TRIAXIAL DEFORMATION OF PAPER UNDER TENSILE LOAD

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Synopsis By way of introduction, the present state of knowledge is reviewed on the deformations of paper under uniaxial tension in the three principal directions—elongation, lateral contraction and change in thickness. Their measurement is briefly described in order to show the relationship between the experimental situation and the definition of the terms by which the deformations are characterised.

Following this, the results of tensile tests with straining cycles are presented. These show, apart from the elongation, the pattern of the cross-sectional changes and their reversible and irreversible components. Using laboratory handsheets, the effects studied were of manufacturing variables such as the grammage, the degree of refining and calendering, also of the furnish and of loading.

These investigations were supplemented by measurements on machine-made papers. It was found that the relationship between plastic elongation and total elongation is almost the same for all papers. There is also a strong similarity between the patterns of lateral contraction of the various papers, for which Poisson ratios $\nu_{xy} = 0{\cdot}16 - 0{\cdot}34$ and $\nu_{yx} = 0{\cdot}04 - 0{\cdot}10$ were found. By contrast, the change in thickness varies considerably from paper to paper and shows negative as well as positive Poisson ratios.

The comparison with results obtained on other sheet-like materials such as packaging foils and various printing substrates demonstrates the peculiarities of the deformation behaviour of paper.

Concluding the article, an attempt is made to arrive at a general formulation of the load/deformation behaviour of paper and it is here that the significance of the investigated fundamental properties of paper for its conversion and use become apparent.

Introduction

SINCE the early days of paper testing, the load/deformation behaviour of paper has received a great deal of attention with the result that, apart from theoretical investigations, a large volume of empirical data on this behaviour

Under the chairmanship of Prof. H. W. Giertz has been published, as shown in review articles.[1, 2] Some articles deal in particular with the plastic components of the elongation; they were obtained in creep experiments[3] and in tensile tests with straining cycles[4] and expressed as a function of the total elongation.

The lateral contraction was not investigated until during the past decade and experimental Poisson ratios between 0·4 and 0·95 have been reported.[5] In 1963, W. Brecht & R. Wanka[6] published the first article on systematic measurements of the lateral contraction of paper as a function of a number of papermaking variables, also commenting on the plastic components. In agreement with isolated earlier results[7] and with more recent results,[8, 9] laboratory handsheets and industrial papers were found to have Poisson ratios between 0·10 and 0·45.

The deformations in the plane of the sheet caused by tensile loads are of great importance for the conversion and the usage of paper and follow the pattern observed on other viscoelastic materials, though this is not true for the change in thickness. A. E. Ranger & L. F. Hopkins[5] considered the usually observed increase in thickness caused by a tensile load to be the result of the lateral contraction that bends fibres that lie in the direction at right-angles to that of the applied load. By contrast, O. E. Öhrn[10] assumes that the tensile load stretches bent fibres that lie in the direction of the applied load and that this causes a loosening of micro-delamination of the paper.

It is the purpose of this article to contribute to the knowledge of the deformation of the cross-section at rightangles to the direction of the applied load. This deformation has only recently been investigated and is therefore less well known. Although paper is considered as a three-dimensional object with regards its deformations, only tensile loads in the plane of the sheet are investigated, thus accepting the essentially two-dimensional structure of paper.

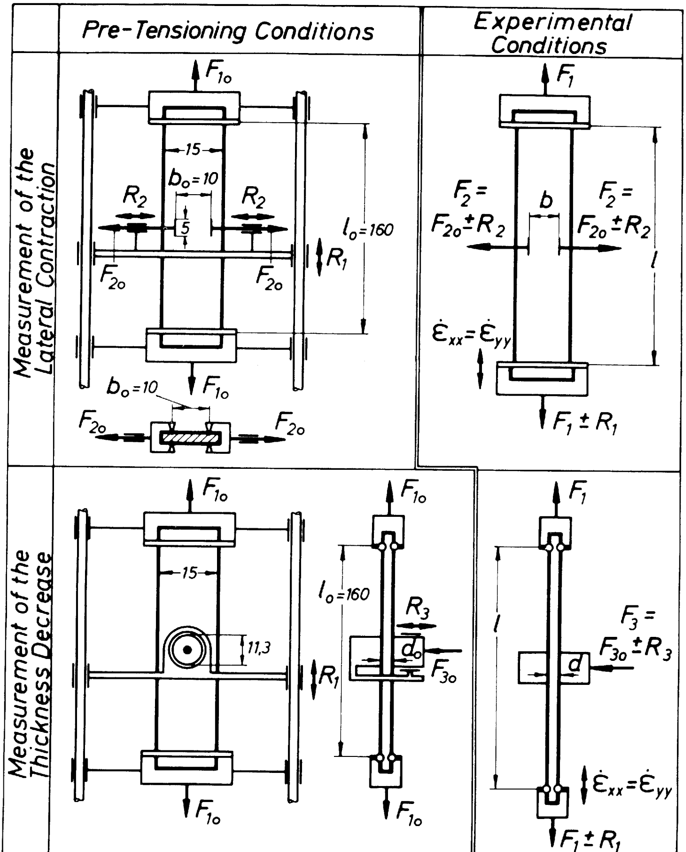
Apparatus, procedure and nomenclature

Apparatus

To facilitate the measurements, a Wolpert tensile tester ZR-E 500 was equipped with additional fittings. The entire system is shown schematically in Fig. 1.

The paper samples are 15 mm wide and the gauge length is $l_o = 160$ mm. They are clamped between cylindrical jaw faces of the two clamps with adjustable force. The upper clamp is connected to an inductive tensile load transducer, which has six ranges between 2 and 100 kp full-scale deflection. The lower clamp can be moved up and down along two pillars with constant speed.

For the measurement of the lateral contraction, two pairs of blades, 5 mm long, grip a length $b_0 = 10$ mm in the lateral direction of the paper sample with the adjustable force. They run in four Teflon bearings 130 mm long with a frictional force $R_2 \leqslant 10$ p. The bearings are mounted on two parallel vibrating rods. The jaws can follow the movement of the sample, as can the thickness gauge, since they are mounted on a cross-member whose weight is counterbalanced and that runs along the vertical pillars with a frictional force $R_1 = 20-50$ p.



Numerical Values

Frictional Forces	$R_1 = 20 \div 50$ p	$R_2 \leq 10$ p	$R_3 = 2$ p	
Pre-Tensioning Forces	CD-Breaking Load $F_{y\,max}$ kp	F_{1o} p	F_{2o} p	F_{3o} p
	< 2,5	50	50	100
	2,5 ÷ 4,5	75	75	100
	> 4,5	100	100	100

Fig. 1—Experimental conditions for the tensile tests with straining cycles (measurements in mm)

The thickness is measured with a pressure of 0·1 kp/cm² between two anvils with an area of 1 cm². The movable anvil runs in a vibrating ground groove, 60 mm long with a frictional force $R_3 = 2$ p.

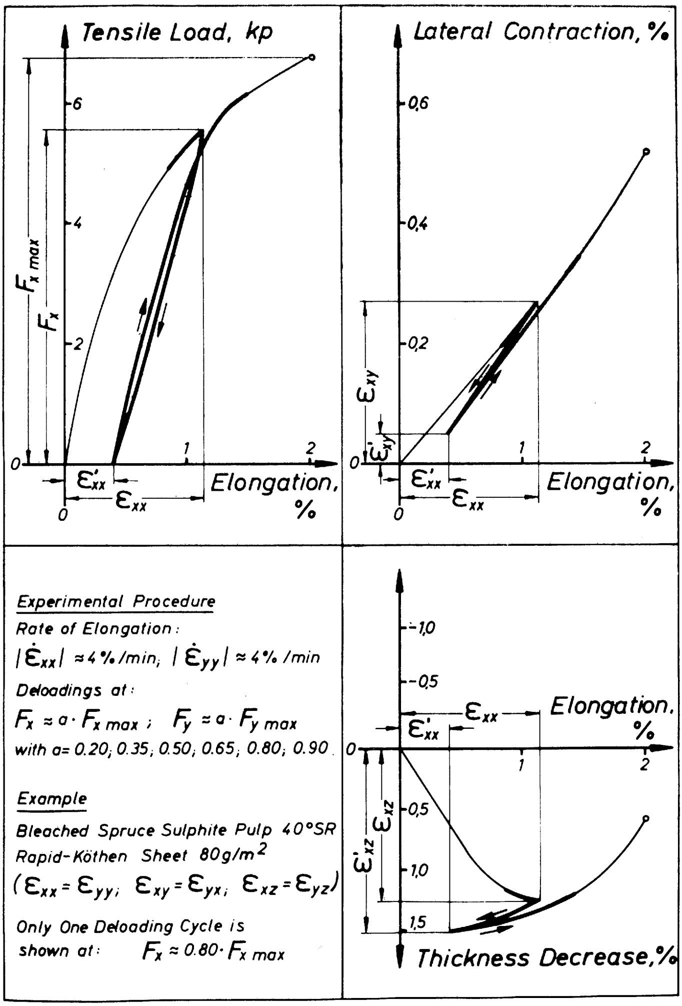


Fig. 2—Recording and calculation of the deformations in tensile tests with straining cycles

The tensile forces are recorded as a function of the elongation. The distance changes of the two sample clamps $(l - l_0)$, the two lateral contraction clamps $(b_0 - b)$ and the two anvils of the thickness gauge $(d_0 - d)$ are measured by means of an inductive strain transducer with an amplification from 1:20 to 1:20 000 and recorded on $X-Y$ plotters.

In order to maintain constant and well-defined initial conditions, preliminary tests were carried out with different types of paper and the pretensioning forces given in Fig. 1 were adopted. The forces occurring during the experiments follow from the pretensioning forces and the frictional forces, which are also given in Fig. 1.

Procedure

The deformations of the sample were measured at 23° C and 50 per cent relative humidity. Six times the samples were loaded to the values shown in Fig. 2, deloaded and immediately loaded again. This simple procedure could be adopted, since the results obtained in this manner agreed well with those of a tensile test without deloading cycles.

The up and down movement of the bottom clamp was controlled so that the rate of elongation was always about 4 per cent/min. Each experiment produced a load/elongation diagram, a lateral contraction/elongation diagram or a thickness decrease/elongation diagram.

The total deformations ϵ at the maximum loads of the cycles and the plastic deformation ϵ' that remained immediately after deloading were read off the diagrams, as Fig. 2 shows. The plastic deformation thus defined includes therefore part of the time-dependent elastic deformation.

Each series of measurements consisted of 15 individual tests. Every time, the total deformation and the plastic deformation were taken from the diagrams for each of the six straining cycles.

Nomenclature

The following definitions are based on the descriptions given above and are supplemented by Fig. 1 & 2. The axes are characterised by conventional indices, the first index referring to the direction of the load and the second index to the direction of the deformation.

Indices

x	Machine-direction
y	Cross-direction
z	Thickness
max	Point of failure

Symbols

W	Grammage in g/m^2 according to DIN 53104
D	Bulk density in g/cm^3 according to DIN 53105
$A = 15 \times d_0$	Cross-sectional area of the sample in mm^2

F_x, F_y	Tensile load in kp (per 15 mm width)
F^*_x, F^*_y	Specific tensile load in $\frac{kp/m}{g/m^2}$ $\left(F^* = \frac{F \times 1\,000}{W \times 15}\right)$
ε	Deformation
ε'	Plastic deformation
ε_{xx}, $\varepsilon_{yy} = 100(l - l_o)/l_o$	Elongation in per cent
ε_{xy}, $\varepsilon_{yx} = 100(b_o - b)/b_o$	Lateral contraction in per cent
ε_{xz}, $\varepsilon_{yz} = 100(d_o - d)/d_o$	Decrease in thickness in per cent
$E_x = \frac{F_x \times 100}{A \times \varepsilon_{xx}}$, $E_y = \frac{F_y \times 100}{A \times \varepsilon_{yy}}$	Modulus of elasticity in kp/mm²
$E^*_x = \frac{F^*_x \times 100}{\varepsilon_{xx}}$, $E^*_y = \frac{F^*_y \times 100}{\varepsilon_{yy}}$	Specific modulus of elasticity in $\frac{kp/m}{g/m^2}$
$\nu_{xy} = \varepsilon_{xy}/\varepsilon_{xx}$, $\nu_{yx} = \varepsilon_{yx}/\varepsilon_{yy}$	Poisson's ratio for the lateral contraction
$\nu_{xz} = \varepsilon_{xz}/\varepsilon_{xx}$, $\nu_{zy} = \varepsilon_{yz}/\varepsilon_{yy}$	Poisson's ratio for the decrease in thickness

Presentation and discussion of the experimental results

Effect of making variables

FIRSTLY, the *effect of the grammage* on the load/deformation behaviour was studied on handsheets between 40 and 200 g/m² that had been made from different pulps on the Rapid-Köthen sheetformer. Fig. 3 and Table 1 show the deformations for a spruce groundwood measured in tensile tests with straining cycles.

TABLE 1—LOAD/DEFORMATION BEHAVIOUR—

Property			*Spruce groundwood 52° SR Rapid-Köthen sheets grammages, g/m²* 40	80	120	160	200
Bulk density	D	g/cm³	0·39	0·44	0·47	0·48	0·49
Specific breaking load	$F^*_{x\,max}$	$\frac{kp/m}{g/m^2}$	1·86	2·20	2·30	2·65	2·74
Extension at break	$\varepsilon_{xx\,max}$	per cent	0·93	1·13	1·18	1·27	1·30
Modulus of elasticity	E_x	$\frac{kp}{mm^2}$	103·00	135·00	142·00	169·00	176·00
Specific modulus of elasticity	E^*_x	$\frac{kp/m}{g/m^2}$	278·00	311·00	313·00	360·00	363·00
Poisson ratios	ν_{xy}	—	0·15	0·16	0·16	0·16	0·21
	ν_{xz}	—	0·25	0·36	0·36	0·50	0·59
Property ratio	$E^*_x/F^*_{x\,max}$	—	149·00	141·00	136·00	136·00	132·00

The breaking load and the elongation at break increase with increasing grammage, producing a family of load/elongation curves as shown in Fig. 3, although the relationship between the plastic elongation and the total elongation is the same for all grammages. Plastic elongations occur only at total elongations above 0·2 per cent, corresponding to a tensile load of about

20 per cent of the breaking load, but then make up a rapidly increasing proportion of the total elongation.

The lateral contraction depends slightly on the grammage and increases approximately linearly with the elongation, corresponding to Poisson ratios $\nu_{xy} = 0{\cdot}15$–$0{\cdot}21$. No plastic lateral contraction occurs unless the elongation in the direction of the applied load leads to a permanent deformation in this direction also. The plastic components of the lateral contraction and the longitudinal elongation develop in similar fashions.

The thickness of handsheets decreases at first with increasing elongation, passes through a minimum at $\epsilon_{xz} = 0{\cdot}2$–$0{\cdot}3$ per cent and exceeds the original value only near the fracture strain. Poisson ratios $\nu_{xz} = 0{\cdot}25$–$0{\cdot}59$ were found. Even at low tensile loads, plastic thickness reductions occur, which increase with increasing elongation to higher values than the thickness reductions measured under tensile load, when they decreased again.

Table 1 seems to indicate that, with increasing grammage, the bulk density of groundwood handsheets also increases noticeably. If the bulk density is corrected for the thickness extrapolated to a grammage of 0 g/m², however, it is found to vary 0·52–0·54 g/cm³ only. The apparent increase of the bulk density is therefore caused by the well-known influence of the surface roughness; this effect decreases with increasing grammage.

The *effect of refining* was also studied on Rapid-Köthen handsheets.

EFFECT OF MAKING VARIABLES

Bleached spruce sulphite pulp Rapid-Köthen sheets 80 g/m², beaten in Jokro mill to freeness, °SR					*Illustration printing paper, grammage 65 g/m², calender compacted to different bulk densities*			
14	*28*	*43*	*56*	*70*				
0·50	0·73	0·79	0·82	0·90	0·55	0·62	0·70	0·90
1·75	5·31	5·96	6·31	6·24	2·63	2·48	2·54	2·55
1·29	1·95	2·07	1·94	1·90	0·87	0·93	1·01	1·02
142·00	488·00	570·00	645·00	605·00	223·00	210·00	249·00	330·00
292·00	689·00	726·00	783·00	697·00	410·00	370·00	362·00	399·00
0·17	0·24	0·25	0·25	0·32	0·23	0·23	0·23	0·23
0·15	0·89	1·05	1·16	1·19	0·26	−0·35	−0·53	−1·00
167·00	130·00	122·00	124·00	112·00	156·00	149·00	143·00	156·00

Fig. 4 and Table 1 show results for a softwood pulp; similar results were obtained for hardwood pulps.

The relationship between the plastic elongation and the total elongation over the entire range of Schopper-Riegler values is given by a single curve, identical with that shown in Fig. 3.

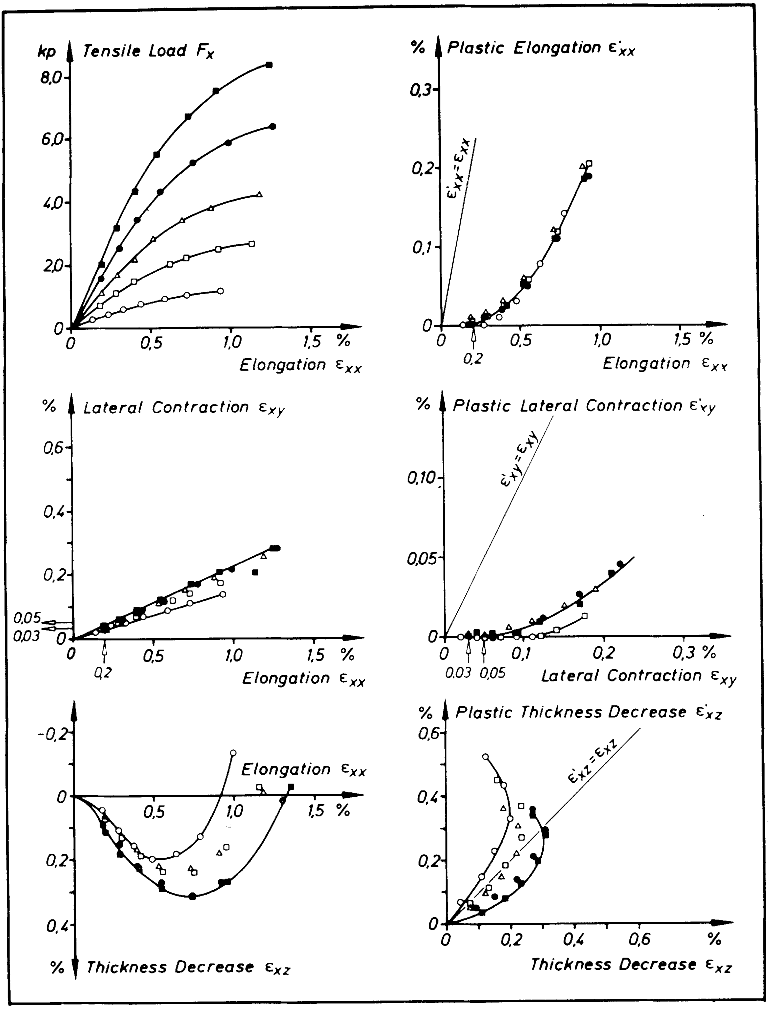


Fig. 3—Deformations in tensile tests with straining cycles
Spruce groundwood, 52° SR, Rapid-Köthen handsheets grammages
○ 40 g/m², □ 80 g/m², △ 120 g/m², ● 160 g/m², ■ 200 g/m²

Contrary to other experimental results,[8] the curve describing the increase of the lateral contraction with increasing elongation is slightly bent downwards. Refining causes the Poisson ratios ν_{xy} to increase from 0·17 to 0·32. With increasing Schopper-Riegler value, the plastic lateral contraction for a given total lateral contraction decreases.

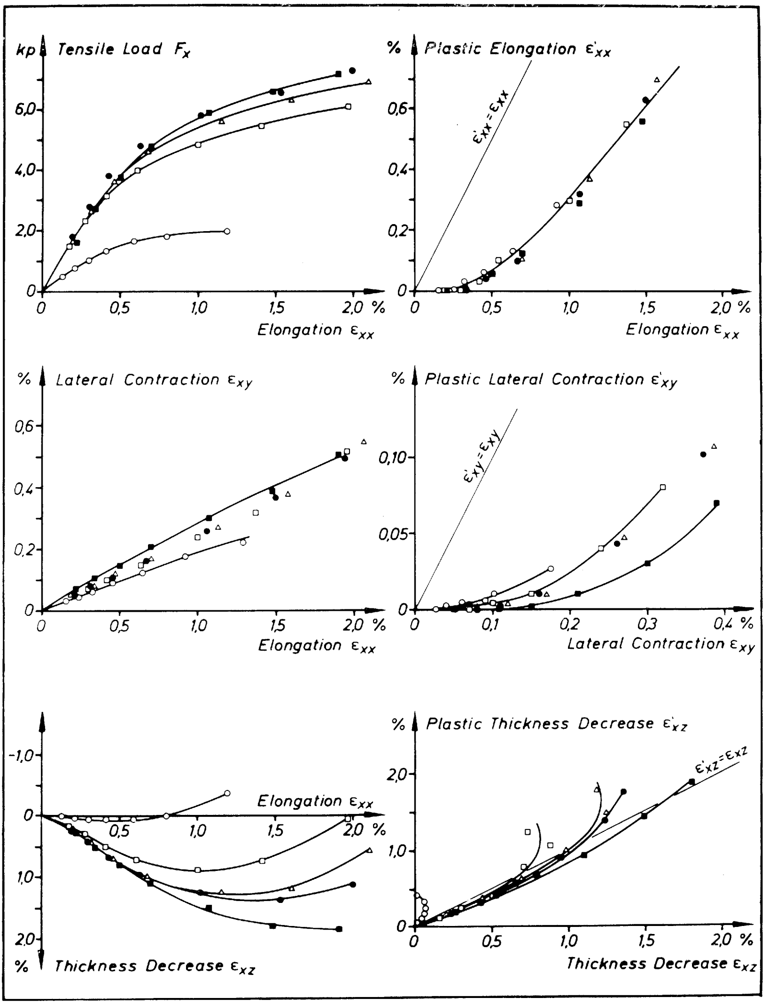


Fig. 4—Deformations in tensile tests with straining cycles
Bleached spruce sulphite pulp, Rapid-Köthen handsheets 80 g/m², beaten in the Jokro Mill to ○ 14° SR, □ 28° SR, △ 43° SR, ● 56° SR, ■ 70° SR

The Poisson ratios ν_{xz} for the reduction in thickness increase with the refining from 0·15 to 1·19. The maximum thickness reduction for highly beaten pulp is 1·9 per cent, which is much larger than that for groundwood. The plastic components of the thickness reduction increase with increasing Schopper-Riegler value as do those of the lateral contraction.

Fig. 5 and Table 1 show the *effect of calendering* for an illustration printing paper whose bulk density *D* was increased in four steps from 0·55 to 0·90 g/cm³. Similar results were found when calendering Rapid-Köthen handsheets.

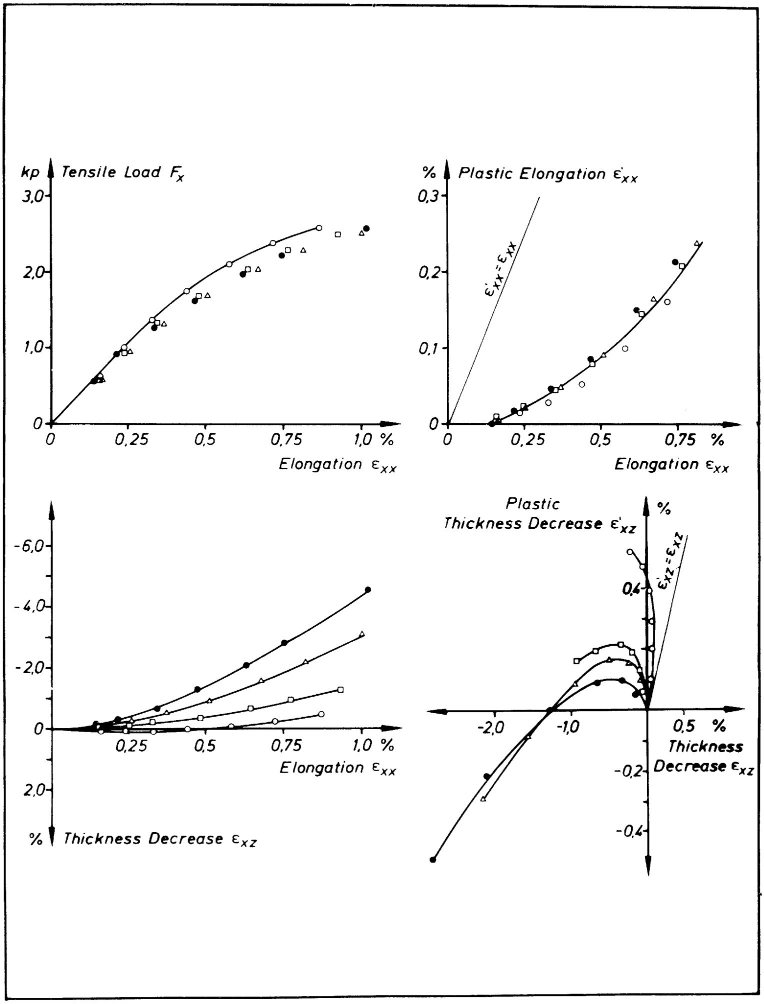


Fig. 5—Deformations in tensile tests with straining cycles
Illustration printing paper, 65 g/m² (industrial paper, machine-direction) compactedby calendering to bulk densities of ○ 0·55 g/cm³, □ 0·62 g/cm³, △ 0·70 g/cm³, ● 0·90 g/cm³

Whereas the effect of calendering on the load/elongation behaviour and on the lateral contraction is negligible, it does affect the pattern of thickness changes. The uncalendered paper suffers at first a thickness reduction and reaches even at a load of 90 per cent of its fracture load a thickness increase of only $\epsilon_{xz} = -0{\cdot}45$ per cent. The heavily calendered paper has a Poisson ratio $\nu_{xz} = -1{\cdot}0$ and a maximum thickness change of $\epsilon_{xz} = -4{\cdot}6$ per cent. The thickness increase is of a plastic nature only for more heavily calendered

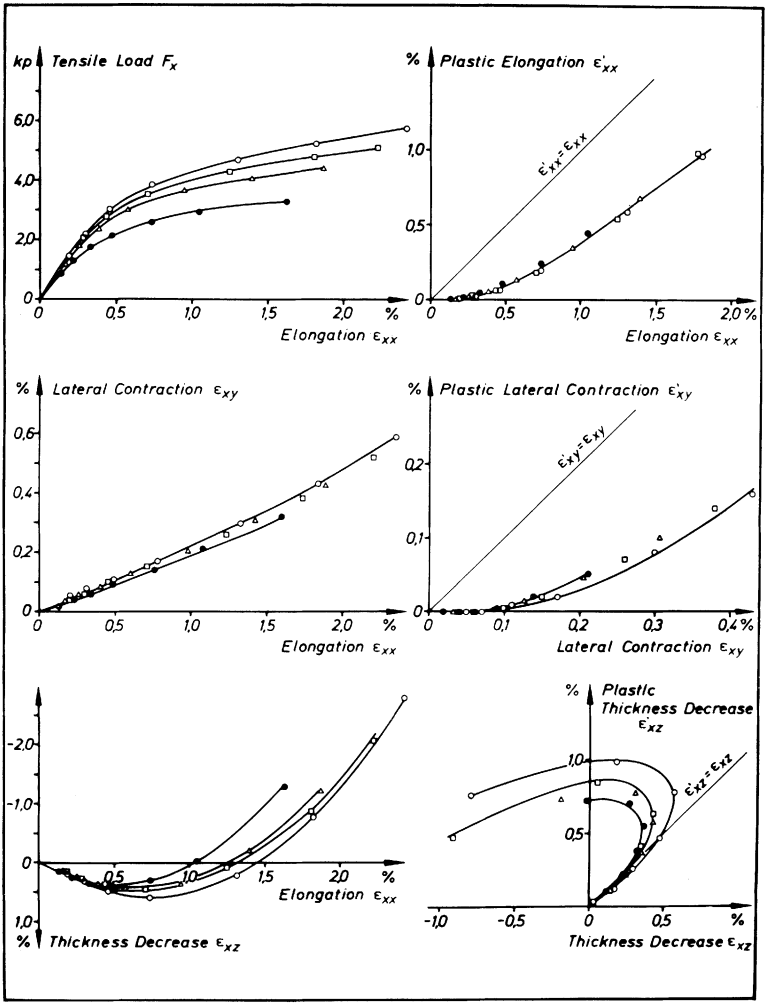


Fig. 6—Deformations in tensile tests with straining cycles
Rapid-Köthen handsheets, grammage of the fibres 80 g/m² bleached spruce sulphite pulp (25° SR)+china clay ○ 0 per cent, □ 7 per cent, △ 14 per cent, ● 25 per cent

papers and even here only for high elongations; all other conditions produced plastic thickness reductions only.

The making variables investigated therefore affect the pattern of thickness changes most of all, to a much lesser extent the lateral contraction and leave the relationship between plastic elongation and total elongation unchanged.

Effect of furnish variables

The *effect of fillers* was again studied on Rapid-Köthen handsheets that had been made from a slightly beaten spruce sulphite pulp with various addition levels of china clay at constant grammage of the fibres.

According to Fig. 6, the plastic elongation as a function of the total elongation is independent of the filler content. Contrary to other results,[6] the inclusion of rigid filler particles in the fibre network leads to a reduction of the Poisson ratio ν_{xy} from 0·25 for the unfilled handsheet to 0·15 for the handsheet with 25 per cent ash content (Table 2). The handsheets containing filler suffer, as do the unfilled sheets, a thickness reduction at small elongations; with increasing elongation, the thickness increases sooner than that of the unfilled sheets.

The *effect of several pulps* on the load/deformation behaviour of handsheets is shown in Fig. 7. The Poisson ratios for the lateral contraction are lower for the filled handsheet and for the sheet made from groundwood than they are for sheets made from 100 per cent beaten pulp, as one would expect. The thickness changes are also in agreement with the results discussed above. As with all other Rapid-Köthen handsheets, no permanent increase in thickness was observed.

The furnish variables investigated have a less pronounced effect on the load/deformation behaviour of paper than have the making variables discussed earlier.

Industrial papers and other sheet-like materials

FIG. 8 shows the difference in the load/elongation behaviour of one of the commercial papers examined when tested in the machine and in the cross directions. Fig. 9 shows the results for a number of printing papers, one packaging paper and one folding boxboard, with the load applied in the machine-direction. This mode of loading is of particular interest for the conversion and usage of these papers.

The strong correlation between plastic elongation and total elongation as discussed previously is found for commercial papers also and is independent of the direction of the applied load.

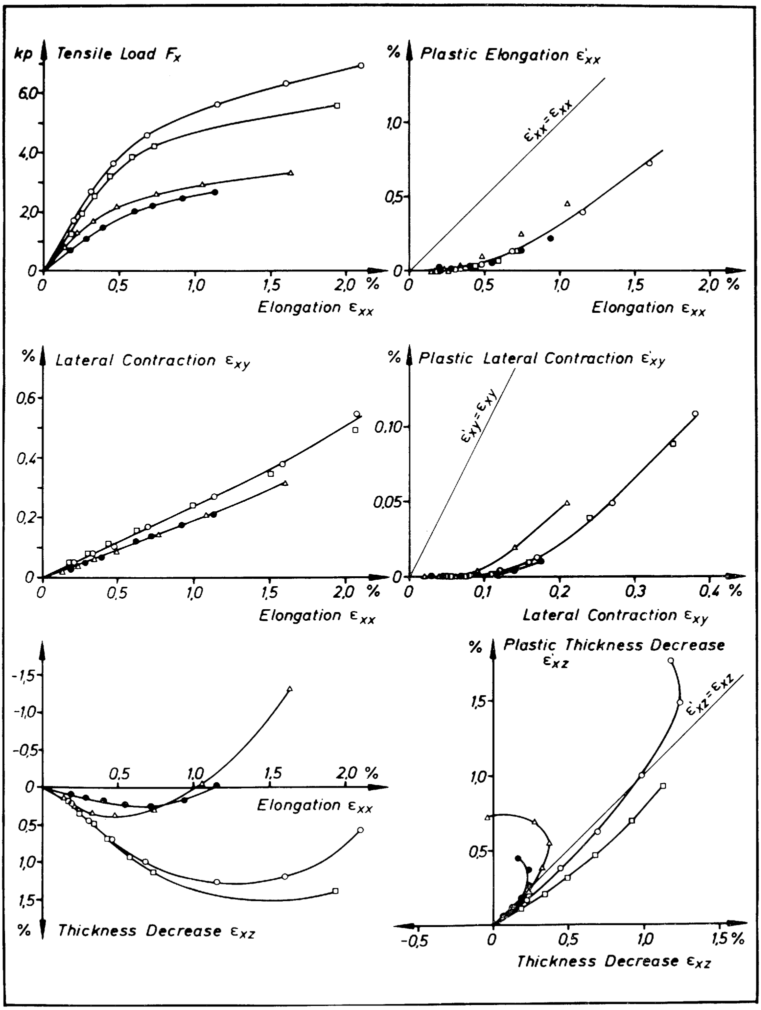


Fig. 7—Deformations in tensile tests with straining cycles Rapid-Köthen handsheets, grammage of the fibres 80 g/m^2 ○ bleached spruce sulphite pulp, 43° SR, △ bleached spruce sulphite pulp, 25° SR +25 per cent ash content, □ bleached beech sulphite pulp, 45° SR, ● spruce groundwood 52° SR

Generally speaking, for the range of grades investigated here, the effect of the grade on the lateral contraction when tested in the same direction is smaller than the differences that occur when the same paper is loaded in the

TABLE 2—LOAD/DEFORMATION BEHAVIOUR: EFFECT OF FURNISH VARIABLES

Property			*Rapid-Köthen sheets with grammage of fibres 80 g/m² bleached spruce sulphite pulp 25° SR and china clay* Ash content, per cent				*Rapid-Köthen sheets with grammage of fibres 80 g/m²*			
			0	7	14	25	*Bleached spruce sulphite pulp 43° SR*	*Bleached spruce sulphite pulp 25° SR +25 per cent ash*	*Bleached beech sulphite pulp 45° SR*	*Spruce groundwood 52° SR*
Bulk density	D	g/cm^3	0·64	0·67	0·67	0·71	0·79	0·71	0·79	0·44
Specific breaking load	$F^*_{x\,max}$	$\frac{kp/m}{g/m^2}$	4·76	3·81	3·05	1·95	5·96	1·95	4·89	2·20
Extension at break	$\varepsilon_{xx\,max}$	per cent	2·42	2·23	1·87	1·63	2·07	1·63	1·94	1·13
Modulus of elasticity	E_x	$\frac{kp}{mm^2}$	400·00	351·00	324·00	250·00	570·00	250·00	486·00	135·00
Specific modulus of elasticity	E^*_x	$\frac{kp/m}{g/m^2}$	613·00	527·00	483·00	352·00	726·00	352·00	614·00	311·00
Poisson ratios	ν_{xy}	—	0·25	0·20	0·19	0·15	0·25	0·15	0·25	0·16
	ν_{xz}	—	0·90	0·74	0·77	0·93	1·05	0·93	1·03	0·36
Property ratio	$E^*_x/F^*_{x\,max}$	—	129·00	138·00	158·00	181·00	122·00	181·00	126·00	141·00

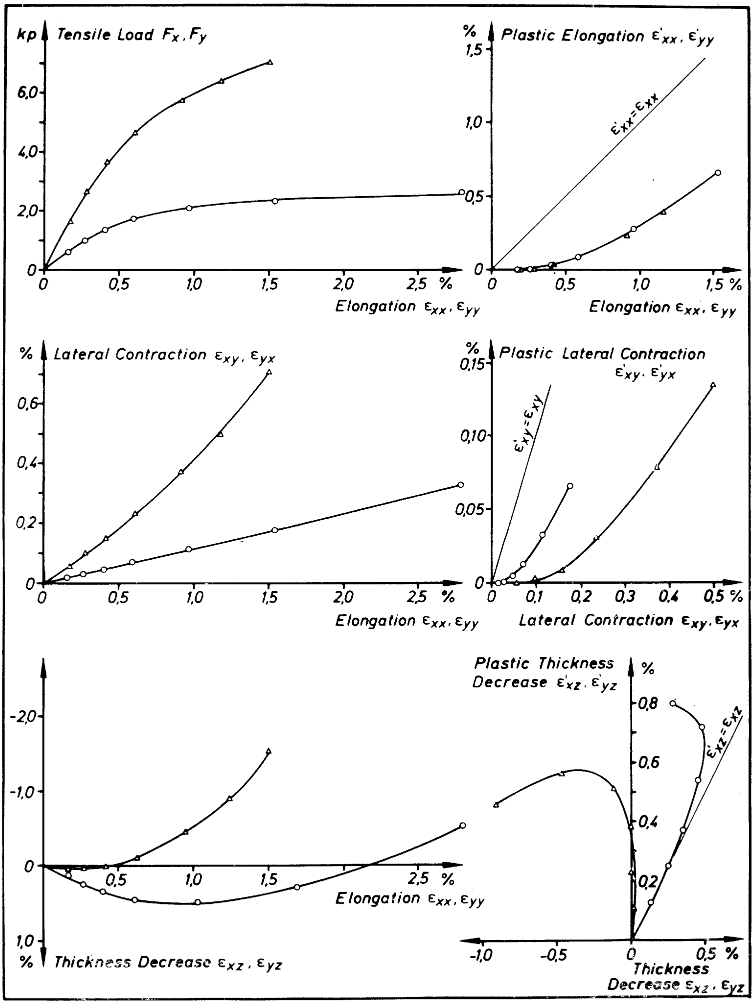


Fig. 8—Deformations in tensile tests with straining cycles
Machine-finished woodfree offset printing paper (IV) grammage 80 g/m², bulk density 0·75 g/cm³ △ machine-direction ○ cross-direction

machine-direction and in the cross-direction. This result is underlined by the Poisson ratios shown in Table 3: $\nu_{xy} = 0{\cdot}16$–$0{\cdot}34$ and $\nu_{yz} = 0{\cdot}04$–$0{\cdot}10$. If the Poisson ratios are not as in this case calculated from the initial slope of the lateral contraction/elongation curve, but for an elongation of 1 per cent, the values are found to agree very well with those reported by W. Brecht &

R. Wanka[6] for machine-made papers and quoted as follows: $\nu_{xy} = 0{\cdot}23$–$0{\cdot}45$ and $\nu_{yz} = 0{\cdot}10$–$0{\cdot}15$.

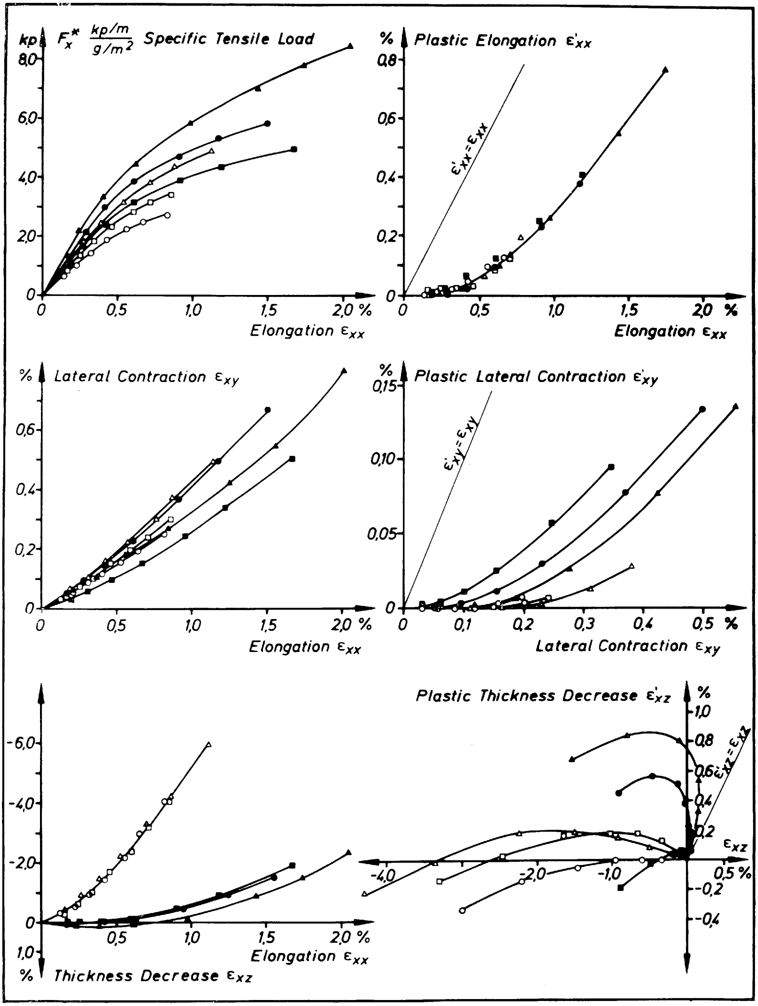


Fig. 9—Deformations in tensile tests with straining cycles
Industrial papers (machine-direction)

○ Uncoated illustration printing paper (I)
△ Coated illustration printing paper (II)
□ Newsprint paper (III)
● Offset printing paper (IV)
▲ Kraft sack paper (V)
■ Folding boxboard (VI)

TABLE 3—LOAD/DEFORMATION BEHAVIOUR OF INDUSTRIAL PAPERS

Property			I *Calendered uncoated illustration printing paper containing groundwood*	II *Calendered coated illustration printing paper containing groundwood*	III *Machine-finished newsprint*	IV *Machine-finished woodfree offset printing paper*	V *Kraft sack paper*	VI *Coated (white top) folding boxboard*
Grammage	W	g/m²	65·00	55·00	52·00	80·00	75·00	280·00
Bulk density	D	g/cm³	0·98	1·04	0·68	0·75	0·64	0·67
Specific breaking load	$F^*_{x\ max}$	$\frac{kp/m}{g/m^2}$	2·73	4·90	3·37	5·80	8·50	4·95
	$F^*_{y\ max}$	$\frac{kp/m}{g/m^2}$	0·94	1·54	1·40	2·16	3·94	1·60
Extension at break	$\varepsilon_{xx\ max}$	per cent	0·83	1·12	0·85	1·50	2·04	1·67
	$\varepsilon_{yy\ max}$	per cent	1·44	2·05	1·69	2·78	4·50	3·07
Modulus of elasticity	E_x	$\frac{kp}{mm^2}$	466·00	690·00	356·00	579·00	542·00	386·00
	E_y	$\frac{kp}{mm^2}$	160·00	208·00	108·00	224·00	238·00	126·00
Specific modulus of elasticity	E^*_x	$\frac{kp/m}{g/m^2}$	468·00	638·00	532·00	765·00	861·00	591·00
	E^*_y	$\frac{kp/m}{g/m^2}$	155·00	189·00	163·00	295·00	374·00	194·00
Poisson ratios	ν_{xy}	—	0·23	0·34	0·27	0·31	0·31	0·16
	ν_{yx}	—	0·09	0·10	0·08	0·10	0·10	0·04
	ν_{xz}	—	−2·28	−2·95	−2·20	0·15	0·31	−0·08
	ν_{yz}	—	−0·59	−0·41	−0·01	0·77	1·36	0·89
Property ratios	$\frac{\nu_{xy} \times E^*_y}{\nu_{yx} \times E^*_x}$	—	0·85	1·01	1·03	1·20	1·35	1·31
	$\frac{E^*_x}{F^*_{x\ max}}$	—	171·00	130·00	158·00	132·00	101·00	119·00
	$\frac{E^*_y}{F^*_{y\ max}}$	—	165·00	123·00	116·00	137·00	95·00	121·00

TABLE 4—LOAD/DEFORMATION BEHAVIOUR OF PACKAGING FOILS AND PRINT SUBSTRATES

Property			XI *Aluminium packaging foil*	XII *Polythene packaging film*	XIII *Printing paper made from synthetic fibres*	XIV *Printing paper made from synthetic fibres*	XV *Printing film made from biaxially stretched polystyrene*	XVI *Printing film made from highly loaded, biaxially stretched polythene*
Grammage	W	g/m^2	40·00	190·00	115·00	100·00	130·00	90·00
Bulk density	D	g/cm^3	2·63	0·91	0·78	1·03	1·04	1·13
Specific breaking load	$F^*_{x\ max}$	$\frac{kp/m}{g/m^2}$	2·28	1·85	5·30	2·60	5·00	7·02
	$F^*_{y\ max}$	$\frac{kp/m}{g/m^2}$	2·02	1·87	3·35	1·47	4·50	5·62
Extension at break	$\varepsilon_{xx\ max}$	per cent	2·26	430·0	5·04	9·64	40·30	53·80
	$\varepsilon_{yy\ max}$	per cent	2·66	440·0	10·50	18·30	34·90	69·90
Modulus of elasticity	E_x	$\frac{kp}{mm^2}$	3 130.00	24·30	259·00	114·00	377·00	314·00
	E_y	$\frac{kp}{mm^2}$	2 910·00	27·40	104·00	33·00	354·00	282·00
Specific modulus of elasticity	E^*_x	$\frac{kp/m}{g/m^2}$	1 250·00	24·50	340·00	102·00	403·00	360·00
	E^*_y	$\frac{kp/m}{g/m^2}$	1 150·00	29·00	148·00	28·00	385·00	343·00
Poisson ratios	ν_{xy}	—	0·31	0·61	0·30	0·33	0·45	0·55
	ν_{yx}	—	0·34	0·75	0·14	0·10	0·44	0·44
	ν_{xz}	—	—	0·71	−1·34	−0·49	−3·57	−4·19
	ν_{yz}	—	—	0·59	−0·25	+0·83	−3·30	−1·86
Property ratios	$\frac{\nu_{xy} \times E^*_y}{\nu_{yx} \times E^*_x}$	—	0·84	0·96	0·93	0·91	0·98	1·19
	$\frac{E^*_x}{F^*_{x\ max}}$	—	548·00	13·00	64·00	39·00	81·00	51·00
	$\frac{E^*_y}{F^*_{y\ max}}$	—	569·00	16·00	44·00	19·00	86·00	61·00

With all commercial papers examined, the plastic component of the lateral contraction was greater when the tensile load was applied in the cross-direction than when it was applied in the machine-direction. The shapes of the curves describing the plastic lateral contraction as a function of the total lateral contraction are equal to the functions $\epsilon'_{xx} = f(\epsilon_{xx})$ and $\epsilon'_{yy} = f(\epsilon_{yy})$, respectively.

In contrast to laboratory handsheets, the machine-made papers, in agreement with other published results,[10, 11] exhibit considerable thickness increases, up to $\epsilon_{xy} = -6$ per cent, particularly at higher levels of extension. The thickness changes for commercial papers show a greater variation than the other deformations caused by a tensile load. This can be seen from the Poisson ratios $\nu_{xz} = -2{\cdot}95$ to $+0{\cdot}31$ and $\nu_{xz} = -0{\cdot}59$ to $+1{\cdot}36$. Over a wide range of elongation, the thickness after deloading is lower than the original thickness; at high levels of extension, however, plastic thickness increases were also observed.

For comparison, the load/elongation behaviour of a number of packaging foils and printing substrates were examined under the same conditions as the commercial papers. The results are given in Table 4 and shown in Fig. 10.

For the load/elongation curves, the specific tensile load F^* (that is, the tensile load divided by the grammage) was used as ordinate. The specific fracture load F^*_{max} is then numerically identical to the breaking length in km. Fig. 10 shows clearly the relatively low extensibility and high breaking load of industrial papers. The values for the modulus of elasticity in Tables 3 and 4 emphasise the relatively uniform load/elongation behaviour of the papers compared with the diversity of the other sheet-like materials. This is true also for the relationship between the plastic and the total elongations. On the other hand, the two classes of materials show a similar elongation/lateral contraction behaviour. The thickness increase under tensile load, so typical for industrial papers, is found again only for printing substrates made from synthetic fibres.

Finally, the effect of the sheet-like structure of the materials and of the method of measurement on their material properties should be noted. For example, the bulk density and the modulus of elasticity of the aluminium foil (see Table 3) are noticeably lower than those normally quoted for the material aluminium ($\gamma = 2{\cdot}70$ g/cm^3, $E = 7\,000$ kp/mm^2). This is caused by thickness variations and by the surface roughness, which for foils becomes extremely significant.

Summary and discussion

The results confirm earlier findings,[3, 4] according to which the development of the plastic elongation as a function of the total elongation is independent of the type of paper and of the direction of the applied load. The values

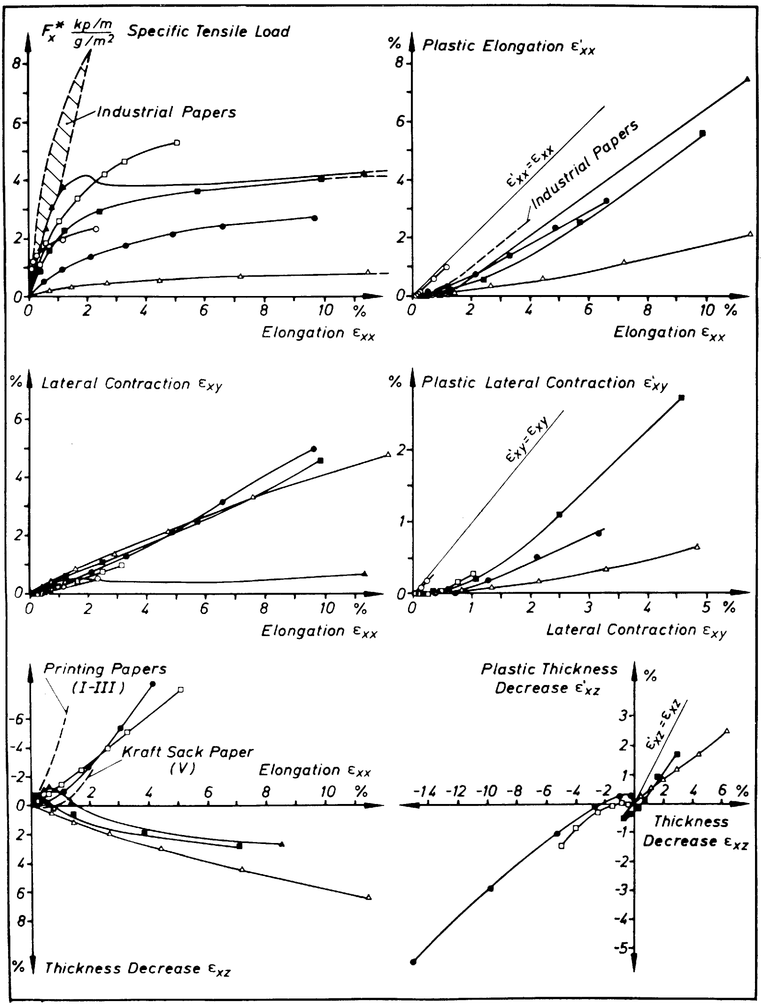


Fig. 10—Deformations in tensile tests with straining cycles packaging foils and printing substrates (machine-direction)

○ Aluminium packaging foil (XI)
△ Polythene packaging foil (XII)
□ Printing paper from synthetic fibres (XIII)
● Printing paper from synthetic fibres (XIV)
▲ Polystyrene printing foil (XV)
■ Polythene printing foil (XVI)

found for the lateral contraction agree well with those known from theoretical considerations. The development of the plastic component of the lateral contraction with increasing total lateral contraction follows a curve with a shape similar to that for the corresponding elongations.

According to the theory of elasticity, the following relationship should hold for anisotropic materials—

$$\frac{\nu_{xy} \times E_y^*}{\nu_{yx} \times E_x^*} = 1.$$

Indeed, the experimental values for this ratio were found to lie in the following ranges—

Industrial papers	0·85–1·35
Packaging foils and printing substrates	0·84–1·19

The thickness changes during the tensile loading was usually positive at first (that is, the thickness decreased); it then passed through a minimum and increased again, frequently to values above the initial thickness. If the thickness did not decrease at the beginning of the elongation, its increase was at first slight and became steeper only with increasing elongation. Both shapes of curves suggest that two opposite processes take place at the same time.

At small elongations, the free fibre segments between two points of contact are oriented, according to a model suggested by O. E. Öhrn;[10] in addition, there are lateral contractions of the individual fibres, which continue during the tensile loading. Both phenomena cause the thickness of the paper to decrease or to prevent its increase. The increase is at first completely elastic as a result of the loosening or micro-delamination of the structure as postulated by O. E. Öhrn[11] and develops plastic components only at higher levels of elongation, caused by the fracture of fibre-to-fibre bonds.

The explanation for the apparent effect of the grammage on the thickness increase (Fig. 3) is the fact that, as a result of the surface roughness, the initial thickness of thin papers is found too high (see *Effect of making variables*). Increasing Schopper-Riegler values cause a reduction of the delamination effect as a result of the crushing and shortening of the fibres and of increased fibre-to-fibre bonding. The thickness of highly beaten handsheets therefore steadily decreases because of the lateral contraction of the individual fibres (Fig. 4). When the sheet had been compacted in dry form (that is, without the formation of new fibre-to-fibre bonds), even a small elongation causes micro-delamination, which immediately reverses a small part of the thickness reduction caused by calendering (Fig. 5). Printing substrates made from synthetic fibres show large thickness increases as one would expect (Fig. 10).

The model suggested by O. E. Öhrn therefore explains at least part of the observed thickness changes during elongation, but there is still room for a more detailed interpretation.

Load/deformation behaviour of paper and its conversion and use

IF ONE wants to derive relevant conclusions for the converter and the user of paper by generalising the results of the investigation of its load/elongation behaviour, the most encouraging starting point is Fig. 10. It shows that such a generalisation should be possible for the material paper, which, in spite of the differences between different grades, covers a much narrower range of properties than the class of packaging foils and other printings substrates. On the other hand, one must always bear in mind that the dimensions of paper cannot be altered by external forces only, but to a large extent by moisture expansion processes.

We know of no practical problems related to the thickness change of paper under a tensile load. Since such thickness increases occur also at pressures higher than those used in these measurements, such relationships cannot be excluded.

The similar shape of the load/elongation curves of the industrial papers examined facilitates many approximate predictions in practice. An example is the so-called areal ratio† for the approximate calculation of the rupture energy of paper from its fracture load and elongation. Similarly, the specific moduli of elasticity of the industrial paper $E_x^* = 468\text{–}861 \dfrac{\text{kp/m}}{\text{g/m}^2}$, are close together, compared with the values $E_x^* = 24{\cdot}5 \dfrac{\text{kp/m}}{\text{g/m}^2}$ for the polythene packaging film and $E_x^* = 1\,250 \dfrac{\text{kp/m}}{\text{g/m}^2}$ for the aluminium packaging foil. On the other hand, the values for the ratio $E_x^*/F_{x\,max}^*$, lying between 101 and 181, show that the method frequently suggested to calculate the breaking load from the modulus of elasticity is reasonably accurate only for a narrow range of grades.

Important conclusions can be deduced from the fact that the plastic elongation that occurs is controlled only by the total elongation, but not by the quality of paper or the direction of the applied load—

1. As long as the total elongation of paper is below 0·2 per cent or as long as the tensile load is smaller than 20 per cent of the breaking load, there is in general no permanent elongation.
2. Furthermore, it is generally found that above about 0·5 per cent total elongation the plastic elongation becomes noticeable, that at 2–2·5 per cent total elongation it is of the same magnitude as the elastic elongation, then increases

† This is the area under the load/elongation curve, divided by the product of the fracture load and elongation

linearly with increasing total elongation, whereas the elastic elongation increases only slightly.

This rule was formulated by W. Brecht, H. J. Knittweis & W. Schmidt[4] and can be extended by saying that, so long as the total elongation does not exceed 0·2 per cent, no plastic lateral contraction occurs. The total lateral contraction can be estimated as approximately 15–35 per cent of the total elongation.

The value of such statements for the design and the operation of printing and converting machines becomes apparent when one thinks of problems such as misregister or inaccurate punching as they increasingly occur in the insetting process or in the usage of data processing papers or even with packaging papers.

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References

1. Van den Akker, J. A., *Tappi*, 1970, **53** (3), 388–400
2. Kubát, J., 'Rheology of paper' in *Rheology, Theory and Applications*, Ed. F. R. Eirich (Academic Press, New York and London, 1969), Vol. 5
3. Brezinski, J. P., *Tappi*, 1956, **39** (2), 116–128
4. Brecht, W., Knittweis, H. J. and Schmidt, W., *Wochbl. Papierfabr.*, 1971, **99** (8), 287–292
5. Ranger, A. E. and Hopkins, L. F., *The Formation and Structure of Paper*, Ed. F. M. Bolam, (Technical Section of the British Paper and Board Makers' Association, London, 1962), 277–310
6. Brecht, W. and Wanka, R., *Das Papier*, 1963, **17** (4), 141–147
7. Rothmann, M., *The Formation and Structure of Paper*, Ed. F. M. Bolam (Technical Section of the British Paper and Board Makers' Association, London, 1962), 316
8. Jones, A. R., *Tappi*, 1968, **51** (5), 203–209
9. Taylor, O. L. and Craven, J. K., *Consolidation of the Paper Web*, Ed. F. M. Bolam (Technical Section of the British Paper and Board Makers' Association, London, 1966), 852–895
10. Öhrn, O. E., *Svensk Papperstidn.*, 1965, **68** (5), 141–149
11. Ihrman, C. B. and Öhrn, O. E., *Consolidation of the Paper Web*, Ed. F. M. Bolam (Technical Section of the British Paper and Board Makers' Association, London, 1966), 410–434

Discussion

Dr D. H. Page The question of the expansion of paper in thickness (Z-direction) is a very interesting one theoretically and the mechanism by which it occurs is something that I have thought about for some time. I think you have added much in experimental expertise, though I think you have still left open how exactly this thickness increase occurs. I have a possible suggestion for a mechanism.

If you take a single fibre and extend it, because it has a finite spiral angle, the fibre tends to take a more round form, a less flattened form. If the lumen is completely bonded, the fibre will not debond and you obtain no thickness increase; if the lumen is not totally bonded, a flattened fibre will tend to come to a more rounded form. Is it possible that the explanation for the thickness increase of the paper is related in this way to the thickness increase of the fibre itself?

Mr H. L. Baumgarten Yes, this may be one of the processes that take place during tensile elongation, but (as I tried to say) there are at least three different processes.

Dr Page This would be a process that would lead to plastic deformation later on in the load/elongation curve and I would expect it from the single fibre data—but does it occur?

Mr Baumgarten At the end of the load/elongation process, we sometimes found a plastic increase in thickness.

The Chairman During drying, the whole sheet is compacted by the Campbell forces and, when bonding takes place, dried-in stresses are created in the Z-direction. When the bonds partly break under loading, then the fibres will spring back again. This was shown by Ranger & Hopkins in their paper at the 1961 symposium.

Under the chairmanship of Prof. H. W. Giertz

Dr L. S. Nordman Have you made any measurements of thickness changes during relaxation or during creep experiments?

Mr Baumgarten We are doing these tests at present, but we have no results that we can give here.

Mr J. F. Waterhouse We have obtained some very similar results to yours on the permanent set characteristics of polymer-reinforced fibre networks. We found that the plastic deformation was unaffected by the type of polymer used, but was mainly controlled by the kind of fibre. Furthermore, the set characteristics were essentially the same for the machine-direction and cross-direction of the paper. These results indicate that the plastic deformation, which I think your results in part show, is almost solely controlled by the plastic behaviour of the fibre itself. Yet, in Fig. 9, your results do not seem to indicate that the plastic deformation depends on the kind of fibre.

Dr N. G. M. Tuck In your graphs, I assume that the thickness decrease shown as being negative is in fact a thickness increase—in effect, a double negative.

Mr Baumgarten That is correct.

Dr S. I. Cavlin Did you ever find that drying conditions, the restraint during drying and so on, were important? Many experimental facts in the current literature show that the shrinkage affects the strain, particularly the plastic strain, to break point.

Mr Baumgarten We have experimented, but it is very difficult to get handsheets dried in different ways without altering the surface properties of the sheet, so we made no thickness measurements on this handsheet.

Dr Cavlin It is possible to change the modulus of elasticity in one direction to at least 50 per cent. Theoretically, this implies that the Poisson ratio also can be changed by about 50 per cent just by restraining the web during its drying.

Mr Baumgarten The relationship between the plastic elongation and total elongation is unchanged by the method of drying.

Dr A. de Ruvo I would like to comment that, when we are investigating different modes of deformation for single fibres, our intention is not to put them into any network theory or any theory in which the characteristics of the fibre are introduced in some very complicated formula. We aim for the limiting strengths of the fibre, then to see which characteristics of paper are similar to the behaviour of the fibre. From that, we hope to evaluate what deformations prevail in the sheet—for example, in spite of the elastic property in tensile deformation of the fibre, it is not sensitive to humidity, although the shear modulus is sensitive to humidity in the same way as the elastic property of the sheet. This indicates to me that the main deformation during elastic strain is in the shear and bending of the fibres.

The Chairman Dr Dodson has presented his paper in a very clear way indeed and it is not the aim of this meeting to discuss the theoretical points in it. We have to discuss how paper behaves when it is used.