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CONDUCTION OF HEAT IN PAPER

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ABSTRACT

The thermal conductivity of paper was measured using a thermoacoustic method based on the propagation of a periodic temperature wave in the medium. Thermal diffusivity and thermal conductivity can be calculated from the resulting phase shift.

The thermal conductivities of sheets prepared from different pulps were measured under standard conditions and at 70 ^{0}C and 70 % RH.

In paper, heat is conducted through both the solid phase and the gaseous phase. In the case of dense paper and at high moisture contents, heat transfer due to diffusion of water vapour makes a major contribution.

The results were used to construct a qualitative physical model for the conduction of heat in paper. In the normal paper density range of 400 - 900 kg/m^3 heat conduction can be explained in terms of layers of air and solid phase connected together in different ways.

At higher densities and higher moisture contents the mechanisms of heat conduction change.

The heat conduction characteristics of paper are better explained using thermal diffusivity calculated in terms of basis weight than by using thermal diffusivity and thermal conductivity.

1. INTRODUCTION

The use of temperature gradients has recently received considerable attention in attempts to delelop calendering. Heat is used to try to plasticize the surface fibres of the paper so as to limit the deformations taking place in the paper to the surface layers. Effective use of temperature gradients requires a quantitative understanding of the factors that give rise to these gradients. The development of a temperature distribution in the thickness direction of the paper during calendering is affected by two factors:

1. The rate of heat transfer from roll to paper.
2. The rate of heat transfer in the paper itself.

We discuss here the effect of paper characteristics and measuring conditions on the transfer of heat in the thickness direction of the paper.

Heat is known to be transferred by three different mechanisms: conduction, convection and radiation. Inside a homogeneous solid, heat can only be transferred by conduction. In a porous substance such as paper, heat is transferred via the solid phase as well as through the interstitial gaseous phase. The following discussion is limited to the conduction of heat through paper.

The measuring system was designed to exclude the effect of convection. The transfer of heat by radiation is inversely proportional to the density of the porous material. In the density range commonly encountered with paper the effect of radiation can be neglected when temperature differences are small (1) (2). It should be remembered that paper is a heterogeneous substance whose thermal conductivity is determined by the characteristics of both the solid and the gaseous phases.

2. MEASUREMENT OF THERMAL CONDUCTIVITY

2.1 General

It was stated above that the temperature distribution in the thickness direction of the paper during calandering is affected by the rates of heat transfer from roll to paper and within the paper itself.

Experimental determination of thermal conductivity is difficult because both these factors are at play, even in laboratory measurements. If we study the effect of calendering on thermal conductivity, the heat transfer in the surface changes at the same time. If the measurements are made using a small compressive force, the resistance of the surface to heat transfer has a major effect on the final result. It is difficult to separate the effects of these two factors.

The term 'apparent thermal conductivity', which includes the resistance of the surface to heat transfer, is often used (3). Apparent thermal conductivity can be determined through steady-state measurements using the temperature difference, heat flow and thickness of the layer of material.

Dynamic methods involve solution of differential equations of heat transfer using the boundary conditions of each case. The equations obtained can then be solved to give the thermal conductivity (4) (5). However, the solution of the group of equations may be somewhat insensitive (5).

In this work a dynamic method was used. The measuring system and treatment of the results were chosen so as to permit measurement of the thermal conductivity of the paper without the error introduced through the heat transfer coefficient.

2.2 Audio-frequency thermal waves

In the following some of the aspects of thermal waves and their use in non-destructive testing on à microscopic scale are described.

Thermal imaging, or thermal testing, has usually been associated with thermal analysis of macroscopic structures like brick walls, insulated water piping, etc. Only recently it has been observed that one can actually build a microscope using short wavelength thermal waves.

Thermal wave imaging has its roots in the so-called photoacoustic effect. An intensity modulated laser beam hits a sample surface in a closed cell or cavity. Part of the laser light is reflected away, part is absorbed by the sample. In accordance with the basic laws of physics, the absorbed part of the light energy is converted into thermal energy, which diffuses in the sample according to the thermal diffusion laws. The periodic heating of the sample surface also creates a periodic heating of the adjacent gas in contact with the sample surface. The periodic heating of the gas in a closed cell creates a periodic pressure, or sound waves. The sound waves can in turn be detected with a microphone inside the cell. If the sample surface is highly light absorbing, then the phase and magnitude of the acoustic signal detected with the microphone have a direct relationship with the thermal properties of the sample surface and, the layer immediately below the sample surface.

Recent conference proceedings in photoacoustic spectroscopy provide excellent guides to the possibilities of the photoacoustic effect and thermal waves [6,].

Acoustic wave equations and thermal wave diffusion equations have certain similarities which may allow the visualization of thermal waves as somewhat similar to overdamped acoustic waves.

The acoustic waves equation has the familiar form:

$$\frac{\partial^2 z}{\partial t^2} = v^2 \frac{\partial^2 Z}{\partial x^2} \qquad (1)$$

(v velocity of sound)

with a periodic undamped solution

$z = \exp(i\omega t \pm ikx)$

which describes wave motion in direction x with a real wave vector k.

A thermal wave diffusion equation can be written as

$$\frac{\partial T}{\partial t} = \alpha = D\frac{\partial^2 T}{\partial x^2} \qquad (2)$$

which has an overdamped wave solution

$$T = \exp(i\omega p \pm k'x)$$

where k' is complex

$$k' = \frac{1 + i}{\sqrt{2}}\left(\frac{\omega\rho C}{K}\right)$$

The thermal wave can also be described as a "temperature wave" in the form of an exponentially decaying sinusoidal wave:

$$T = T_0 \exp\left(-\sqrt{\frac{\omega}{2D}}\, x\right) \sin\left(\omega t - \sqrt{\frac{\omega}{2D}}\, x\right) \qquad (3)$$

The term $\sqrt{\omega/2D}$ describes the phase shift of the wave as it propagates through the sample, Here,

$$D = K/\rho C \qquad (4)$$

is thermal diffusivity, where K is thermal conductivity, C specific heat and ρ density. $\omega = 2\pi f$ is the periodic excitation (like the modulation frequency of the laser beam). Furthermore, the "penetration" or range of the thermal waves may be given as $\mu = \sqrt{2D/\omega}$. (At a frequency of 10 Hz and typical thermal values for paper, a penetration depth of the order of 100 µm is obtained.)

The instrument and measurement principle described in the following were developed at the Department of Physics, University of Helsinki, by R. Rajala and M. Luukkala.

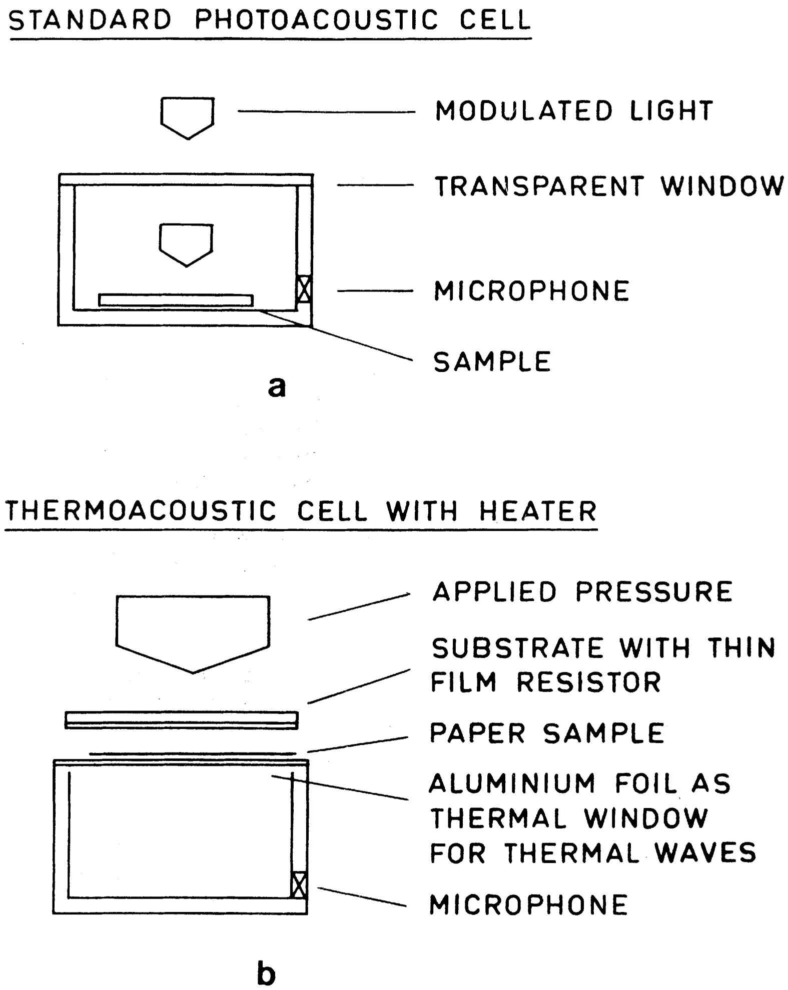


Fig 1—The principle of photoacoustic method

In standard photoacoustic work, the sample is placed at the bottom of the cell and the signal is recorded with a microphone. In this way, the signal is obtained from the front surface, the surface which the incident light beam strikes (see Figure 1a). However, in the present case the transmitted thermal wave is measured by generating a thermal wave at one side of a paper sample and detecting it at the opposite side with various thermal detectors. The transmitted thermal wave undergoes a phase shift $\Delta\phi$ that is directly proportional to the thickness Δx of the sample:

$$\Delta\phi = \Delta x\sqrt{\omega/2D} \qquad (5)$$

The method is best illustrated in Fig. 1b. A conventional PA cell has an optically transparent window, whereas in the present case we have an optically non-transparent but themally conductive window made of thin aluminium foil. The thermal wave can easily propagate through the aluminium window, enter the enclosed cell and create a periodic pressure there which is then picked up with the microphone as sound. The polished Al window reflects all visible light. The paper sample is then placed flat against the aluminium window with a slight pressure so that a good thermal contact between the sample and the window is obtained.

The thermal wave can be generated in a number of ways, from which we have selected the resistance heating method. As in photoacoustics, the thermal wave might have been generated by directing a modulated laser beam upon the free top surface of the sample. However, paper, and in particular white paper, is a scatterer of light rather than an absorber of light. Thus, only a small fraction of the incoming light would be converted into thermal energy on the top surface of the paper. Besides, due to small light attenuation in the paper, a poorly defined themal wave-front would be obtained.

For a thermal wave source we have a flat thin film resistor on a ceramic substrate which is placed flat against the top side of the paper. The thin film is made by evaporting a chromium layer onto a ceramic substrate in such a way that a thin "neck" is formed to act as a point-like source. This flat construction can take a substantial mechanical load, thus permitting the mechanical (or thermal) contact between the paper sample surfaces to be maximized.

The resistance of the planar resistor turned out to be about 50 ohms, which is quite suitable for a transistorized modulated power supply. The modulation frequency of the power supply could be varied between 1 - 50 Hz, but it turned out that about 10 Hz is most suitable for the present paper samples.

The thermal signal that is obtained from the microphone in the modified photoacoustic cell is amplified and then fed to a standard phase lock detector that obtains its reference signal from the periodic heater. Monitoring the changes in the phase angle gives information of the variations in D, which reflects the basic thermal properties of the sample, in accordance with the equation (5).

2.3 Application of the method to paper

The periodic temperature wave undergoes a phase shift both in the layer of material being studied and elsewhere in the measuring system. The magnitude of the phase shift is determined by the interfaces heating resistor - paper, paper - thermal detector and by the thermal detector itself.

The effect of these interfaces is small when the material being studied has a very smooth surface. In this case the phase shift caused by the system can be approximated by performing a test without the material. The heating resistor is simply pressed against the thermal detector and the phase shift measured.

Measurement of uncalendered paper means that there are two interfaces between the rough paper and measuring system. Their contribution cannot be estimated using the blank test described above. The problem was solved by making measurements for a series of laboratory sheets with basis weights ranging from 30 to 120 g/m^2. This gives a series of different phase shifts measured using the same interfaces. According to equation (5), Δphase shift is directly proportional to Δx.

The slope is $k = \sqrt{\omega/2D} = \sqrt{\frac{\Pi \cdot f}{D}}$.

The thermal diffusivity is thus obtained from equation:

$$D = \frac{\Pi \cdot f}{k^2} \qquad (6)$$

The thermal conductivity is obtained from equation (4).

In this study, the specific heat of paper was not measured; instead it was taken from the literature (5) to be 1.45 kJ/kg^{0}K.

The method used to measure the density of the paper is important when comparing the thermal conductivities of calendered and uncalendered sheets. Determination of paper density is known to involve an error due to surface roughness, especially with thin and rough papers (7).

As mentioned above, measurement of thermal diffusivity is based on the change in phase shift with changes in the thickness (basis weight) of the paper. This means that density cannot be measured in the normal way; instead density has to be determined from the ratio Δ thickness/Δ basis weight. The principle involved is that as the basis weight is increased the thickness of the layer lying between the surfaces of the paper increases; the phase shift is then measured as a function of the thickness of this core layer. The density determined in this way is therefore that of the core layer, not that of the whole paper.

As already pointed out, the temperature wave undergoes not merely a phase shift but also a decrease in amplitude. This naturally means that measurements can only be made up to a certain paper thickness. Beyond this the amplitude of the temperature wave is so small that it cannot be reliably measured. The limit with the equipment used in this study was c. 120 µm. Above this limit the reproducibility of the measurements was poor.

From the point of view of the application of the results, it would be interesting to perform the measurements using a very small compressive force. The only force pressing the paper against the calander roll is its own tensile stress. It was impossible to arrange this in practice, as the intensity of the temperature wave passing through the paper was then too small.

For this reason the measurements were made using the same pressure as that used in standard thickness measurement (100 kPa).

3. EXPERIMENTAL PROGRAMME

Several different pulps were compared, and the effects of degree of beating, calendering and atmospheric conditions on the thermal conductivity of the paper were studied. Only laboratory sheets (recirculation of white) were used. The pulps studied are shown in Table 1.

Table 1. Pulps studied

1. SGW 100 CSF
2. PGW 100 CSF
3. TMP 100 CSF
4. SGW 70 CSF
5. SGW 48 CSF
6. bleached pine kraft, 20 ^{0}SR
7. bleached birch kraft, 22 ^{0}SR
8. bleached birch kraft, 48 ^{0}SR
9. bleached birch kraft, 69 ^{0}SR
10. 30 % bleached pine kraft, 20 ^{0}SR, 70 % SGW, 76 CSF

Measurements were made on both calendered and uncalendered sheets. The sheets were calendered in a laboratory calander (two passes at a linear pressure of 80 kN/m, velocity 7 m/min).

Measurements were made in standard conditions (23 ^{0}C, 50 % RH) and in an air-conditioning chamber at 70 ^{0}C. The average temperature of the heating resistor during the measurements was c. 20 ^{0}C above ambient temperature. The temperature of the paper rises to that of the heating resistor during the measurement.

Several further experiments were carried out to investigate the factors affecting heat transfer and the heat transfer mechanisms.

4. RESULTS

4.1. Effect of calendering and degree of beating on thermal conductivity

The thermal diffusivities and thermal conductivities calculated from the results are shown in Table 2. The measurements made for mechanical pulps 1–4 were treated as if from one pulp, as the differences between these pulps were negligible. On the other hand, the scatter in the results obtained for the bulkiest pulps was fairly wide (greatest thicknesses were c. 190 µm), and combining the results gives more reliable estimates for the quantities being studied.

Table 2. Thermal diffusivity and thermal conductivity at 23 ^{0} C and 50 % RH.

Pulp	Density, kg/m^3 $\frac{\Delta\text{basis weight}}{\Delta\text{thickness}}$	Thermal diffusivity m^2/s.10^{-6}	Thermal conductivity W/m^{0}K
Uncalendered			
1-4	403	0.072	0.042
5	592	0.074	0.064
6	699	0.063	0.064
7	749	0.058	0.063
8	894	0.059	0.077
9	968	0.078	0.110
10	451	0.061	0.040
Calendered			
1-4	638	0.058	0.054
5	757	0.057	0.063
6	992	0.070	0.100
7	971	0.072	0.101
8	1003	0.090	0.134
9	1114	0.100	0.162
10	713	0.056	0.058

If density and thermal conductivity increase in the same proportion during calendering, thermal diffusivity will remain constant. The results show that in the case of mechanical pulps calendering causes a decrease in thermal diffusivity, whereas the reverse is true of chemical pulps. In other words, with

Table 3. Effect of humidity and temperature on thermal conductivity.

Pulp	Density kg/m^3	D' $kg/s.m^4$	Thermal conductivity $w/m.^0K$	Moisture content %
Calendered sheets 23 ^0C, 50 % RH				
1,3	634	0,022	0,051	8,0
5	757	0,034	0,066	8,0
8	971	0,067	0,101	6,0
9	1034	0,095	0,133	6,4
70 ^0C, 70 % RH				
1,3	611	0,044	0,104	15,1
5	701	0,074	0,154	15,8
8	979	0,141	0,209	10,8
9	1090	0,157	0,209	10,0
Uncalendered sheets 23 ^0C, 50 % RH				
8	749	0,032	0,061	6,0
9	894	0,046	0,075	6,4
70 ^0C, 70 % RH				
8	741	0,085	0,166	10,1
9	934	0,128	0,199	11,9

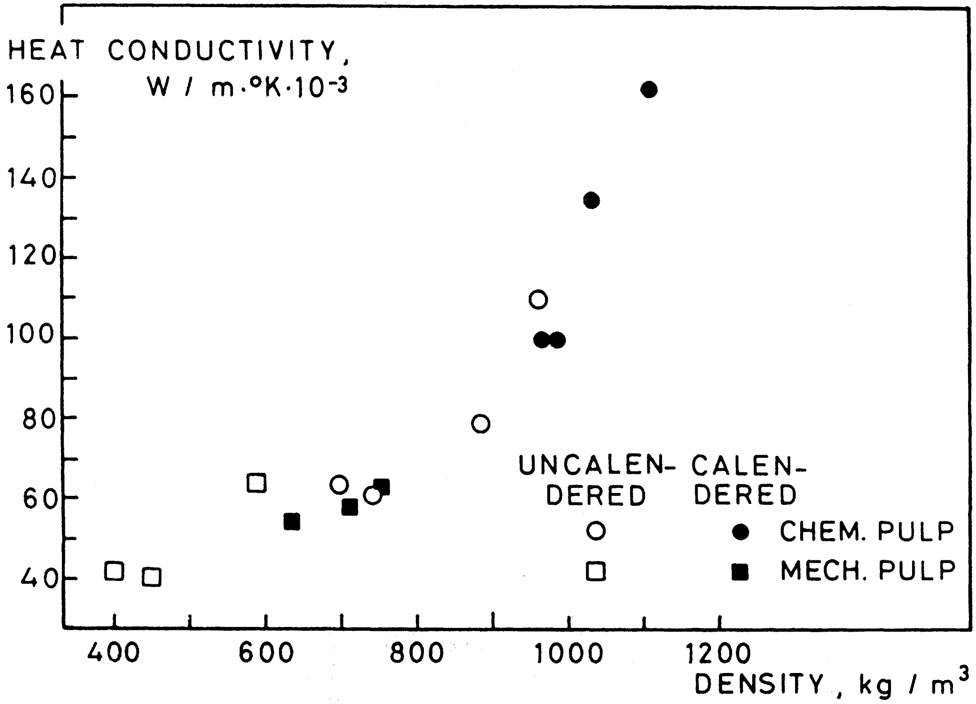


Fig 2—Heat conductivity as a function of density

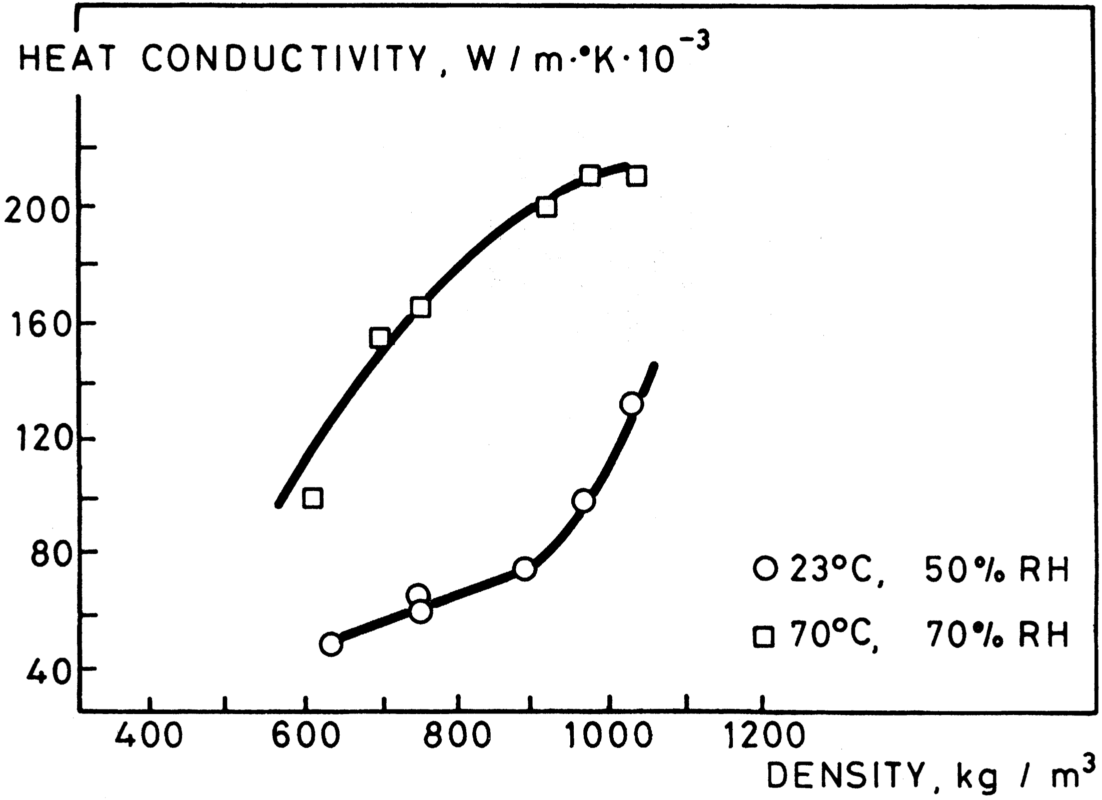


Fig 3—The effect of moisture and temperature on heat conductivity

mechanical pulps thermal conductivity increases less than density during calendering.

Figure 2 shows thermal conductivity as a function of density. It can be seen that density explains most of the variation in thermal conductivity.

4.2 Effect of humidity and temperature on thermal conductivity

The problems involved in measuring paper thickness have already been mentioned under the discussion of density determiantion. Varying humidity and temperature introduces the problems of swelling of the paper due to moisture and changes in compressibility due to both humidity and temperature, to add to the problem of surface roughness. The operation of the thickness gauge at high temperature also causes problems.

At the same time, variation of temperature and conduction of heat through a layer of a certain thickness is not the most practical way of describing the characteristics of the paper. From the practical point of view it is often more important to know the conduction of heat through a layer defined in terms of its basis weight. This gives a clearer picture of what is required to form a temperature gradient. For this reason, these results were calculated using a method that differs from that used previously. The method is described in Appendix 1.

The results of the measurements are presented in Table 3. Both D' and thermal conductivity are greatly increased by increasing humidity and temperature. As shown in Figure 3, the results cannot be explained in terms of density differences.

5. MECHANISMS AFFECTING CONDUCTION OF HEAT THROUGH PAPER

5.1. General

Several theories have been put forward to describe the conduction of heat through porous materials. Some require a detailed knowledge of the structure of the materials (8). Such theories could not be used in this study. There are also theories that do not require a knowledge of the structure of the material; instead they require just the properties of the components and, in some cases, the proportions by volume of the components.

A simple model for a porous material consists of plate-like layers of solid phase and gaseous phase (fig. 4). The plates are either in the direction of the heat flow (I) or perpendicular to it (II) (9).

In these two cases the thermal conductivities are:

$$K_I = (1-\gamma)K_S + \gamma K_G \tag{7}$$

$$K_{II} = \frac{1}{\dfrac{1-\gamma}{K_S} + \dfrac{\gamma}{K_G}} \tag{8}$$

(γ is the fraction of pores)

If the porous material has a structure in which the contribution from structure II is a and that from structure I is 1-a, the thermal conductivity of the material is

$$K = \frac{1}{\dfrac{1-a}{K_I} + \dfrac{a}{K_{II}}} \tag{9}$$

Parameter a thus diminishes as the connections formed by the solid phase between layers perpendicular to the direction of heat flow increase.

Use of formula (9) requires knowledge of the thermal conductivity of a non-porous fibrous material.

The purpose of the experiments described in the following was to investigate the mechanisms affecting the conduction of heat, and to use the results to formulate a model for the thermal conductivity of paper.

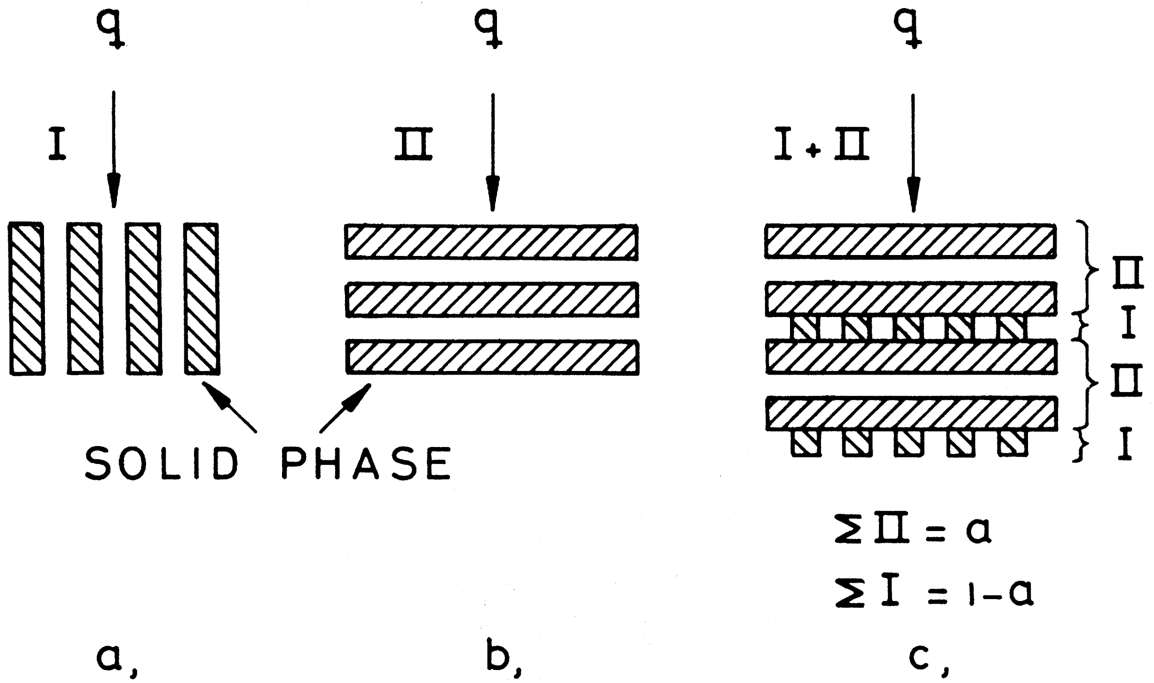


Fig 4—Model structure for a porous substance

5.2. Effect of inter-fibre bonding on heat conduction in paper

As already pointed out, in paper heat is conducted through both the fibre network and the gaseous phase. The question arises as to the role of the inter-fibre bonds. To investigate this, experiments were carried out to determine the effect on thermal conductivity of breaking the bonds, and the thermal conductivity of a single, continuous sheet was compared with the thermal conductivity of a combination of two separate sheets made from the same pulp.

The results from this last experiment (calendered mechanical pulp) are shown in Figure 5. When the compressive force is 150 kPa (1.5 times that used in thickness determinations) the interface causes a clear, additional lag in the phase shift. The interface between the sheets thus retards the transfer of heat. The interface corresponds to a layer of paper of c. 3 g/m^2. Even a small pressure (460 kPa) is enough to remove the effect of the interface completely (accuracy of phase shift measurement is c. ± 1 ^{0}).

Another experiment was carried out in which the bonds in a woodfree paper were broken by pulling the paper c. 40 times over the edge of a thin plate (bending 180 ^{0}). The dresult was a drop of c. 30 % in tensile strength. Calendering was used to restore the paper's original density and roughness. The treatment had no effect on thermal conductivity.

These experiments clearly showed that the inter-fibre bonds do not affect thermal conductivity of paper.

5.3 Thermal conductivity of the solid phase of paper

The thermal conductivity of the solid phase of paper was determined using very thin paper grades produced from highly beaten chemical pulp. The papers used in the measurements are described in Table 4.

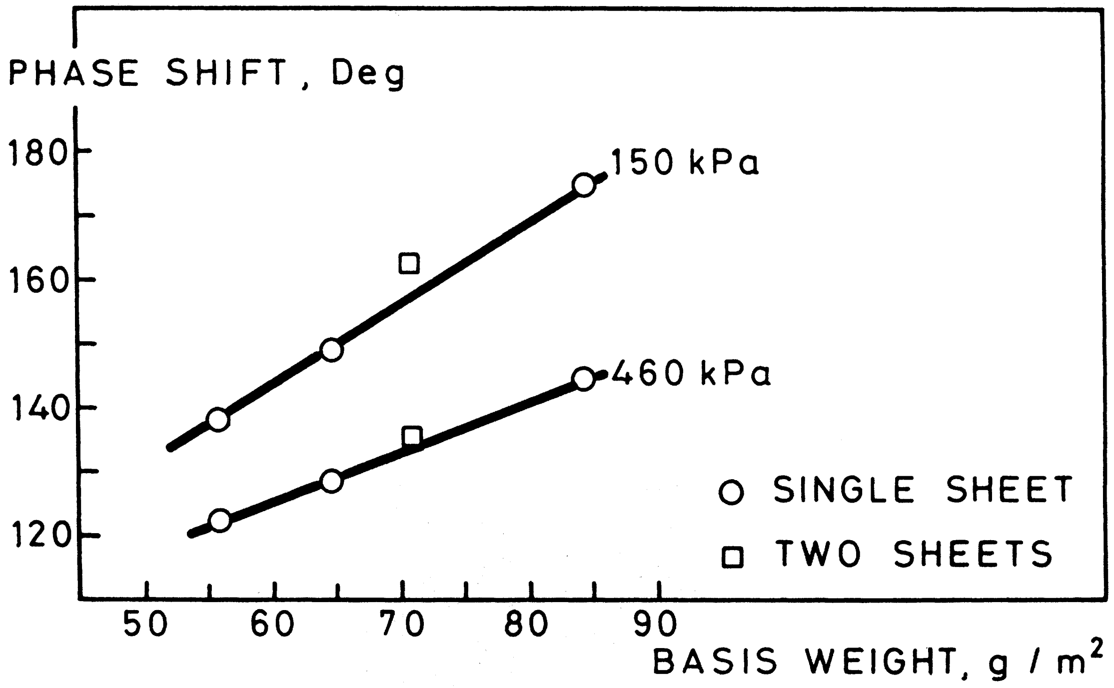


Fig 5—The effect of unbonded boundary on heat conduction

Table 4. Papers used to determine the thermal conductivity of the solid phase of paper.

Paper	Basis weight g/m^2	Density kg/m^3
1. One-time-carbonizing (uncalendered)	13,6	680
2. One-time carbonizing (calendered)	13,6	1070
3. Capacitor tissue	12	1200
4. Capacitor tissue	8,4	1200

The sample used in the determinations consisted of 1-8 layers. The compressive force was 460 kPa. The results obtained for papers 3 and 4 showed that the number of interfaces in the paper stack had no effect on the thermal conductivity. This experiment thus confirmed the results obtained for the role of inter-fibre bonds.

Carrying out measurements with different numbers of layers in the stack permits calculation of Δphase shift/Δbasis weight, which can then be used to calculate D'. Since dense papers like those used here are compressed very little at 460 kPa, the values of thermal diffusivity and thermal conductivity calculated using standard thickness and density are presented in Table 5, though these are somewhat higher than the real values.

Table 5. Thermal properties of high-density papers.

Paper sample	D' kg/s.m^2	Thermal diffusivity m^2/s.10^{-6}	Thermal conductivity W/m^{0}K
1	0.171	0.367	0.365
2	0.164	0.145	0.224
3	0.155	0.108	0.187
4	0.161	0.112	0.195

These results are somewhat surprising. The thermal conductivity of paper 1 is about twice that of papers 3 and 4, though its density is much lower. The difference is even more striking for thermal diffusivity. On the other hand, D' is constant within the measuring acccuracy. The result cannot be explained in terms of the solid phase - gaseous phase models presented earlier.

To understand the factors that affect the thermal conductivity of paper it is necessary to examine the conduction of heat in the gaseous phase in more detail.

5.4 Conduction of heat in the gaseous phase

According to the definition of thermal conductivity, the flow of heat through a layer of thickness d is given by

$$q = \frac{K \cdot \Delta T}{d} \qquad (10)$$

Equation (10) does not hold completely for a gas sandwiched between two plane surfaces. The heat flow is obtained from equation (11).

$$q = \frac{K_G \cdot \Delta T}{d + 2g} \qquad (11)$$

where g is the temperature jump distance, the magnitude of which depends on the properties of both solid and gaseous phases (mechanism 1.). g is of the same order of magnitude as the mean free path of a molecule in the gaseous phase. The mean free path for nitrogen at 45 ^{0}C is c. 0.1 μm (10).

When the distance between the surfaces is less than the mean free path of a gas molecule the heat transfer mechanism changes. The gas molecules no longer lose their energy on colliding with other gas molecules, but instead bounce back and forth between the two surfaces, giving up their energy direct rather than via other molecules (mechanism 2.). Knudsen has proposed the following equation for heat flow in such a situation (10):

$$q = \frac{1}{2} A(\gamma + 1) \quad \frac{CvP}{\sqrt{2\pi\ RT}} \quad , \text{ where} \qquad (12)$$

A = constant, depending on the properties of the gas and solid material
Cp = specific heat of the gas at constant pressure
Cv = specific heat of the gas at constant volume

$$\gamma = \frac{Cp}{Cv}$$

It is important to note here that the heat flow is independent of the distance between the surfaces (providing the distance is sufficiently small).

If the surface of the solid phase is damp, some heat may also be transferred by evaporation and condensation (mechanism 3). The thermal conductivity resulting from this, K_{diff}, is inversely proportional to the partial pressure of air. For a mixture containing water vapour and a very small proportion of air K_{diff} may be several hundred times higher than the thermal conductivity of air.

Using this as our basis, we can now put forward a qualitative physical model for the conduction of heat in paper that explains the results obtained.

5.5 Physical model for heat conduction in paper

The results shown in Table 5 (D' independent of pore volume, K diminishes as pore volume decreases) cannot be ecxplained by means of structure I described above. Structure II, on the other hand, has precisely these characteristics if the thermal conductivity of the gaseous phase is much greater than of the solid phase and if the flow of heat through the gas layer is independent of the thickness of the gas layer, for the layer thicknesses occurring in the porous structure. From what has been said above, this is quite natural. The pores in a very dense paper are small and, because of capillary condensation, they contain a greater proportion of water than larger pores. In such small pores the value of K_{diff} is high.

Figure 6 shows three different layer structures. According to the model presented here, the flow of heat through all three structures is the same as long as heat transfer takes place according to mechanisms 2 and 3. The thermal resistance of a layer of given basis weight is thus independent of density. From this it follows that, calculated in the normal way, thermal diffusivity and thermal conductivity cannot be applied as concepts to dense paper, whose thermal properties are better described using the quantity D'. For high-density papers D' is constant at around 0.16 $kg^2/s.m^4$ over a fairly wide density range. It can be used to calculate the thermal conductivity of the solid phase of paper, taking the density to be the density of the fibre wall (1500 kg/m^3), since only the solid phase poses any resistance the flow of heat. The thermal conductivity of the solid phase is thus 0.157 W/m^0K. Sanders and Forsyth (11) obtained roughly the same value for different papers using a pressure of c. 120 bar.

Figure 7 presents the thermal conductivities of paper (K_S = 0.157 W/m^0K) calculated from equations (7) and (8), together with the measurement results. The results show that in the density range 400-900 kg/m^3 the thermal conductivity of paper can be explained using a structure in which the contributions from structure I (bridges formed by solid phase) increases from c. 0.3 to c. 0.6 (Fig. 4b). The resulting physical model for paper appears to be quite a realistic one. Because the heat flow mechanism in the gaseous phase of small pores containing water vapour is quite different from that assumed in the derivation of formulas (7), (8) and (9), this model cannot be used to describe paper of high density. In the high density range, paper behaves as shown in Figure 6.

Values of D' for the measurements presented in Table 2 are given in Table 6. Calendering increases D' for both mechanical and chemical pulps. As can be seen from the figure 8, within the limits of accuracy of the measurements density explains the differences in D' for measurements made at the same temperature and humidity. At higher temperature and humidity the relationship between density and D' is different. The highest values of D' are very close to the limit presented above.

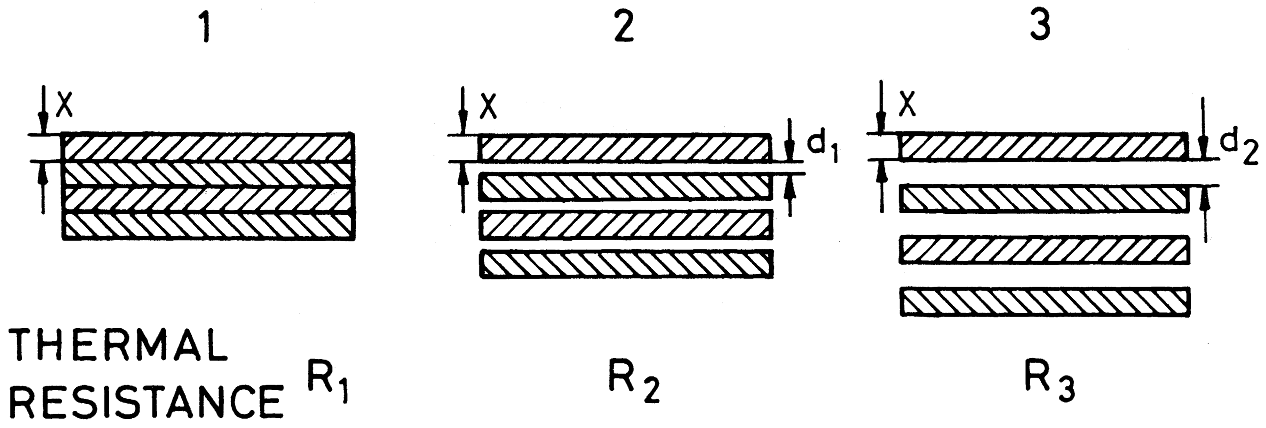


Fig 6—Heat conduction in dense paper. $R_1 = R_2 = R_3$ if heat transfer occurs via mechanisms 2 and 3 (page 14)

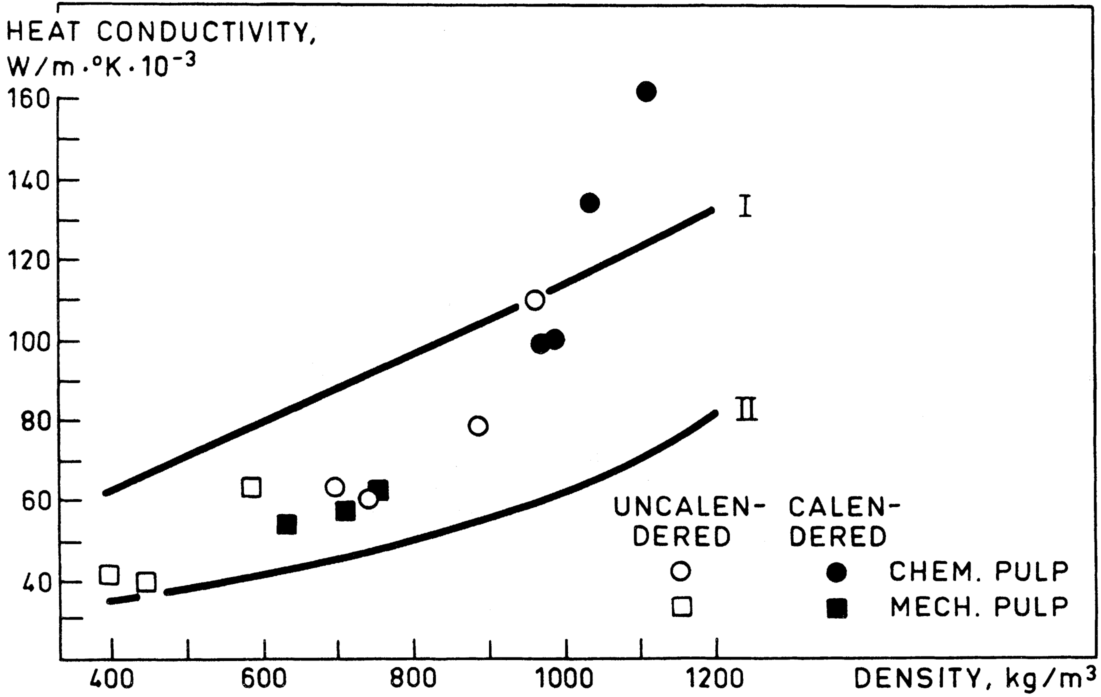


Fig 7—Comparison of observed heat conductivities with those calculated from equations (7) and (8) (curves I and II respectively)

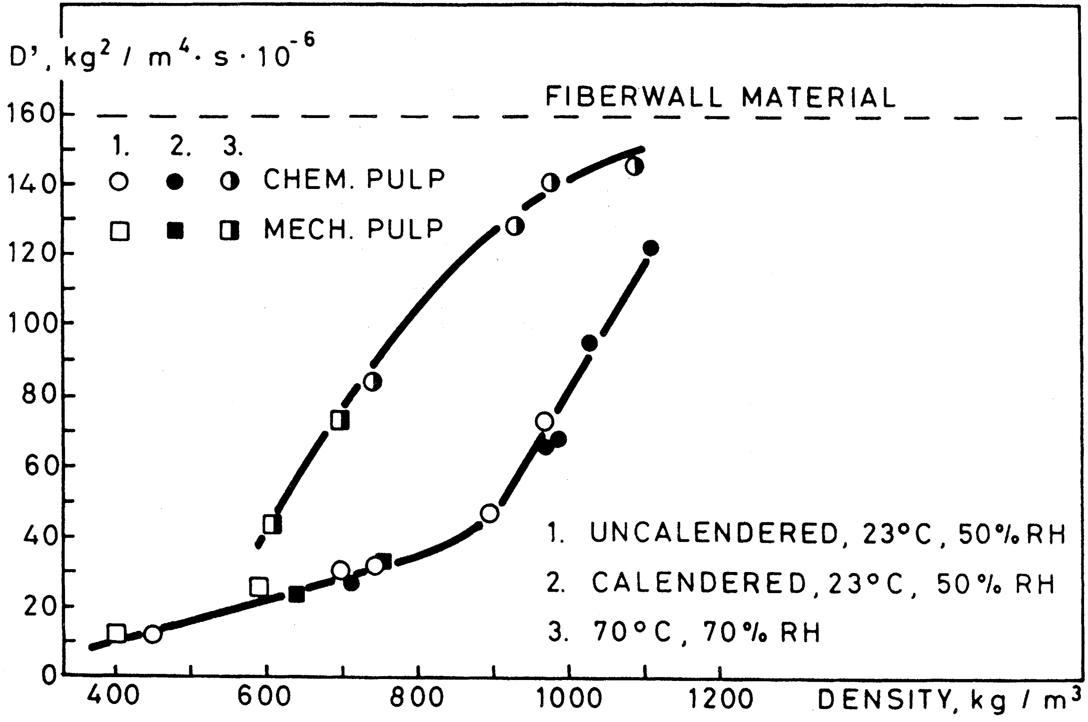


Fig 8—D′ as a function of density

The thermal conductivity of air increases with temperature, but decreases with humidity (in the absence of evaporation and condensation). The effect of temperature calculated from formula (10) is far smaller than the accuracy of measurement.

In other words, it does not explain the results obtained. The real reason is presumably the flexibility of the fibres and the greater contribution from mechanisms 2 and 3. At constant paper density, softening of the fibres would appear to result in fewer air gaps to hinder the passage of heat via mechanism 1.

Preliminary experiments to investigate the effect of compressive force have shown that, at constant density, thermal conductivity is higher when it is achieved using a high compressive force. Compression naturally affects the largest gaps between the fibres first.

Table 6. Values measured for D' in standard conditions.

	UNCALENDERED SHEETS		CALENDERED SHEETS	
Pulp	Density kg/m^3	D' $kg^2/s.m^4$	Density kg/m^3	D' $kg^2/s.m^4$
1-4	403	0,012	638	0,023
5	592	0,026	757	0,033
6	699	0,031	992	0,069
7	749	0,032	971	0,067
8	894	0,047	1034	0,096
9	968	0,073	1114	0,124
10	451	0,012	713	0,028

CONCLUSIONS

1. Conduction of heat in paper takes place via both the solid and gaseous phases.

2. A model based on the properties of the solid and gaseous phases estimates well the thermal conductivity of paper over the normal density range (600 - 900 kg/m^3).

3. The model does not, however, take into account the heat conduction mechanisms characteristic of paper. Heat can be conducted through the gaseous phase by three different mechanisms, depending on the distance between fibres and the role of water. From this it follows that, calculated in the normal way, thermal diffusivity and thermal conductivity are not very suitable for describing the thermal properties of paper, particularly of high-density paper. The quantity D' is more suitable for this.

4. No inter-fibre bonding or even any contact between fibres is required for efficient transfer of heat between fibres. In the case of hygroscopic materials, a sufficiently small air gap already produces maximum heat conduction.

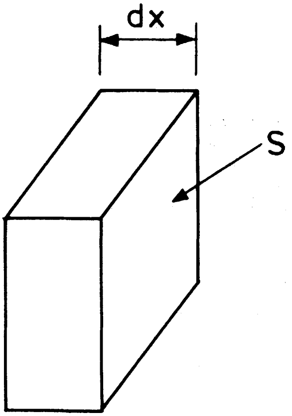
5. The variation in thermal conductivity observed in a standard atmosphere at a constant compressive force can be well accounted for by changes in density. However, thermal conductivity also depends on how the density is achieved.

6. Conduction of heat through the gaseous phase depends on the porosity (density), but also on the length destribution of the distances between fibres. The smaller the proportion of large air gaps, the better heat is conducted.

7. At constant density, an increase in moisture content and temperature or in external pressure reduces the proportion of large, insulating air gaps. At the same time the conduction of heat through diffusion of water vapour increases. These factors give rise to a clear increase in the thermal conductivity of paper.

8. For the solid phase of paper produced from chemical pulp D' = 0.160 $kg^2/m^{4}\,{}^0K$ and thermal conductivity = 0.157 W/m^0K.

9. To derive a quantitative model to describe the thermal conductivity of paper requires knowledge of the distribution of the distances between fibres in the z-direction and of the partial pressures of air and water vapour in pores of different sizes.



APPENDIX 1

Equation (2) was derived using a volume element of known density. The differential equation describing the conduction of heat can also be derived by taking a basis weight element instead of a volume element.

Basis weight element, dx', with surface area s.

Incoming heat flow = J_E
Outgoing heat flow = J_E
Element mass dm = Sdx'
Increase in heat
content of element in unit time

$$\frac{dq}{dt} = Sdx' \left(C\frac{\sigma T}{H}\right), \tag{1'}$$

where C = specific heat.

The increase in heat content can also be expressed by means of heat flows

$$\frac{dq}{dt} = - \frac{\delta J_E}{\delta x'} \, Sdx' \tag{2'}$$

Combining (1') and (2') we have

$$C \frac{\delta T}{\delta t} \qquad \frac{\delta J_E}{\delta x'} \tag{3'}$$

Thermal conductivity is defined by

$$J_E = -K \frac{\delta T}{\delta x} \tag{4'}$$

$[x]$ = length unit

x is related to x' as follows: $x' = \sigma x$.

Therefore $\delta x = \frac{1}{\sigma} \delta x'$

Substituting in equation (4') we have

$$J_E = -K\sigma \frac{\delta T}{\delta x'} \qquad (5')$$

Combining (3') and (5') we obtain

$$\frac{\delta T}{\delta t} = \frac{K\sigma}{C} \frac{\delta^2 T}{\delta x'^2} \qquad (6')$$

Putting $\frac{K\sigma}{C} = D'$

$$\frac{\delta T}{\delta t} = D' \frac{\delta^2 T}{\delta x'^2} \qquad (7')$$

We result is thus an equation of exactly the same form as equation (2). In this case, wowever, the relationship between the properties of the material and D' is different. The change does not affect equation (3). Basis weight is naturally used as x in place of thickness. The symbol D' is used below to represent the thermal diffusivity calculated in the manner presented.

The liearity of the phase shift was just as good for basis weight as for thickness.

The value of 1.45 kJ/kg^{0}K for the specific heat of paper was used in all cases. This is not an unreasonable assumption, despite changes in the moisture content. The moisture content of the paper was so low in every case that the fibres contained no free water. If the specific heat of paper increased with water content, the thermal conductivities of the dampest samples would have been higher than presented here.

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Transcription of Discussion

Conduction of Heat in Paper

by I. Kartovaara, R. Rajala, M. Luukkala and K. Sipi

Marchessault Reprographic companies are interested in thermal conductivity because of their interest in thermal transfer printing and condenser type paper is sometimes used as a substrate for ink donor film. Your instrument sounds like a photo-acoustic spectrometer and one can use these instruments to make such measurements also. We have had many discussions concerning the influence of pores vs the solid and an obvious material to test is cellophane which is solid cellulose. Have you tried cellophane as a model in your experiments?

I. Kartovaara I tried to obtain non coated cellophane samples but there appeared to be none available.

Marchessault I think it would be interesting to add data on cellophane to your measurements as a comparison to paper. The data we have comparing cellophane to condenser paper was roughly what we expected it to be for non-porous cellulose.

Baum Since machine made paper is directional and is usually treated as an orthotropic material, there are three thermal conductivities for paper just as there are three electrical conductivities. I am not sure that when you were looking at heat induction in the plane of the sheet, you were not looking at the different thermal conductivities in the machine and thickness directions.

Kartovaara No, we have only been measuring conductivity in the thickness direction.

Prof R.H. Atalla IPC, Appleton, U.S.A.

The difference between the results at 23°C and 50% R.H. and 70°C and 70% R.H. suggests that you are at the threshold of activating a different mode for transmission of heat, an intra molecular mode of transmission, because at 70°C, you are approaching the point at which the torsional vibrations of the pyranose ring are extremely highly populated. In regeneration of cellulose, as one approaches that temperature, one begins to get different conformations coming out of solution. As one goes higher and higher in temperature, and also in calendering, the distortions of the pyranose ring become very significant. I think Dr. Hattulla of FPPRI discussed the introduction of disorder as one goes to even higher temperatures and the torsional vibrations may contribute to the disordering mechanism. Under such conditions, the torsional vibrations will participate in heat conduction.

Dr. R.T. Kerekes University of British Columbia, Vancouver, Canada

You pointed out in your introduction that when you transfer heat to paper from a solid surface, as in calendering, there are two resistances to heat transfer. There is a contact resistance and then conduction within the material itself. The former is influenced by surface roughness of the latter by wet density. We developed a test to measure thermal conductivity as well as contact resistance. Could you calculate a value of contact resistance from the intercept shown in one of your figures which shows a finite thickness at zero basis weight assuming that this thickness is all air? The second point I would like to raise is that you found increased heat transfer to a web under load as opposed to one not under load, but at the same density. Could it not be that in transferring heat under pressure you may be decreasing the contact resistance whereas in the unloaded condition, you have a higher contact resistance, because although the density is the same in both cases, the surface roughness can differ considerably.

Kartovaara The way we calculate the results should eliminate these effects of roughness because the contact resistance between the heating element and the material have no effect on the measurement, because we only use the slope. The intercept is not used at all in any of our calculations. Referring to your first question, we thought about that but because we are not measuring heat flows, we are only measuring phase shift in the temperature variation. We don't know anything about the amplitude of the temperature variation of the heat flow. Therefore this method cannot be used to measure the heat transfer co-efficient.

Dr. B.D. Jordan PPRIC, Pointe Claire, Canada

In almost every material you can think of there is a relationship between thermal and electrical conductivity and if we are going to rule out the contribution of ionic transport in this model, it would be simple enough to carry out the experiment to measure the conductivity at the same time that you measure the thermal conductivity. Have you done this?

Kartovaara No, we have not.

Prof B. Steenberg Royal Institute of Technology, Stockholm, Sweden

Have you calculated the amount of heat conducted to a sheet in a calender through heat conductivity to the amount of heat which is evolved inside the sheet due to shear forces, causing friction inside the sheet?

Kartovaara No, but I think we have done experiments which show clearly that the temperature increase of the paper in a single nip between two rolls is practically negligable compared to the heat transfer you can get from very hot rolls with temperatures of 100°C or higher. I have not actually done any accurate measurements or calculations for that.

Steenberg Do you agree that it would be of some interest especially in view of the effect of the high temperature and high humidity in a region where you have a very high heat conductivity, which is the case in super-calendering of glassine?

Kartovaara Experience shows that this type of heat generation is very small.

Dent You are talking about conduction in the gas phase, but from your diagrams, if you go to lower density, all your conductivities are reducing to very low values, which is what I would anticipate the gas phase conduction would be. The extra heat transfer beyond the fibre phase conduction must be mass transfer evaporation related to liquid motion within the system rather than heat transfer by conduction especially in the gas phase.

Kartovaara The heat conduction through evaporation and condensation is normally included in the concept of heat conduction, so that is part of that concept.

Atalla I might suggest that if one were to carry out the experiment at a number of different temperatures, the activation energy might give some clues as to the mechanisms involved.

Kartovaara There were several practical difficulties in this type of work, because the thermal acoustic cell is very sensitive to noise and any kind of noise disturbs the measurement cell. For example, if you make the measurements in a chamber where you have increased temperature and humidity, you must stop the equipment and then perform the measurements and this is so tedious, that we have not done very many of these measurements.