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MASS DISTRIBUTION AND SHEET PROPERTIES OF PAPER

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Synopsis Three terms that refer to the distribution of local grammage in the plane of a sheet of paper are introduced—the intensity, macroscale and microscale of mass distribution. The measurement of these quantities by an optical method and by beta radiography is discussed. Data are obtained by a specially designed microdensitometer and analysed by a frequency analyser. Results are presented as wavelength spectra of mass distribution.

Experimental spectra for handsheets and machine-made sheets are compared with theoretical spectra for sheets composed of fibres having randomly distributed orientations and positions. It is shown both theoretically and experimentally that it is important to use a small measuring area.

A poor mass distribution is demonstrated to have a negative influence on the scattering coefficient and opacity of paper. It is shown how the mass distribution and other paper properties affect the visual appearance of multi-ply board and the look-through and print-through of paper. The concept of mass distribution is extended to the distribution of ink on paper. The connections between the mass distribution of ink and the prints are discussed.

Finally, the adverse effect of a poor mass distribution on the strength properties of handsheets and machine-made sheets is demonstrated using recent (partly unpublished) experimental data by Cavlin & Rudström.

EVALUATION OF MASS DISTRIBUTION

DEFINITIONS

Introduction

UNEVENNESS is a characteristic of many materials, not least of paper. One of the chief aims of the papermaker is to optimise the uniformity of the sheet. A sheet with increased small-scale grammage uniformity will usually be stronger and possess better optical and printing properties; also a better retention of fines and additives would be achieved, as well as a lower consumption of coating slurry to obtain a certain level of quality.

Under the chairmanship of Dr H. Corte

Although the central problems involved in achieving a high degree of homogeneity have been dealt with extensively in the literature, there is a great deal of confusion regarding the techniques for the analysis and description of this property.

In the past, the only way of estimating the degree of homogeneity achieved has been to look at the light transmitted through the sheet of paper. In this way, the trained papermaker could distinguish between a 'good', 'fair' and 'wild' formation. Sometimes, other parameters were also assessed subjectively. A well-closed sheet would normally have superior properties.

Many instruments have been designed to emulate the ability of the papermaker to judge a sheet by transmitted light. Reasonably good correlations have been achieved between subjective and instrumental rankings of the look-through of various grades of paper.

It is a well-known fact, however, that the measured rankings and correlations hold only for one particular grade. The correlations are completely upset when, for instance, a newsprint sheet is compared with an unbleached kraft bag sheet.

The basic property used in this work for describing the unevenness of paper is the grammage and its distribution in the plane of the sheet: uneven distribution is the main cause of an uneven look-through. The look-through is influenced also by the optical properties of the paper, notably by the opacity (this is discussed in a later section), by the directionality and colour of the light and by the properties of any backing material that may be used.

The objective property to be used is thus the local grammage or local 'mass density' as it is termed by Corte.⁽¹⁾ The mass distribution is thus the basic property that is the subject of this work. The look-through of paper and how it is influenced by mass distribution and other paper properties is discussed in the section on visual perception (page 51).

In the past, the term formation has been used to signify the mass distribution, as well as look-through or the actual forming process. In this work, it is used as a general term in accordance with the ISO definition—'*Formation*: The manner in which the fibres are distributed, disposed and intermixed to constitute the paper.'

Since paper is produced from a suspension of fibres that tend to form flocs, the term flocculation is inevitably associated with the process of paper forming. Flocculation refers to the local variations of concentration in a fibre suspension, not to the evenness of the finished sheet. In the authors' opinion, however, it is also correct to refer to dense or 'heavy' areas in a sheet as flocs.

Mass distribution

THE variation of the local grammage w in a sheet of paper is stochastic. A

measure of the intensity of the variations is the standard deviation $\sigma(w)$. The standard deviation is a particularly convenient measure of the intensity when electrical signals are used, because it is equal to the RMS value of the signal, a quantity that can be measured directly. Sometimes, it is preferable—or even necessary—to normalise the standard deviation by dividing it by the mean grammage \overline{w} . This ratio is usually termed the coefficient of variation of the local grammage V(w). In this work, it is called the intensity of the mass distribution (or just mass distribution) and is denoted by the symbol F—

Power spectra

For work on the correlation between mass distribution and sheet properties, it is necessary to have a more comprehensive description of the variations than that afforded by a single number.

Such a description is provided by power spectra; a method long used in turbulence research.⁽³⁾ Some of the mathematical basis of power spectra is given below. One of the main reasons for performing this seemingly complex type of analysis is that it permits a simple visualisation of the data.



Fig. 1—Lumps of pulp of different size and weight in a paper sheet

In turbulence, the velocity field is considered to consist of a mean flow with a number of superimposed eddies of different geometrical sizes and intensities. In the same manner, a sheet can be considered to be composed of a number of lumps of pulp (flocs) of different size and weight (Fig. 1).

A power spectrum describes the contribution to the variance from different frequency intervals (Fig. 2). All phase information is lost in a power spectrum.

This means that two variations of the same frequency give the same contribution to the spectrum irrespective of their phase relationship. In the present case, this means that a floc of a certain size and intensity will give the same contribution to the spectrum irrespective of its position in the sheet.



Fig. 2—The frequency power spectrum

The frequency spectral density P(n) can be integrated to give the variance—

The spectral density P(n) can be calculated directly from a signal with a normal distribution of amplitude⁽⁴⁾—

where w' is the deviation of the local grammage from the mean and T is the measuring time or from its autocorrelation function $C(\tau)$ —

$$P(n) = 4 \int_{O}^{\infty} C(\tau) \cos (2\pi n\tau) \cdot d\tau \qquad (4)$$

In equations (3) and (4), P(n) is defined for positive frequencies only. This requires that P(n) = P(-n), which is the case for all physical processes considered.

An extensive description of the power spectrum and its properties has been given by Norman & Wahren.⁽⁵⁾

Wavelength spectra

The interpretation of the spectral distribution of mass density is made easier if the wavelength is chosen as the independent variable instead of the frequency. By defining the wavelength l as—

a spectrum can be obtained that is independent of the scanning speed \bar{u} . This definition implies that for a sine curve of frequency n, l is the distance between two consecutive peaks. For a floc in a finished sheet, this means that its wavelength is equal to twice its geometrical size. For a stochastic signal (with normal amplitude distribution), however, there is no correlation between wavelength and floc separation, which has been suggested by Burkhard, Wrist & Mounce.⁽⁶⁾

Their results were based on measurements on model signals, the amplitude distributions of which were not approximately Gaussian. Hence, their results were not based on a material suitable for this type of analysis.

In practice, it is found that the signals derived from measurements on paper are stochastic in nature and that their amplitude distribution is always sufficiently close to Gaussian to make spectral analysis a valid tool.



Fig. 3-Frequency and wavelength spectra

The wavelength spectral density E(l) may be calculated from the frequency spectral density P(n), using the condition that the variance (or power) within a

given wavelength range should be equal to that within the corresponding frequency range (Fig. 3).

It follows that-

This transformation has not been performed for the results obtained with previous formation meters. Normally, the RMS values from the individual filters are presented without even considering the different band widths. Such procedures may be acceptable for the comparison of results. Yet, when quantitative evaluation of the results is required for correlation with other sheet and suspension parameters, the results may become very confusing.

According to equation (2), the square of the intensity of the mass distribution F^2 is equal to the area under the spectral density curve. By introducing a variable upper limit of integration, the characteristic curve of the mass distribution F(l) can be defined as—

The symbol F previously used for mass distribution is thus merely a simplified way of writing the limit $\lim F(l)$.

The characteristic curve indicates that the mass distribution F may be progressively built up from the contributions from flocs of increasing size. The way in which the characteristic curve is obtained from the wavelength spectrum is shown in Fig. 4.



Fig. 4—Relation between the wavelength spectrum and the characteristic curve

The total variation in grammage for wavelengths smaller than l may be read directly from the characteristic curve. If the total variation within the wavelength interval l_1 to l_2 is required, the following relation may be used—

If there are any *periodic* components in the signal, they will show up as discrete pulses or peaks superimposed on the power spectrum. An example is the spectrum for a sheet with wire mark. In this case, the wavelength corresponding to the superimposed peak is equal to the distance between the wire marks.

Scales

IN TURBULENCE research, a microscale λ and a macroscale L are defined. These are a measure of the average and the largest eddies, respectively, that occur in a turbulent flow field. These parameters apply equally well to the variation of mass in a sheet of paper. They provide a condensed description of the shape of the spectrum.

$$\frac{1}{\lambda^2} = -\frac{1}{2F^2\bar{u}^2} \cdot \left[\frac{\mathrm{d}^2 \mathbf{C}(\tau)}{\mathrm{d}\tau^2}\right]_{\tau=0} = \frac{2\pi^2}{F} \cdot \int_{0}^{\infty} \frac{\mathbf{E}(l)}{l^2} \mathrm{d}l = \frac{\pi^2 \cdot N_o^2}{2\bar{u}^2} \quad . \tag{9}$$

The term N_{\circ} is the number of times the grammage signal crosses its mean per unit time⁽⁴²⁾ and can be evaluated by a simple electronic device.

$$L = \frac{\overline{u}}{F^2} \int_{O}^{\infty} \mathbf{C}(\tau) \mathrm{d}\tau = \frac{1}{4F^2} \lim_{l \to \infty} l^2 \cdot \mathbf{E}(l) \quad . \qquad . \qquad (10)$$

INSTRUMENTAL EVALUATION OF SPECTRA

Analogue signal

The power spectrum may be obtained by frequency analysis of an analogue signal that is proportional to the local grammage. All components that do not have a frequency within the selected frequency band may be filtered out electronically by a frequency analyser. The width Δn of the frequency band may be either constant over the whole frequency range or proportional to the centre frequency of the band.

The signals considered in the analysis of the mass distribution in sheets of paper are stochastic and cover a large frequency range. In this case, a satisfactory resolution is obtained with a reasonable number of filters if the band width is taken to be proportional to the centre frequency. Fortunately, this method is the easiest and fastest in practice. The band width Δn is then given by the relationship—

For $B = \frac{1}{3}$ (that is, one third octave), b = 4.32. This band width Δn (thus also the ratio between two consecutive centre frequencies n) is within 0.3 per cent of 2 dB. In practice, a band width of 2 dB is chosen, since each decade of frequency is then covered by ten equal steps. The slight adjustment in scales is theoretically insignificant and just as practical as Bach's 'Das wohltemperierte Klavier'.

The RMS value of the signal passing through the filter is measured. If the RMS value is V(n) volts, the power within the frequency band is $V^2(n)$ and the relative power per unit frequency P(n) is given by the expression—

where \overline{V} is the mean value of the signal. The spectral distribution within the whole frequency range is obtained by repeating the measurement for a number of centre frequencies.

The error in the observed value depends on the band width Δn and the analysis time T according to—

where ϵ is the relative standard error.

The observed frequency spectrum may be transformed into a wavelength spectrum by means of equations (6), (11) and (12)—

$$E(\overline{u}/n) = \frac{bn}{\overline{u}} \left[\frac{V(n)}{\overline{V}} \right]^2 \qquad . \qquad . \qquad . \qquad (14)$$

As mentioned earlier, when using filters with Δn equal to one third octave, the ratio between two consecutive centre frequencies corresponds to 2 dB. This implies that an increase in amplification of 1 dB per filter, plus a shift of the entire spectrum, will perform the transformation from frequency to wavelength spectrum.

The characteristic F(l), defined in equation (7), may also be calculated directly from the values $V(n_i)$ obtained in the frequency analysis—that is, without first calculating the wavelength spectrum.

$$\mathbf{F}(\overline{u}/n_k) = \sqrt{\sum_{i=k}^{l_{\max}} \{\mathbf{V}(n_i)/\overline{V}\}^2} \qquad . \qquad . \qquad (15)$$

Mass distribution and sheet properties

Digital signal

Most frequency analysers cannot be used for determining spectra at very low frequencies, because their lower frequency limit is usually a few Hertz. An alternative method for determining the frequency spectrum is by numerical calculation from a digital grammage signal. Digital recording is necessary when the measuring methods have long time constants—for example, grammage measurements with a beta-gauge. The grammage is recorded at equidistant points and stored on tape. All further processing is done by computer. The power spectrum is calculated from equation (3), using for instance the fast Fourier transform (FFT).⁽⁷⁾ It can be calculated also from the autocorrelation function using equation (4).

MASS DISTRIBUTION IN RANDOM SHEETS

WHEN the wavelength power spectrum of a sheet of paper is evaluated, a number of factors have to be considered. Some of these are—

- 1. How well formed is the sheet compared with some standard sheet?
- 2. How does the size of the measuring area affect the spectrum?
- 3. How does anisotropic fibre orientation affect the spectrum when the sheet is scanned in different directions?

The random sheet (that is, a sheet consisting of randomly scattered fibres) is a suitable standard on which to base comparisons. The spectra of random sheets can be evaluated theoretically. The problem has been treated by Corte,⁽⁸⁾ but he calculated only the variance, not the power spectrum.

Isotropic sheets

THE method of calculating the wavelength power spectrum of random sheets is based on the general theory of two-dimensional spectral analysis of stochastic processes.⁽⁹⁾ The effect of a finite measuring area is taken into account and the two-dimensional spectrum is transformed to give the physically realisable one-dimensional spectrum for the chosen scanning direction. Finally, this spectrum is transformed from the mathematically convenient wave number spectrum into the more useful wavelength spectrum.

The method is generally applicable. It is first applied to an isotropic random sheet, consisting of randomly distributed rectangular fibres of length L and width B (Fig. 5).

The equal weight of each fibre is assumed to be evenly distributed over the rectangular area. The grammage at any point is therefore directly proportional to the number of fibres covering that point. The distribution of fibres is random with respect to the position of the fibre centres, as well as to the

orientation of the fibres. The random sheet is scanned along a straight line using a circular measuring area of arbitrary diameter ϕ .



Fig. 5-Randomly distributed rectangular fibres

The analysis results in equation (16), which describes an isotropic spectrum —that is, a spectrum independent of the scanning direction. The expression is rendered dimensionless by expressing all lengths relative to the fibre width B.

$$E\left(\frac{l}{B}\right) = \frac{1}{n_m} \cdot \frac{4}{\pi^2} \cdot \frac{L}{B} \left(\frac{B}{l}\right)^2 \cdot \frac{1}{2} \cdot \frac{1}{M} \left(\frac{y \cdot \frac{B}{B}}{y \cdot \frac{B}{B}}\right)^2 \cdot \frac{1}{M} \left(\frac{y \cdot \frac{L}{B} \cdot \cos t}{y \cdot \frac{B}{B} \cdot \cos t}\right)^2 \cdot \left[\frac{\sin \left(y \cdot \sin t\right)}{y \cdot \sin t}\right]^2 \cdot dt$$

$$\int_{\pi \frac{B}{l}} \frac{1}{\pi \frac{1}{N}} \cdot \frac{1}{M} \left(\frac{1}{N} \cdot \frac{B}{N}\right)^2 \cdot \frac{1}{N} \cdot \frac{1}{N$$

The value n_m is the mean number of fibres covering a point and J_1 is a first order Bessel function.

It may be seen that the spectral density at a given wavelength is inversely proportional to n_m . This makes the transformation to different grammages very simple. If the wavelength spectrum is plotted on a diagram with logarithmic scales, a change in grammage requires only a vertical shift of the entire spectrum. This fact also facilitates the computation of spectra for fibres of non-uniform dimensions.

According to equation (2), the intensity of mass distribution F may be calculated by integrating the expression for the spectral density from zero to infinite wavelength. In the idealised case of an infinitesimally small measuring

area, the triple integral in equation (16) can be evaluated analytically. The result is $F = 1/\sqrt{n_m}$. The same result can be arrived at, rather heuristically, by considering the number of fibres at a point to have a Poisson distribution.



In general, equation (16) has to be numerically evaluated (Fig. 6). This figure shows the spectra for different values of the ratio of the measuring diameter ϕ to the fibre width B.



When ϕ/B is equal to zero, the 'real' spectral density of the sheet is obtained. Increasing the size of the measuring area is seen to reduce the apparent spectral density rapidly at small wavelengths and in such a way that the maxima of the curves occur at a wavelength that is about twice the diameter of the measuring area (compare with Fig. 30). The corresponding characteristic curves on the right show that, even at the relatively small measuring diameter of about four fibre widths (about 100 microns), only half the actual intensity of the mass distribution is recorded.

The influence of the length to width ratio of the fibres is shown in Fig. 7. Since the mean number of fibres covering a point n_m will be unchanged if the fibres are divided into an arbitrary number of parts, this figure can also be used to estimate the effect of changing the fibre length by cutting.

The four sheets represented in Fig. 7 have the same actual intensity of mass distribution, since n_m is unchanged, but the mass distribution recorded with a finite measuring area decreases as the length to width ratio decreases.

Anisotropic sheets

SUPPOSE that the fibre orientation is not random, but follows some given distribution. Any angular distribution of the fibres can be used, but, in order to simplify the calculations, the distribution shown in Fig. 8 is assumed.



The final result consists of two terms, the isotropic term in equation (16) and a correction term proportional to the anisotropy parameter e. Typical results are shown in Fig. 9.

It is clear that the spectra for the machine and cross directions cross one another; also that both the 'real' and the recorded intensities of mass distribution are independent of the scanning direction. Therefore nothing may be concluded about fibre orientation from the mass distribution values alone.



METHODS OF SENSING GRAMMAGE

THERE are three principal methods of sensing the local grammage—

1. Weighing 2. Absorption of light 3. Absorption of beta-rays

These will now be discussed in some detail.

Weighing

THIS method has not been used extensively to determine mass distribution. The main problem is to cut the test pieces to a given size with sufficient accuracy. To achieve an accuracy of 4 per cent in the local grammage, test samples 1 mm square would have to be cut to an accuracy of 20 μ m edge length—that is, the width of a single fibre. It is therefore a useful method only when evaluating the mass distribution for larger measuring areas. It is possible to calculate the power spectrum from the measured values (see section *Digital signal*, p. 15), but it is impracticable, because this requires a very large number of measurements and is not used.

Light absorption

THE most common method of determining formation is to measure the variations in the light transmitted through the sheet.^(6, 10-21)

Two instruments have been used extensively for laboratory measurements —the Thwing-Albert meter⁽¹⁰⁾ and the QNS/M meter.⁽⁶⁾

The latter has been used in studies of sheetforming,⁽²²⁾ kraft paper formation,⁽²³⁾ evaluation of a multi-grade head $box^{(24)}$ and wire mark in newsprint.^(25, 26) A few instruments are designed for on-line measurement, ^(13, 17, 19, 20) but these have not yet been widely accepted.

The instruments are generally claimed to give an objective value of the look-through, but in no case has the effect of the basic optical properties of the sheet been investigated thoroughly.

Basic equations

When formation is measured by the light transmission method, it is necessary to consider the basic equations relating the optical properties of a sheet. (27, 28)

Kubelka⁽²⁸⁾ derived the following basic equation relating the transmitted light intensity I to the incident light intensity I_0 and the optical properties of a sheet—

$$A = \ln \frac{1}{\overline{T}} = \ln \frac{I_0}{\overline{I}} = \ln \left(\frac{a}{\overline{b}} \sinh bsw + \cosh bsw \right) \quad . \qquad (17)$$

where $A = \ln\left(\frac{1}{T}\right)$, absorbance (normally absorbance is defined as ${}^{10}\log\frac{1}{T}$, but to get simpler formulas $\log\frac{1}{T}$ is used throughout this paper)

 $a = \frac{1}{2}(1/R_{\infty} + R_{\infty})$

$$b = \frac{1}{2}(1/R_{\infty}-R_{\infty})$$

 R_{∞} = the reflectivity of the sheet

s = the scattering coefficient of the sheet

For this equation to be valid, the incident light must be diffuse. Directional incident light is diffused as it passes through the paper. Hence, the absorbance of directional light is very high for thin, light-scattering materials such as lightweight paper. As the grammage is progressively increased, the absorbance of directional incident light approximates more and more to that of diffuse light. In theory, diffuse detection is not imperative. As the transmitted light is perfectly diffuse, even directional detection will give correct transmission values. In practice, however, it is found that, owing to the surface geometry, the exiting light is not diffuse when measured for very small surface elements such as are used in the measurement of formation by light transmission. In conclusion then, care should be taken to use perfectly diffuse incident light and as diffuse a light detection principle as possible. All the formation meters mentioned above^(6, 9-20) employ more or less directional incident light and directional detection.

It should also be remembered that light transmission measurements can be interpreted in terms of grammage only if the fibres possess relatively uniform light absorption characteristics. It is not usually possible therefore to evaluate the distribution of mass density by optical methods in sheets composed of more than one component. Examples are sheets composed of two entirely different pulps, sheets that contain fillers and coated papers. Since papers are usually made from mixtures of different pulps, it would appear that beta-ray absorption should be used in place of optical techniques. It will be demonstrated later (Fig. 33), however, that results using the optical technique are not influenced significantly by mixing different pulps, even when the components differ greatly as for newsprint furnish. In this case, the optical technique outlined below is generally useful. When fillers or coatings have been used, the beta-ray absorption technique produces more quantitatively correct results.



Equation (17) is shown graphically in Fig. 10. It is seen that there is a linear relationship between the absorbance and the grammage (w of bsw) only when $R_{\infty} = 0$, that is, only for perfectly black sheets. This is unsatisfactory, because it means that the method of assessing the relative variations in grammage will necessarily be complex in all practical cases. A method of starting at a high value of bsw seems attractive, since all curves in Fig. 10 have the slope 1 for large values of bsw. A proportional relationship would be achieved, see the transformed origin in Fig. 10.

A practical way of attaining the desired transformation is to put the sheet in contact with a backing material, through which the incident light is transmitted. If the reflectance of the backing material is R_b , as measured from the side in contact with the paper, it can be shown that the ratio of the intensity of the transmitted light to the intensity of the incident light (measured off the backing material without paper) is changed from that given by equation (17) into—

$$A = \ln\left(\frac{a - R_b}{b} \sinh bsw + \cosh bsw\right) \qquad . \qquad . \qquad (18)$$

This formula is illustrated in Fig. 11.



Fig. 11—Graphical representation of equation (18)

In this case, there is a proportional relationship between bsw and $\ln\left(\frac{1}{T}\right)$ if $\frac{a-R_b}{b} = 1$, that is, if $R_b = R_\infty$. This means that, if the sheet is backed by some material with the same reflectance as the reflectivity of the sheet itself, a proportional relationship will be achieved and equation (18) will reduce to—

It follows that-

$$\frac{\mathrm{d}A}{A} = \frac{\mathrm{d}(bsw)}{bsw} \qquad . \qquad . \qquad . \qquad . \qquad (20)$$

If bs is assumed to be constant, then---

In other words, the coefficient of variation of grammage (the intensity of the mass distribution) is equal to the coefficient of variation of absorbance, if the parameter *bs* is assumed to be constant.

Influence of scattering coefficient

Local variations in scattering coefficient—Owing to a variable pressure distribution during pressing and calendering, the fibre bonding (therefore the local scattering coefficient) also changes. Modification of the distribution of fines also causes scattering variations.

As seen from equation (17), variations in scattering coefficient cannot be separated from grammage variations by optical methods. This is why beta-ray absorption has to be used when large variations in the local scattering coefficient are expected (see section *Beta-ray absorption*, p. 29). The influence of local variations in the scattering coefficient on the absorbance may be evaluated as follows. It follows from Fig. 11 that, provided the difference between the reflectance of the background and the reflectivity of the sheet is small, a proportional relationship as in equation (19) is a good approximation. Differentiating equation (19) yields—



Fig. 12—The weighting function φ_0 in equation (22)

The weighting function ϕ_0 is shown in Fig. 12. It may be seen that the influence of local variations in the scattering and absorption coefficients are

included in the measured signal, but that their contribution is small when the reflectivity of the paper is low.

The above analysis is approximate, since any difference between the reflectivity of the sheet and the reflectance of the background is not accounted for. For large variations in scattering coefficient, the case can be treated using

equation (18) and introducing $a = 1 + \frac{s}{k}$, $b = \sqrt{2\left(\frac{k}{s} + \frac{k}{s}\right)}$ and $sw = \frac{s}{k} \cdot kw$.

The transformed equation is shown graphically in Fig. 13.



Since different values of k/s give straight lines through the origin, a weighting function ϕ [compare ϕ_0 in equation (22)] defined as—

is given by the slope of the lines. In Fig. 14, ϕ is shown as a function of the reflectivity of the sheet and the relative deviation in the scattering coefficient. For small $\Delta s/s$ values, ϕ is equal to ϕ_0 in Fig. 12.

It is obvious from the above (Fig. 12 & 14) that the lower the R_{∞} (higher k/s), the lower is the sensitivity to variations in the scattering coefficient. In order to minimise the influence of variations in the scattering coefficient on the measured mass distribution, the colour of the transmitted light should be chosen to give the maximum k/s. For uncoloured sheets, this will generally occur in the blue regime, as the absorption coefficient of cellulose increases at

decreasing wavelengths. If the local variations in grammage and scattering coefficient are independent of one another, the following relationship can be deduced from equations (19) and (23)—

$$V^{2}(A) = V^{2}(w) + \varphi^{2} \cdot V^{2}(s) = F^{2} + \varphi^{2} \cdot V^{2}(s) . \qquad (24)$$

That is, the coefficient of variation of the absorbance is equal to the coefficient of variation of the grammage plus a term that is approximately proportional to the coefficient of variation of the scattering coefficient.

It may be expected that the local pressure depends on the local grammage during wet pressing. Therefore it is not obvious that the local variations in the grammage and the scattering coefficient can be assumed to be independent. On the contrary, investigation into these matters might be a profitable area for research into the structure of paper. For this case, it is found that—

$$\mathbf{V}^{\mathbf{2}}(\mathbf{A}) = [\mathbf{V}^{\mathbf{2}}(w) + \varphi^{\mathbf{2}} \cdot \mathbf{V}^{\mathbf{2}}(s) + \varphi^{\mathbf{2}} \cdot \mathbf{V}^{\mathbf{2}}(\mathbf{A}) \cdot \mathbf{V}^{\mathbf{2}}(s)(1 + \varphi^{\mathbf{2}}) + 2\varphi \cdot \varphi \cdot \mathbf{V}(\mathbf{A}) \cdot \mathbf{V}(s)]/[1 + \varphi \cdot \varphi \cdot \mathbf{V}(\mathbf{A}) \cdot \mathbf{V}(s)^{\mathbf{2}}] .$$
 (25)

Here, ρ is the coefficient of correlation between the local grammage and the scattering coefficient.

The influence of the pressure during wet pressing on the scattering coefficient of laboratory sheets has been investigated. A special arrangement allowed the pressure to be increased well above the standard for sheetmaking. The scattering coefficient was calculated from two reflectance measurements.

Pressure, bar	0	5.6	25.8	52.3
Bleached sulphite, 19° sR	0	0	-4.3	-18.8
Bleached sulphite, 49° sR	0	-22.4	-34.0	-44.6
Unbleached kraft, 20° sR	0	-11.9	-24.6	-33.6
Unbleached kraft, 48° sr	0	7.8	-23.9	-32.4

TABLE 1—INFLUENCE OF THE PRESSURE DURING WET PRESSING ON THE SCATTERING COEFFICIENT Changes in the scattering coefficient are expressed as percentages

Groundwood is much less sensitive than chemical pulp in the range investigated. It is a well known fact, however, that the scattering coefficient decreases drastically during calendering at a critical pressure level (calender blackening).

Influence of the sheet thickness on the attainable resolution—It was mentioned earlier that light is scattered during its passage through a sheet and the influence of this on the linearity of the measurement was discussed. It was pointed out that it was necessary to use diffuse light. Even so, the successive scattering of light limits the resolution that can be achieved in this type of measurement. The limited resolution causes errors in the measurement of small-scale variations. The thicker the sheet and the higher the scattering coefficient, the larger are the errors.

Fig. 15—Light diffusion photographed in a microscopeLeft—An 0.2 mm diameter light beamRight—Light beam after passing through newsprint

An experiment has been performed to estimate the magnitude of this effect. Incident light was arranged as a 0.2 mm diameter parallel light beam and the light transmitted through the sheet was observed through a microscope. Photographs were taken (Fig. 15) and analysed in a microdensitometer. The results are presented in Fig. 16 as the relative enlargement of the light beam.

The area of the transmitted light beam has been arbitrarily defined as the area with a light intensity greater than half the centre intensity.

Results for groundwood, sulphite and kraft pulps are shown in Fig. 16. Light diffusion seems to be proportional to grammage for the chemical pulps, whereas the trend is less clear for groundwood pulps.

Fig. 31 shows the influence of the sheet thickness on the wavelength spectrum for newsprint samples.

Decreasing the scattering coefficient by immersing the sheet in a liquid with an index of refraction close to that of cellulose increases the measuring resolution considerably. Even water (index 1.33 compared with 1.5 for cellulose) has a considerable effect (see Fig. 17 & 18). The diameter ratio for the wet sulphite sample has been plotted in Fig. 16. It falls on the line for unbleached kraft.

Measurements such as those discussed above lead to the conclusion that the attainable resolution is limited to approximately the thickness of the sheet when measurements are made on normal untreated papers. Hence, if sheets of entirely different grammages and/or thicknesses are to be compared, the size of the measuring aperture should be chosen to correspond to the thickness of the thickest sheet.

Fig. 18—Influence of water on light diffusion in 110 g/m² bleached sulphite (19° sR)

Influence of the surface structure

The structure of the sheet surface has been shown to influence the wavelength spectrum considerably at small wavelengths. This is mainly due to nondiffuse detection (see section *Microdensitometer*, p. 33). The basic equation (17) assumes a homogeneous sheet with an even surface, in which case detection of transmitted light may be directional. In a practical case, however, the uneven surface causes local variations, which are poorly recorded if diffuse detection is used. A practical way of accomplishing diffuse detection is to put a thin scattering layer on top of the sheet.

Beta-ray absorption

As mentioned in section *Light absorption* (p. 19), there are serious limitations to the use of light transmission for the evaluation of mass distribution in sheets that have components with different optical characteristics or sheets with local variations in the scattering coefficient. Both these disadvantages are avoided when beta-rays are used. The wavelength of beta-radiation is so large that the rays are not scattered when they are transmitted through a sheet. The absorption of beta-rays depends only on the total mass, not on the composition of this mass.

Three useful radiation sources are listed in Table 2.

Isotope	Energy, maximum, keV	Penetration maximum, g/m²	Half-life years
Krypton-85	700	2 250	10
Promethium-147	200	400	4.5
Carbon-14	150	240	5 500

TABLE 2—BETA-RAY SOURCES

In ordinary equipment for grammage measurement, Kr-85 is a useful source of radiation. Relatively large measuring areas have to be used in order to get sufficient statistical accuracy in on-line measurements. A measuring area of 25 cm^2 is a normal value. This is many orders of magnitude larger than the area required for mass distribution measurements. Therefore, mass distribution measurements have to be performed on stationary or very slowly moving webs.^(29, 30)

In both these investigations,^(29, 30) promethium-147 was used and the measuring area was 1 mm². The long measuring times that were required as a result of the low scanning speeds made calculation of power spectra impractical.

Beta-radiography

Instead of measuring local beta-ray transmission in a large number of small areas, a beta-radiograph can be made. A source with a uniform strength over its area is placed in contact with the sheet to be tested and the transmitted radiation is recorded with a roentgen film. In principle, the resolution of the method therefore depends on the resolution of the microdensitometer used to analyse the variations in the local transmission of the roentgen film. Several investigators have examined small-scale grammage variations using betaradiographs.⁽³¹⁻³⁴⁾ Since only one exposure is required, it is not essential to have a very strong source, so far as time consumption is concerned. C-14 is a suitable isotope and a source that uses it may be obtained from The Radiochemical Centre.⁽³⁵⁾ The source consists of C-14 embedded in a clear plastic plate that is 1 mm thick and a few hundred cm² in area. Safety precautions need not be too rigid, even if the maximum strength obtainable is used.

In this investigation,⁽³⁶⁾ the size of the source was 100 mm \times 150 mm and the strength was 8.4 mCi. It was mounted in the bottom of a light-sealed box, one side of which could be opened to allow the test sheet and the film to be inserted. The space above the source was restricted to slightly more than 130 mm \times 180 mm, which was the size of the standard roentgen film that was used. The film and the test sheet were pressed tightly against the source by a stiff plate actuated by a mechanical spring. The plate was covered with one sheet of foam rubber and one thin rubber sheet to give an even pressure distribution.

Fig. 19-Radiograph of newsprint

The test sheet was inserted in a frame made from a double-folded plastic sheet. Along one side a number of calibration strips made from cellulose film and aluminium foil with known grammages were arranged to enable the relationship between grammage and film absorbance to be determined for each individual exposure. Fig. 19 shows a beta-radiograph with calibration strips. The radiographs were examined in the scanning microdensitometer, described under *Measuring equipment* (p. 33).

Relationship between grammage and film absorbance—Radioactive decay is a statistical process, but the radiation for a large number of particles can be considered as a continuous wave with a constant amplitude and energy flux. For thin samples—

$$T = e^{-\mu w}$$
 (26)

where T is transmission factor,

 μ is absorption coefficient.

This is true so long as the approximations are valid.

In practice, the absorption of the radiation follows the above expression until the grammage reaches 3 or 4 times the half thickness. At higher grammages, the rate of absorption increases so that the maximum range is about 7 or 8 times the half thickness.

The half thickness is a function of the energy of the incident radiation and, for C-14, it is about 30 g/m². Therefore, the limit of exponential absorption may be expected to be between 90 g/m² and 120 g/m². The maximum range should be 210–240 g/m².

The measured quantity is the absorbance $A = \ln\left(\frac{1}{T}\right)$ of the developed film. So long as the linear range of the film is used, the film absorbance is proportional to the amount of radiation transmitted by the sample, thus to the exposure time t and the transmission factor T. Therefore, the relationship between the variables is—

$$A = c \cdot t \cdot e^{-\mu w} + A_{\infty} \qquad . \qquad . \qquad . \qquad (27)$$

which can be written as-

$$\ln(A - A_{\infty}) = \ln ct - \mu w$$
 . . . (28)

Here, A_{∞} is the absorbance of the unexposed parts of the film—that is, for infinitely high grammage. It can be determined on an unexposed part of the film.

Equation (28) has been experimentally verified (Fig. 20). From this plot, c and μ can be evaluated—

$$c = 3.6 \text{ h}^{-1}$$

 $\mu = 0.032 \text{ m}^2/\text{g} = 320 \text{ cm}^2/\text{g}$

The data in Fig. 20 have been replotted in Fig. 21 to give the exposure time required to achieve the absorbance levels 2, 3 and 4, respectively, for varying grammages.

Fig. 20—Film absorbance as a function of grammage and exposure time

Fig. 21—Required exposure time as a function of grammage for different absorbances

MEASURING EQUIPMENT

Microdensitometer

THE following specific requirements necessitated the development of a special microdensitometer-

- 1. Diffuse incident light for transmission measurements.
- 2. The reflectance of the diffuse light source should be able to be matched with the reflectivity of the test sheet.
- 3. High light intensity to allow measurements on heavy sheets.
- 4. Accurate focusing should be possible over the entire sheet.
- 5. The size of the measuring area and the spectral response should be easy to change.
- 6. The scanning speeds should be compatible with the standard frequency analysers.
- 7. Scanning in a defined direction of the sheet.
- 8. Measurement on any sheet size smaller than the maximum.
- 9. Signal conditioning with simple calibration.
- 10. The use of the instrument would be extended if it could measure with reflected light.

The mechanical design of the microdensitometer is shown in Fig. 22. The sheet to be examined is placed between a thin translucent plastic sheet and the outside of the teflon cylinder 2. The latter has a diameter of 100 mm and a length of 200 mm. The cylinder is illuminated from the inside by the lamp 9. The teflon material diffuses the illumination and also has the advantages that it can be machined to a cylindrical shape and can withstand the heat from the lamp.

The sheet surface is projected by the lens 5 on the aperture 4. The aperture 4 determines the shape of the measuring area, which is normally circular and 0.1 mm in diameter.

A filter is place on top of the aperture to select the desired wavelength range. The filter and the aperture are easy to change. The light transmitted through the aperture is recorded by the photomultiplier 3. The cylinder is rotated with a peripheral speed of 1 m/s. The measuring equipment and the lamp are mechanically connected and are moved sideways with a speed of 1 cm/s by means of the rotating screw and its driving belt 1. The sheet is therefore scanned along a spiral path, which has a slope of 1 in 100 to the direction of rotation. This gives a distance of about 3 mm between the scanning lines. The axial motion is restricted by two adjustable limit switches. Compressed air is supplied through the tube 11 and blown towards the lamp from the nozzle 10.

The track-hold unit 7 is included so that measurement ceases at sheet joints. Therefore, sheets need not cover the entire periphery. A lamp and a photodiode are placed on each side of the gap of the housing 7. The translucent ring 8 is fastened to the cylinder and rotates in the gap. A black tape applied to the ring cuts off the illumination of the diode when it passes through the gap and thus activates the hold state. The signal freezes at the value it has when the light is cut off. The maximum size of the sheet that can be examined is restricted by the dimensions of the cylinder to about 310 mm \times 180 mm. Any sheet smaller than these dimensions can be examined by manipulation of the length of the tape governing the hold period.

Fig. 22-The microdensitometer: mechanical arrangement

It is impossible to combine a large aperture with a great depth of field using ordinary optics. The depth of field is required to allow for the deviations from concentricity in the cylinder (see Fig. 32 for influence of focusing). The problem has been temporarily solved by using standard lenses and putting a scattering layer on top of the sheet to be examined. Ideally, this layer scatters all light leaving the measuring area, independent of its original direction, so that a certain percentage enters the lens system. This facilitates the use of optics that have a sufficient depth of field. The complete arrangement is shown in Fig. 23.

The measuring diameter normally used for the mass distribution measurements is 0.1 mm. A smaller scanning area would mean that insufficient light is supplied to the photomultiplier when measurements are made on sheets with a high absorbance. The noise from the photomultiplier would then obscure the variations of the grammage signal at small wavelengths. Another limitation to the minimum size of the measuring area is that the diameter should be larger than the sheet thickness if the light absorbance is to represent the grammage within the measuring area alone (see section on the influence of sheet thickness on the attainable resolution, p. 26).

Fig. 23-The microdensitometer: optical arrangement

An evenly exposed photographic film is inserted between the sample and the cylinder so that the right reflectance value from the teflon cylinder is achieved. This film decreases the reflectance by a factor of T^2 , where T is the transmission of the exposed film.

On-line measurements

Optics similar to those in Fig. 23 have been used for on-line measurements. The running web is then in contact with the measuring side of the optics. Focus will therefore always be on the sheet surface, independent of the sheet thickness. A scattering layer cannot be used, as it would be worn by the running web. Light collection with fibre optics that have a large aperture may give a more diffuse measurement.

The optical background to the running web can be a small plate made from a diffuse material with a suitable reflectance.

Signal conditioning

Light absorbance

THE relationship between light absorbance and grammage was derived earlier-

$$A = 1m(I_0/I) = bsw$$
 (19)

The absorbance signal is obtained by subtracting the logarithm of the photomultiplier current from a reference signal. The reference signal is set equal to $\ln I_o$ by zeroing the difference when measuring on the uncovered

Fig. 24—Signal conditioning for light transmission measurements

cylinder—that is, when the grammage equals zero (see Fig. 24). The absorbance signal is then passed through the track-hold unit, which compensates for the gap between the sheet ends. The signal is normalised with respect to its mean value using—

which is a good approximation for small relative variations. This normalising method can be applied only to signals that are proportional to the quantity under consideration.

The output signal is fed directly to the frequency analyser and the mass distribution spectrum is measured without further calibration.

Beta-radiographs

The relationship between radiograph absorbance and grammage was derived on page 31 (grammage and film absorbance)—

$$\mu w = \ln \operatorname{ct} - \ln(A - A_{\infty})$$
 (30)

The signal conditioning is shown in Fig. 25. The first electronic units, including the track-hold unit, are identical with those used for light transmission analysis (see previous section).

Fig. 25—Signal conditioning for beta-radiography measurements

The signal from the first differential amplifier is zeroed on an unexposed part of the radiograph (with absorbance A_{∞}) by adjusting the reference. This makes the signal to the track-hold unit equal to $A - A_{\infty}$. This signal is passed through a logarithmic amplifier, after which, according to equation (30), the relationship between grammage and signal can be represented by a straight line. Adjusting the second reference to give zero output from the second differential amplifier on a fully exposed part of the radiograph (grammage zero) will shift the straight line vertically to pass through the origin. The normalising unit in Fig. 24 can then be used.

An exposure time for the radiographs of 1.5 h will give proportionality between signal and grammage up to about 100 g/m², using the calibration method above. This upper limit was predicted above (p. 31) considering the half thickness. Using the microdensitometer described in the microdensitometer section, longer exposure times will give absorbances at zero grammage so high as to cause deviations from the linear relationship.

For high grammages, longer exposure times and individual calibration is required. This is done as illustrated in Fig. 26. The output voltage e is measured for the three calibration steps closest to the mean grammage \overline{w} of the test sample. The intercept Δe is used for normalisation of the data according to—

Fig. 26-Calibration of beta-radiographs

Evaluation and presentation of wavelength spectra

THE output signal is fed to an automatic frequency analyser, General Radio 1921 Real-Time Analyzer, where it is filtered through 45 $\frac{1}{3}$ -octave filters connected in parallel and covering the frequency range 3–80 000 Hz. For each channel, the attenuation is set equal to ± 25 dB in 1 dB steps. This makes the transformation from frequency to wavelength spectrum very simple (see section *Analogue signal*, p. 13). The result is presented on a storage oscilloscope and punched on tape (Fig. 27). For further processing, a tape reader transfers the data to a Hewlett Packard 9100 B calculator with plotter.

Fig. 27—Equipment to measure and record wavelength spectra

Standard diagrams are mounted in the plotter and the wavelength spectrum and the characteristic curve are drawn automatically (Fig. 28). The logarithmic scales used in the diagrams are chosen so that the modulus of the characteristic is twice the modulus of the spectral density. This means that a transformation of the results from grammage w_0 to pw_0 , which will require a 1/p

Fig. 28—Standard diagram for plotting wavelength spectrum and characteristic curve: results for sample in Fig. 19

shift of the spectrum, at the same time shifts the characteristic curve $\sqrt{1/p}$ as required by equation (7). The following variables are listed in the table in the upper right corner—

- a. Mass distribution F (per cent) taken from the end value of the characteristic curve.
- b. Microscale λ (mm) automatically calculated by the calculator according to equation (9).
- c. Macroscale L (mm) calculated using the asymptotic value c of the spectrum.

$$L = c/F^2$$
 (per cent) (32)

Compare this with equation (10).

d. Transmission factor T (for light transmission measurements only), calculated from the absorbance measured after the differential amplifier (Fig. 24).

EXPERIMENTAL RESULTS

Light transmission

SOME of the limitations discussed earlier have been investigated experimentally. Measurements on sheet samples of widely differing size result in the same wavelength spectrum, but the statistical errors are larger the smaller the sample size (Fig. 29).

Fig. 29—Measurements on newsprint samples of widely differing size

Fig. 30—Influence of measuring area on the recorded wavelength spectrum

The influence of the diameter of the measuring area on wavelength spectrum for random sheets was calculated above (Fig. 6). Fig 30 shows the results of measurements made with four different measuring areas. The observation that the wavelength at the maximum of the spectrum is twice the diameter of the measuring area seems to cover also real sheets.

The influence of sheet thickness on the resolution is demonstrated in Fig. 31. Measurements were made on 1, 2, 3 and 4 newsprint samples, respectively and the results normalised to a grammage of 50 g/m^2 (see section on wavelength spectra). The results show that the same resolution is obtained for wavelengths above 10 mm (5 mm floc size); for smaller values, the resolution decreases with increasing sheet thickness.

The influence of focusing is demonstrated (Fig. 32). Measurements were performed with the optics focused on the sheet surface and with deviations of 0.4, 0.9 and 1.2 mm, respectively. Like sheet thickness, focusing affects the record spectrum at small wavelengths. When an 0.1 mm diameter measuring area is used, correct focusing is obtained by adjusting the distance between the detecting system and the sheet surface until the maximum output from the filter corresponding to a wavelength of 0.2 mm is achieved.

Fig. 31—Influence of sheet thickness on resolution (newsprint samples)

Fig. 32—Influence of focusing on resolution: figures on curves indicate deviation between plane of focus and sheet surface

Fig. 33 shows a comparison between light transmission and beta-radiography measurements of mass distribution on a newsprint sample. In the light transmission measurement, the background was chosen to give the right reflectance

Fig. 33—Comparison between light transmission and beta-radiography

Fig. 34—Wavelength spectra for various surface roughnesses: linear scales for spectral density⁽³⁷⁾

value and a blue filter was used to obtain a maximum value of k/s. The light transmission measurement is more sensitive to surface roughness than the radiography measurement. The wire mark therefore shows up more distinctly in the light transmission spectrum when the measurement is performed on the wire side.

Light reflection

THE equipment for reflectrometric measurements (item 10, p. 33, Fig. 23)

Fig. 35—Light reflection from creped papers with different impact angles⁽³⁸⁾

was used in two investigations for the characterisation of the structure of sheet surfaces.

Söderström⁽³⁷⁾ compared the coefficient of variation of reflected light to the results obtained with a number of commercial surface roughness testers. There was a strong correlation with the profile depth as measured with a Hommel Perth-0-Meter, in which a diamond stylus follows the surface. The correlation with surface roughness measured by the Bendtsen method was not so good.

The advantage of characterising the surface by variations in light reflection is that a comprehensive description of the surface structure is obtained from frequency analysis of the reflected light signal. A measure of the geometrical size of surface disturbances is obtained. Results are shown in Fig. 34 for four different papers.

Hollmark⁽³⁸⁾ analysed the structure of crepe papers. Among other things, the sizing and the impact angle of the doctor were varied. The effects of such variations on the creping process were characterised by examination of the papers by light reflection. A few results are shown in Fig. 35.

Beta-radiographs

THE influence of sheet thickness on the resolution of beta-radiographic measurements is illustrated in Fig. 36. Radiographs were made of two newsprint samples; first individually, then with the two sheets placed together.

Fig. 36—Beta-radiography measurements for one and two newsprint samples, respectively

After normalising to a common grammage, it is found that the resolution in the latter case is not significantly worse than that for the single measurements. The resolution is therefore much less influenced by sheet thickness when using beta-radiographs than when using the light transmission technique.

The surface roughness of the test samples, however, has a considerable effect on the resolution: this is demonstrated in Fig. 37. One beta-radiograph was obtained for an uncalendered newsprint sample; another was obtained after calendering. The more uniform mass distribution that might be expected in the small floc size range after calendering was obscured by the fact that *the more even sheet surface resulted in a much more detailed radiograph*. The unevenness at small floc sizes even appeared to increase with calendering because of this effect.

Random sheets versus real sheets

HANDSHEETS were carefully formed from a fractionated pulp with comparatively uniform fibre dimensions.⁽³⁹⁾ Mass distribution spectra were measured on radiographs of these handsheets. In Fig. 38, the spectrum for a 39 g/m² sheet made of bleached sulphite pulp is compared with the spectrum

 $n_m = 4.40$ L/B = 44 $B = 21 \ \mu m$

for the corresponding random sheet (calculated from equation (16) with $n_m = 4.4$, $B = 21 \ \mu m$, L/B = 44 and $\phi/B = 4.7$). The sheet thickness was 87 μ m before calendering; after calendering, it was 50 μ m. This has a strong influence on the resolution, as discussed above. It is obvious, however, that an even greater decrease in sheet thickness could not raise the spectrum to the level of the random sheet, as it is not significantly changed above 10 mm wavelength. It is obvious that the real handsheet is superior to the random sheet within the whole wavelength range investigated. This is not surprising, considering the way in which the sheets were formed—a well dispersed, highly diluted fibre suspension is drained evenly through a wire. The fibres may be assumed to be randomly distributed in the suspension. During the drainage, local thin spots in the accumulating fibre mat will have a lower than average resistance to flow, causing an increased flow in this area and a corresponding increase in fibre accumulation. This would cause a more even mass distribution in the formed sheet than in the random sheet. This was suggested by Wrist⁽⁴⁰⁾ without experimental and theoretical support as early as the 1961 Oxford symposium.

Fig. 39—Sheet with data as in Fig. 38 Left—Handsheet Right—Random sheet

The effect may be ascribed partly to local deformations of the fibres during the consolidation process. Using the terminology of Norman,⁽⁴⁷⁾ this causes a good 'forming consolidation'.

A radiograph of the handsheet and the corresponding random sheet are compared in Fig. 39. The latter was drawn by a computer, displayed on an oscilloscope screen and photographed. The random sheet, which contains the same number of fibres as the handsheet, has many more holes. This may explain the differences in spectral densities (Fig. 38). Fig. 40 shows a 20 g/m², dry laid non-woven sample, which has a strong fibre orientation. The term $n_m = 2.0$ and the mean fibre width $B = 20 \ \mu m$. The wavelength spectra for the machine and the cross directions are plotted in Fig. 41, together with the corresponding spectra for random sheets. Maximum fibre orientation (maximum value of the anisotropy parameter *e*) has been chosen for the random sheet and the distribution of fibre orientation is shown in Fig. 8. The non-woven sheet is more uneven than the random sheet over the whole wavelength range both in the machine and the cross directions. This is to be expected, since the non-woven sheet has not been formed in a drainage process like a handsheet.

INFLUENCE OF MASS DISTRIBUTION ON SHEET PROPERTIES THE MASS DISTRIBUTION AND THE OPTICAL PROPERTIES OF PAPER AND BOARD

BECAUSE of the non-linear relations between grammage and transmittance and between grammage and reflectance, an uneven distribution of grammage leads to an increase in the transmittance and a decrease in the reflectance. The extent of the influence can be evaluated numerically using the Kubelka-Munk equations and a suitable distribution function for the mass distribution. If the average transmittance and reflectance of the uneven sheet are designated by \overline{T} and \overline{R} , respectively, it is found that—

$$\overline{T} = \int_{O}^{\infty} \frac{b}{a \sinh bsw + b \cosh bsw} \varphi(w) dw \quad . \qquad . \qquad (33)$$

$$\bar{R}_{\circ} = \int_{O}^{\infty} \frac{\sinh bsw}{a \sinh bsw + b \cosh bsw} \varphi(w) dw \quad . \qquad . \qquad . \qquad (34)$$

A suitable distribution function for the mass distribution $\phi(w)$ is the Poisson distribution. Numerical evaluation by a computer is then possible, provided the integrations are replaced by summations.

In order to give a qualitative impression of the nature of the influence of mass distribution on the optical properties of paper, a simplified case is treated below. It is assumed that the transmittance is given by the simple formula derived for a special case in the basic equations section, (p. 20).

$$T = e^{-bsw}$$
 (19)

This equation holds also when the grammage is measured by beta-ray absorption (if *bs* is put equal to the absorption coefficient). Consequently, the following also applies to errors in the measurement of grammage by beta-ray absorption.

Fig. 40—Anisotropic non-woven sample: grammage 20 g/m², $n_m = 2.0, B = \mu m$

Fig. 41—Wavelength spectra for anisotropic non-woven sample (Fig. 40) and corresponding random sheet with e = 1.0

The application of the Poisson distribution to the variation in grammage leads to-

This expression may be transformed and expanded into the series-

$$\ln(\overline{T}/T) = \frac{F^2}{2}(\ln T)^2 + \frac{F^4(\ln T)^3}{6} + \dots \qquad (36)$$

The series converges very rapidly. The second term (and subsequent evennumbered terms) is negative. The first term is identical to the result obtained if a Gaussian distribution is assumed instead of a Poisson distribution. The difference between the results obtained using these two distributions is very small so long as the intensity of the mass distribution is not too large.

If the above series is truncated after one term and the scattering coefficient is evaluated from one transmission (absorbance) and one reflectivity measurement, the following expression for the relative decrease in the effective scattering coefficient is obtained—

Since $bsw = A_{\infty}$ (that is, the absorbance of the paper when it is in contact with a backing material of the same reflectance as the reflectivity of the paper), it is seen that the relative importance of a good mass distribution increases with the light absorbance of the paper. The term F^2 , however, is quite small for well-formed papers. For example, newsprint has an absorbance value slightly greater than unity and an estimated total intensity of mass distribution of some 35 per cent (including all wavelengths down to the wavelength of light, but only as extrapolated from measured wavelength spectra). This leads to a decrease of only 6 per cent in the effective scattering coefficient. Owing to the use of the simple form of the transmittance formula, the above result is too low, but it yields the desired approximate relationship. It may be concluded that the influence of mass distribution on the *average* optical properties of paper is quite small, except in extreme cases. At very low grammages, the influence may be considerable. This may account for some apparent failures to verify the Kubelka-Munk equations experimentally.

A poor mass distribution affects the visual appearance more when a dark surface is covered by a paper or a layer of fibres. An example is multi-ply board with a waste paper centre. This case may be treated using the Kubelka-Munk formula—

$$bsw = Arctgh \frac{a-R}{b} - Arctgh \frac{a-R_g}{b}$$
 . . . (38)

where R_g is the reflectance of the background (that is, the waste paper centre) and R is the reflectance of the composite. In order to simplify the final expression, the parameter—

$$a_R = \frac{1}{2} \left(\frac{1}{R} + R \right)$$
 (39)

is introduced. The coefficient of variation of reflectance is assumed roughly to correspond to the visual perception of unevenness (see following section, p. 51) and may then be calculated from—

Accordingly, the coefficient of variation of reflectance from the surface is proportional to the intensity of the mass distribution (F) of the top layer. It is also a function of the optical properties of the material in the top layer and of the backing.

Fig. 42—The predicted visual impression of unevenness at any given formation of the top layer of a multi-layer board represented by the parameter ξ , shown as a function of the scattering power *sw* of the top layer and as a function of the reflection factor of the product: the reflectivities of the materials in the top and bottom layers are indicated by the figures in the numerator and denominator, respectively

The implications of the formula may be illustrated by the following example. The parameter $\xi = V(R)/F$ is a measure of the extent to which a certain unevenness in the mass distribution is amplified by the optical properties of the top layer and of the backing (or centre layer of the multi-layer board).

This parameter has been calculated for two boards having centre layers with reflectance factors of 0.4 and 0.5, respectively and covered by top layers having reflectivities of 0.8 and 0.9, respectively and various average grammages. The results are shown in Fig. 42 & 43. Fig. 42 shows the apparent visual unevenness (the parameter ξ) as a function of the scattering power *sw* of the top layer, also as a function of the average reflectance of the composite. For a given mass distribution intensity *F*, there is a certain grammage at which the perception of non-uniformity of reflectance of the composite is a maximum, in spite of the fact that the reflectance factor (luminance factor) of the surface increases smoothly with increasing grammage of the top layer.

FIg. 43—The predicted visual impression of unevenness at any given formation of the top layer of multi-layer board (this is the same data as shown in Fig. 42, taking into consideration the decrease in F value with increasing grammage)

It was shown in the section on isotropic sheets (p. 15) that the intensity of the mass distribution normally decreases with increasing grammage. If it is assumed that F is inversely proportional to the square root of the average grammage of the top layer, the previously shown relationships are shifted somewhat, as shown in Fig. 43. The parameter ξ/\sqrt{sw} represents the visual perception of unevenness, taking into account the variation of F with grammage.

Both diagrams show that, provided the flocculation tendency of both top layer pulps is the same, the brighter pulp gives a less even appearance of the product when the two are compared at an equal reflectance factor (luminance factor or brightness). The differences are very small, however, especially at low grammages, which is the region of particular interest. The diagrams also show that an increase in the reflectance factor of the backing (that is, covering the centre layers with a layer of brighter pulp) greatly reduces the impression of non-uniformity of the final product. This method is of course the one normally used.

SOME THOUGHTS ON THE VISUAL PERCEPTION OF UNEVENNESS OF PAPERS

This section concerns the subjective judgment of the unevenness of papers (that is, the look-through) and how an objective measure of this property may be obtained, but these ideas have not been verified experimentally to the extent that any degree of reliability can be stated. They are presented here only as possible methods of obtaining some desired 'subjective' measures, as these are in all probability closely related to the kind of information contained in the wavelength spectra of the mass distribution.

An uneven distribution of grammage in a sheet of paper can be observed by transmitted light. It then appears as a cloudiness, which is called a poor look-through. There are many unknown factors of human perception involved in the measurement of look-through. A number of investigators (see section on light absorption method of sensing grammage, p. 19) have related instrumental evaluations of look-through to subjective rankings of lookthrough by panels and obtained some satisfactory correlations. It is not the purpose of the present paper to review or expand on this type of approach. Instead, the discussion will be limited to discussing a number of factors that should be considered when trying to relate the distribution of mass density to the look-through.

The eye is sensitive to light and light intensity variations and to colour, but not to grammage. Therefore, two papers having the same intensity and spectra of mass distribution, but different colour and other optical properties (such as opacity) will be perceived differently.

It is not known how different colours influence the perception of nonuniformity. It seems safe to assume, however, that if agreement between an instrumental method and subjective assessment is to be obtained the *spectral distribution of the light* to be used for the measurement should be given by an incandescent lamp and a luminance filter—that is, the green or Y-filter used in conventional reflectometers. This filter has characteristics that approximate the sensitivity of the human eye-brain combination.

It was pointed out previously that in the measurement of mass distribution by light transmission, perfectly *diffuse incident light* should be used. This condition also applies when judging the look-through of paper sheets. If directional light is used, such as when examining a sheet of paper by transmitted light from one distant lamp, then the sensitivity to very thin areas in the sheet becomes very pronounced. In certain cases (for example, when the sheet is examined for pinholes), this technique can be useful; when trying to form an all-over subjective rating of the look-through of a sheet, diffuse incident light should be used. This greatly increases the repeatability of the assessment and also provides a basis for identifying the properties of the paper that influence the relationship between mass distribution and look-through.

The optical geometry on the measuring side of the paper specimen should of course be similar to that of the human eye. This means that the measurements should be performed with a *small spatial angle*. When only one 'subjective' quantity is to be measured the *size* of the measuring aperture should probably be related to the resolution of the human eye.

It may be safely assumed that, for a given shape of the mass distribution spectrum, the perception of unevenness increases monotonously with the F value. (The relationship need not be linear of course.) Furthermore, the judgment of look-through should not be influenced appreciably by the mean intensity of the incident light so long as it is chosen within reasonable limits. The eye adapts to the average luminance of the inspected surface, at least if the latter covers the whole field of view or is the only source of light. This reasoning leads to the conclusion that the eye perceives only the deviations of the intensity of the transmitted light from the average intensity. Therefore, a convenient measure of the intensity of perception of unevenness is the coefficient of variation of the transmitted light intensity—

The same result can be arrived at in a different way. The human eye and brain respond to light and light intensity variations in a very complex way. The well-known Weber-Fechner's law states that the response is proportional to the logarithm of light intensity. A simple measure of the 'subjective' intensity of light variations should therefore be the standard deviation or RMS value of the logarithm of the light intensity—

Therefore, this approach also leads to the conclusion that the coefficient of variation of transmitted light intensity is an important measure of 'subjective unevenness'. Equation (42) holds only for moderate variations of light intensity.

All *formation meters* presented in the literature (pp. 19–20) have relied on the above measure or on closely related ones—for example, the rectified average value has sometimes been used instead of the RMS value. The normalisation of the RMS value with respect to the mean has often been accomplished by automatic standardisation of the transmitted light intensity or a related parameter. Authors have then shown the validity of this quantity as a measure of look-through. It is therefore natural to conclude that the coefficient of variation of the intensity of the transmitted light bears a strong relationship to the look-through of papers.

The following discussion refers to the *influence of other paper properties*, including mass distribution, on the coefficient of variation of the intensity of transmitted light. The influence can be calculated from the Kubelka-Munk equations, starting, for instance, from equation (17) of this paper. The coefficient of variation of transmitted light is found to be—

$$V(I) = F \cdot \frac{1}{2} \frac{1 + R_{\infty}^2 - 2\Omega R_{\infty}^2}{1 - R_{\infty}^2} \ln \frac{1 - \Omega R_{\infty}^2}{1 - \Omega} = F \cdot bsw \left[1 + \frac{2R_{\infty}^2}{e^{2bsw} - R_{\infty}^2} \right].$$
(43)

Here, Ω is the opacity (= R_0/R_∞) and R_∞ is the luminance factor of the paper. The formula is shown graphically in Fig. 44. For higher values of opacity than shown in the diagram, the coefficient of variation asymptotically approaches the value—

$$V(I) = F \cdot bsw = Fw\sqrt{k^2 + 2ks}$$
 (44)

If the paper is placed against a background having the same reflectance as the reflectivity of the sheet, the same formula holds.

Fig. 44—Ratio of coefficient of variation of transmitted light and intensity of mass distribution as a function of sheet opacity and reflectivity

Impressions of unevenness (or at least readings from formation meters) are therefore proportional to F and to some function of the paper properties (approximately the opacity). The higher the opacity, the worse is the look-through when the distribution of mass density is the same.

A further contribution towards a better 'subjective' measure of the visual perception of unevenness of mottled surfaces was proposed by Olle Andersson⁽¹⁵⁾ in connection with work on the look-through of paper. He hypothesised that the visual perception of unevenness of a sheet of paper observed by transmitted light was proportional to the RMS value of the spatial derivative of the logarithm of the light intensity—

He verified this hypothesis by asking a number of people to judge the unevenness of a series of papers by pair comparisons. A similar expression, the rectified average value of the spatial derivative of the reflectance variations, was proposed and used by Poulter⁽⁴¹⁾ as a measure for the mottle of solid prints.

Since it seems possible that measures such as these may offer advantages in obtaining better 'subjective' measures, some further study is required. For this purpose, the more elaborate and more tractable expression proposed by Andersson is chosen. Suppose the electrical signal e is proportional to the intensity of the transmitted light—

$$e = k_1 I = k_1 I = k_1 T I_0$$
 . . . (46)

The unevenness given by Andersson's expression then becomes-

$$u = \text{RMS}\left[\frac{d(\log I)}{dx}\right] = \text{RMS}\left[\frac{d(\log e)}{dx}\right] \quad . \quad . \quad (47)$$

It is known from the theory of stochastic processes⁽⁴²⁾ that the RMS value of the derivative of a stochastic signal is closely related to the intensity and micro scale λ of the signal, so that the above expression becomes—

It was stated in the section isotropic sheets that the size of the measuring aperture determines the position along the wavelength axis of the maximum of the wavelength spectrum of the mass distribution of random sheets. In fact, the micro scale of the measured spectrum becomes nearly equal to the diameter of the measuring aperture. The above formula then simplifies to—

where ϕ is the diameter of the measuring aperture. The formula clearly

indicates that the size of the measuring aperture is a most important parameter. It also shows that, for well-formed sheets, the above hypothesis of Andersson, as well as the one proposed by Poulter, should rank the sheets in the same order as the simpler coefficient of variation of transmitted light.

The above statement is not necessarily true for real sheets. Our measurements show that in most commercial sheets the micro scale is often appreciably (often several times) larger than twice the size of our measuring aperture, which has a diameter of 0.1 mm. Hence, Andersson's (and probably also Poulter's) measures may in some cases grade papers better than the coefficient of variation of transmittance alone.

This can be so, however, only if the size of the measuring aperture is sufficiently small.

For more detailed and significant information on the perception of unevenness, reference should be made to a paper by J. Merchant.⁽⁴³⁾ He presents the hypothesis that the human visual sense samples the spatial power spectrum of the input image, just as the aural sense samples the temporal power spectrum of the input sound. The justification for his hypothesis is the fact that the sensitivity of the retina (except at the fovea) to form or pattern in the input image is very much poorer than is suggested by the corresponding upper cutoff spatial frequency of the retina. This property is characteristic of power spectrum sensitive devices. In developing his hypothesis, Merchant proposes that the human retina samples the spatial average of the luminance function over the studied area, as well as sampling certain weighted averages of the spatial power spectrum within this area.

He points out that this method of sampling introduces ambiguities into the perception of the object, except when the latter is very simple—for example, one sharp line. Merchant's hypothesis seems very plausible and it helps to explain some familiar observations—rather to put words and numbers to them.

An observer of a paper easily perceives regular disturbances such as wire and felt marks and streaks. In a wavelength spectrum, such disturbances are represented by quite sharp and high peaks at wavelengths corresponding to the distance between the disturbances. It is therefore clear that when such peaks occur (and they do) they should be treated separately and be given special weight in any composite figure that is intended to represent the subjective impression of unevenness.

The wavelength spectra of random sheets (see pp. 15–19) have a rather smooth and simple form. This is also observed in practice for well-formed sheets, but many commercial and laboratory-made papers possess wavelength spectra of formation that contain a 'hump' in a particular range of wavelength. This hump is not necessarily a maximum, but only a deviation from the smooth and regular appearance of spectra of random sheets. According to Merchant's hypothesis, an observer perceives such variations in a rather vague and unspecified way. If he does not study the object very carefully (using foveal vision), he only sees irregularities of certain sizes. Depending on their geometrical size, he may perceive them as dots, grains, flocs, blobs or clouds, for instance.

Such irregularities may be disturbing when looking through a sheet of paper. Hence, such humps in the measured spectra should also be given special weight. (This statement presupposes, of course, that not only the common reader, but also papermakers, printers and advertising people do not normally study the structure of paper very carefully. For such studies, only the complete spectrum will suffice.)

Having discussed in particular the peaks and humps in wavelength spectra and having stated that, apart from such irregularities the spectra always have approximately the same shape, it remains to discuss the general level of the spectrum stripped of these irregularities. There are reasons to believe that the level of the spectrum is not very significant. One reason is apparent from Merchant's hypothesis. If the spectrum does not contain any humps or peaks, the observer possibly perceives nothing but general background noise. If the noise level is within the normal range, he will only perceive it as 'paper'.

An illustration of this is given below. As part of a reconstruction and modernisation programme, foils were installed on a medium speed papermachine producing magazine paper. The sheet had a very streaky appearance on start-up (perception = streaky = bad). It was soon found that the streaks were due to a twisted forming board and maladjustments in the head box. As these problems could not be eliminated immediately, the papermakers decided to remove the first two foil boxes immediately following the head box, substituting three table rolls for them. The machine then produced saleable paper until the basic cause of the streaks was remedied and the foils put back into service.

Our measurements showed that the streaky paper produced on start-up after the reconstruction had a significantly better formation than the one produced with the table rolls in position right after the head box, yet also had a hump at wavelengths approximately between 2 and 10 cm. This hump was easily perceived by the eye in an otherwise well-formed sheet. When the table rolls were replaced, the hump was drowned in the general noise of the poor formation of the resulting sheet.

The above example serves to show that, within limits, deviations from a regular spectral shape are much more easily perceived as irregularities than the general level of the variations. It also shows that any simple general measure, such as the one based on Andersson's hypothesis, may easily lead to erroneous conclusions.

Summing up, it is suggested that a good measure of the subjective perception of unevenness of paper observed by transmitted light, should be based on the measurement of wavelength spectra of absorbance. Special emphasis should be given to peaks and humps in the spectrum. Less attention should be given to the general level of the spectrum. This hypothesis can be verified or rejected only by carefully conducted psychophysical experiments. At present, we do not have any reliable results of such experiments.

If nothing else, this part of our paper may serve to demonstrate some of the complexities involved in the human perception of unevenness of paper. It indicates also some of the dangers involved in trying to work with only visual assessment (even if scientifically performed) of the results of changes to materials, processes and procedures. For technical and scientific work, only 'objective' measures—for instance, those presented in other sections of this paper—should be used.

MASS DISTRIBUTION, PRINT-THROUGH AND UNEVENNESS IN SOLID PRINTS

Print-through

WHEN a solid print is applied to one side of a paper, the other side of the paper may become mottled in appearance and its reflectance factor may be lowered. These are the results of print-through. The term is used here in a general sense. It is usually obvious that the mottle is closely related to the distribution of mass in the plane of the sheet. Heavy areas in the paper appear brighter than thin areas.

There are many factors involved, however. Larsson⁽⁴⁴⁾ has listed these as-

Show-through	influenced by paper opacity
Strike-in	influenced by pigment separation
Oil separation	influenced by oil separated from the ink.

Measurements are necessary to distinguish how the various factors influence the mottle on the reverse side of a printed sheet of paper. A measurement made with the attachment for reflectance measurements in the apparatus described under *Microdensitometer* can be used to relate the mottle to the mass distribution of the paper, which is mounted on the rotating cylinder with a black backing material (heavily exposed photographic film). The coefficient of variation of the light intensity reflected from the paper is measured. The results may be interpreted in terms of the local grammage of paper between the scanned surface and the black backing or the black ink. For this interpretation, equation (40) is used with $R_g = 0$. The explicit solution in this case is—

$$\mathbf{V}(R_{\rm o}) = F' \cdot \frac{1-\Omega}{\Omega} \cdot \frac{1-\Omega R_{\infty}^2}{1-R_{\infty}^2} \ln \left\{ \frac{1-\Omega R_{\infty}^2}{1-\Omega} \right\} \quad . \qquad . \qquad (50)$$

In this formula, Ω is the opacity (= R_o/R_∞) when R_o is the reflectance factor on the reverse side of the print and F' is the coefficient of variation of grammage between the unprinted surface and what is sensed as the 'black backing'. For a printed paper, the 'black backing' is the print. In this way, the distribution of ink-free paper can easily be evaluated and presented in the form of a wavelength spectrum. This appears to be closely related to the spectrum of mass distribution, but there is insufficient experimental material at the time of writing to allow definite conclusions to be made. Oil from the printing ink is usually extracted by a suitable solvent to avoid any influence that an uneven distribution of the oil may have on the results.

Mass distribution of an ink fllm

The concept of mass distribution is not necessarily limited to paper. It may, for instance, be applied to an ink film printed on a sheet of paper. This provides an objective measure of the variation in the ink film thickness (or the ink film 'grammage') and its distribution in the plane. The variations in the quantity of ink on the paper cause variations in the reflectance of the printed surface. For a given mass distribution of the ink, the corresponding reflectance variations will depend on the optical properties of the ink and the paper. The mass distribution of the ink is the most objective measure with which the behaviour of different inks can be compared, irrespective of their optical properties.

The reflectance variations from a solid print may be used to evaluate the mass distribution of the ink film. Use is then made of another reflectance attachment to the scanning microdensitometer, which gives diffuse incident illumination. Use is again made of equation (40) for the evaluation of the results in terms of mass distribution. For a black ink on an appreciably brighter sheet of paper, the formula simplifies to—

$$R = R_{i\infty} + (R_p - R_{i\infty}) \exp(-s_i w_i / R_{i\infty}) \quad . \quad . \quad . \quad (51)$$

Here, the index *i* refers to the ink and the index *p* to the paper. Accordingly, w_i is the 'grammage' of the ink. It is necessary that the ink be on top of the paper for this simple formula to be valid—that is, on top of the uppermost fibre at each point. This certainly is not absolutely true, but it is considered a good approximation in many cases. Reference is made here to a paper by Lyne & Madsen.⁽⁴⁵⁾ These authors concluded that the penetration of the ink pigment into the paper is very limited in letterpress printing. Only those fibres that contact the ink film on the plate become blackened. Use is also made

of the concept of a scattering and an absorption coefficient for the description of printing inks, which has been shown to be valid by Tollenaar.⁽⁴⁶⁾ The above formula can then be developed further to yield the coefficient of variation of reflectance from a solid print as a function of the intensity of the mass distribution of the ink film $V(w_i)$. Introducing the parameter—

$$x = s_i w_i / R_{i\infty} = w_i \sqrt{k_i^2 + 2k_i s_i}$$
 (52)

it is easily shown that-

In this formula, x is a measure of the amount of ink on the paper. The formula shows how the coefficient of variation of reflectance is affected for a given value of the unevenness of the print $V(w_i)$. It is the coefficient of variation that most earlier investigators have measured. It appears that the coefficient of variation of reflectance is affected not only by the unevenness of the print, but also by the amount of ink on the paper, the properties of the ink and the properties of the paper. For a diagrammatic representation, $V(R)/V(w_i)$ is plotted as a function of x in Fig. 45. The graph shows that the

Fig. 45—Ratio of reflectance variations to variations in amount of ink on the print as a function of average ink level and blackness of the ink

coefficient of variation of reflectance increases to a maximum, then decreases when an increasing average amount of ink is transferred to the paper *with constant relative non-uniformity*. Qualitatively, this may be understood to mean that at low ink levels a certain percentage non-uniformity of the distribution of the ink is easily measured against the fairly bright background of the lightly coloured print. On the other hand, at high ink levels, the same percentage non-uniformity is not measured as easily because of the lower efficiency of extra ink due to the negative exponential term in the formula. The figure also shows that a blacker ink (low $R_{i\infty}$) gives higher reflectance variations than a less black ink and that inks of different blackness ($R_{i\infty}$) give maxima at different ink levels. Inspection of the formula shows that the ink level where the maximum occurs is only a function of the ratio of the reflectivities of the ink and the paper. It may be concluded from these observations that reflectance variations measured as a function of the ink level can show a maximum even if the relative unevenness of the amount of ink on the paper $V(w_i)$ is constant. Measurements of reflectance variations have shown that the maxima occur regularly. This does not mean that the intensity of the mass distribution of the ink film is absolutely constant (it is not), but it varies much less than is perceived by an observer.

Wavelength spectra of the mass distribution of the ink can be measured to obtain further information about the distribution of the ink on the paper. Such measurements show that there is generally a close relationship between the detail structure of the paper and the mottle of the print. Since both can be measured and expressed in the same terms, quantitative research into this area is possible and is now in progress. An example of some typical results is shown in Fig. 46.

The influence of calendering and other finishing operations will be reported elsewhere.

MASS DISTRIBUTION AND THE STRENGTH PROPERTIES OF PAPER

WHEN evaluating the effect of mass distribution on strength properties, the main problem is to separate the effect of other variables. One of these, termed *forming consolidation* by Norman,⁽⁴⁷⁾ is related to the microstructure of the sheet. A high forming consolidation is defined as one that leads to a high local breaking length. This may also be called a well-closed sheet. Fibres that are arranged evenly possess a relatively large area that is suitable for bonding to other fibres. A more disordered microstructure exposes a smaller area for bonding within the sheet.

It has been pointed out (see *Random sheets versus real sheets*) that, when making a sheet from a very dilute suspension, the fibres will have time and incitement to arrange themselves, during sheet consolidation, in a structure that is far more even than even a random structure. Therefore, such a sheet may be expected to be stronger than a random sheet or a sheet that has a more uneven mass distribution. It is not necessary, however, that a sheet with an uneven mass distribution also displays a disordered microstructure. Yet this may often be the case when conventional forming methods are used. Therefore, a sheet with an uneven mass distribution, which has poor strength properties due to local stress concentrations, normally also has a disordered microstructure that in turn decreases bonding.

The influences of mass distribution and of forming consolidation have now been discussed. Two further points need to be considered when the effect of forming conditions on the sheet strength is studied—

- 1. All properties must be compared at the same sheet density.
- 2. The effect of fibre orientation must be considered.

The determination of fibre orientation is very time consuming, but this may be avoided by using the results of Cavlin & Engman.⁽⁴⁸⁾ They found that the geometrical mean of breaking length in the machine and cross directions is independent of fibre anisotropy. This was verified for a large range of fibres and anisotropy ratios, using laboratory sheets made in the French rotating former 'la Formette Dynamique'.

At the 1965 Cambridge conference, Norman⁽⁴⁷⁾ attempted to separate the effects of mass distribution and forming consolidation. In order to achieve this, he defined a specific breaking length as the average of the local breaking length at a number of randomly selected areas. The local breaking length was measured on specially formed test pieces, including a 3 mm wide waist. Assuming that the specific breaking length is independent of the mass distribution, the effect of forming stock concentration on forming consolidation was calculated as the ratio of the measured specific breaking length to the specific breaking length of a standard handsheet made from the same pulp. The value of the investigation was limited by the fact that the sheet density was not considered when the strength data were evaluated. Norman showed conclusively, however, that forming consolidation decreased with increasing stock concentration, both for handsheets and for machine-made sheets.

Rudström⁽⁴⁹⁾ investigated the influence of mass distribution on strength properties for laboratory sheets, paper made on an experimental paper-machine and paper made on an industrial machine.

The laboratory sheets were formed at three different grammages—40, 120 and 220 g/m². Different mass distributions were obtained by using time lags of 0, 60 and 300 s between agitation and drainage in the sheet mould. Unbleached kraft pulp was used, beaten to 15° , 25° and 35° sR, respectively. Mass distribution spectra are shown in Fig. 47–49. The sheets were pressed to three different density levels. The results of the evaluation of the strength properties are shown in a condensed form in Fig. 50–52. The geometric mean breaking stress $\overline{\sigma}$ was divided by the sheet density ρ to make them proportional to the breaking length.

The mass distribution in the 40 g/m² sheets was more uneven, the longer the

time lag. The strength decreased significantly when the time lag increased from 0 to 60 s, but decreased only slightly for greater time lags (for the 15° sR sheet, it was unchanged.)

Fig. 50-52-Strength values for handsheets⁽⁴⁹⁾

Time lag	0 s	60 s	300 s	
15° sr	0	*	-+-	
25° sr	\triangle	T	×	
35° sr	35° sr □		∇	

Increasing the time lag from 60 to 300 s caused sedimentation before drainage, which in turn resulted in unchanged or improved mass distribution at 300 s for the 120 and 220 g/m² sheets. The strength properties were affected in a similar way for the 120 and 220 g/m² sheets. The unbeaten pulp (15° sR) produced sheets with strengths that were not significantly affected by changes in the mass distribution. The beaten pulps 25° and 35° sR) yielded significantly higher strengths without time lags and no change in strength when the time lag was increased from 60 to 300 s (no change in mass distribution either).

Unbleached kraft pulp beaten to 24° sR was used to make the papers on the experimental papermachine. Sheets with grammages of 65, 105 and 130 g/m^2 were produced and the mass distribution was changed by increasing the consistency in the head box from 0.29 to 0.76 per cent. The mass distribution spectra are shown in Fig. 53–55. The sheets produced from the higher consistencies contained a large number of flocs of wavelength 5 mm and above (the mass distribution spectra of the laboratory sheets were more even).

Fig. 53-55-Wavelength spectra for machine-made sheets⁽⁴⁹⁾

The industrial machine was large, a modern sack paper machine, producing paper of 75 g/m² at 530 m/min from a 25° sR unbleached kraft pulp at 0.2 per cent consistency. The mass distribution was changed by increasing the head box consistency in steps to 0.4 per cent. The mass distribution spectra are shown in Fig. 56. The specific strength of the machine-made sheets is shown in Fig. 57. The decrease in strength, with increasing consistency, of the industrial sack paper is significantly greater than that of the comparable 65 g/m²

experimental machine paper, in spite of a greater change in head box consistency (and thus mass distribution) in the latter case. In the 105 and 130 g/m² papers, the deteriorating mass distribution caused a maximum of 40 per cent decrease in specific strength.

Lyne investigated the effect of mass distribution on strain variation in newsprint⁽⁵⁰⁾ and the effect of mass distribution on tearing strength.⁽⁵¹⁾ He found that the Elmendorf tear and the in-plane tear were affected in opposite directions.

Cavlin⁽⁵²⁾ considers the strength of a sheet of paper to be determined principally by a large number of weak spots. Fig. 58 shows the distribution of weak spots in a test sheet. Test areas of different sizes include a different number of weak spots and therefore the smallest one, which contains the lowest number of faults, should exhibit the largest strength values. It may be concluded that the side of the test area should be approximately equal to the mean distance between the weak spots if these are to be avoided in some of the test areas.

Fig. 58—Distribution of weak spots in measuring areas of different sizes⁽⁵²⁾

It is vital that the rate of strain is kept constant and that the shape of test samples of different sizes is the same. This results in the same stress distribution during straining and also minimises any effects of this on the comparison between different sample sizes in case the faults are anisotropic in nature. Square samples were chosen.

Some characteristic strength distribution curves are shown in Fig. 59. They illustrate that the mean strength and the standard deviation (derived from the slope of the curve) decreases when the measuring area is increased. The curves

follow the Weibull statistical distribution functions⁽⁵³⁾ and are chosen for best fit to the experimental data. The minimum test area used was 0.1 cm^2 , since the boundary effect from clamping is too large for smaller test areas.

Fig. 59—Characteristic strength distribution curves⁽⁵²⁾

Kraft paper samples (100 g/m² and freeness of the pulp 20° sR) were made on an experimental papermachine. The paper was produced at a high consistency to give a poor mass distribution. Laboratory sheets were also made from the same pulp using the French sheetformer. The laboratory sheets were made with the same anisotropy as the machine-made sheets and were pressed to the same density. The wavelength spectra of the mass distribution are shown in Fig. 60. The spectral densities differ by nearly a factor of 10 at 30 mm wavelength, whereas they are the same at wavelengths below 2 mm.

Fig. 60—Wavelength spectra for a 'aboratory sheet and a machinemade sheet (52)

The influence of the size of the test area is shown in Fig. 61. The strength of the machine-made sheet is 57 per cent of that of the laboratory sheet for a large test area (the macro-scale regime). The difference is decreased to 10 per cent at 0.1 cm^2 measuring area (the micro-scale regime). This measuring area corresponds to a floc size of about 3 mm (6 mm wavelength). It is only at wavelengths smaller than 2 mm that the sheets are of comparable evenness with respect to the mass distribution. This corresponds to a measuring area of 0.01 cm^2 , which cannot be achieved in practice. Hence, comparisons of forming consolidation in this range is impossible.

Strengths increase markedly as the test area is decreased. The transition range is less well defined for the machine-made paper. The area at which the strength starts to increase with decreasing test area is closely related to the mean distance between the weak spots in the sheet. The decrease in strength of the machine-made paper in the macro-scale range is an indication of uneven mass distribution in the large wavelength range. The same effect is much less pronounced in the laboratory sheet.

A comparison was also made between the laboratory sheet and a wellformed machine-made sheet made from the same pulp. Differences of less than 10 per cent were obtained, even with large test areas. This suggests that an even machine-made sheet is not necessarily weaker than a laboratory sheet (compared at equal anisotropy and sheet density).

The effect of mass distribution on the characteristic strength distribution is shown in Fig. 62 for the micro and macro scale regimes respectively. The standard deviation of the strength values is higher in the micro-scale range. This is also the case when the pressing and beating conditions are varied. Large variations in mass distribution tend to shift the entire curve to a lower

Fig. 62—Characteristic strength distribution curves in the micro-scale and macro-scale regimes⁽⁵²⁾

------ Laboratory sheet

strength range. This is also the case for sheets of lower density produced with less pressing and beating.

In conclusion, Cavlin⁽⁵²⁾ suggests that a considerable part of the strength of paper is determined by heterogeneities in the state of bonding in the micro-scale regime. In a well-formed sheet, however, the main variables controlling paper strength and elasticity are the average sheet density and the fibre orientation (which control the force distribution in the micro-scale regime).

The investigations described above $^{(47, 49-52)}$ all show that the strength properties are determined to a large extent by the mass distribution. Further work is required to determine the relative influence of stress concentration effects and forming consolidation effects. This could be achieved by extending the techniques described in the investigations $^{(47, 49-52)}$ by the determination of variations in scattering coefficient (bonded area) using light transmission methods; and by the determination of grammage variations using beta-radiography.

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References

- 1. Corte, H., Paper Maker (London), International number 1971, 81
- 2. Wahren, D., Svensk Papperstidn., 1967, 70 (21), 725
- 3. Hinze, J. O., Turbulence (MacGraw-Hill, New York, 1959)
- 4. Blackman, R. B. and Tukey, J. W., *The Measurement of Power Spectra* (Dover Publications, New York, 1958)
- 5. Norman, B. and Wahren, D., Svensk Papperstidn., 1972, 75 (20), 807
- Burkhard, G., Wrist, P. E. and Mounce, G. R., Pulp & Paper Mag. Can., 1960, 61 (6), T319
- 7. IEEE Trans. Audio and Electroacoustics, Vol. AU-15, No. 2, June 1967
- 8. Corte, H. and Dodson, C. T. J., Das Papier, 1969, 23 (7), 381
- 9. Häglund, L., Norman, B. and Wahren, D., 'The power spectrum of the basis weight of random sheets', International Paper Physics Conference, Montreal, 1971
- Davis, M. N., Roehr, W. W. and Malmstrom, H. E., Paper Trade J., 1935, 101 (4), 31
- 11. Brecht, W. and Wesp, A., Das Papier, 1952, 6 (17/18), 359
- Favis, D. V., Robertson, A. A. and Mason, S. G., Can. J. Tech., 1952, 30 (10/11), 280
- 13. Williams, D. J., Appita, 1955, 9, 209
- 14. Robertson, A. A., Pulp & Paper Mag. Can., 1956, 57 (4), 119
- 15. Andersson, O. and Sundevall, K., Svensk Papperstidn., 1960, 63 (6), 167
- 16. Westra, H. A., Papierwereld, 1960, 14 (10), 591, 625

- 17. Hansson, G., Svensk Papperstidn., 1965, 68 (20), 721
- 18. Karttunen, P., Electricity in Finland, 1966, 39 (12), 363
- 19. Tuomenoja, A. T., Paper Trade J., 1969, 153 (11), 42
- 20. Kallmes, O., Paper Trade J., 1971, 155 (43), 46
- 21. Williams, D. J., Appita, 1972, 25 (6), 440
- 22. Howe, B. I. and Cosgrove, J. C., Can. Pulp & Paper Ind., 1964, 17 (11), 30
- 23. Howe, B. I. and Cosgrove, J. C., Paper & Timber (Finland), 1963, 45 (4a), 191
- 24. Howe, B. I., Hopkins, J. A. and Stokes, T. W., Pulp & Paper Mag. Can., 1970, 71 (1), T14
- 25. Duchnicki, T. S., Pulp & Paper Mag. Can., 1965, 66 (12), T599
- 26. Pye, I. T., Pulp & Paper Mag. Can., 1971, 72 (5), T167
- 27. Kubelka, P. and Munk, F., Z. Techn. Physik, 1931, 12, 593
- 28. Kubelka, P., J. Opt. Soc. Amer., 1948, 38 (5), 448; 1954, 44 (4), 330
- 29. Balodis, V., Appita, 1965, 18 (5), 184
- 30. Aaltonen, P. and Simola, J., Paper & Timber (Finland), 1971, 53 (9), 479
- 31. Attwood, D. and Parker, J. R., Paper Tech., 1962, 3 (5), 435
- 32. Tydeman, P. A., B.P. & B.I.R.A. Bull., June 1965, 8
- 33. Van Ooij, W. J., Papierwereld, 1969, 24 (3), 67
- 34. Corte, H., Das Papier, 1970, 27 (5), 261
- 35. The Radiochemical Centre, Amersham, Bucks., England
- Norman, B. and Smith, D., *Beta-Radiography of Paper* (Internal Report, STFI, PA, 1972)
- Söderström, P. O., Ytjämnhetsmätning med reflekterat ljus (Thesis work, STFI, PA, 1971)
- Hollmark, B., Studium av kräppningsprocessen på en kräppningsmaskin, STFI Medd. No. 144 (1972)
- 39. Htun, M. T., Jämförelse av massfördelningen i laboratorieark med randomark (Thesis work, STFI, PA, 1971)
- 40. Wrist, P. E., 'Dynamics of sheet formation on the Fourdrinier machine', Formation and Structure of Paper, Ed. F. M. Bolam (Technical Section, British Paper & Board Makers' Association, London, 1962), 839
- 41. Poulter, S. R. C., Applications of microdensitometry in printing quality studies: Measurement of print uniformity and factors affecting it, PATRA Printing Laboratory Report 65 (April 1965)
- 42. Liepman, H. W., Helv. Phys. Acta, 1949, 22, 119
- 43. Merchant, J., J. Opt. Soc. Amer., 1965, 55 (10), 1 291
- 44. Larsson, L. O., *Physical background and some terms used to describe print quality*, Tidningspappersbrukens Forskningslaboratorium Report No. 7: 3 (1967)
- 45. Lyne, L. M. and Madsen, V., Pulp & Paper Mag. Can., 1965, 66 (3), T175
- 46. Tollenaar, D., 'Ink distribution and density of prints', 14th EUCEPA Conference, Budapest, 1971
- Norman, R. J., 'Dependence of sheet properties on formation and forming variables', *Consolidation of the Paper Web*, Ed. F. M. Bolam (Technical Section, British Paper & Board Makers' Association, London, 1966), 269
- 48. Cavlin, S. and Engman, C. (STFI, PA), to be published

- 49. Rudström, L., Formationens inverkan på papperets styrkeegenskaper, Meddelande STFI No. 83-86 (1971)
- 50. Lyne, M. B. and Hazell, R., 'Formation testing as a means of monitoring strength uniformity', *Fundamental Properties of Paper related to its Uses* (this volume), 74-100
- 51. Lyne, M. B., Jackson, M., Ranger, A. E. and Trigg, A. R. V., 'The evaluation and optimisation of the in-plane tearing strength of paper', *Fundamental Properties of Paper related to its Uses* (this volume), 269–398
- 52. Cavlin, S. (STFI, PA), to be published
- 53. Weibull, W., A statistical theory of the strength of materials, Ingenjörsvetenskapsakademiens handlingar No. 151 (1939)

Transcription of Discussion

Discussion

The Chairman The theme of the 1961 symposium at Oxford was The Formation and Structure of Paper. Its announcement at the 1957 Cambridge symposium, the first of them all, caused a number of research activities in various parts of the world—which is indeed one of the declared aims of these symposia.

It is difficult to guess how many research projects would have remained dormant or would never have been undertaken without the challenge and call of our symposia. There is no doubt that in many cases they had the function of a catalyst and exerted a beneficial influence on the rate of progress.

In any case, by 1961, some of the research activities had produced their first results. We, the listeners, were presented with an impressive array of new concepts. Some of them were put forward as tentative working hypotheses, some of them were proclaimed with the power of conviction. What is more, some of the concepts have been pursued further up to the present day and have yielded a new crop of results. We will be listening to these today.

If I single out three of the concepts that we discussed twelve years ago, it is only because of their relevance to our first session today and does not reflect in any way on the others. The first is the concept of the uneven structure of paper in the plane of the sheet and, in particular, the concept of randomness and the question whether paper is more uniform or less uniform than random. I think it is now accepted by most that the non-uniform distribution of fibres in the plane of the sheet is the one single structural property that distinguishes paper more than any other from other sheet-like materials. Take three familiar objects—a linen cloth, regenerated cellulose film and a sheet of paper. All three can consist (and often do) of one hundred per cent cellulose and it is only their different structures that make them such entirely different materials. The first part of our session today will deal with the measurement and the consequences of the uneven areal mass distribution of paper.

The second concept is that of the layered structure of paper, a mere working hypothesis twelve years ago, but in the meantime recognised as a fundamental structural feature of paper. This will be dealt with in the first two papers after lunch.

Under the chairmanship of Dr H. Corte

Mass distribution and paper properties

The third concept, in some ways related to the second, is the nature and extent of fibre bonding and, in particular, the question of optical contact. The last two papers this afternoon will present some new ideas, also some new experimental discoveries in this area. The last paper will provide too a kind of link to the two sessions tomorrow.

Prof. D. Wahren If the work presented in our paper had been done solely for the purpose of seeing whether a real sheet is more or less uneven than a random sheet I agree with you; of course, this is just one of the things that we can do when we have these techniques available. I think that your way of making the assessment is very elegant and I will certainly try it out myself.

The only information obtained, however, is whether or not the number of flocs longer than the scanning length exceeds that of a random sheet. Our method gives a detailed picture of the floc size distribution in the real sheet and the random sheet, respectively.

Prof. J. Silvy This contribution is about the extensive use of the Kubelka-Munk theory to deal with the influence of large variations of the scattering coefficient on the intensity of the mass distribution, especially when the conclusion is to use a light with a high absorption—that is, the range where the validity of the Kubelka-Munk theory is doubtful.

I am sure that you know well the limit of this theory as it is applied in the field of paper studies, but may I focus it quickly by two illustrations. The first is from a work published by Dr L. S. Nordman in 1965 of the Finnish Paper Research Institute in his extensive research programme on the measurement of relative bonded area in correlation with variations of the scattering coefficient of more or less bonded sheets. Here we can see the influence of the colour of the light and proof is given of the dependence between coefficients S and K. The S value diminishes when the absorption K increases—is this explained theoretically?

In the second illustration, we have a comparison made by Klier of the Coating Center in Bethlehem, Pennsylvania, between the rigorous theory of radiative transfer in diffusing media by Chandrasekhar and the results of the Kubelka-Munk approximations. Here, links are established between the Kubelka-Munk apparent absorption and the scattering coefficients of the true absorption coefficients α (the η curve) and the true scattering coefficient σ (the χ curve). We can see that K is not linearly linked to the true absorption a nor is S to σ , consequently K/S to a/σ . We see that theoretically S as estimated by Kubelka-Munk diminishes as K/S increases and the discrepancy is more and more important as absorption of light in the medium becomes higher. (See Klier, K., J. Opt. Soc. Amer., 1972, **62** (7), 882.)

Discussion

Prof. Wahren I will try to answer what is really not a question. We have been looking into these matters in depth and I am very well aware of the facts shown in your two illustrations. We are content if in calibration we get proportionality between grammage and the signal. The Kubelka-Munk equations may be inexact, but it is normally a matter of only a few per cent of the absolute value. After calibration (or after automatic standardisation, which is a feature of our method), the errors are much smaller. The scattering coefficient varies with the wavelength of the light. We do not use monochromatic light, but well-filtered light. We have found that this is good enough. White light cannot be used, however, if any reasonable degree of accuracy is to be obtained. Your remarks are relevant in general, but not particularly to our measurements.

 $Dr \ D. \ Atack$ I am gratified that our prepared contribution has elicited such a lengthy clarification in the form of an addendum. Unfortunately, we received the addendum only this morning and have not had time to study it. From a brief perusal of it, we feel that it does not in any way affect the validity of our original remarks. We shall discuss the issue in more detail in a forthcoming publication.