293^2 \tag{A3}$$

This model is not suitable for the analysis of the mechanics of paper drying since it does neglect the moisture-dependency of the material parameters during drying. The model is therefore applied in analysis of the dry paper after drying. Furthermore, the model does not predict accurate stress levels after drying, since it neglects relevant types of material behaviour of paper materials, such as viscosity and plasticity. However, the principal aim here is not to predict accurate stresses in the individual plies, but to determine the contributions of free shrinkage from each ply to the free shrinkage of the multiply board. In such predictions, the ratios between predicted stresses in different plies, rather than the absolute values of the stresses in the plies, are of relevance. The viscous behaviour of the individual plies, which has a large impact on the stresses in the individual plies, is however expected to have a small effect on the ratios of stresses between plies. Furthermore, plasticity is assumed to have a limited influence on the stresses in the individual plies during free drying of the multiply board.

A multiply structure is composed of two or more individual plies. When a multiply structure is dried freely, the individual plies generally exhibit different amounts of free shrinkage strain. The interaction between the plies, where plies that strives to shrink much are restrained by plies that strive to shrink less, gives rise to elastic strains and consequently to a non-uniform through-thickness profile of in-plane stresses in the multiply board. Under such circumstances, a ply with large potential for free shrinkage will be in a state of tension, while a ply with small potential for free shrinkage will be subjected to compression. Since no outer forces are acting on a multiply board during free drying, the sum of the forces in the individual plies must vanish. This condition can be expressed as,

$$\sum_{i=1}^{n} \sigma^{i}_{MD} w_i = \sum_{i=1}^{n} \sigma^{i}_{CD} w_i = 0 \tag{A4}$$

where n denotes the number of individual plies, σ^{i}_{MD} and σ^{i}_{CD} express the stresses in the in-plane principal material directions, respectively, for ply number i, and w_i is the basis weight of this ply. There is one strain state only, which satisfies the condition in Equation (A4), when the plies in the board are constrained to exhibit equal in-plane strains as the entire multiply board. This strain state consequently corresponds to the free shrinkage strain of the multi-ply structure, ε^{fs}_{Lam}.

The equation system defined by Equations (A1) and (A4) is reduced to Equation (A5) in the special case when all plies are either isotropic or when they exhibit equally large freely dried stiffness anisotropy. In Equation (A5), E_i and ε^{fs}_{i} denotes the tensile stiffness index and free shrinkage strain of ply number i, respectively, in one of the in-plane principal material directions (MD or CD). In other words, the free shrinkage strain in the in-plane principal material directions of the multiply board turns out to be uncoupled in these two special cases. This expression further constitutes an excellent approximate solution of the free shrinkage of the multiply board, also for situations when neither of these special cases applies.

$$\varepsilon_{Lam}^{fs} = \frac{\sum_{i=1}^{n} w_i E_i \varepsilon_i^{fs}}{\sum_{i=1}^{n} w_i E_i} \tag{A5}$$

The solution of the expression in Equation (A5) involves the use of a relation between the tensile stiffness index and the total accumulated strain in the individual plies, e.g. the use of Equation (3), which causes some implications. Firstly, the tensile stiffness index of an individual ply, E_i, becomes a function of the free shrinkage strain of the multiply board, ε_{Lam}^{fs}, which makes the evaluation of Equation (A5) more difficult. Secondly, Equation (3) predicts that the tensile stiffness index of an individual ply will decrease linearly with the total accumulated strain, even in situations when the total accumulated strain becomes lower than the free shrinkage strain of the ply. Consequently, the tensile stiffness index of the ply is predicted to be lower than E^{fs} when freely dried under compression. This second implication may cause that the tensile stiffness index of a ply in a multiply board, which exhibit small potential for free shrinkage, can be predicted to have a very low or even negative tensile stiffness index as a consequence of its interaction with other plies exhibiting large potential for free shrinkage. This non-physical effect may be avoided by re-expressing the relation between tensile stiffness index and accumulated total strain as in Equation (A6),

$$E = E^{fs} + (E^{r} - E^{fs})\left(1 - \frac{\varepsilon}{\varepsilon^{fs}}\right)\theta\left(1 - \frac{\varepsilon}{\varepsilon^{fs}}\right) \tag{A6}$$

where θ denotes the Heaviside's step function with the characteristics given by,

$$\theta = \begin{cases} 1, \; when\left(1 - \dfrac{\varepsilon}{\varepsilon^{fs}}\right) \geq 0 \\ 0, \; when\left(1 - \dfrac{\varepsilon}{\varepsilon^{fs}}\right) < 0 \end{cases} \tag{A7}$$

The expression in Equation (A6) coincides with Equation (3) for total accumulated strains which are larger than the free shrinkage strain, but deviates from Equation (3) by predicting a constant tensile stiffness index that is equal to the freely dried tensile stiffness index, E^{fs} for accumulated total strains that are smaller than the free shrinkage strain. The insertion of Equation (A6) into Equation (A5) yields the expression given by Equation (A8). The restrained dried basis weight, instead of the real basis weight, has been used in this expression, since the basis weight are present in all terms and since the plies in the board are constrained to exhibit equal in-plane strains as the entire multiply board, which causes all relative changes in basis weight to be proportional. The free shrinkage strain of the multiply board is evaluated by solving this second order equation.

$$(\varepsilon^{fs}_{Lam})^2 - 2\frac{\sum_{i=1}^{n}(E_i^r - E_i^{fs})w_i^r\theta\left(1-\frac{\varepsilon^{fs}_{Lam}}{\varepsilon_i^{fs}}\right)+\sum_{i=1}^{n}E_i^{fs}w_i^r}{\sum_{i=1}^{n}(E_i^r - E_i^{fs})\frac{w_i^r}{\varepsilon_i^{fs}}\theta\left(1-\frac{\varepsilon^{fs}_{Lam}}{\varepsilon_i^{fs}}\right)}\varepsilon^{fs}_{Lam} + \frac{\sum_{i=1}^{n}E_i^{fs}\varepsilon_i^{fs}w_i^r+\sum_{i=1}^{n}(E_i^r - E_i^{fs})\varepsilon_i^{fs}w_i^r\theta\left(1-\frac{\varepsilon^{fs}_{Lam}}{\varepsilon_i^{fs}}\right)}{\sum_{i=1}^{n}(E_i^r - E_i^{fs})\frac{w_i^r}{\varepsilon_i^{fs}}\theta\left(1-\frac{\varepsilon^{fs}_{Lam}}{\varepsilon_i^{fs}}\right)} = 0 \quad \text{(A8)}$$

The numerical intensity of the expression in Equation (A8) makes it cumbersome to use in engineering predictions of the anisotropic properties of multiply boards. The complexity of the expression is mainly caused by the presence of the tensile stiffness index in Equation (A5), which in turn is dependent on the total accumulated strain. A reasonable approximation could therefore be to simply replace E_i by the freely dried tensile stiffness index E_i^{fs} (see Equation (A9)) or the restrained dried tensile stiffness index E_i^r (see Equation (A10)) of the individual plies in Equation (A5), instead of using the relation for the tensile stiffness index given by Equation (A6).

$$\varepsilon^{fs}_{Lam} = \frac{\sum_{i=1}^{n} w_i E_i^{fs}\varepsilon_i^{fs}}{\sum_{i=1}^{n} w_i E_i^{fs}} \quad \text{(A9)}$$

$$\varepsilon_{Lam}^{fs} = \frac{\sum_{i=1}^{n} w_i E_i^r \varepsilon_i^{fs}}{\sum_{i=1}^{n} w_i E_i^r} \tag{A10}$$

A comparison of these three different expressions for predicting the free shrinkage strain of the multiply boards that are studied in this work are shown in Table A1.

Table A1 Predictions of the free shrinkage strain of the multiply boards using Equations (A5), (A9) and (A10).

Laminate	1	2	3	4	5	6	7
MD, using Equation (A5)	−2,62	−1,96	−4,73	−3,70	−3,10	−5,24	−4,57
MD, using Equation (A9)	−2,62	−1,92	−4,44	−3,55	−2,94	−5,16	−4,57
MD, using Equation (A10)	−2,62	−1,97	−4,77	−3,82	−3,27	−5,22	−4,57
CD, using Equation (A5)	−2,78	−3,52	−5,03	−6,46	−7,37	−7,37	−8,50
CD, using Equation (A9)	−2,78	−3,38	−4,69	−5,87	−6,94	−7,21	−8,50
CD, using Equation (A10)	−2,78	−3,47	−5,02	−6,27	−7,23	−7,29	−8,50

These results indicate that Equation (A10) yields excellent engineering predictions of the free shrinkage strain of the studied multiply boards. The approximate solution in Equation (A10) may be further simplified by inserting the restrained dried basis weight and by realising that the restrained dried tensile stiffness index of the multiply board can be evaluated directly as a weighted average value of the restrained dried tensile stiffness indices of the individual plies. Equation (A11) shows the proposed approximate expression for determining the free shrinkage strain of the multiply board.

$$\varepsilon_{Lam}^{fs} = \frac{\sum_{i=1}^{n} w_i^r E_i^r \varepsilon_i^{fs}}{w_{Lam}^r E_{Lam}^r} \tag{A11}$$

Equation (A11) states that the product of the restrained dried basis weight of the laminate, w_{Lam}^r, the restrained dried tensile stiffness index of the laminate, E_{Lam}^r, and the free shrinkage strain of the laminate, ε_{Lam}^{fs}, is equal to the corresponding product for each individual ply, summed over all plies.

Equation (A11) highly facilitates the calculation of the free shrinkage strain of a multiply laminate. This relation has therefore been used for predicting the free shrinkage strain of multiply boards throughout this work.

Transcription of Discussion

PREDICTIONS OF ANISOTROPIC MULTIPLY BOARD PROPERTIES BASED ON ISOTROPIC PLY PROPERTIES AND DRYING RESTRAINTS

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Jean-Francis Bloch EFPG-INPG

Thank you for this interesting presentation. I do have some comments and some questions. First, in Figure 3 in the text, which you also used in your presentation, I have a question concerning the definition of the axes. You have used the label "fibre anisotropy". Do you have in mind fibre orientation anisotropy?

Torbjörn Wahlström

Yes. Actually in the text, we say fibre orientation anisotropy.

Jean-Francis Bloch

My second point is: what is your definition of fibre orientation anisotropy? I suspect that you consider the number of fibres in the machine direction and the number of fibres in the cross direction.

Torbjörn Wahlström

Yes, at least as a conceptual description.

Jean-Francis Bloch

So, I would like to make you aware of work familiar to many people here in this field. I am referring to Van Den Akker, Corte, Kallmes, Silvy and Dodson. The point is that you also have to consider the length of the fibre. I mean, just to give you an example: if you have three long fibres in the machine direction, three short fibres in the cross direction, you will consider it as isotropic and obviously mechanical properties will be anisotropic. Therefore, I do think that maybe this has to be considered. That was my first comment.

Torbjörn Wahlström

The method could have been described better, but it does give you a length weighting.

Jean-Francis Bloch

The second comment is: what is your conclusion on Figure 3 about the relationship between fibre orientation anisotropy and tensile stiffness anisotropy? Do you consider it as linear, not linear, what is your conclusion about this figure?

Torbjörn Wahlström

I consider it as linear, meaning that the tensile stiffness index anisotropy for a standard paper is equal to the fibre orientation anisotropy. I must say I am very surprised that there are not more data existing about fibre orientation and how that affects properties. Considering the amount of people working on predicting fibre orientation, it is quite surprising that the link to the paper properties is so weak. We have studied four furnishes in this study and found that the relation, Figure 3 in the proceedings, is okay. It is not perfect data, but as said earlier, to be used in an engineering approach it seems to be good enough.

Jean-Francis Bloch

If the Chairman allows me, I have a last question. Do you consider that fibre orientation depends on drying?

Torbjörn Wahlström

No.

Jean-Francis Bloch

I mean it is well known that for, let us say, little beating, there is no effect, but for very high beating, for example, tracing paper, you may have some influence of drying. I mean shrinkage may modify the orientation but only for a very high beating levels. Do you think this is true?

Torbjörn Wahlström

Not so much work has been done in this area but, for example, Steenberg made experiments on a greaseproof paper machine using a highly refined furnish as you say, and they used a press draw around 20% and still could not see any effect on the fibre orientation. I cannot imagine that there would be any significant effect on properties or the fibre orientation from drying shrinkage or stretch.

It is, of course, a cornerstone in this model that the fibre orientation is constant through the papermaking process.

Joel Panek Iggesund Paperboard

I have a question about your bending stiffness prediction and how good would that be if there was not an overprediction in the thickness. I do not consider that an overprediction of bending stiffness from 25% to 100% is good enough. But if the thickness was corrected, would you have better bending stiffness predictions?

Torbjörn Wahlström

Yes. The relative overprediction is around 10% and that would typically give 30% overprediction in bending stiffness.

Joel Panek

I think it is positive because there is a consistent overprediction, so it should be able to be taken into account and I think the CD data did not show significant scatter. So, I am just wondering if you did get a better prediction of the thickness with that, would you get a better prediction of the bending stiffness.

Torbjörn Wahlström

Of course, we would get better prediction of bending stiffness, but I do not know if that is the whole story here.

Marit Van Lieshout Paperlinx

Tjorbörn, from the beginning you say that the properties are only the sum of properties of the plies. Is there no effect of how these plies are bound together?

Torbjörn Wahlström

Not really, of course they have to be bound together; otherwise, the shrinkage will not be transferred from one ply to another. In my world, if they are not bound together, the paper will fall apart. Perfect bonding between the plies is assumed here. For example, if one ply shrinks 8% and the other 6%, then the shrinkage of the multi-ply board will be in between 6% and 8%.

Marit Van Lieshout

So this is how it should be if the bonding was perfect?

Torbjörn Wahlström

It is not an issue here, I mean the bonding is good enough. Take validation experiments as an example. We took a handsheet made of a middle ply furnish from a liquid packaging board machine containing CTMP, with a handsheet made of unbleached sulphate. They were couched together in the wet state and they attached enough to each other not to delaminate during the free shrinkage trial which is quite critical. You have to get good enough plybonding, otherwise, they will separate during drying.