Our study will focus on the low consistency refining kinetics of a pulp suspension at a given level of applied specific energy. The mechanical treatment on fibres caught in the confined zones of gap clearance (bar crossings) reveals some heterogeneity. It can be experimentally verified that the physical effects on fibres are not the same whether the confined zones are close to the internal radius (input side) or close to the external radius (output side). One must also account for the dynamic sliding motion of the confined zones through the rotation of the rotor plate (or cone) in front of the stator. Hence, each bar crossing has its own sliding velocity.

In order to predict the radial variability of the cutting effect on fibres, through engineering parameters that can be easily determined (radial coordinate; angular velocity; width of bars, width of grooves, average bar angle, both for rotor and stator patterns), a theoretical understanding of the radial distribution of the pressure must be undertaken. The pressure is locally applied on the pulp pads in confined zones of the gap clearance.

The best homogeneous results are obtained either with a cylindrical or a conical refiner where the bars are parallel to the
rotation axis. The degree of variability on the cutting effect on fibres is increased with an increase of the bar inclination versus the radial direction and a decrease of the ratio between the internal and external radius.

**INTRODUCTION**

This research concerns the refining kinetics of pulp suspensions in the low consistency regime. The refining trials are performed under constant net power and constant angular speed. This analysis is developed through a physical description of the forces encountered in an industrial refiner, forces which can contribute to the refining effects on fibres. In particular, we are interested in determining radial variability of the cutting effects on fibres, resulting from different local force intensities.

Some results, already obtained in the hydro-mechanical theory [1] will be recalled. The radial coordinate will not be considered initially. Then, the description will be given, in a general sense, in order to apply for disc and conical refiners. This will lead to the formula of the global friction coefficient of the couple: cellulose materials and metal of bars.

However, as this description is not in accordance with the experimental results obtained on the cutting effects on fibres, an extension will be proposed allowing for the influence of the radial coordinate on the physical description.

Taking into account the dynamic sliding motion and the velocity of rotor bars, it will be possible to determine the net power distributed on the working zone of an industrial refiner, versus the radial coordinate. The engineering parameters will be precised and their influence on the variability of the desired cutting effect on fibres analysed and discussed. A local extension of the reference specific edge load will be proposed and the expression of the corresponding local effective pressure on fibres postulated.

**HOW TO DETERMINE THE GLOBAL FRICTION COEFFICIENT?**

**Demonstration [1]**

The following analysis is general and can be extended to the case of industrial refiners running in continuous mode. If the refining trials are performed, in discontinuous mode (or batch), under constant net power $P_{net}(W)$ and constant angular velocity $\omega(rad.s^{-1})$, then studying the influence of the net specific energy consumption $E_m(J.kg^{-1})$ (where $m_s$ is the dry mass of pulp) is analogous to studying the refining kinetics through the equation:
In the gap clearance of an industrial refiner, the surfaces (rotor/stator) facing each other are fitted with bars (though it is not necessary to have bars to get refining action on fibres). Bars do not cover the complete area and some zones correspond to the inter-crossing bars that compose the gap clearance and others (rotor bar/stator groove; rotor groove/stator bar; rotor groove/stator groove) constitute the complementary area. The effective fraction of the area is easily calculated by the expression:

$$\xi = \frac{a_R \cdot a_S}{(a_R + b_R) \cdot (a_S + b_S)}$$

where \((a,b)\) are the width of bars and grooves respectively and the corresponding subscript \(R, S\) stands for the rotor and the stator respectively.

Now, let us define the space average mechanical pressure \(P_m(t)\) exerted on the compressed fibrous pads in the inter-crossing areas of the bars of a refiner (see Figure 1) and the space average hydraulic pressure \(P_h\) elsewhere.

**Figure 1** Configuration of a conical and a single disc refiner.
small surface between the radius $\rho$ and $\rho + dp$, the elementary normal force can be calculated according to the following expression:

$$dF_n(t) = \overline{P_m(t)} \cdot \frac{2\pi \rho . dp}{\sin(a_c)} + \overline{P_h(t)} \cdot (1 - \zeta) \cdot \frac{2\pi \rho . dp}{\sin(a_c)}$$  \hspace{1cm} (3)

Equation (3) applies under both refining and non-refining conditions. In the case of non-refining conditions, it is experimentally measured that the average hydraulic pressure is obtained anywhere (inter-crossing or their complementary areas). The Equation (3) then becomes:

$$dF_n^0 = \overline{P_h(t)} \cdot \frac{2\pi \rho . dp}{\sin(a_c)} + \overline{P_h(t)} \cdot (1 - \zeta) \cdot \frac{2\pi \rho . dp}{\sin(a_c)}$$  \hspace{1cm} (4)

If the gap clearance $\bar{e}(t)$ is less than a typical value already defined [2], i.e. the critical gap clearance $\bar{e}_c$, then the elementary normal force $dF_n(t)$ is greater than the reference value $dF_n^0$ and the net (or effective) elementary force is defined accordingly:

$$dF_{n \text{eff}}(t) = dF_n(t) - dF_n^0 = \overline{P_m(t)} - \overline{P_h(t)} \cdot \frac{2\pi \rho . dp}{\sin(a_c)}$$  \hspace{1cm} (5)

The average mechanical pressure is a total pressure exerted on both the solid and the liquid phase. In the pressing of food materials by Körmendy and co-authors [3] or the wet pressing of paper materials by Nilsson and co-authors [4], the concept of total pressure is clearly defined. The total pressure is made up of a hydraulic pressure (on the liquid phase) and of an effective pressure, transmitted through the solid phase, in the compressed zones (inter-crossing areas of bars). The effective pressure is the expression between brackets in Equation (5). From this observation, the net friction force $dF_{n \text{eff}}(t)$, a differential quantity, can be assessed if a global friction coefficient $f(t)$ is postulated as well. As the morphology of the solid components of the pulp suspension is varying through the refining kinetics, the global friction coefficient is obviously a function of time, then:

$$dF_{f \text{eff}}(t) = f(t) \cdot dF_{n \text{eff}}(t) = f(t) \cdot \overline{P_m(t)} - \overline{P_h(t)} \cdot \frac{2\pi \rho . dp}{\sin(a_c)}$$  \hspace{1cm} (6)

In this first description of the forces exerted between rotor and stator surfaces, in an industrial refiner, the physical quantities $\overline{P_m(t)}$ and $\overline{P_h}$ are averaged
on the corona located between the radii \( \rho_i \) and \( \rho_e \) (see Figure 1). Then, the integration of both Equations (5) and (6) can be undertaken noting a new engineering parameter for simplification:

\[
k = \frac{\rho_i}{\rho_e}
\]  

(7)

After integration over the complete corona and replacing this ratio by the new parameter \( k \), it leads to:

\[
\begin{align*}
F_{n}^{\text{eff}}(t) &= \left[ \overline{P_m(t)} - \overline{P_h} \right].\xi.\pi.\rho_e^2(l - k^2) \sin(a_c) \\
F_{f}^{\text{eff}}(t) &= f(t).\left[ \overline{P_m(t)} - \overline{P_h} \right].\xi.\pi.\rho_e^2(l - k^2) \sin(a_c)
\end{align*}
\]

(8)

These effective elementary forces in (5) and (6), or the resulting power consumed by friction, are responsible for the refining effects to occur. The differential friction power can be determined through the following equations:

\[
dP_f^{\text{eff}}(t) = dF_f^{\text{eff}}(t). \omega. \rho = f(t).\left[ \overline{P_m(t)} - \overline{P_h} \right].\xi.2\pi\omega.\rho^2.d\rho \sin(a_c)
\]

(9)

If one postulates that this friction power, integrated over the complete corona, is the net power consumed for the refining effects, then an average radius \( \langle \rho \rangle \) can be calculated according to:

\[
P_f^{\text{eff}} = P_{net} = f(t).F_n^{\text{eff}}(t).\omega.\langle \rho \rangle
\]

(10)

then:

\[
\langle \rho \rangle = \frac{2\rho_e(l - k^3)}{3(l - k^3)}
\]

(11)

From Equations (8) and (10), it is possible to extract the global friction coefficient \( f(t) \) as indicated:
One may observe that this expression leads to a non-determination of the global friction coefficient when the net power equals zero since the average total pressure is equal to the average hydraulic pressure and no refining effects are visible on fibres.

In the paper industry, the cutting effect on fibres is often related to an intensity index called the specific edge load. In a previous work, we have generalized this concept through the *reference specific edge load* since this index correctly accounts for the influence of engineering variables on the cutting effect on fibres [5]:

\[
C_0^s = \frac{3P_{net}(a_R + b_R)(a_S + b_S)\sin(a_c)}{2\pi\omega \rho^3(l - k^3)}
\]  

If we combine Equations (12) and (13), it leads to a fundamental result [2] that gives the physical nature of the reference specific edge load:

\[
\overline{P_m}(t) - \overline{P_h} = \frac{C_0^s}{f(t) a_R a_S}
\]  

**Commentaries**

This Equation (14) reveals that only average quantities can give birth to the reference specific edge load, a quantity of an equally average nature. However, the refining kinetics can be developed through a hydrodynamic description, considering a complex fluid of an equivalent apparent viscosity, as done in reference [6]. It can be demonstrated that the hydraulic pressure is a function of the space coordinate through the refiner, as in any lubricated bearing [7]. In this hydrodynamic description ([6]), the use of local variables does reflect the physical reality, which is not the case for averaged quantities.

By comparison, in a conical or a disc refiner, the mechanical pressure should be a function of the radial coordinate in the direction of pulp motion, from the internal to the external radius. This suggests that the reference specific edge load should be a local quantity or, at least, a function of the radial coordinate: \( C_0^s(\rho) \). If we are able to evaluate this local quantity, then the mechanical pressure \( P_m(\rho,t) \) will be defined by an extension of Equation (14) accordingly:

\[
P_m(\rho,t) - \overline{P_h} = \frac{C_0^s(\rho)}{f(t) a_R a_S}
\]
A question remains to be solved: how to determine the local reference specific edge load? Let us suppose that an elementary refiner has a working area comprised between the radii $\rho$ and $\rho + d\rho$. In that case, the local reference specific edge load should be defined as follows:

$$C_s^0(\rho) = \frac{dP_f^\text{eff}(\rho,t)}{dL_c^0(\rho)}$$

(16)

where the quantity, in the denominator, represents the elementary cutting speed given by the expression:

$$dL_c^0(\rho) = \frac{2\pi \rho}{a_R + b_R} \cdot \frac{2\pi \rho}{a_S + b_S} \cdot \frac{\omega}{2\pi} \cdot d\rho$$

(17)

If the effective power is dissipated by friction, as calculated by Equation (9), then it can be shown that the local reference specific edge load does not appear to be a function of the radial coordinates $\rho$. From this statement, the calculation of the power dissipated must be undertaken differently. In a next section, particular attention will be paid to the sliding velocity of moving bars in the rotating motion of the rotor in front of the stator.

**THE EXPERIMENTAL CAMPAIGN**

Before considering the new calculation of the power consumed, an experimental campaign was undertaken with an industrial double disc refiner (16″ or 0.406 m diameter) running in hydra cycle mode. In order to approach the influence of the radial coordinate on the refining effects on fibres, a simple approach was chosen. The influence of the location of a working corona was investigated on chosen pulps.

For that purpose, 3 coronas were studied, one close to the inlet or to the internal radius (Z1), one medium (Z2) and the last one, close to the external radius (Z3) or the periphery, as illustrated on Figure 2 where only one disc sector is shown.

Naturally, the 3 coronas were chosen with the same Reference Specific Edge Load in order to compare the refining effects on fibres with the same intensity and with the same effective power applied. From a practical point of view, a working zone is fitted with bars manufactured with an over height of 1 mm in the corona for stator and rotor discs. After the refining trials, we verify that only the working bars were worn, that is to say, only these bars, belonging to the corona analysed, were active in the refining process.

Radial Distribution of the Pressure in an Industrial Refiner
2 pulps: a bleached softwood Kraft pulp and a bleached hardwood bisulphite pulp and 2 geometries: one with a more prone “cutting” pattern and another with a “fibrillating” pattern were used for the trials.

However, for the sake of simplicity, only the results obtained on the bleached softwood Kraft pulp with a “fibrillating” geometry (sum of the grinding angles equals 30°) will be presented. The experimental results [8] are summarized in Table 1 and Table 2 respectively, for the average weighted fibre length $\bar{L}_f \text{ in mm}$ and the Schopper-Riegler ($^\circ$SR) pulp slowness.

What conclusions can be drawn from these tables?
First, it can be seen that the cutting effect on fibres is increasingly pronounced from the internal (Z1) to the external corona (Z3). Second, the pulp

![Figure 2](image.png)

**Figure 2** Schematic representation of the 3 coronas.

<table>
<thead>
<tr>
<th>$E_m (kWh.t^{-1})$</th>
<th>0</th>
<th>15.2</th>
<th>30.3</th>
<th>45.5</th>
<th>60.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z1</td>
<td>1.67</td>
<td>1.65</td>
<td>1.58</td>
<td>1.54</td>
<td>1.50</td>
</tr>
<tr>
<td>Z2</td>
<td>1.67</td>
<td>1.64</td>
<td>1.49</td>
<td>1.46</td>
<td>1.43</td>
</tr>
<tr>
<td>Z3</td>
<td>1.67</td>
<td>1.63</td>
<td>1.46</td>
<td>1.42</td>
<td>1.30</td>
</tr>
</tbody>
</table>

*Table 1* Average weighted fibre length (in mm) – $Lc^o = 20.2 \text{ km/s} – Cs^o = 0.9 \text{ J/m}$ for 3 different locations $Zi$ of the working zone in a double disc refiner
slowness is the highest for the medium corona (Z2). These results can be explained if we assume that the main refining effects on fibres (cutting and fibrillation) occur simultaneously at an equal level. In the internal corona (Z1), the cutting effect is weaker and the fibrillating effect may increase more than on the external corona (Z3) where the power is more dissipated to cut the fibres. The conclusions can be obtained for both pulps (softwood and hardwood) and both geometries. Following various investigations, it seems to reflect a general behaviour.

These findings are in total accordance with previous research by the same authors [9]. It was demonstrated in [9] that the (reference) specific edge load has to be chosen and adapted to optimize the $\theta_{SR}$ kinetics of the fibrous raw material to be refined, see Figure 3 for example where the $\theta_{SR}$ is shown versus the reference specific edge load, for any specific energy, expressed in kWh/t.

For a given net energy per unit mass of pulp, if the applied value of $C_0$ is too low for the fibrous material (at the left side of the optimum), then the $\theta_{SR}$ kinetics is slow (case of Z1); if the $C_0$ is adapted (on the optimum) then the $\theta_{SR}$ kinetics is optimized (case of Z2); if the $C_0$ applied is too strong for the fibrous material (at the right side of the optimum), then the $\theta_{SR}$ kinetics is slowed down (Z3) compared to the optimized case. All these observations are confirmed by Table 2. The red curve on Figure 3 represents the locus of the adapted values of the reference specific edge load, for the raw material to be refined.

So, in order to simulate the influence of the radial coordinate on the refining intensity (the cutting effect on fibres), an experimental campaign was undertaken with 3 different coronas. It proved that the radial coordinate needs to be included in any description of a refining intensity concept. It is the purpose of the following paragraph to propose a calculation of the net power dissipated by friction taking into account the sliding velocity of moving bars.

### Table 2  Pulp slowness ($\theta_{SR}$) – $L_{c0} = 20.2$ km/s – $C_0 = 0.9$ J/m for 3 different locations $Z_i$ of the working zone in a double disc refiner

<table>
<thead>
<tr>
<th>$E_m$ (kWh/t)</th>
<th>0</th>
<th>15.2</th>
<th>30.3</th>
<th>45.5</th>
<th>60.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z1</td>
<td>14</td>
<td>16.5</td>
<td>19</td>
<td>21.5</td>
<td>25</td>
</tr>
<tr>
<td>Z2</td>
<td>14</td>
<td>17</td>
<td>20</td>
<td>23</td>
<td>26.5</td>
</tr>
<tr>
<td>Z3</td>
<td>14</td>
<td>16</td>
<td>18.5</td>
<td>21</td>
<td>24</td>
</tr>
</tbody>
</table>
KINEMATICS OF THE ROTOR BARS AND SLIDING VELOCITY

Analysis the kinematics (case of a single disc refiner)

The radial coordinate is now concerned and the case of a single disc refiner is chosen to illustrate the kinematics. On Figure 4, we can see 2 positions of a given rotor bar at time $t$ and time $t + dt$. Our purpose is to determine the local sliding velocity of this given moving bar since it reveals a displacement, materialized by the distance $AB$, during the time $dt$.

Since distance $OA$ represents the coordinate $\rho$ and distance $OB$ the coordinate $\rho + dp$, it is possible to estimate $dp$ by the distance $AC$. The physical meaning of distance $AD$ is the sliding displacement of the point $A$ on the rotor bar during time $dt$. These sliding displacements $AB$ and $AD$ are
differential quantities that can be calculated considering the two local angles \( \varphi_s \), \( \varphi_r \), respectively, of the given stator bar (rotor bar respectively) versus the radial direction \( \vec{OC} \). In the two rectangular triangles \( ACB \) and \( ACD \), the following differential equations can be easily obtained:

*Figure 4* Motion of a given rotor bar in front of a given stator bar for the case of a disc refiner.
\[
\begin{align*}
\begin{cases}
  dp = AB \cos(\varphi_s) = ds \cos(\varphi_s) \\
  dp = AD \cos(\varphi_r) = dr \cos(\varphi_r)
\end{cases}
\end{align*}
\] (18)

Since the measure of the angle DOB is given by \(\omega dt\), then the distance DB can be determined by the followings, considering the first order in the developments:

\[
(\rho + dp) \omega dt = ds \sin(\varphi_s) + dr \sin(\varphi_r) \equiv \rho \omega dt
\] (19)

By combining the previous set of Equations (18) and (19), and noting \(\gamma\) the local crossing angle of the given bars (see Figure 4), we obtain:

\[
\rho \omega dt = ds \frac{\sin(\varphi_r + \varphi_s)}{\cos(\varphi_r)} = ds \frac{\sin(\gamma)}{\cos(\varphi_r)}
\] (20)

If the fibres (to be impacted) cover the stator bars as it is intended, it seems logical to pay greater attention to the sliding velocity of a given rotor bar on a given stator bar. The sliding velocity is calculated as follows:

\[
V(\rho, \gamma, \alpha) = \frac{ds}{dt} = \omega \rho \cos(\varphi_r) \frac{\omega \sqrt{\rho^2 - \rho_x^2 \sin^2(\varphi_r)}}{\sin(\gamma)} = \frac{\omega \sqrt{\rho^2 - \rho_x^2 \sin^2(\alpha)}}{\sin(\gamma)}
\] (21)

The last expression introduces the local angle \(\alpha\) of the rotor bar concerned. However, everything being equal, the sliding velocity is not the same if the first rotor bar is considered or the last one on the same sector angle \(\theta\). In order to calculate the effective power dissipated by friction, one should consider all bars of the rotor versus all bars of the stator. We chose another approach, postulating that the geometrical angular parameters could be considered by their average values. In which case, the “average” velocity \(V(\rho, \bar{\gamma}, \bar{\alpha})\) is an increasing function of the local radius \(\rho\).

The problem of assessing the effective power dissipated by friction during the bar motion can be approached more easily.

**Distribution of the cumulated effective power**

Under the previous assumptions, the net power can be expressed by the following equation with an unknown average angle \(\lambda(t)\) between the tangential force \(dF_{eff}^{\varphi} (\rho, t)\) and the average sliding velocity \(V(\rho, \bar{\gamma}, \bar{\alpha})\):
Radial Distribution of the Pressure in an Industrial Refiner

\[ dP_{f}^{\text{eff}} (\rho, t) = dF_{f}^{\text{eff}} (\rho, t). \frac{\omega.\sqrt{\rho^{2} - \rho_{e}^{2}.\sin^{2}(\bar{a})}}{\sin(\gamma)}.\cos(\lambda(t)) \] (22)

By replacing the differential friction force with Equation (6) and putting \( \alpha_c = \pi / 2 \) for the case of a single disc refiner, another expression can be obtained:

\[ dP_{f}^{\text{eff}} (\rho, t) = \{ f(t).[\bar{P}_n(t) - \bar{P}_h].\xi.2\pi\omega\}.\frac{\rho.\sqrt{\rho^{2} - \rho_{e}^{2}.\sin^{2}(\bar{a})}}{\sin(\gamma)}.\cos(\lambda(t)).dp \] (23)

Then, the expression between brackets can be replaced with the help of Equation (12) resulting in:

\[ dP_{f}^{\text{eff}} (\rho, t) = \frac{3. P_{net}}{\rho_{e}^{2}(1 - k^{2})}.\frac{\cos(\lambda(t))}{\sin(\gamma)} . \rho.\sqrt{\rho^{2} - \rho_{e}^{2}.\sin^{2}(\bar{a})}.dp \] (24)

In this equation, an angle remains an unknown; this can be solved through boundary conditions since the power is distributed from \( \theta \) on the internal radius \( \rho_i \) until \( P_{net} \) on the external radius \( \rho_e \). The Equation (24) allows us to calculate the distribution of the cumulated effective power dissipated by friction from the internal radius \( \rho_i \) until a local radius \( \rho \):

\[ P_{\text{eff}} (\rho) = \int_{\rho_i}^{\rho} dP_{f}^{\text{eff}} = \frac{P_{net}}{\rho_{e}^{2}(1 - k^{2})}.\frac{\cos(\lambda(t))}{\sin(\gamma)}.\{ [\rho^{2} - \rho_{e}^{2}.\sin^{2}(\bar{a})]^{3/2} - [\rho_i^{2} - \rho_{e}^{2}.\sin^{2}(\bar{a})]^{3/2} \} \] (25)

After simplifications and introducing the engineering ratio \( k \) defined by Equation (7), the unknown angle can be obtained from the boundary limits, this leads to the general result for the cumulated effective power from the internal until the local radius:

\[ P_{\text{eff}} (\rho) = P_{net}.\frac{\left[ \left( \frac{\rho}{\rho_e} \right)^2 - \sin^2(\bar{a}) \right]^{3/2} - [k^2 - \sin^2(\bar{a})]^{3/2}}{\cos^2(\bar{a}) - [k^2 - \sin^2(\bar{a})]^{3/2}} \] (26)
It also leads to the definition of the \( \lambda \) angle and of the constant \( A(k, \alpha) \) as follows:

\[
\cos(\lambda) = \frac{(1 - k^3)}{\cos^3(\alpha) - [k^2 - \sin^2(\alpha)]^{3/2}} \cdot \sin(\gamma) = A \cdot \sin(\gamma)
\]

(27)

This angle is not time dependent and the time \( t \) must be erased from previous Equations from (22) to (25). In order to give a physical meaning to this result, let consider radial bars for the rotor disc, then the lambda angle is easily determined, it complements the average crossing angle \( (\hat{\lambda} = (\pi / 2) - \hat{\gamma}) \).

**Distribution of the reference specific edge load**

In the general case of conical or disc refiners, the local reference specific edge load can be obtained by the following expressions, deduced from Equations (16), (17) and from Equation (25) to (27):

\[
C_{0s}(\rho) = \frac{dP_{eff}(\rho)}{dL_c(\rho)} = \frac{(1 - k^3)}{\cos^3(\alpha) - [k^2 - \sin^2(\alpha)]^{3/2}} \cdot \frac{\sqrt{(\rho/\rho_e)^2 - \sin^2(\alpha)}}{(\rho/\rho_e)}
\]

(28)

It can be demonstrated and observed through Figure 5 that the local reference specific edge load \( C_{0s}(\rho) \) is an increasing function of the radial
coordinate $\rho$. The variability of the predetermined cutting effect on fibres can be assessed, for example, by the ratio $R$ of the maximum value (obtained on the external radius) to the minimum value (obtained on the internal radius):

\[
R = \frac{C_0(\rho_e)}{C_0(\rho_i)} = \frac{(\cos \bar{a}) k}{\sqrt{k^2 - \sin^2 \bar{a}}} 
\]  
(29)

From a theoretical point of view, a refiner for which $k = 1$ gives no variability to the local reference specific edge load ($R = 1$) or no variability on the cutting effect on fibres. This refiner with an internal radius equals to the external one is a cylindrical refiner.

As this technology is not wide spread in the Paper Industry, from both theoretical and practical points of view, it is more useful to evaluate the importance of the variability for conical and disc refiners.

If a conical refiner is used with a mean bar angle $\alpha$ equal to 0, then the ratio $R$ is equal to 1 whatever the value of $k$ is, since a simplification occurs. No variability is expected for the cutting effect on fibres in that case.

For a disc refiner with a typical value of $k$, the ratio of the local reference specific edge loads is seen to be increasing with the mean bar angle $\bar{a}$. Equation (28) reveals the existence of a limiting value for this angle where the ratio $R$ can reach high numerical values:

\[
\bar{a}_{\text{lim}} = \arcsin(k) 
\]  
(30)

For example, the ratio $R$ is equal to 1.57 with $\alpha = 15^\circ; \theta = 30^\circ; k = 0.60$; $\bar{a} = 30^\circ; \bar{a}_{\text{lim}} = 37^\circ$

Hence, the local reference specific edge load on the external radius is 57% higher than that of the internal radius.

If the refining trials of the experimental campaign, with 3 different coronas, are re-analysed, we understand now why the cutting effect on fibres is increasing from $Z1$ to $Z2$ and $Z3$. The Equation (28) and Figure 5 reveal that the local reference specific edge load is an increasing function of the radial coordinate; it means that fibres are being subject to increasing “cutting” indexes.

The knowledge of the local reference specific edge load is sufficient to determine the pressure. The radial pressure $P_m(\rho, t)$ is given by the extension of Equation (14) for the case of a “theoretical” refiner with a running area comprised of radii $\rho$ and $(\rho + d\rho)$:
The same conclusions apply accordingly. It is demonstrated that the relative pressure is an increasing function of the radial coordinate. The difference between the maximum and minimum pressure is given by:

\[ P_m(\rho, t) - P_h = \frac{C_0(\rho)}{f(t) a_R a_S} \]  

This difference is increasing with the mean angle \( \bar{\alpha} \), but is decreasing with \( k \).

**CONCLUSION**

The most homogeneous results on the cutting effect on fibres should be obtained with a refiner where both internal and external radii are identical, i.e. a cylindrical refiner. In the case of conical refiners where all bar angles are equal to 0, the total pressure applied on the pulp pads, located in the inter-crossing areas of bars, is uniform versus the radial coordinate.

In the case of disc refiners, the total relative pressure, or the local pressure transmitted through the solid phase, is an increasing function of the radial coordinate exemplified by Equations (28) and (31). There is still a difference in the pressure applied which can be assessed between the internal and the external radius.

These theoretical findings are experimentally confirmed on an industrial scale, with disc refiners, where the cutting effect on fibres is increasingly pronounced from the internal to the external radius, i.e. from the inlet to the outlet in the pulp motion.

These results are an extension of the interpretation of the curves \( ^{o} \text{SR} \) versus reference specific edge load for any given net specific energy. If the intensity \( C_{s}^{o} \) is too low (case of Z1 on the internal radius) then the \( ^{o} \text{SR} \) refining kinetics is not optimized and is slowed down. If the intensity is perfectly adapted (case of Z2 on the medium location on the corona) then the \( ^{o} \text{SR} \) kinetics is optimized. Finally, if the reference specific edge load is too high (case of Z3 on the external radius) then again the \( ^{o} \text{SR} \) kinetics will not be optimized.

So, what would be the best evolution for \( C_{s}^{o} \)? Certainly not an increasing
function of the radial coordinate obtained with the current patterns of disc refiners.

On the contrary, as the pulp is refined from the internal to the external radius, the best refining kinetics should be obtained with a decreasing function of the local $C_s\degree$ versus the radial coordinate. This is explained through the locus of the maxima on Figure 3: the more the fibres are refined, the less are values of the intensity to be applied.

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REFERENCES

Transcription of Discussion

RADIAL DISTRIBUTION OF THE PRESSURE IN AN INDUSTRIAL REFINER

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Juha Salmela \hspace{1em} VTT Processes

Do you think it would be possible or beneficial, instead of using straight bars, to use some kind of bending or changing the angle of those bars, so that you could keep the angle constant?

Jean-Claude Roux

Yes, that is one solution of the analytical problem. You can maintain, for example, a constant crossing angle of the bars on all the paths between the internal radius to the external radius. What I did not mention in the presentation is that, from an analytical result, you can obtain such spiral or logarithmic curves in angular coordinates.

Juha Salmela

So, what do you think would be the result of that?

Jean-Claude Roux

I think that when you master the effect and when you have validated the model, a very big variety of curves can fit the physical equations. So, I have no
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preference, very often I receive the question what do you think of conical or disc refiners: which is the best? No, old technology can be good, if you master all the engineering parameters on the effect on fibres. So, I have no precise idea on what is the best. Probably when you keep this crossing angle constant, it can give an interesting solution, for example, for the variability of second order effects. It is just one solution.

*Warren Batchelor*  
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In your derivation here are you assuming that each time a rotor and stator bar cross over each other, that they have a mat of fibres trapped underneath them at that point? What would happen if that were not the case?

*Jean-Claude Roux*

In fact it is very difficult to describe these phenomena. So what I am assuming is that I follow a bar, which is rotating and which is crossing over other bars. As the situation is very different for each crossing, we model each one. It means that we have plenty of points, plenty of velocities, because it is very different for each crossing and furthermore, it changes with time. So, we consider instead the average value of, for example, the rotor grinding angle and then we consider that the rotor bar is moving against a stator bar and we consider that the scenario is exactly the same. This is because we would like to have some analytical tool, which can be validated and which is not far from the physical reality of the phenomenon, which in fact is very complicated as you mentioned.

*Derek Page*

If I could ask one question, which I am sure you have heard before. What do you do with a refiner, which definitely refines there is no question about it, but does not have any bars? I would like to see a more general theory that would apply to that. I am talking about a Lampén mill for example, which beats and does not have bars.

*Jean-Claude Roux*

You are perfectly right. Probably, I should have said at the beginning of my talk that it is a typical case of refiners with bars, but I have been working with bars for a long time. I promise the audience that I am also studying
the problem with no bars, only normal force and friction force, with my colleagues who are also present here.

_Derek Page_

You mean if I manage to come back in 4 years time, I’ll get the answer?

_Jean-Claude Roux_

You remember that I presented some refiner studies 8 years ago and then also 4 years ago in a review paper. So, step-by-step we are advancing, but the problem is very complex.