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FACTORS THAT AFFECT THE STIFFNESS OF PAPER

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Synopsis Use of paper for structural purposes has prompted research toward better utilisation of materials, determining potential stiffness and procedures to attain such stiffness.

Stiffness is dependent on thickness and a new definition of thickness is proposed that facilitates the quantification of paper stiffness with traditional engineering concepts without introducing errors caused by surface roughness.

Perhaps equally important as thickness in achieving potential stiffness is specific gravity, because the modulus of elasticity will usually vary as the cube of specific gravity if densification occurs while paper is wet.

The most efficient means for control of stiffness is restraint during drying, with a potential for threefold increases in modulus of elasticity. Even larger increases in stiffness in the order of five or sevenfold are awaiting a better means of controlling fibre orientation.

A new method is being developed for quantifying interfibre bonding that will be independent of sheet grammage and additives.

Moisture is seen as the greatest obstacle to making high performance structural fibre products because of the inability of these products to maintain reasonable levels of stiffness when wet, even when treated with synthetic resins. In spite of this problem, the potential high stiffness and strength-to-weight ratios that are available in wood fibre makes the creation of new structural products from pulp fibres inevitable.

Introduction

THE versatility of woodpulp fibres as a raw material is exemplified by a range of products that extends from soft fluffy tissue to dense, stiff countertops. Thus, the value and utility of wood fibre products is demonstrated by our ability to increase or decrease stiffness over an extremely wide range. Yet, in spite of practical and technical know-how that has allowed us to make such divergent products of wood fibre, sizeable gaps in our knowledge prevent

Under the chairmanship of Prof. H. W. Giertz us from making the most efficient use of a truly remarkable resource. This report surveys the factors we consider important to the control of paper stiffness.

One of the problems of our technology has been to think of stiffness and strength as synonyms. This view is helped along by the fact that these two factors will often correlate well, because both depend on specific gravity, though we do not believe there is necessarily a fundamental relationship between stiffness and strength. For example, these properties do not always change proportionally, because they do not have the same dependence on restraint during drying.[1]

Both stiffness and strength are important factors, but it is more frequently the presence or lack of stiffness that limits our choice of materials. For example, in many product-converting operations, strength is secondary; what actually determines the ultimate productivity of the operation is the inherent quality of stiffness.

What then is the maximum stiffness that can be achieved with a hydrogen-bonded cellulose fibre system? It was predicted[1] that, by aligning all fibres in the same direction and carefully controlling shrinkage during drying, we could achieve an elastic modulus of about $1{\cdot}5 \times 10^6$ lb/in², with specific gravity of about 0·4. This is a fairly impressive stiffness estimate for paper at any density, because it is equivalent to modulus of elasticity values for wood parallel to the grain at that specific gravity. Unfortunately, making paper with all fibres in parallel alignment is not an easy matter, so proof of this prediction is still waiting. To establish the efficacy of these predictions, however, a determined effort was made to maximise known factors affecting tensile strength and stiffness.

Test results from a few small handsheets indicate that even higher stiffness is attainable. These handsheets, made at a specific gravity of 0·75, had a tensile strength of 38 000 lb/in² and an elastic modulus of $3{\cdot}8 \times 10^6$ lb/in². In these samples, tensile strength in a principal direction was about six times as high as the strength perpendicular to that direction.

These values substantially exceed the specific strength and stiffness or strength/stiffness to mass ratio of all common structural materials including wood.

These data are illustrated in Fig. 1. Except for plastic paper and structural steel, strength and stiffness values for non-cellulose materials (shown as square symbols) are taken from *Science* magazine.[2] The values for steel are from Mark's *Mechanical Engineers' Handbook*.[3] The values for synthetic paper were our own test values. When available, a rough estimate of cost per pound is shown following each symbol.

This graph shows the relative superiority of kraft paper over plastic paper substitutes. The predicted specific modulus of elasticity value for unidirectional fibre-oriented paper (U-paper) was roughly comparable with such common structural materials as wood, aluminium and steel. The strength/specific gravity level greatly exceeds structural materials, except those for unidirectional graphite and boron fibres in an epoxy matrix.

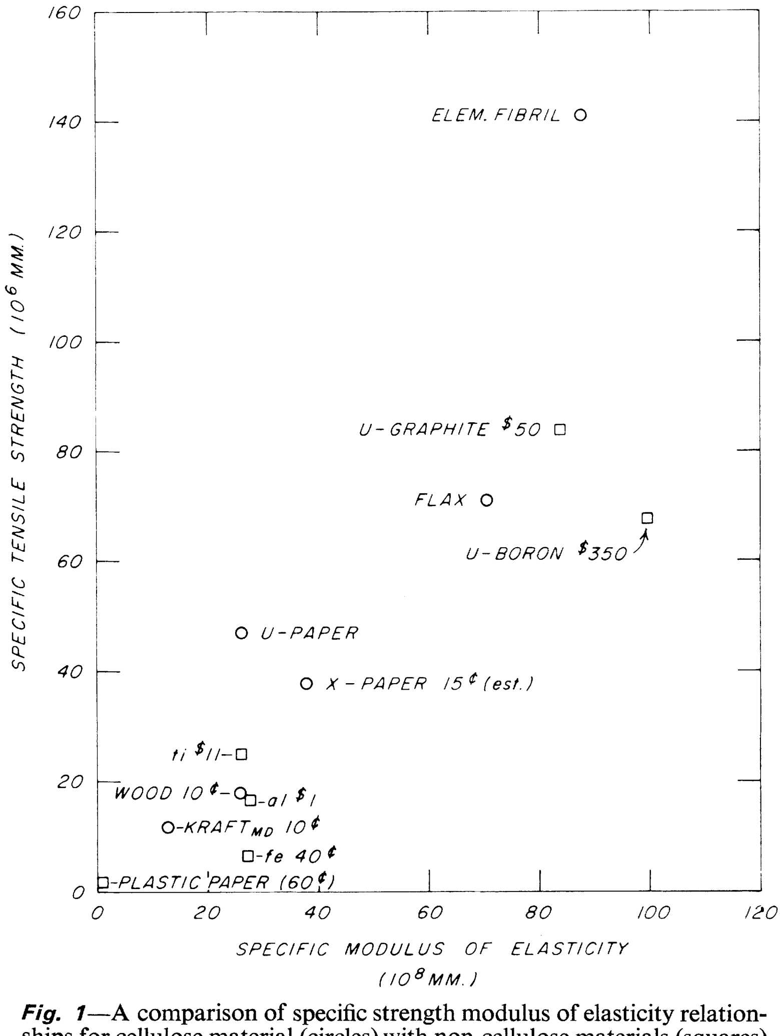

Fig. 1—A comparison of specific strength modulus of elasticity relationships for cellulose material (circles) with non-cellulose materials (squares)

Test results of the experimental directional paper samples (X-paper) indicate that our predictions for ultimate stiffness are possibly too conservative.

They indicate that we can make paper stiffer and stronger than the wood from which it was made. Quite logically we should expect this result because, as the lignin fraction is replaced with a more cellulosic fraction, we obtain an inherent improvement in the strength/stiffness to mass ratio. There must, of course, be sufficient interfibre bonding to utilise this increased fibre stiffness.

Values on Fig. 1 for flax were obtained from Mark's book.[4] He also published the illustrated estimate of specific stiffness and strength for elementary fibrils of cellulose.

Using the specific and elastic modulus of kraft paper (Fig. 1) as our current level of achievement, we anticipate our next challenge will be in the vicinity of X-paper. This level should be reasonably easy to achieve. The highest level will involve utilising the potential properties of the elementary fibril.

Over the years, wood has been increasingly divided into thinner or smaller pieces, which are then reconstituted into structural products such as laminated beams, plywood, particleboard and fibreboard. The motivation for this trend has been to achieve products of greater uniformity and versatility. Now the time has come to give more serious consideration to making structural building products from pulp fibres. The high stiffness-to-weight ratio and low cost of wood fibre are strong incentives for creating new products of wood and pulp fibre.

Quantitative assessment of stiffness

THIS paper is an effort to quantify stiffness in terms of its most important factors. Ultimately, we believe these factors should be unified so a more realistic appraisal can be made of the relative importance of each. With the exception of thickness, which we must treat first, we have tried to present these factors in order of importance.

Thickness

One of the most effective and direct means of changing paper stiffness is by changing thickness. Although this should be a straightforward matter, it has been clouded by the way we defined thickness and how we tried to measure stiffness. In this paper, we use the traditional engineering definition of the product of elastic modulus and moment of inertia or EI. Both E and I contain units for paper thickness; E is inversely proportional to changes in thickness and I varies as the cube of thickness.

Obviously, any error in thickness will greatly influence calculated bending stiffness. According to the present state of the art, such errors are inevitable. Thickness values as determined by standard methods, whether they be TAPPI, APPITA or SCAN, are going to be in error, because these methods cannot deal effectively with surface roughness. As these methods now stand, they result in values that are always too high when judged by standards presented here. This bias is the result of assuming that surface portion of the sheet is characteristically the same as the rest of the sheet.

Instead of defining thickness as the distance between two platens when paper is clamped under a specified pressure, it will have a more correct and useful value if we define it in terms of performance.

Specifically, thickness can be defined by the following equation—

$$t = \sqrt{\frac{12(EI)}{(EA)}} \quad \quad (1)$$

where EI is bending stiffness and EA is extensional stiffness.

This equation provides an estimate of thickness that we will call effective thickness, a definition based on performance and adherence to the principles of engineering mechanics. An important consideration in determining thickness by this procedure is that the fibre properties across the sample thickness should be relatively constant. Obviously, this method would not work as well with cylinder board as with pulp handsheets or single-ply Fourdrinier board. Test results showed that the possible presence of a density gradient was not a problem for the materials selected.

In addition to visual estimation of thickness, a seecial mercury pyknometer was constructed to obtain a value of thickness calculated from volume displacement. The pyknometer gave thickness values that were very close to the effective thickness values obtained from use of equation (*1*) and data obtained from bending and tension tests. Because of its simplicity and ease of operation, the mercury pyknometer was used as a reference standard for estimating the error in single sheet thickness with a TAPPI standard micrometer. Furthermore, the mercury displacement method is applicable to papers having non-uniform density.

Comparing standard thickness values with effective thickness really examines the amount of surface roughness. Consequently, the error in measurement of sheet thickness with a standard micrometer is always positive. In these tests, the magnitude of error expressed as a percentage of effective thickness was appreciable and significant. For handsheets of softwood kraft pulp (60 g/m^2), the error in single sheet thickness was 17 per cent. The same amount of surface roughness on a half weight sheet (30 g/m^2) produced errors in thickness of 33 per cent.

The range of error in thickness using a standard micrometer on a small assortment of papers and boards was generally 5–25 per cent. Certain very coarse corrugating media, however, will produce errors in thickness as high as 82 per cent.

Such errors as these will be cubed when *EI* is predicted. The net result is a tendency greatly to overestimate the potential stiffness of paper products.

On the other hand, if we accept the effective thickness as the true thickness of paper and board, there is no reason that we should not be able accurately to predict stiffness from the modulus of elasticity. There is also no necessity of avoiding use of terms for stiffness and strength that are related to paper thickness, as we do when we compare paper on the basis of their extensional stiffness. The greatest obstacle to the adoption of engineering methods for paper evaluation is overcome by using effective rather than standard thickness values.

Restraint during drying

Of all the factors that influence stiffness, control of restraint during drying seems the most practical tool available to the papermaker. Whereas increasing the specific gravity can have greater influence on stiffness, it appears more economical and efficient first to maximise stiffness through control of restraint during drying.

Restraint during drying is expressed as the linear change in dimension (as a percentage of the initial dimension) that a wet web is allowed to undergo as it is dried to equilibrium weight at 50 per cent relative humidity. This approach was adopted because wet-to-dry dimensions are easy to obtain. We have long suspected, however, that a more powerful measure of restraint during drying would involve simultaneous measurement of the shrinkage force that develops during drying. This would simplify data interpretation, for effects of restraint during drying can vary with fibre orientation and level of inter-fibre bonding. For example, increasing fibre orientation or the fibre bonding level will increase modulus of elasticity for each percentage change in wet-to-dry dimension of the test sample.

With various pulp furnishes, if specific gravity and fibre orientation are held constant, it is possible to achieve a threefold increase in modulus of elasticity.[5] Magnitude of the increase is greater at higher levels of fibre orientation; when the results are expressed as a percentage of the modulus of elasticity for webs that were dried unrestrained, the increase was a fairly constant 300 per cent.

This is illustrated in Fig. 2, which shows elastic modulus for handsheets made from a bleached softwood kraft pulp beaten in a Valley beater to a freeness of 450° CSF. The data are presented for five levels of fibre orientation, which is expressed as the cotangent of the average angle of fibre orientation relative to the test direction of the web.

The webs were dried in special drying frames that exerted a uniaxial stress in the test direction. Negative values of dimensional change indicate the amount of shrinkage that was allowed during drying. Positive numbers indicate the stretch that was applied to the wet webs before being dried. The starting point of each curve is the modulus of elasticity for completely unrestrained webs.

The advantage of controlling stiffness by restraint during drying is that large increases can be obtained without adding fibre. Not only stiffness, but strength is improved in the process. The only disadvantage is a loss in total stretch, which decreases in proportion to the increase in stiffness.

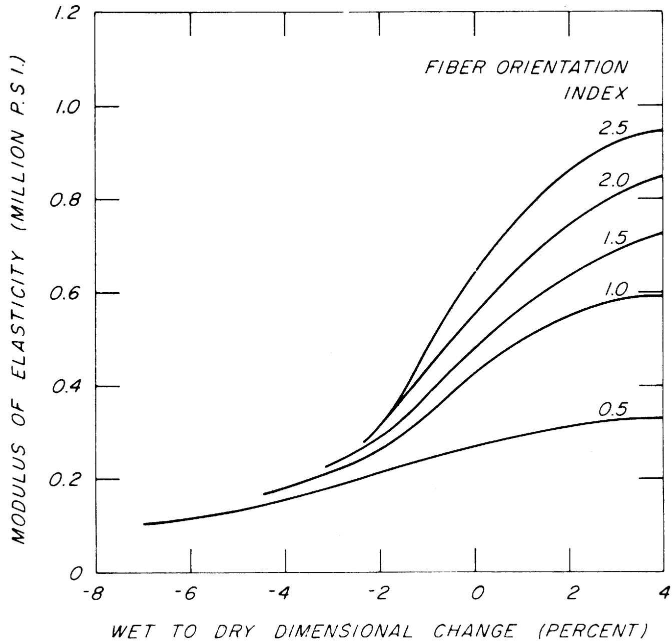

Fig. 2—Effect of restraint during drying on modulus of elasticity of kraft pulp handsheets at five levels of fibre orientation: negative values indicate shrinkage; positive values indicate stretch; point at beginning of each curve is unrestrained

Fibre orientation

Changing fibre orientation is another potentially inexpensive way of increasing the stiffness of fibre products. As things stand, while good control of fibre orientation has been obtained with handsheets,[6] most papermachines have limited control over fibre orientation. If orientation is to be fixed at some level, most product requirements seem to dictate a preference for less, not more fibre orientation. Thus, we have a natural bias away from high stiffness.

In our previous research,[1] we predicted that, at a specific gravity of 0·4, the maximum value for the elastic modulus might be $1{\cdot}5 \times 10^6$ lb/in^2 (Fig. 3). The value assumes that all fibres lie in the same direction and a high level of restraint is applied during drying. We are now estimating that this figure is closer to $2{\cdot}0 \times 10^6$ lb/in^2 (for this specific gravity) based on our exploratory investigations into high levels of fibre orientation. Research is underway to obtain data to broaden our experimental basis for this curve.

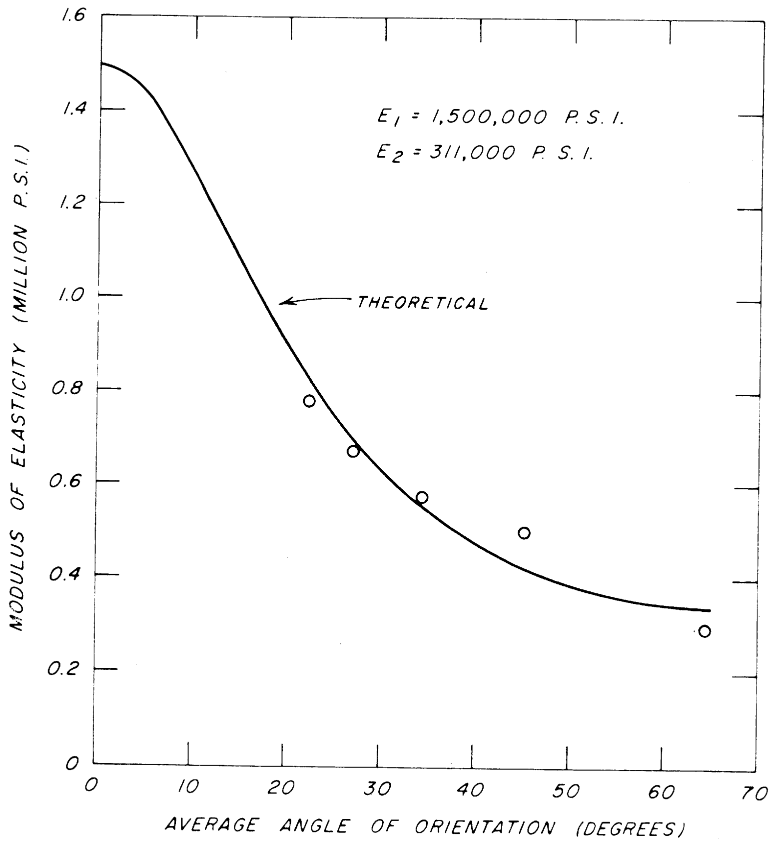

Fig. *3*—Effect of fibre orientation on modulus of elasticity of kraft pulp handsheets that were restrained from shrinking during drying

In general, it appears that we can expect a five to sevenfold increase in modulus of elasticity or stiffness over the total range of potential fibre orientation. Approximately half of this increase appears to occur in the last 20° of change as we approach the potentially most directional sheet.

Achievement of the high stiffness levels that are potentially available in woodpulp fibres is going to require more versatility than is available in paper-machines today. Chances are that, to be done, it will require a radically different head box design or methods of web forming.

Specific gravity

The relationship between specific gravity and stiffness turns out to be one of the more complicated variables. Even after the influence of thickness is dealt with, there remain such problems as moisture control at time of pressing and springback after pressing.

Even though control of specific gravity offers the greatest opportunity for large changes in stiffness, it is often one of the least desirable avenues for change because of material costs, loss of other properties and problems with sheet crushing.

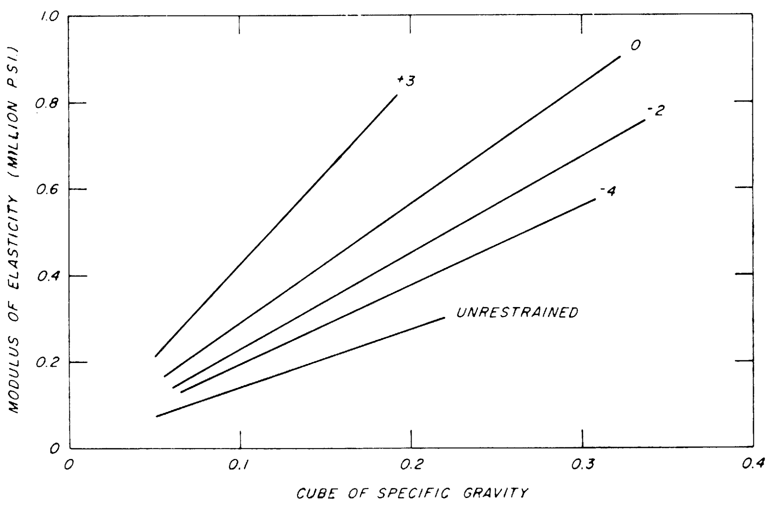

Fig. 4—Effect of specific gravity on modulus of elasticity of pulp handsheets: numbers indicate percentage linear change in dimension (positive, stretch; negative, shrinkage) that was allowed to occur during drying

Nevertheless, Fig. 4 is evidence of possible increases in modulus of elasticity as specific gravity of pulp handsheets changes with species (fibre morphology), pulping process and degree of beating.

These data suggest that stiffness of pulp handsheets is controlled by density and restraint during drying (orientation of fibres was not a factor, since fibre orientation was random). The data are for 15 different furnishes representing kraft, sulphite, semi-chemical and groundwood pulps. Both hardwood and softwood species are represented, as well as a wide range of refining in a laboratory beater. Curves are not based on data when the pressing conditions were varied; such a relationship would not hold if some samples were pressed when wet and others densified when dry.

The modulus of elasticity of handsheets can be estimated from specific gravity and drying restraint data alone, regardless of fibre morphology or differences in pulping processes. This relationship does not hold for strength properties. Strength is occasionally correlated with stiffness of lumber and other wood products, but it is an especially dangerous assumption to make about paper products.

Modulus of elasticity is shown in Fig. 4 to vary as the cube of specific gravity. Examination of the top line for webs that were stretched 3 per cent and held shows the large effect that changing specific gravity can be expected to have. In raising the specific gravity from 0·36 to 0·57 (0·047 to 0·185 on graph), we increase elastic modulus four times. Another example is that, at this level of restraint, a difference in specific gravity between 0·52 and 0·54 results in a 15 per cent difference in elastic modulus.

Some of the pitfalls to be avoided in looking at specific gravity as a fundamental variable have already been indicated. Chief among them is the moisture content at the time the sheet is compacted. For example, calendering dry paper can significantly increase specific gravity without changing the specific modulus of elasticity (elastic modulus divided by specific gravity). On the other hand, even small amounts of moisture increase the specific modulus of elasticity. In general, the specific modulus will increase, upon densification, with additional moisture up to about 40 per cent. After this level is reached, additional amounts are less important, although the relationship between expelled moisture, original fibre moisture control, springback and water returned from the press blotters will cause the modulus of elasticity to vary even at the same specific gravity.

Interfibre bonding

The problem with specifying specific gravity as a fundamental parameter has been a failure adequately to quantify interfibre bonding for each level investigated. A specific gravity/bonding index would be extremely useful for predicting stiffness as well as many other properties. As it stands, we know of no method available that measures interfibre bonding or that distinguishes between intrafibre and interfibre bonding.

Several years ago, we initiated research on shrinkage force during drying, hoping it would provide an index of interfibre bonding. Since this shrinkage force depends on interfibre linkages to transmit web consolidation forces, we hoped it would lead to an index of interfibre bonding. Research based on the measurement of web shrinkage force has been completed by Von L. Byrd of the U.S. Forest Products Laboratory and is soon to be published.

Web shrinkage was definitely superior to light scattering (specific surface area) as a tool for measuring bonding changes. Light scattering failed to measure interfibre bonding changes (Fig. 5), because it is more sensitive to changes in sheet grammage, fibre furnish, internal structure and surface modification than to actual changes in interfibre bonding. Web shrinkage is not limited by such restrictions.

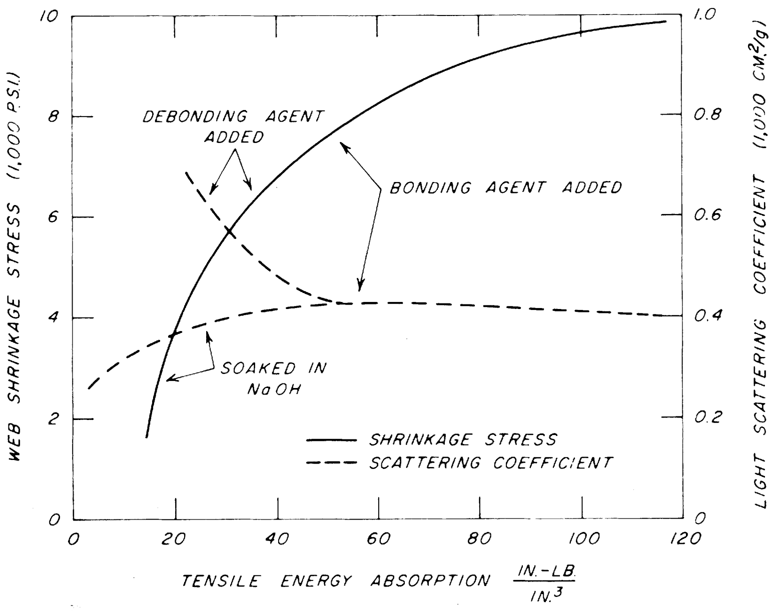

Fig. 5—Shrinkage stress and scattering coefficient for a variety of pulp handsheets shown as a function of tensile energy absorption: note the opposite effects of debonding agent and sodium hydroxide on light scattering coefficient

Actual interfibre bonding changes were calculated from simultaneous measurement of web shrinkage and moisture content, enabling detection of bonding changes during such manufacturing processes as web consolidation and pressing.

This technique is a measure of an intrinsic pulp property (calculation of actual hydrogen bond formation from simultaneous measurement of web shrinkage stress and water removal) and it is therefore not subject to the limitation of fibre type, brightness, sheet moisture content and sheet grammage suffered by indirect measurements such as specific surface area (light scattering). This technique is applicable to machine-made papers and offers promise of a dynamic interfibre bonding measurement during papermaking.

Moisture content

Before we can realise the full potential of pulp fibres, we need to know how to maintain their stiffness over long periods at high moisture levels. Although this is a major obstacle, we still need to know what performance we can expect without this protection.

Although considerable literature deals with the relationship of various strength properties and moisture contents, only a small amount of data exists on the relationship between elastic modulus and equilibrium moisture content.

The relationship between moisture and elastic modulus for a typical 42 lb softwood kraft linerboard is shown in Fig. 6. These data show a linear relationship between 4 and 14 per cent. moisture content for both machine and cross directions. Over this range, which is equivalent to increasing the relative humidity from 22 to 85 per cent, the modulus of elasticity was reduced by about 50 per cent in both principal directions of the linerboard.

Part of the reduction in stiffness by the softening of fibre bonds from humidity will not be regained in the redrying cycle. The permanent loss or stress relaxation effect of 5 days at 90 per cent rh is only 10–20 per cent of the increase obtained from restraint during drying. The really important and catastrophic losses occur only on complete soaking in water; even then, it is likely that small residual stiffness increases from restraint during drying will always remain with paper.

Temperature

The effect of temperature on elastic modulus is difficult to specify clearly because of the almost inextricable way in which moisture control and relative humidity are involved with temperature.[8] For example, as temperatures increase from 60° to 120° F at constant 50 per cent rh, elastic modulus decreases about 7 per cent. For these same conditions, however, the equilibrium moisture content (EMC) was reduced from 7·35 per cent to 6·05 per cent. Thus, the reduction in modulus of elasticity due to temperature must be adjusted for a 1·3 per cent change in moisture content, which by itself tends to raise the modulus of elasticity. The total or net change in elastic modulus due to temperature alone is linear with temperature and amounts to an average loss of about 18 per cent for the temperature change specified.

Long exposure to high temperatures (155° F) will substantially reduce tensile strength, but have no effect on the stiffness of paper.

Resin additives

The paper industry has successfully used various synthetic resins in maintaining the strength and integrity of fibre products that are expected to become wet while in use. Examples are towels, concrete forms, corrugated boxes, bags and filter papers, yet industry has not been equally successful in maintaining the stiffness of these products in either their dry or wet state.

Some resins modestly increase the stiffness of dry paper products, particularly when fibre bonding levels are low. If bonding in paper is already at a reasonable level, however, it is easier to improve stiffness by adding more fibre instead of resin.

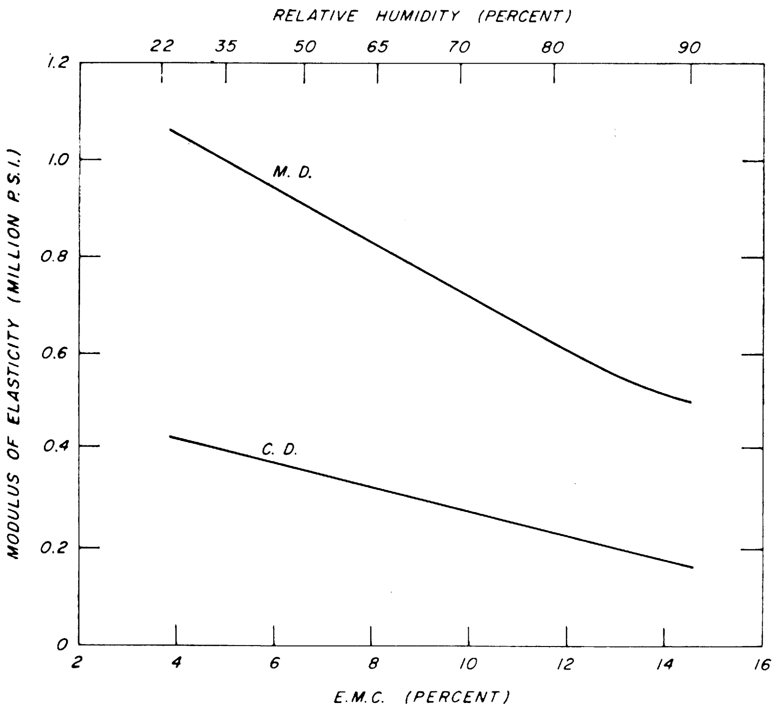

Fig. 6—Effect of moisture on modulus of elasticity of kraft linerboard in the machine and cross directions

Improving the stiffness of wet fibre products is even more difficult. One experiment,[9] using papers containing respectively 20 and 35 per cent alcohol-soluble phenolic resin, showed these papers reached moisture equilibrium within about 2 days when exposed to the range of moisture conditions shown in Fig. 6. The loss of modulus of elasticity with increasing EMC was about the same for resin-treated papers or for untreated paper.

This does not mean, however, that the resin had no effect: at the same relative humidity, the EMC of resin-treated paper was a few per cent lower than that of untreated paper. What these findings suggest is that resin additives do not increase the stiffness of wet paper, but act only to reduce the EMC; for equal increases in EMC, the losses in stiffness of paper will be the same.

Summary

THE utility of the majority of fibre products depends on their inherent stiffness. Efficiency in design demands an adequate understanding of the factors discussed, as well as their relative importance. For the papermaker, we suggest that bonding, restraint during drying and fibre orientation are the factors most useful in controlling stiffness.

In terms of effecting major changes, we rank in decreasing order of effect the variables of—(*1*) fibre orientation, (*2*) density/fibre bonding, (*3*) restraint during drying, (*4*) thickness, (*5*) moisture, (*6*) temperature and (*7*) chemical additives.

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Discussion

Dr J. E. Luce The stiffness increases that accompany extreme fibre orientations are very interesting, but one wonders at the usefulness of a paper with all its stiffness in one direction. What happens to the cross-direction stiffness when you obtain this extremely high machine-direction stiffness?

Mr V. C. Setterholm The properties of the paper in the other direction will be reduced the same as is shown in Fig. 3. I think that the uses are fairly obvious. We considered a structural sandwich, for example—that is why in part I emphasised pulp fibres. We have a potential here for new areas of development and our own and other laboratories are looking at structural applications.

The Chairman I have one question on the technique. Everyone agrees that it is extremely difficult to measure the thickness of paper in a proper way. You suggest calculating the thickness by dividing the bending stiffness by the extensional stiffness. Have you checked the correctness of this? For instance, when you measure the stiffness of a machine-made paper in both machine-direction and cross-direction, do you obtain the same thickness?

Mr Setterholm Yes, actually a paper on the subject is being published, part of it being presented at the last Gordon Research conference. We verified the method with a special mercury pyknometer as well as by examination under a microscope.

The Chairman Have you, by way of another example, given the paper mechanical conditioning? The thickness does not change much, but the bending thickness changes significantly, whereas the extensional stiffness remains more or less unchanged.

Under the chairmanship of Prof. H. W. Giertz

Mr Setterholm If EI or EA is changed by such treatment, it must be reflected in a thickness change. Obviously, by changing the properties of the paper by mechanical treatment, we then have a new material.

The Chairman If the treatment is as severe as you have indicated, I presume that you have broken some bonds and have thickened the sheet.