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DYNAMIC COMPRESSION OF SATURATED FIBRE MATS

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AS a means of access to the more complicated phenomena of wet pressing, we have observed and interpreted compression dewatering in an extensively simplified case. It appeared best to begin with a system characterised by a single spatial co-ordinate; a system containing synthetic fibres which would be non-swelling and nearly identical, filtration-formed into a uniform bed, and mechanically conditioned; a system for which one could expect to find stable and reproducible compressibility and permeability properties. It also appeared that we could construct a theory of the behaviour of such a system, and develop effective numerical procedures by which to obtain the predicted response to rapidly varying applied stress. Experiments and theory were to include conditions in which the increasing stress on the solid portion of the system would induce significant non-uniformity.

It was suspected that the relationship between solids concentration and stress on the fibre system, represented by a static compressibility function, might not be adequate under conditions of rapidly varying stress. In the absence of experimental evidence, we arranged the theory to admit a solids concentration which depends on the local stress history, but have used a static compressibility function in the evaluation of predicted response. Comparison with experiment has confirmed that a time-dependent compressibility effect is present, and that it leads to appreciable departures from results expected on the basis of measurements of static compressibility.

The theory supposes that the solid and fluid components of the system are, separately, incompressible. The continuity conditions are, accordingly,

$$(\partial U/\partial z) = (\partial \varepsilon/\partial t) \quad \text{and} \quad (\partial U'/\partial z) = -(\partial \varepsilon/\partial t), \quad . \quad . \quad (1)$$

where ε is the porosity, and U and U' are the flow rates of the solid and of the fluid, respectively (volume per unit area per unit time). If we assume, further,

Under the chairmanship of Dr J. D. Peel that inertial effects would represent, at most, a small correction, there is a simple relationship between the gradients of solid stress and of fluid pressure:

$$(\partial p/\partial z) = -(\partial p'/\partial z), \qquad (2)$$

in which p is the solid stress and p' is the fluid pressure. With respect to completed experiments, estimates show this assumption to have been applicable except for a very brief interval at the beginning of the compression.

Provisionally, we assume that the solids concentration $c(z, t)$ and the solid stress $p(z, t)$ are directly related, as in the formula

$$c = c_0 + c_1 p^N. \qquad (3)$$

For use in calculation, the parameters in equation (*3*) were determined by a least-squares adjustment, from measurements on the mat to be used in the compression experiment.

The model assumes that the fluid pressure gradient and the flow rates are connected as in

$$a(U'(z, t)) - [(\varepsilon/(1-\varepsilon)]U(z, t)) = -(\partial p'/\partial z). \qquad (4)$$

The flow resistance a depends on the porosity and characteristics of the solid component of the system.

A description implied by these assumptions, as shown by Nelson[1] and Emmons,[2] is contained in the equation

$$-(\partial c/\partial t) = U'(0, t)(\partial c/\partial z) - (\partial/\partial z)[ca^{-1}(\partial p/\partial z)]. \qquad (5)$$

The independent variables are z and t, where z is the spatial co-ordinate, and the system extends from $z = 0$ to $z = L$; the superficial velocity at $z = 0$ is $U'(0, t)$.

The grammage is assumed to be constant, but the applied-stress schedule will lead to a decreasing mat thickness L. To avoid the complication of a moving boundary, we convert to a system of co-ordinates in which the independent variables are w and t, where w is the cumulative mass:

$$w = \int_0^z c(z', t)\, dz'. \qquad (6)$$

The equation which results is

$$-(\partial c/\partial t) = c^2 a^{-1}(\partial p/\partial w)(\partial c/\partial w) - c(\partial/\partial w)[c^2 a^{-1}(\partial p/\partial w)]. \qquad (7)$$

Scrutiny of the argument leading to equation (*7*) shows that it applies, not only to the situation represented by equation (*3*) or an alternative of its kind, but also when $c(z, t)$ is the outcome of present and previous values of the local solid stress. In the cases which we have reduced to computational procedures, however, c is a function which depends only on the local and current value of p. For these, we rearrange equation (*7*) in the form

$$-(\partial p/\partial t) = c^2 a^{-1}(\partial p/\partial w)^2 - c(\mathrm{d}c/\mathrm{d}p)^{-1}(\partial/\partial w)[c^2 a^{-1}(\partial p/\partial w)]. \qquad (8)$$

The intended conditions are that the range of w is fixed; that the boundary at $z = 0$ is impermeable to either component; and that the mat is confined at $z = L$ by a piston which offers negligible resistance to fluid flow. Thus the initial condition is that $c(z, 0)$ or $p(z, 0)$ have assigned values, and the boundary conditions turn out to be that $(\partial p/\partial z) = 0$ at $z = 0$ for all values of t; and that $p(L, t) = P(t)$ be the given applied stress.

We have developed and tested numerical treatments of the problem consisting of equation (*8*) and its initial and boundary conditions. Although non-linear, the equation has a qualitative resemblance to the equation of heat condition. The most effective procedure has been a generalisation of the Crank–Nicolson method.

The numerical results consist of tables showing the solid fraction and the solid stress as functions of w, at values of t which increase in equal intervals.

In the limiting case of a mat which remains nearly uniform during the experiment (or a portion of it), it can be shown that the fluid pressure difference becomes

$$p'(L, t) - p'(0, t) = (a/2)L(\mathrm{d}L/\mathrm{d}t), \qquad (9)$$

in the application of which the connection between L and the applied stress involves $c = W/L$, which must be the predictable outcome of the stress history. For comparison with experiment, we introduce the static compressibility function, equation (*3*).

The compression apparatus consisted of a cylindrical cup, 76·2 mm in diameter, mounted on a flat plate, with a closely-fitting permeable piston inserted into the cup. Thrust from a pneumatic actuator was applied to the piston through a linkage which included a strain gauge load cell. The fluid pressure at the plate was measured by a small transducer located beneath the surface and communicating with it by several small pressure taps, in an arrangement which prevented direct contact of the sensing element with the fibres. Piston displacement was determined with a linear motion potentiometer. The transducer signals, after amplification, were recorded photographically by a multichannel oscilloscope.

The fibres were cut from dacron monofilaments, and had a density of 1·38 g/cm^3. The average length was 3·65 mm and the average diameter 23·1 μm. The fibres were dispersed in hot distilled water, under vacuum, and the suspension diluted to 0·1 g/l with distilled water. Filtration forming over a period of about 40 min resulted in a mat about 20 mm thick at a grammage of about 2 kg/m^2. The water-saturated mat was then transferred to the plate of the compression apparatus.

The maximum stress to be attained in the dynamic compression was applied to the mat for 2 min, after which the pneumatic pressure was released and

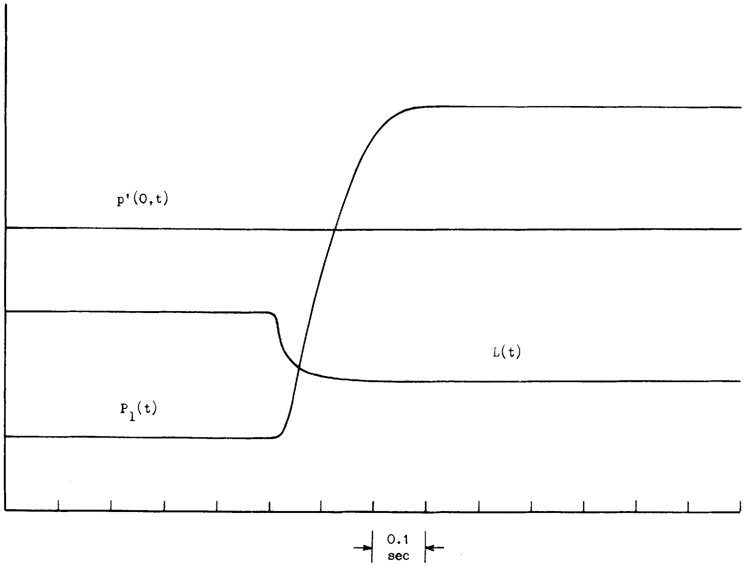

Fig. 1—Dry compression experiment

the mat (still water-saturated) subjected only to the weight of the piston (1·576 kg) for 2 min. The mechanical conditioning of the mat consisted of 12 such cycles. Upon completion of tests with the wet mat, it was removed and dried overnight at 105° C. The dry mat was weighed and replaced on the plate for mechanical conditioning as before.

Static compressibility data for both wet and dry mats was obtained in the compressibility apparatus. At the end of a 15 min period at constant load following each increase, the mat thickness was measured for increasing loads within the range of interest. The fitted static compressibility parameters differed somewhat for wet and dry mats (the units are g/cm³ for the solids concentration, and Pa for the stress):

Mat	c_0	c_1	N
Wet	0·2231	0·01255	0·2360
Dry	0·1544	0·03212	0·1701

In a separate apparatus, a mechanically-conditioned and water-saturated mat was clamped and sealed between two permeable septa at a fixed separation. Distilled water, deaerated and filtered, passed through the compressed mat slowly, under a pressure drop small compared to the average stress sustained by the mat. Flow rate, pressure drop, mat thickness, and temperature

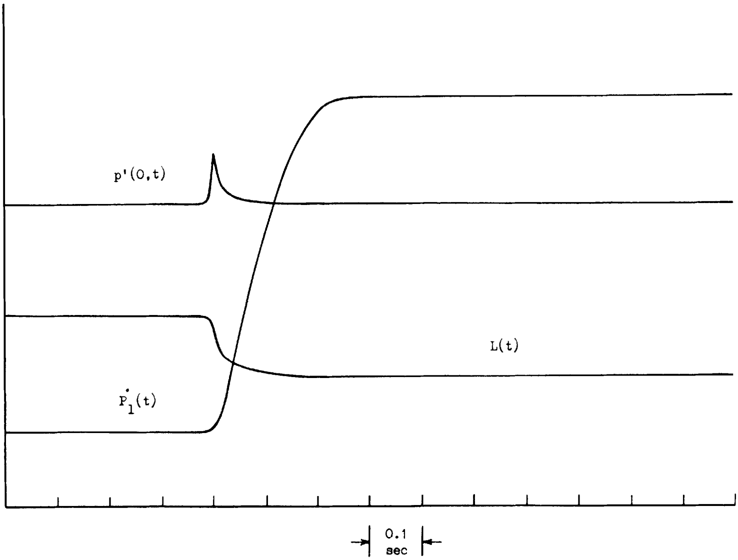

Fig. 2—Wet compression experiment

were recorded. A fitted empirical formula, giving the flow resistance as $a = \mu/K$, is

$$K = (6{\cdot}395\,\varepsilon^{11{\cdot}3} + 0{\cdot}573\,\varepsilon^{2{\cdot}34}) \times 10^{-10}\,\mathrm{m}^2. \qquad (10)$$

Let us consider, as an example, a dry (Fig. 1) and a wet (Fig. 2) compression experiment. The same mat served for each, the wet tests being performed first. The figures are tracings of original oscilloscope records. The curves marked P_1 represent the apparent applied stress, and are to be corrected for the inertial effect of the piston; the correction is derived from the curves marked L, which show the displacement. The fluid pressure curves are marked $p'(0, t)$. Typical numerical values follow.

Mat	L (initial) mm	L (final) mm	P_1 (initial) kPa	P_1 (final) kPa	$p'(0, t)$ (maximum) kPa
Wet	6·222	3·625	6·881	3 683	14·49
Dry	6·826	3·272	6·153	4 764	—

The fluid pressure remained constant during the dry compression experiment. Evidence of a time-dependent response to applied stress appeared when changes in the measured thickness L were compared with the changes which

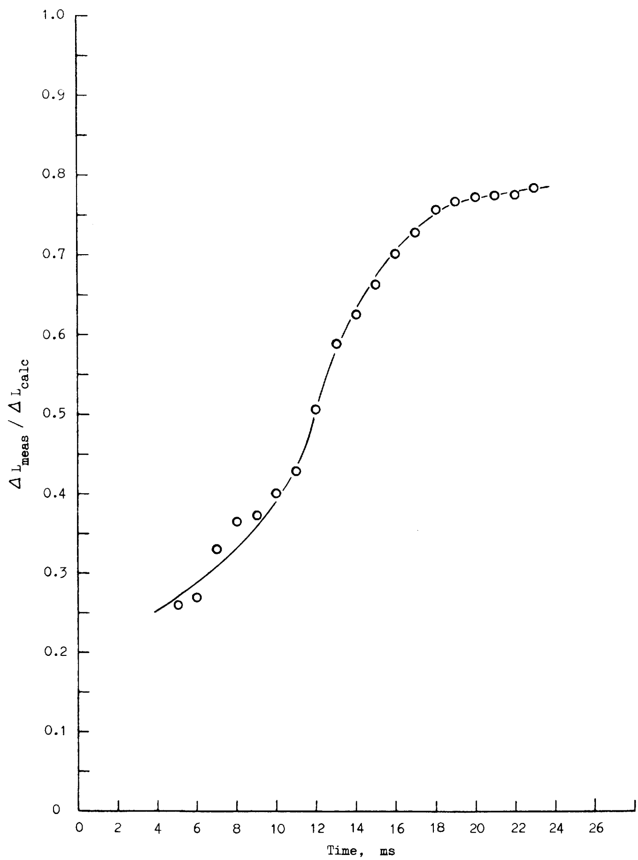

Fig. 3—Time-dependent response in dry compression

would be computed from the static compressibility function and the (corrected) applied stress, as in Fig. 3.

The wet compression experiment, under conditions which also resulted in approximate spatial uniformity in the mat, shows a sharp maximum in the fluid pressure $p'(0, t)$, which coincides with the period of most rapid change in L. Fig. 4 shows computed and measured values of the fluid pressure, as functions of the time. The measured values have a delayed (and higher) maximum, which we consider to reflect a time-dependent response in the wet mat to applied stress.

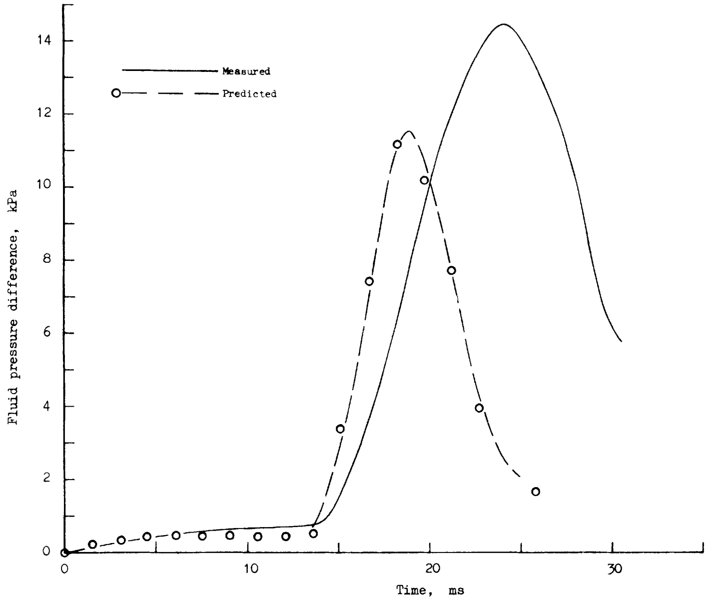

Fig. 4—Comparison of measured and predicted fluid pressure

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