

Dynamic Test of Laminated Veneer Lumber Elastic Modulus and Its Probability Distribution

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The vibrational frequency method was used to measure the elastic modulus of laminated veneer lumber (LVL), and the feasibility of using Weibull distribution to analyze the elastic modulus data of LVL was considered. Samples were randomly selected as test pieces at the factory. The sponge support structure was used to realize the free beam state, and the modal test results verified the accuracy of realizing the free beam. Under transient excitation, the elastic modulus of the specimen was obtained by testing the first-order bending frequency. The Weibull distribution fitting test, Weibull distribution K-S test, and normal distribution K-S test were used for the test data. The probability of LVL elastic modulus was calculated under a given value. The results showed that the LVL elastic modulus did not obey the two-parameter Weibull distribution ($E_u=0$). The LVL elastic modulus fit to the three-parameter Weibull distribution (E_u) was greater than half of the minimum test value and the normal distribution. When 9 GPa and 8 GPa were used as the setting values of E_u , the calculated probability value was relatively stable. At this time, E_u was 81% and 92% of the minimum elastic modulus 9.815 GPa. Therefore, it was recommended to use 80% to 90% of the minimum value of the measured data as the setting value of the position parameter E_u . The three-parameter Weibull distribution and the normal distribution calculated LVL elastic modulus have the same probability under the given value.

Keywords: Laminated veneer lumber; Elastic modulus; Weibull distribution; Normal distribution

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INTRODUCTION

Laminated veneer lumber (LVL) is a structured wood-based panel made of material from small-diameter logs. Compared with other sawn timber, its main advantages are reflected in its low material variability and high overall mechanical properties of the structure (Wang and Dai 2013; Boccadoro *et al.* 2017; Schuler 2017). As an engineering structural material, its structural mechanical properties and reliability are notable (Xue 2013; Divino 2016; Nasir *et al.* 2019; Purba *et al.* 2019).

The most commonly used statistical analysis of data is based on the normal distribution. This paper used the frequency method to accurately measure the elastic modulus (Wang *et al.* 2012, 2018, 2019; Xie *et al.* 2019; Wargua *et al.* 2020) and applied the Weibull distribution to analyze LVL elastic modulus data. The linear fitting and K-S methods were applied to the elastic modulus data of the sub-sample. The results showed that in addition to the three-parameter Weibull distribution, the elastic modulus of LVL obeys the normal distribution.

EXPERIMENTAL

Transient Excitation Frequency Method for Measuring LVL Elastic Modulus

Test principle

According to the beam transverse bending theory, the relationship between the first-order bending frequency of a free beam and the elastic modulus is as follows (Wang *et al.* 2015),

$$E = 0.9462 \frac{(\rho f_1^2 \cdot L^4)}{h^2} \quad (1)$$

where E is the modulus of elasticity of the specimen (Pa), ρ is the air-dry density of the specimen ($\text{kg}\cdot\text{m}^{-3}$), f_1 is the first-order bending natural frequency of the specimen (Hz), L is the length of the specimen (m), and H is the thickness of the test piece (m).

By testing the first-order bending frequency f_1 of the free beam, the elastic modulus value is calculated by Eq. 1.

Specimen and test methods

A total of 149 LVL specimens were used, with the dimensions of 2700 mm (l) \times 770 m (b) \times 30 mm (h). The air-dry density of specimens (ρ) was 480 to 618 kg/m^3 , and the moisture content (MC) was 9.5 to 14%.

Using the transient excitation vibration method, the LVL specification material specimen was struck with a rubber hammer to generate lateral free vibration. The frequency spectrum of the specimen was tested, and the first-order bending frequency of the specimen was identified from the spectrogram. The block diagram of LVL specification material specimen and test system is shown in Fig. 1.

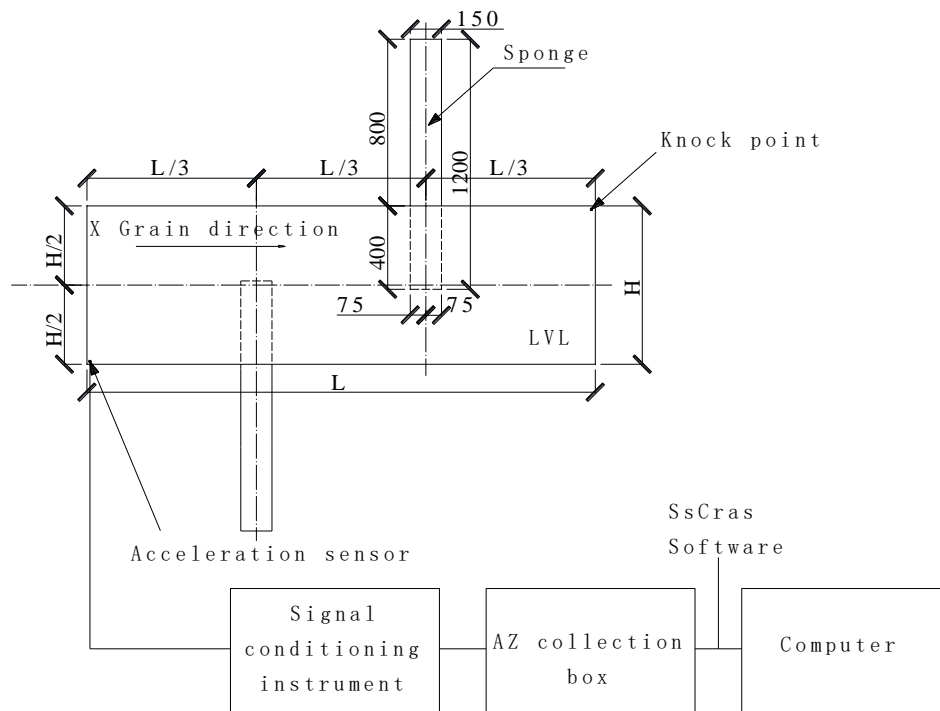


Fig. 1. Block diagram of the fundamental frequency test system of the LVL test piece

Experimental protocol

The results of the modal test on the No. 2 specimen show that the ratio of the second, third, and fourth-order frequencies to the first-order frequency in the first four-order frequencies and the position of the first four-order mode nodes were completely consistent with their theoretical analytical solutions. Thus, the suspension tested in the article achieved free support. Therefore, a test plan for testing the first-order bending frequency of the specimen by the transient excitation method was proposed (Fu 2002; Wang *et al.* 2012), as follows:

1) Acquisition parameter settings: analysis frequency, 200 Hz; FFT length, 8192; acquisition mode, free running; one data block; voltage range, 5000 mV; and magnification of the conditioner, 100. The filter frequency was set to 200 Hz to prevent frequency aliasing, *i.e.*, to ensure that the aliasing frequency (false frequency) of the third-order natural frequency did not appear on the spectrum.

2) The accelerometer was installed at the corner of the test specimen, and the rubber hammer hit the opposite corner of the test piece.

Figure 2 shows the frequency spectrum of LVL No. 2 specimen. It is not difficult to read the first-order bending frequency from the spectrogram; the fundamental frequency is 19.63 Hz.

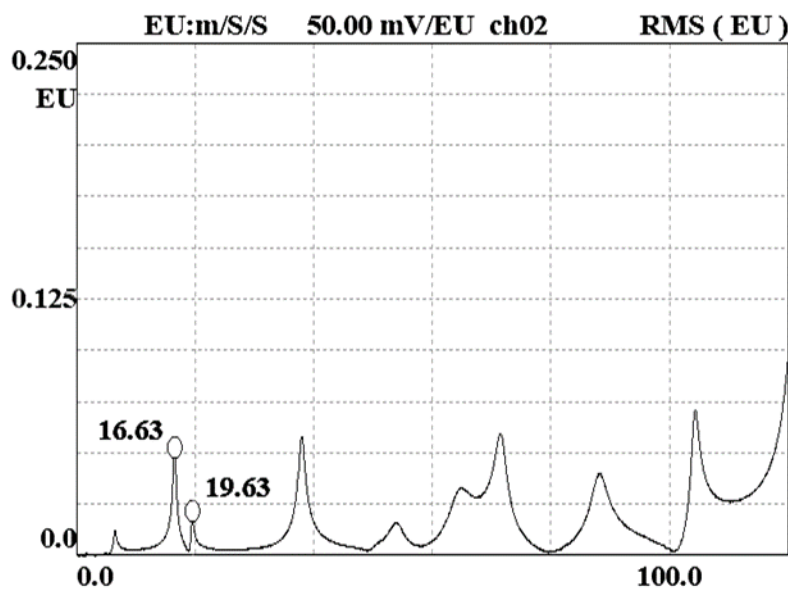


Fig. 2. Free board sponge placement spectrum

The elastic modulus values of 149 LVL finished test pieces were measured by beam transverse bending vibration theory (Fig. 4). To illustrate the correctness of the test, the suspension method (Wang *et al.* 2015, 2018, 2019) was used for verification with a total of 20 samples. This method has proven to be accurate (Wang *et al.* 2015, 2018, 2019). Figure 3 shows that the elastic modulus measured by the two test methods has a good correlation.

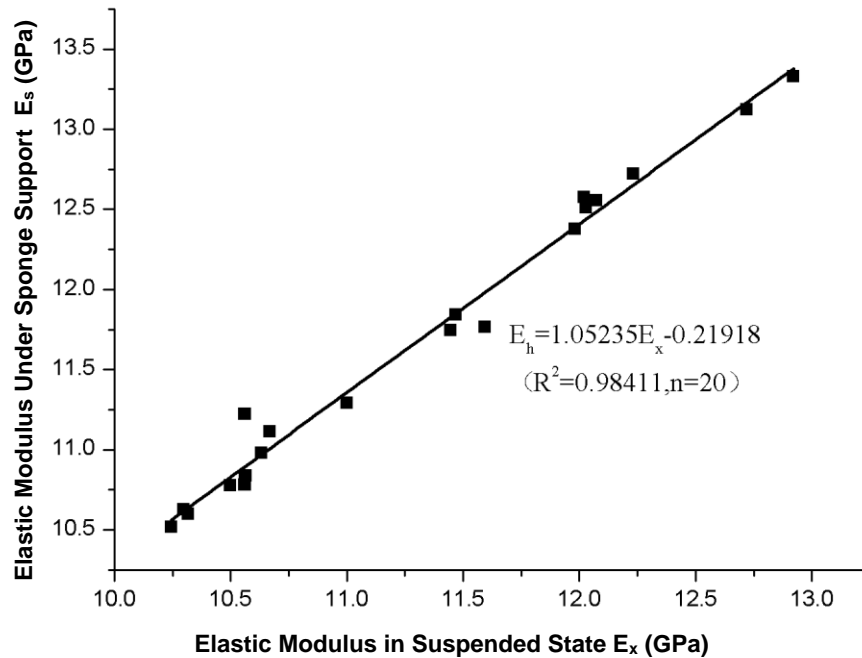


Fig. 3. Correlation between elastic modulus in suspended state and elastic modulus under sponge support

Probability Distribution of LVL Elastic Modulus

Weibull distribution and its numerical characteristics (Weibull and Swenden 1951)

Given the elastic modulus E of LVL is a random variable ζ , which conforms to the following three-parameter Weibull distribution function $F(E)$,

$$\text{Pro}(\zeta \leq E) = F(E) = 1 - \exp\left(-\left(\frac{E - E_u}{E_0}\right)^m\right), \quad E > E_u \quad (2)$$

$$\text{Pro}(\zeta \leq E) = F(E) = 0$$

where m and E_0 are the distribution parameters, m is the shape parameter, E_0 is the scale parameter, and E_u is the location parameter, which represents the overall minimum elastic modulus.

According to the definition of the overall average μ and standard deviation σ , and with the aid of the gamma function $\Gamma(x)$, Eqs. 3 and 4 are generated.

$$\mu = E_0 \Gamma\left(1 + \frac{1}{m}\right) + E_u \quad (3)$$

$$\sigma = E_0 \sqrt{\Gamma\left(1 + \frac{2}{m}\right) - \Gamma^2\left(1 + \frac{1}{m}\right)} \quad (4)$$

Weibull distribution fit test

The measured values of the elastic modulus E of the sub-samples were arranged from small to large to form an order statistic E_i , $i = 1, 2, 3, \dots, 148, 149$. The corresponding probability value is $i/(n + 1)$. If the order statistic E_i has the same value and the number of

repetitions is k , the corresponding probability value is $(i * k)/(n + 1)$. The parameter n is the sample size, $n = 149$. For a given E_i value, Eq. 2 shows that (Gumbel 1958):

$$P(\zeta \leq E_i) = 1 - \exp\left[-\left(\frac{E_i - E_u}{E_0}\right)^m\right] \quad (5)$$

Substituting the order statistic E_i and the corresponding probability value $i/(n + 1)$ into Eq. 5, after logarithmic operation, produces Eq. 6.

$$\log_{10}\left(\log_{10}\frac{n+1}{n+1-i}\right) = m \log_{10}(E_i - E_u) + \log_{10}\left(\frac{\log_{10} e}{E_0^m}\right) \quad (6)$$

According to the measured minimum elastic modulus $E = 9.819$ GPa, set E_u to be 0, 2.0, 2.5, 4.5, 6.0, 8.0, and 9.0 GPa, respectively. In the Weibull coordinate system $\log_{10}(E_i - E_u) - \log_{10}\left(\log_{10}\frac{n+1}{n+1-i}\right)$, the value of $\left(\log_{10}(E_i - E_u), \log_{10}\left(\log_{10}\frac{n+1}{n+1-i}\right)\right)$ is calculated. If the test point falls on a straight line, it is considered that the elastic modulus value can conform to the Weibull distribution, where m is determined by the slope of the straight line, and E_0 is the intercept between the straight line and the coordinate axis.

The scatter plot of each measuring point in the Weibull coordinate system is shown in Fig. 4 ($E = 9$ GPa).

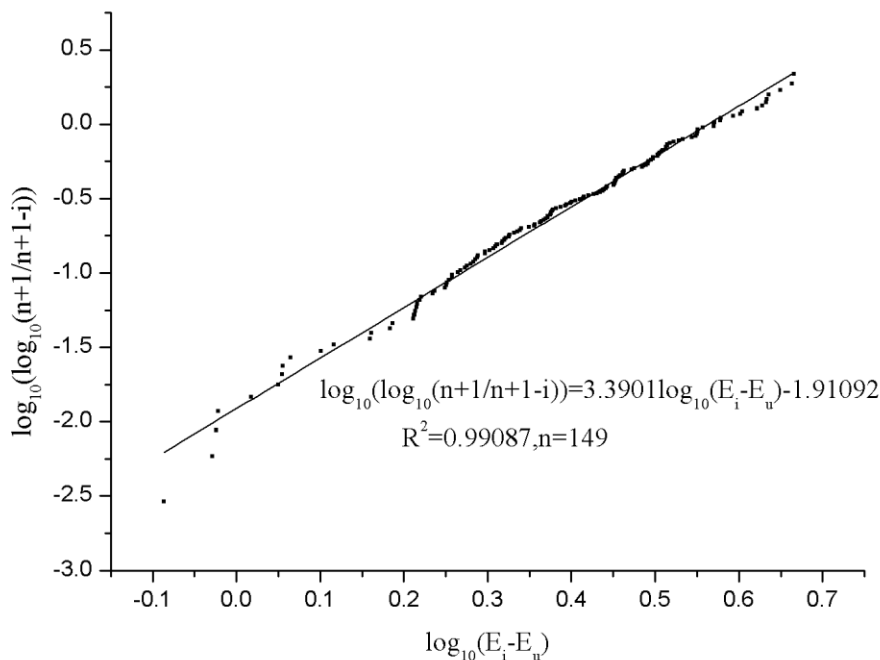


Fig. 4. Fitting test of Weibull distribution of elastic modulus of LVL

When $E_u = 9.0$ GPa, $R^2 = 0.99087$, $n = 149$, indicating that the fitting effect is good. Therefore, the elastic modulus of LVL in the Weibull coordinate system can be fitted with a straight line.

From the intercept of the straight line and the ordinate axis and the slope of the straight line, $m = 3.3901$, $E_0 = 2.86$ GPa, the distribution function of LVL elastic modulus is as follows.

$$F(E) = P(\zeta \leq E) = 1 - \exp\left(-\left(\frac{E - 9}{2.86}\right)^{3.3901}\right)$$

Substituting $m = 3.3901$ and $E_0 = 2.86$ GPa into Eqs. (2) and (3), the overall average m and standard deviation s are estimated to be: $m=11.568$ GPa; $s=0.837$ GPa; the coefficient of variation is 7.2%. For a sample with a capacity of 149, the sample average is 11.573 GPa, and the standard deviation is 0.848 GPa.

After the setting of the E_u value, the Weibull parameters obtained by linear fitting and the corresponding estimated LVL overall elastic modulus mean and standard deviation are shown in Table 1.

Table 1. Possible Weibull Parameters and their Numerical Characteristics Estimated from Fitting of Experimental Data in Weibull Coordinate System

Location Parameter (GPa)	Shape Parameter	Scale Parameter (GPa)	Correlation Coefficient	Mean Value (GPa)	Standard Deviation (GPa)
0	16.296	11.95	0.9883	11.569	0.874
2	13.497	9.94	0.9430	11.565	0.866
2.5	12.796	9.44	0.9443	11.567	0.863
4.5	9.982	7.43	0.9510	11.567	0.852
6	7.853	5.92	0.9587	11.570	0.841
8	4.946	3.89	0.9768	11.569	0.826
9	3.390	2.86	0.9909	11.568	0.837

K-S test of Weibull distribution and normal distribution

There are two types of random variable distribution type testing: known parameter estimation distribution type; unknown parameter, first estimate the parameter, and then test the distribution type. Among many inspection methods, the K-S method is the most commonly used one.

Test the hypothesis H_0 : LVL overall elastic modulus obeys Weibull distribution, H_1 : LVL elastic modulus does not obey Weibull distribution.

At present, the K-S test for testing hypothesis H_0 belongs to the case of unknown parameters. First, the parameters should be estimated. The estimated values of the possible Weibull parameters are calculated from Table 1. One of the possible parameters corresponds to one of hypothetical known distribution function. Secondly, the difference between the hypothetical known distribution function and the empirical distribution function in the order statistics was calculated. The maximum value after taking the absolute value is the test statistic D_n value in the K-S method. Finally, according to the significance level α and the sample size, the critical value table of the Weibull distribution K-S test when the parameters are unknown was checked to obtain the critical value of the test

statistic $D_{n,\alpha}$, if $D_{n,\alpha} > D_n$, accept H_0 , otherwise reject H_0 (Wang *et al.* 2015). The results of K-S test are shown in Table 2.

Using the sample mean and standard deviation as the estimated values of the population mean and standard deviation of the normal distribution, the K-S test was performed on the known distribution function of this hypothesis. The steps for calculating D_n are the same as above. The test results of the normal distribution are listed in Table 2.

The critical value d of D_n is obtained by looking up the table (Liang and Liu 1989; Wu *et al.* 1990; Ceraldi *et al.* 2001; Wu *et al.* 2001), based on the sample size and significance level α .

Weibull distribution:

$$\alpha=0.05, n=149, D_{n,\alpha} = D_{149,0.05} = 0.888/\sqrt{149} = 0.07274$$

(The parameters are unknown)

Normal distribution:

$$\alpha=0.05, n=149, D_{n,\alpha} = D_{149,0.05} = 0.886/\sqrt{149} = 0.07258$$

(The parameters are unknown)

Table 2. Possible Weibull Parameters and their Numerical Characteristics Estimated from Fitting of Experimental Data in Weibull Coordinate System

Possible Weibull distribution parameters			Value of D_n for K-S	Reject or accept H_0
E_u (GPa)	m	E_0 (GPa)		
0	16.296	11.95	0.09641	Reject
2	13.497	9.94	0.09238	Reject
2.5	12.796	9.44	0.09234	Reject
4.5	9.982	7.43	0.08798	Reject
6	7.853	5.92	0.08307	Reject
8	4.946	3.89	0.06621	Accept
9	3.390	2.86	0.04577	Accept
Normal distribution X=11.573GPa, S=0.848GPa			0.04897	Accept

The results in Table 2 show that: 1) the LVL elastic modulus does not obey the two-parameter Weibull distribution ($E_u=0$); 2) the LVL elastic modulus obeys the three-parameter Weibull distribution where the position parameter E_u is greater than half of the minimum test value; and 3) the LVL elastic modulus obeys the normal distribution.

Probability calculation of elastic modulus at a given value

The three-parameter Weibull distribution and normal distribution accepted by the K-S test are used to calculate the probability of elastic modulus at a given value (Li and Hu 2004). The results are shown in Table 3.

When 9 and 8 GPa were used as the setting values of E_u , the calculated probability value was relatively stable. At this time, E_u was 81% and 92%, respectively, of the minimum elastic modulus 9.815 GPa. Therefore, it was recommended to use 80% to 90% of the minimum value of the measured data as the setting value of the position parameter E_u . The three-parameter Weibull distribution and the normal distribution calculated LVL elastic modulus have the same probability under the given value.

Table 3. Probability of Overall Elastic Modulus of Veneer Lumber Under Given Value

Weibull distribution parameters			Probability of MOE (MPa) of LVL for a given MOE value						
E_u (GPa)	m	E_0 (GPa)	10	10.5	11	11.5	12	12.5	13.5
8	4.946	3.89	0.0366	0.1062	0.2417	0.4473	0.6827	0.8720	0.9961
9	3.390	2.86	0.0278	0.1055	0.2560	0.4675	0.6894	0.8608	0.9902
Normal distribution : X=11.573GPa, S=0.845GPa			0.0318	0.1029	0.2496	0.4657	0.6927	0.8628	0.9885

To explain the probability characteristics of the calculation of the three-parameter Weibull distribution and the normal distribution, the probability distribution curve and probability density curve of the elastic modulus of LVL are given in Figs. 5 and 6.

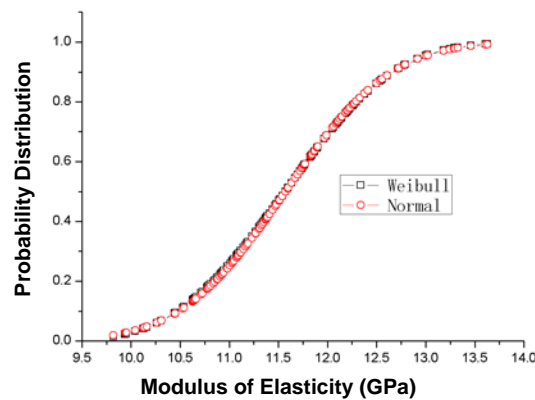


Fig. 5. Probability distribution curve of elastic modulus of veneer laminated wood

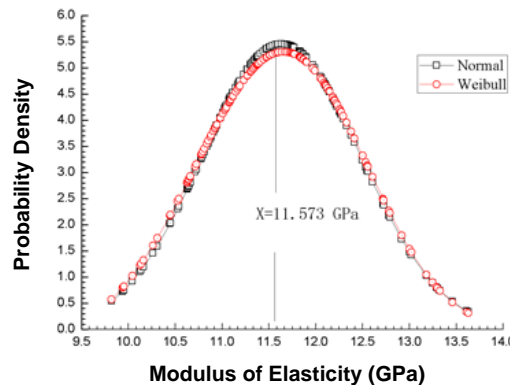


Fig. 6. Probability density curve of elastic modulus of veneer laminated wood

Figures 5 and 6 show that 1) when the elastic modulus does not exceed 11 GPa, the normal probability distribution curve is slightly higher than the Weibull probability distribution curve, and when it exceeds 11 GPa, the opposite is true; 2) the Weibull probability density curve is slightly asymmetric with respect to its mean line (11.568 GPa), and the normal probability density curve is symmetric with respect to the mean line.

CONCLUSIONS

1. The modal test verifies that the test device in the article realizes the support condition of the free suspension of the test piece, and the test plan of the frequency method is proposed. The first-order bending and first-order torsional vibration natural frequency testing were used to calculate the elastic modulus of LVL.
2. The transient excitation frequency method to test the elastic modulus of LVL has the advantages of simplicity, speed, good repeatability, and high accuracy. Moreover, the frequency spectrum of LVL is basically the same in the state of free hanging and sponge support, and sponge can be used as the support condition of the test.
3. The elastic modulus of LVL obeys the normal distribution.
4. The probability that the elastic modulus of LVL calculated by the three-parameter Weibull distribution and the normal distribution is basically equal at a given value (accurate to the second decimal place).

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