# Joint Decision on Wooden Pallet Lease Pricing and Purchase Volume under Recycling and Reusing Mode in the Chinese Market

Jiali Cai, Jinzhuo Wu,* Hui Wang, and Tongtong Wu

A closed-loop wooden pallet rental system was considered in this study, which involved a wooden pallet manufacturer, a wooden pallet leasing company, and several customers. To solve the joint optimization problem of pallet lease pricing and purchase volume under recycling and reusing mode, a mathematical model was constructed with the objective of maximizing the profit of pallet renters under deterministic and stochastic market demand. Meanwhile, comparative analyses were conducted on the optimal pricing, optimal order quantity, and expected profit under the two leasing modes of considering and not considering maintenance. On this basis, sensitivity analyses were performed on some parameters of the two modes. Results showed that pallet renters can adopt the maintenance strategy to increase profits by lowering the rental price appropriately regardless of the market demand. The wood pallet rental supply chain can be more efficient when the pallet residual value, recovery integrity rate, and utilization rate are higher and the out of stock cost and inventory holding cost are lower. Under the maintenance mode, a lower repairing price and higher reparability rate resulted in a more favorable maintenance mode.

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*Contact information: College of Engineering and Technology, Northeast Forestry University, Harbin 150040, China; *Corresponding author: wjz@nefu.edu.cn*

## INTRODUCTION

Pallets are widely used assembly equipment in production, transportation, warehousing, and distribution that play an important role in many aspects of logistics operations. In recent years, the pallet industry in China has been developing rapidly. The annual production of pallets in China in 2020 was about 340 million pieces, with a year-on-year growth of 13.3%; the market holdings of pallets reached 1.55 billion pieces, with a year-on-year growth of 6.9% (Sun and Wang 2021). It is known that the logistics cost in China has been high, much higher than that in developed countries. This is mainly due to the lower level of standardized logistics operations and lower scale efficiency in China. According to the experience of logistics operation, the recycling of pallets is conducive to significantly improving logistics efficiency and reducing logistics costs. Currently, pallets are widely used in both storage and transportation in China, and the major mode of pallet pooling is pallet rental. In 2020, the scale of pallet pool in China exceeded 28 million pieces, with a year-on-year growth of 12% (Sun and Wang 2021). Although China's pallet recycling has reached a certain scale, it is still in the development stage, and there are still many problems in the actual operation process. Therefore, it is necessary to study the issues related to lease pricing and order quantity of pallets for leasing to promote the circular sharing of pallets, improve the efficiency of logistics operation, and reduce the cost of logistics operation.

Currently, most of the studies related to pallet leasing are focused on the operation mode (Wang and Gu 2014; Wang 2018; Gu *et al*. 2019), leasing benefits (Bengtsson and Logie 2015; Carrano *et al*. 2015; Tornese *et al*. 2016, 2018), pallet leasing optimization of supply chain management (Kesen and Alim 2019; Liu *et al*. 2019; Ren and Gao 2019; Chen *et al*. 2020; Ren *et al*. 2020), and so on. Theoretically, there were three types of pallet management, including transfer of ownership, pallet exchange, and pallet pooling (Harris and Worrell 2008). Based on the operational practice in China, the business modes of pallet leasing can be divided into five types, namely pallet rental mode, pallet exchange mode, pallet sales mode, pallet buy-back rental mode, and pallet financing rental mode (Wang 2018). During the process of circulation, there may exist some differences among the pallets in appearance, degree of depreciation, and usage intensity. Therefore, relatively fair and reasonable pricing standards and an accurate billing system are indispensable to achieve efficient pallet rental. Thus far, several studies have been conducted to analyze the issue of pallet rental pricing from the aspects of cost and market demand. For example, Xu (2011) developed a lease optimization pricing model by taking the lease demand and inventory control of pallets into consideration and conducted a sensitivity analysis on the aspect of pallet return rate. Zhao (2012) introduced the idea of revenue management, simulated the pricing strategies of three pallet leasing modes with different circulation characteristics, and further analyzed the coordination of the models. Wu *et al*. (2015) proposed a new pricing method based on revenue management and established the mathematical model from the two aspects of market execution price and actual execution price of key customers. The results of the case study showed that this method can increase the enterprise's revenue by 5% compared to the previous year. Hu and Zheng (2020) constructed a revenue distribution model for the pallet rental supply chain based on the newsboy model and obtained the optimal wholesale price and revenue sharing rate at the state of Pareto optimization through simulation analysis. With regard to the study on reverse supply chain pricing, Yang *et al*. (2018) reconstructed the demand functions of new products and remanufactured products under three different recycling and remanufacturing channel structures, established the corresponding manufacturing and remanufacturing optimization models, and obtained the optimal production and pricing decision. For the dual channel closed-loop supply chain model composed of remanufacturers, retailers, and recyclers, Hosseini-Motlagh *et al*. (2019) studied the optimal pricing decision of enterprises when online channel demand was interrupted under centralized and decentralized decision-making respectively and coordinated the supply chain by using two pricing contracts to improve the environment while increasing the profits of the enterprises. Chen *et al*. (2019) analyzed pricing strategy of the closed-loop supply chain in which retailers were responsible for recycling and manufacturers were responsible for remanufacturing and pointed out that the profit of the whole supply chain system was the highest under the joint pricing strategy. Based on the uncertainty of customer perceived value and recycling quality, Dong *et al*. (2021) analyzed the pricing of new products and remanufactured products and the optimal value of market demand and profits of both parties in the closed-loop supply chain under three modes, as well as the impact of recycling quality and customer perceived value on decision variables.

It is known that pallets can be divided into wooden pallets, metal pallets, plastic pallets, cardboard pallets, and bamboo pallets according to the material. Among them, the plastic pallets and wooden pallets are most widely used. Compared with the irreparable nature of the damaged plastic pallet, the actual wear and tear of wooden pallets is much lower due to the fact that broken wooden pallets can be put back into the pallet pool for use after simple repair and maintenance. However, the existing literature on pallet rental has rarely addressed the economic issues of wood pallet rental supply chain with consideration of the repair and refurbishment of wooden pallets. Therefore, combined with the reality of the wooden pallet leasing industry, this paper investigates the joint optimization problem of single-cycle order quantity and pricing for wooden pallet rental by considering maintenance. The objectives of this paper are to: (1) Construct joint optimization models for wooden pallet rental pricing and order quantity for the two scenarios of considering pallet maintenance or not under deterministic and stochastic market demand; (2) Compare the optimal pricing, optimal purchase volume and expected profit in different scenarios to find out the key influencing factors; and (3) Put forward some business suggestions for the wooden pallet renters based on the analysis results.

## EXPERIMENTAL

### Problem Description and Hypotheses

In this paper, a profit model was constructed for a closed-loop wooden pallet rental supply chain, which consisted of a wooden pallet manufacturer, a wooden pallet leasing company, and several customers. The closed-loop wooden pallet rental system is shown in Fig. 1.

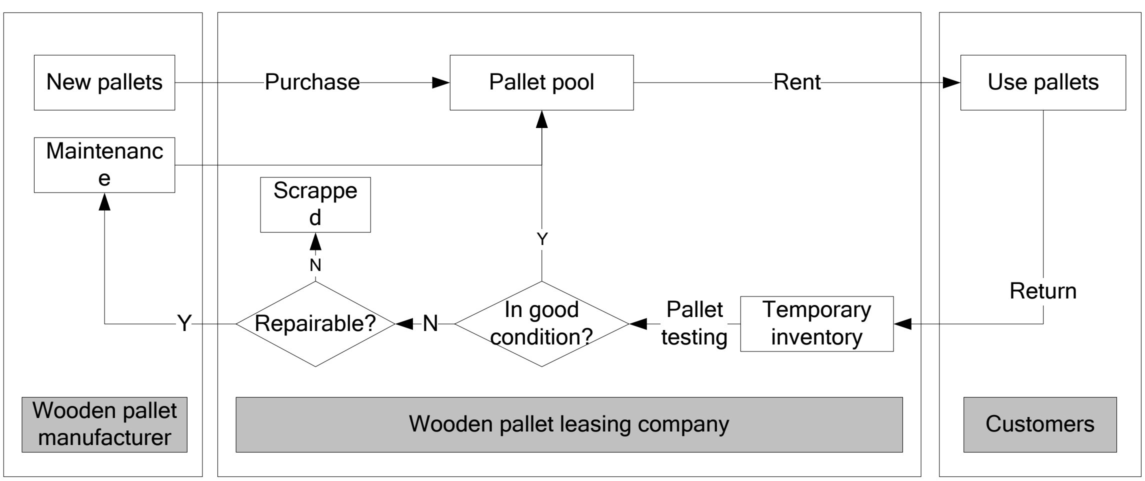



**Fig. 1.** The closed-loop wooden pallet rental system

The operation of the wooden pallet rental supply chain was as follows: The wooden pallet leasing company only purchases pallets at the beginning of the lease period. Customers arrive randomly during the rental period, and the pallet leasing company meets the customers' rental needs by taking pallets from the pallet pool in the order of their arrival. The customer returns the pallets after use and the pallet leasing company inspects all returned wooden pallets. If the returned pallet is intact, it is returned directly to the pallet pool. For broken pallets, if the damage degree is within the repairable range, it is repaired and the pallet leasing company bears the repairing costs; otherwise, it is scrapped. After that, the tested pallets and the repaired pallets can be leased again in the next lease period.

The re-leasing problem of the returned pallet in the second round was not considered in this study.

The hypotheses and definitions of the relevant parameters in this paper are as follows:

- It is assumed that the pallet leasing company can accurately predict the integrity proportion and breakage proportion of the returned pallets during the lease period based on the historical lease data. Let the pallet recovery integrity rate be α, the broken pallet reparability proportion is β, then the pallet loss proportion is γ (including lost and scrapped pallets) can be expressed as: $\gamma = 1 - \alpha - \beta (0 \le \alpha, \beta, \gamma \le 1)$.
- For per 1,000 wooden pallets, the purchase cost is $w$, the out-of-stock cost is $g$, the holding cost is $h$, the maintenance cost is $m$, and the residual value of per 1,000 pallets at the end of the lease period is $s$, $m \le s \le w$.
- The number of pallets purchased by the rental company at the beginning of the period is $q$, the duration of each pallet rented by the customer is $T$, and the total number of pallets rented during the whole lease period is $Y$. The decision-making goal of the wooden pallet leasing company is to determine the optimal rental price $p^*$ and the optimal pallet purchase volume $q^*$, so as to achieve the maximum leasing profit $Z^*$.

The relevant parameters are summarized in Table 1.

**Table 1.** Description of the Model Parameters

| Parameter | Unit | Parameter Definition |
|---|---|---|
| $a$ | PCS | The maximum market demand for pallet rental |
| $b$ | / | The elasticity coefficient of market demand |
| α | / | The integrity proportion of the returned pallets |
| β | / | The reparability proportion of the broken pallets |
| γ | / | The loss proportion of pallets (including lost pallets and scrapped pallets) |
| $g$ | CNY | The daily out-of-stock cost per 1,000 pallets |
| $h$ | CNY | The daily holding cost per 1,000 pallets |
| $w$ | CNY | The purchase cost per 1,000 pallets |
| $m$ | CNY | The maintenance cost per 1,000 pallets |
| $s$ | CNY | The residual value per 1,000 pallets at the end of the lease period |
| $k$ | / | The cost coefficient of pallet maintenance |
| $l$ | / | The price coefficient of the pallet residual value at the end of the lease period |
| $T$ | CNY | The duration of each pallet rented by the customer |
| $Y$ | PCS | The total number of pallets rented during the entire lease period (1,000 pallets) |

## Maximizing Wooden Pallet Leasing Profit Under Deterministic Market Demand

*Profit model and solutions with consideration of maintenance*

When the market demand is deterministic, it can be expressed as $Y = D(p) = a - bp (a > 0, b > 0)$ (unit: 1,000 pallets), where $a$ is the maximum volume demanded and $b$ is the elasticity coefficient of the demand. It was assumed that there is no shortage of stock under deterministic market demand and that the pallet holding cost is constant. Therefore, the out-of-stock cost $g$ and the holding cost $h$ per pallet were not considered in constructing the profit model. The optimal inventory strategy when considering maintenance was as follows: Firstly, a portion of the rental order is met with wooden pallets purchased at the beginning of the rental period. Secondly, when the demand cannot be met by the existing pallets in the pallet pool, the unfulfilled rental orders are met with the pallets returned in good condition from the first-round rental and repaired pallets. From $D(p) - q = (\alpha + \beta)q$, the order quantity of pallets was obtained as $q = (a - bp) / (1 + \alpha + \beta)$. Thus, the leasing profit model can be expressed as:

$$Z = pT(1 + \alpha + \beta)q - wq - m\beta(1 + \alpha + \beta)q + sq(\alpha + \beta)^2 \quad (1)$$

**Theorem 1:** Under a deterministic market demand, there exists a unique optimal leasing price $p^*$.

**Proof:** Calculating the first derivative of Eq. 1 with respect to $p$, presents Eq. 2 as follows:

$$\partial Z(p) / \partial p = aT + mb\beta - 2bpT + b[w - s(\alpha + \beta)^2] / (1 + \alpha + \beta) \quad (2)$$

If Eq. 2 is zero, then the optimal price is as follows:

$$p^* = a / 2b + m\beta / 2T + [w - s(\alpha + \beta)^2] / 2T(1 + \alpha + \beta) \quad (3)$$

Calculating the first derivative of Eq. 1 with respect to $p$, with $p = p^*$, presents Eq. 4 as follows:

$$\partial^2 Z(p) / \partial p^2 = -2bT < 0 \quad (4)$$

Therefore, $Z$ achieves the maximum value at $p^*$, and the optimal pallet purchase amount $q^*$ is:

$$q^* = (a - bp^*) / (1 + \alpha + \beta) \quad (5)$$

And, the value of the expected profit $Z^*$ is as follows:

$$Z^* = (a - bp^*)\{p^*T - m\beta - [w - s(\alpha + \beta)^2] / (1 + \alpha + \beta)\} \quad (6)$$

*Profit model without considering maintenance and solution comparison*

When maintenance is not considered, $\beta = 0$, $m = 0$, and the rest of the assumptions are the same as above. Then, the optimal price can be expressed as:

$$p_0^* = a / 2b + (w - s\alpha^2) / 2T(1 + \alpha) \quad (7)$$

The optimal purchase volume is obtained as:

$$q_0^* = (a - bp_0^*) / (1 + \alpha) \quad (8)$$

The maximum profit is as follows:

$$Z_0^* = (a - bp_0^*)[p_0^*T - (w - s\alpha^2) / (1 + \alpha)] \quad (9)$$

Comparing the optimal price solutions with and without maintenance, presents Eq. 10 as follows:

$$\Delta p = p^* - p_0^* = \beta[(m - 2sw) + (m - 2s)(2\alpha + \beta + \alpha\beta + \alpha^2)] / 2T(1 + \alpha)(1 + \alpha + \beta) \quad (10)$$

Because $w > s > m$, then $m - 2sw < 0$, $m - 2s < 0$, and $\Delta p < 0$. Based on the above equation, it is clear that the leasing price of wooden pallets decreases under the maintenance strategy.

Comparing the procurement volume and expected profitability of the wooden pallet rental supply chain with and without considering maintenance, the results are as follows:

$$\Delta q = (a - bp^*) / (1 + \alpha + \beta) - (a - bp_0^*) / (1 + \alpha) \quad (11)$$

$$\Delta Z = (a - bp^*)\{p^*T - m\beta - [w - s(\alpha + \beta)^2] / (1 + \alpha + \beta)\} - (a - bp_0^*)[p_0^*T - (w - s\alpha^2) / (1 + \alpha)] \quad (12)$$

Clearly, after adding the business of repairing wooden pallets, the impact on procurement volume and profit could not be observed intuitively from Eqs. 11 and 12. Therefore, the impact of whether or not to repair damaged wooden pallets recovered by leasing on the opening purchase volume and expected profit of the leaser was analyzed in the subsequent calculations.

## Maximizing Wooden Pallet Leasing Profit Under Stochastic Market Demand

*Profit model and solutions considering maintenance*

When the market demand is stochastic, it can be expressed as $Y = D(p) + \varepsilon = a - bp + \varepsilon (a > 0, b > 0)$, where $\varepsilon$ is an additive random demand factor, a random variable defined on $[A, B]$ with mean $\mu$. When considering repairing the damaged wooden pallets returned from the lease, pallets that can be rented again are divided into intact pallets and pallets that can be leased again after maintenance. The quantities of the two kinds of pallets are called the leased-back intact volume and the leased-repairable pallet volume, respectively. Let $t = q - D(p)$. According to different market demands $\varepsilon$, the leasing situations can be divided into the following three scenarios:

(1) When $A < \varepsilon \leq t$, the first-turn rental demand can be fully satisfied. The rental quantity is $X = X_1 = D(p) + \varepsilon$, the inventory is $t - \varepsilon$, the leased-back intact quantity is $R(p, \varepsilon) = \alpha[D(p) + \varepsilon]$, the leased-back repairable pallet quantity is $M(p, \varepsilon) = \beta[D(p) + \varepsilon]$, and the quantity of pallets at the end of the lease period is $t - \varepsilon + R(p, \varepsilon) + M(p, \varepsilon)$.

When $\varepsilon > t$, the demand is partially satisfied. The rental company first meets the customer demand with the existing pallets, and the remaining demand is met by the returned pallets that are in good condition and the repaired pallets that can be put into use again. In this case, the first-turn rental quantity is $X_1 = D(p) + t$, the unfulfilled demand is $\varepsilon - t$, the first leased-back intact quantity is $R(p, t) = \alpha[D(p) + t]$, and the repairable pallet quantity after first leasing is $M(p, t) = \beta[D(p) + t]$.

(2) If $0 < \varepsilon - t \leq R(p, t) + M(p, t)$, *i.e.*, when $t < \varepsilon \leq + R(p, t) + M(p, t)$, the remaining rental demand can be fulfilled by the pallets recovered in good condition from the first rental return and pallets that can be put back into use after repair from the rental return. The second-turn rental quantity is $X_2 = \varepsilon - t$, the second-turn inventory is $R(p, t) + M(p, t) - (\varepsilon - t)$, the second leased-back intact quantity is $R(p, \varepsilon - t) = \alpha(\varepsilon - t)$, the repairable pallet quantity after second leasing is $M(p, \varepsilon - t) = \beta(\varepsilon - t)$, and the quantity of pallets at the end of the lease period is $R(p, t) + M(p, t) - (\varepsilon - t) + R(p, \varepsilon - t) + M(p, \underline{\varepsilon} - t)$.

(3) If $\varepsilon - t > R(p, t) + M(p, t)$, *i.e.*, when $\varepsilon > t + R(p, t) + M(p, t)$, the remaining rental demand cannot be met by pallets recovered in good condition from the first rental return and pallets that can be put back into use after repair from the rental return. The second-turn rental quantity is $X_2 = R(p, t) + M(p, t)$, the out of stock volume is $\varepsilon - t - R(p, t) - M(p, t)$, the second leased-back intact quantity is $R(p, R(p, t) + M(p, t)) = \alpha[R(p, t) + M(p, t)]$, the repairable pallet quantity after the second leasing is $M(p, R(p, t) + M(p, t)) = \beta[R(p, t) + M(p, t)]$, and the quantity of pallets at the end of the lease period is $R(p, R(p, t) + M(p, t)) + M(p, R(p, t) + M(p, t))$.

Therefore, the segmented leasing profit model can be expressed as follows:

$$Z = \begin{cases} (pT-w)(D(p)+\varepsilon) - hT(t-\varepsilon) - mM(p,\varepsilon) - hT\big(t-\varepsilon+R(p,\varepsilon)+M(p,\varepsilon)\big) + s\big(R(p,\varepsilon)+M(p,\varepsilon)\big), A<\varepsilon\le t \\ (pT-w)(D(p)+t) - gT(\varepsilon-t) - mM(p,t) + pT(\varepsilon-t) - hT(R(p,t)+M(p,t)-\varepsilon+t) - mM(p,\varepsilon-t) \\ \quad +s\big(R(p,t)+M(p,t)-\varepsilon+t+R(p,\varepsilon-t)+M(p,\varepsilon-t)\big), \quad t<\varepsilon\le t+R(p,t)+M(p,t) \\ (pT-w)(D(p)+t) - gT(\varepsilon-t) - mM(p,t) + pT\big(R(p,t)+M(p,t)\big) - gT\big(\varepsilon-t-R(p,t)-M(p,t)\big) \\ -mM\big(p,R(p,t)+M(p,t)\big) + s\Big(R\big(p,R(p,t)+M(p,t)\big)+M\big(p,R(p,t)+M(p,t)\big)\Big), t+R(p,t)+M(p,t)<\varepsilon\le B \end{cases} \tag{13}$$

When $\varepsilon$ is generally distributed within [$A$, $B$], it is difficult to derive the analytical solutions of optimal pricing and optimal purchase quantity from the above equation. In the following, the optimal inventory and pricing strategy when $\varepsilon$ follows a uniform distribution within the range of ($-\delta$, $\delta$) is considered. At this point:

$$f(\varepsilon) = 1 / 2\delta \tag{14}$$

Then, the expected profit $E(Z)$ can be expressed as follows:

$$E(Z) = \frac{1}{2\delta}\Big\{(pT-w)\Big[\int_{-\delta}^{t}(a-bp+\varepsilon)\,d\varepsilon + \int_{t}^{\delta}(a-bp+t)\,d\varepsilon\Big] + pT\Big[\int_{t}^{t+(\alpha+\beta)(a-bp+t)}(\varepsilon-t)\,d\varepsilon + \int_{t+(\alpha+\beta)(a-bp+t)}^{\delta}(\alpha+\beta)(a-bp+t)\,d\varepsilon\Big] - hT\Big[\int_{-\delta}^{t}(t-\varepsilon)\,d\varepsilon + \int_{-\delta}^{t}(t-\varepsilon+(\alpha+\beta)(a-bp+\varepsilon))\,d\varepsilon + \int_{t}^{t+(\alpha+\beta)(a-bp+t)}((\alpha+\beta)(a-bp+t)-\varepsilon+t)\,d\varepsilon\Big] - gT\Big[\int_{t}^{\delta}(\varepsilon-t)\,d\varepsilon + \int_{t+(\alpha+\beta)(a-bp+t)}^{\delta}(\varepsilon-t-(\alpha+\beta)(a-bp+t))\,d\varepsilon\Big] - m\Big[\int_{-\delta}^{t}\beta(a-bp+\varepsilon)\,d\varepsilon + \int_{t}^{\delta}\beta(a-bp+t)\,d\varepsilon + \int_{t}^{t+(\alpha+\beta)(a-bp+t)}\beta(\varepsilon-t)\,d\varepsilon + \int_{t+(\alpha+\beta)(a-bp+t)}^{\delta}\beta(\alpha+\beta)(a-bp+t)\,d\varepsilon\Big] + s\Big[\int_{-\delta}^{t}(\alpha+\beta)(a-bp+\varepsilon)\,d\varepsilon + \int_{t}^{t+(\alpha+\beta)(a-bp+t)}((\alpha+\beta)(a-bp+t)-\varepsilon+t+(\alpha+\beta)(\varepsilon-t))\,d\varepsilon + \int_{t+(\alpha+\beta)(a-bp+t)}^{\delta}(\alpha+\beta)^2(a-bp+t)\,d\varepsilon\Big]\Big\} \tag{15}$$

**Theorem 2:** Under the condition of giving the rental price, if $\frac{\partial^2 E(Z)}{\partial t^2} < 0$, there exists a unique optimal purchase quantity $q^*$.

**Proof:** Calculating the first derivative of Eq. 15 with respect to $t$, presents Eq. 16 as follows:

$$\partial E(Z) / \partial t = \{(\delta - t)[(pT - m\beta)(1 + \alpha + \beta) - w + gT(\alpha + \beta + 2) + s(\alpha + \beta)^2] + (\alpha + \beta)(1 + \alpha + \beta)(a - bp + t)[m\beta + s(1 - \alpha - \beta) - (p + h + g)T] - 2hT(t + \delta)\} / 2\delta \tag{16}$$

If Eq. 16 is zero, then the optimal price can be expressed as follows:

$$t^* = \{(\alpha + \beta)(a - bp)[(1 + \alpha + \beta)(g + h + p)T - \beta m - s] + (\alpha + \beta)^2[(a - bp)(\alpha s + \beta s - \beta m) - \delta s] + \delta(1 + \alpha + \beta)[\beta m - (g + p)T] + \delta[(2h - g)T + w]\} / \{\beta m + w - (\alpha + \beta)^2 [T(g + h + p) - (\beta + 1)(m - s) + \alpha s] - (\alpha + \beta)[(2g + h + 2p)T - s + (\alpha - \beta)m] - (2g + 2h + p)T\} \tag{17}$$

Calculating the second derivative of Eq. 15 with respect to $t$, presents Eq. 18 as follows:

$$\partial^2 E(Z) / \partial t^2 = \{(\alpha+\beta)^2 (m\beta - s - (p+h+g)T) + (\alpha+\beta)[2m\beta + s - (2p-h-2g)T] - s(\alpha+\beta)^3 + w + m\beta - (2h+p+2g)T\} / 2\delta \tag{18}$$

It is known that with given price $p$, if $\partial^2 E(Z) / \partial t^2 < 0$, when $t = t^*$, the expected profit achieves the maximum value $Z^*$, and the optimal purchase volume is as follows:

$$q^* = a - bp^* + t^* \tag{19}$$

**Theorem 3:** With the given $t$, there exists a unique optimal pricing $p^*$ when the discriminant of $\partial E(Z) / \partial p = 0$ is greater than zero and has a positive number of solutions.
**Proof:** Calculating the first derivative of Eq. 15 with respect to $p$, presents Eq. 20 as follows:

$$\partial E(Z) / \partial p = \{(\alpha+\beta)^2(a-bp+t)[2bT(p+h+g) + 2bs(\alpha+\beta) - (a-bp+t)T - 2m\beta b] + 2(t-\delta)(\alpha+\beta)[(bg-a+2bp-t)T - m\beta b + bs(\alpha+\beta)(bp-a-\delta)] + 2b(\alpha+\beta)(t+\delta)(hT-s) + 2T\delta(2a-4bp+t) + 4b\delta(w+m\beta) - (\delta^2 + t^2)T\} / 4\delta \tag{20}$$

It is known from Eq. 20 that $\partial E(Z) / \partial p$ is a second-order polynomial about $p$. When the discriminant is greater than zero, $\partial E(Z) / \partial p = 0$ has solutions $p_1$ and $p_2$. If $p_1 < p_2$, then finding the third-order partial derivative of $p$ with respect to Eq. 15, we get

$$\partial^3 E(Z) / \partial t^3 = -(3T(\alpha+\beta)^2 b^2) / 2\delta < 0 \tag{21}$$

That is, the function of $\partial E(Z) / \partial p$ is a parabola with an opening downward and the following results can be obtained:

$$\begin{cases} \partial^2 E(Z)/\partial p^2|_{p=p_1} > 0 \\ \partial^2 E(Z)/\partial p^2|_{p=p_2} < 0 \end{cases} \tag{22}$$

If $p_2 > 0$, it is the unique extreme value point of $E(Z)$ at $(0, +\infty)$. That is, $p^* = p_2$ is the only optimal pricing such that the expected profit $E(Z)$ achieves the maximum value.

According to Theorems 2 and 3, the joint optimal calculation of pricing and purchasing volume for a wood pallet rental supply chain considering maintenance under stochastic market demand is as follows:

(1) Solve the equation set:

$$\begin{cases} \partial E(Z)/\partial t = 0 \\ \partial E(Z)/\partial p = 0 \end{cases} \tag{23}$$

(2) Take the set of solutions ($p^*$, $t^*$) where $p$ is the largest positive solution among the solutions of Eq. 23.

(3) The optimal price is $p_0^*$ and the optimal purchase volume $q^* = a - bp + t^*$.

*Profit model without considering maintenance*

When maintenance is not considered, β = 0, $m$ = 0, and the rest of the assumptions are the same as above. Then the expected profit without considering maintenance is as follows.

$$E(Z_0) = \frac{1}{2\delta}\{(p_0T - w + \alpha s - \alpha hT)\int_{-\delta}^{t_0}(a - bp_0 + \varepsilon)\,d\varepsilon + (p_0T + hT + \alpha s - gT - s)\int_{t_0}^{t_0+\alpha(a-bp_0+t_0)}(\varepsilon - t_0)\,d\varepsilon + (p_0T - w + \alpha s -$$

$$\alpha hT)\int_{t_0}^{t_0+\alpha(a-bp_0+t_0)}(a-bp_0+t_0)\,d\varepsilon+(p_0T-w+\alpha p_0T+\alpha^2 s+\alpha gT)\int_{t_0+\alpha(a-bp_0+t_0)}^{\delta}(a-bp_0+t_0)\,d\varepsilon-2hT\int_{-\delta}^{t_0}(t_0-\varepsilon)\,d\varepsilon-2gT\int_{t_0+\alpha(a-bp_0+t_0)}^{\delta}(\varepsilon-t_0)\,d\varepsilon\} \quad (24)$$

Calculating the first derivative of Eq. 24 with respect to $t$, the result is as follows:

$$\partial E(Z_0)\,/\,\partial t_0=\{\alpha(1+\alpha)(a-b\,p_0+t_0)(s-\alpha s-p_0T-hT-gT)-2hT(t_0+\delta)+(\delta-t_0)[\,p_0T(1+\alpha)-w+gT(\alpha+2)+s\alpha^2]\}\,/\,2\delta \quad (25)$$

Calculating the first derivative of Eq. 24 with respect to $p$, the result is as follows:

$$\partial E(Z_0)\,/\,\partial p_0=\{\alpha^2(a-bp_0+t_0)[2bT(p_0+h+g)+2\alpha bs-(a-bp_0+t_0)T]+2\alpha(t_0-\delta)[(bg-a+2bp_0-t_0)T+\alpha bs\,(bp_0-a-\delta)]+2\alpha b(t_0+\delta)(hT-s)+2\delta T\,(2a-4bp_0+t_0)+4b\delta w-(\delta^2+t_0^2)T\}\,/\,4\delta \quad (26)$$

Then, the joint optimization calculation of pricing and purchasing volume for a wood pallet rental supply chain without considering maintenance under stochastic type market demand is as follows:

(1) Solve the equation set:

$$\begin{cases}\partial E(Z)/\partial t_0=0\\ \partial E(Z)/\partial p_0=0\end{cases} \quad (27)$$

(2) Take the set of solutions ($p_0$*, $t_0$*) where $p$ is the largest positive solution among the solutions of the system of Eq. 27.

(3) The optimal price is $p_0$* and the optimal purchase volume $q_0^* = a - bp_0^* + t^*$.

## Data Sources and Model Realization

Based on the market survey data related to the wooden pallet rental business in the Chinese Market, the input parameters of the model were set as follows: the integrity rate of pallet recovery is $\alpha = 0.8$, the reparability rate of broken pallets is $\beta = 0.1$, the purchase cost per 1,000 pallets is $w = 160{,}000$, the average maintenance cost per 1,000 pallets $m$ is 20% of the purchase cost, the residual value per 1,000 pallets at the end of the lease period $s$ is 70% of the purchase cost. Besides, assumed that the maximum market demand for pallet rental is $a = 100$, the elasticity coefficient of market demand is $b = 0.2$, the duration of each pallet rented by the customer is $T = 180$.

The software Maple 2021 was used for model derivation. Maple is math software that combines the world's most powerful math engine with an interface that makes it extremely easy to analyze, explore, visualize, and solve mathematical problems. The software Matlab 7.0 was used to conduct sensitivity analysis on the model parameters and make plots.

# RESULTS AND DISCUSSION

## Optimal Strategy Under Deterministic Market Demand

By substituting the input parameters into Eqs. 3 and 5 through 9, the results of optimal pricing, optimal purchase volume, and maximum profit with and without maintenance under deterministic market demand were derived, which are shown in Table

2. According to Table 2, the rental price of wooden pallets decreased 6.76%, the purchase volume increased by 16.5%, and the profit increased 51.2% under the deterministic market demand when considering maintenance and reuse. Thus, it can be seen that the pricing decreased and the profit increased when this strategy was adopted, which is a “win-win” situation for both pallet leasing companies and customers. The following is an analysis of the effect of these three variables on the optimal strategy: the maintenance cost per 1,000 pallets $m$, the residual value of 1,000 pallets at the end of the lease period $s$, and the duration of pallets rented by the customer $T$.

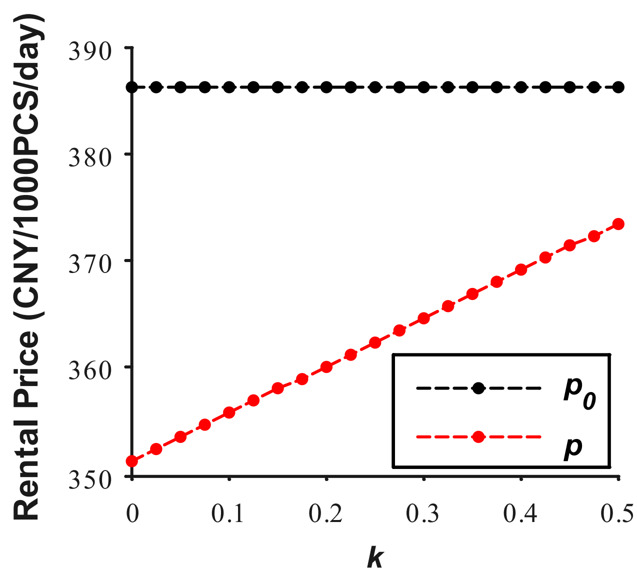
**Table 2.** Comparison of Optimal Strategies Under Deterministic Market Demand

| Maintenance Strategy | Optimal Rental Price (CNY/1,000PCS/day) | Optimal Purchase Volume (1,000 PCS) | Optimal Profit (CNY) |
|---|---|---|---|
| **No maintenance Mode** | 386.30 | 12.634 | 465,427.2 |
| **Maintenance Mode** | 360.18 | 14.718 | 703,832.7 |

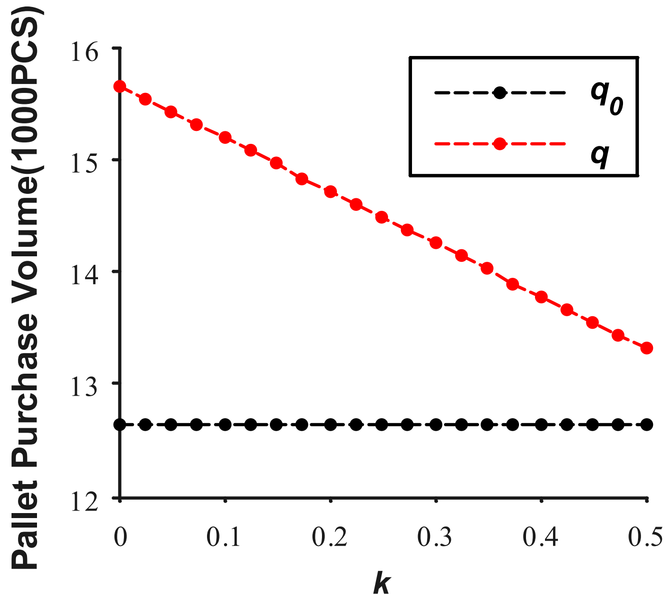
*Influence of maintenance cost on optimal strategy*

The maintenance cost of a pallet is related to the cost of the pallet. Let the cost coefficient of pallet maintenance be $k$, *i.e.*, $m = k \cdot w (0 \leq k \leq 1)$. When $k$ increased from 0 to 0.5, the comparison results of optimal pricing, optimal purchase volume, and expected profit with and without maintenance are shown in Fig. 2. Clearly, the maintenance cost had no effect on the optimal strategy when maintenance was not considered. When considering maintenance, the rental price of pallets was positively proportional to the maintenance cost, the pallet purchase volume and the expected profit was inversely proportional to the maintenance cost. Therefore, it is necessary to control the maintenance cost if the maintenance strategy is considered in the actual business operation.

(a)



(b)




(c)

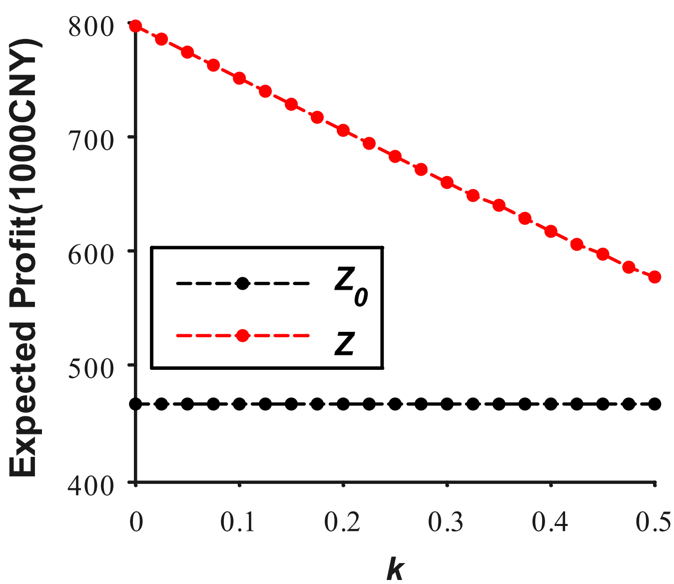
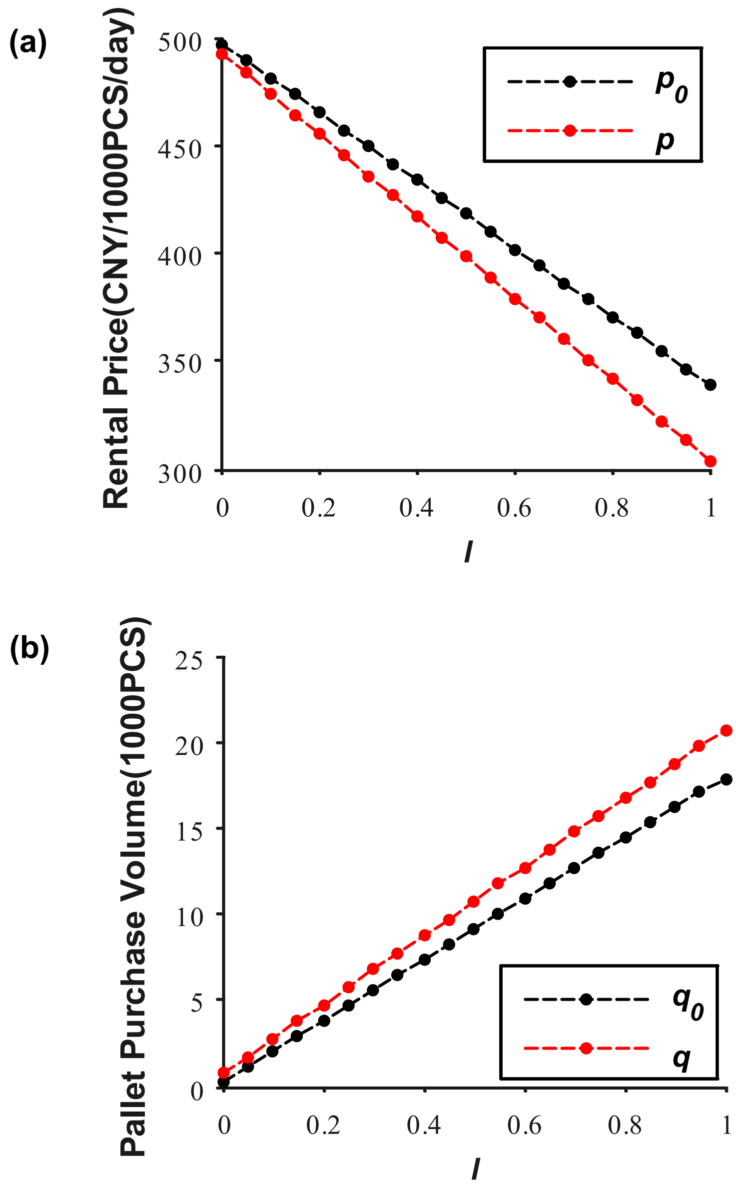



**Fig. 2.** Influence of maintenance cost on optimal strategy; (a) Change in rental price ($p$,$p_0$), (b) Change in purchase volume ($q$,$q_0$), and (c) Change in expected profit ($Z$,$Z_0$)

*Influence of pallet residual value on optimal strategy*

The residual value of the pallets at the end of the lease period is also a factor that affects the optimal strategy. The residual value of pallets at the end of the lease period can be expressed as a function of the pallet purchase cost. Let the price coefficient of the residual value of the pallet at the end of the period be $l$, $s = l \cdot w (0 \leq l \leq 1)$. When $l$ increased from 0 to 1, the comparison results of optimal pricing, optimal purchase volume, and expected profit with and without maintenance are shown in Fig. 3. It is shown that the pallet leasing price decreased 31.8% when not considering maintenance. When the maintenance strategy was adopted, the pallet leasing price decreased 38.4%, and the increase of expect profit was more significant when the residual value of the pallet was higher. Clearly, a higher residual value of the pallet resulted in greater benefits that can be brought to the wooden pallet rental supply chain by considering maintenance. Therefore, pallet rental companies can choose high quality pallets when making orders, which can bring higher economic benefits to enterprises.



(c)

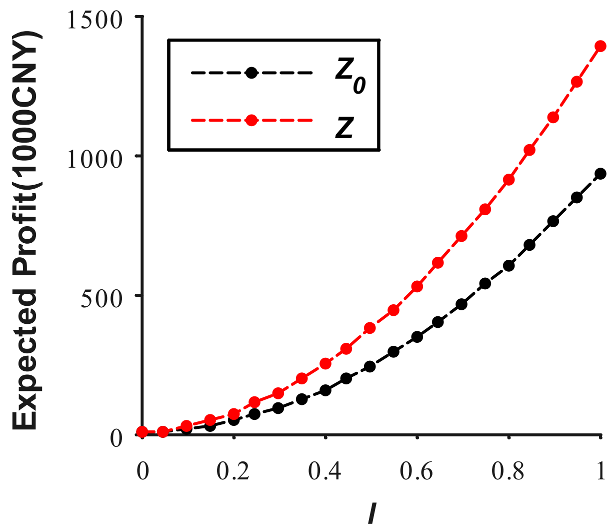
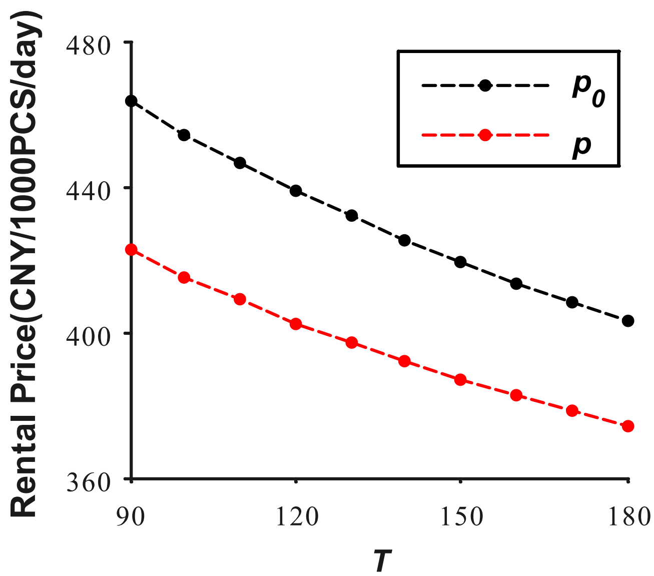



**Fig. 3.** Influence of pallet residual value on optimal strategy; (a) Change in rental price ($p$,$p_0$), (b) Change in purchase volume ($q$,$q_0$), and (c) Change in expected profit ($Z$,$Z_0$)

*Influence of pallet leasing duration on optimal strategy*

The pallet utilization rate plays a critical role in the operation of the pallet leasing supply chain. When the duration of each pallet rented by the customer $T$ increased from 90 to 180, the comparison results of the leasing price, purchase volume, and expected profit under the scenarios of with and without maintenance are shown in Fig. 4.

(a)



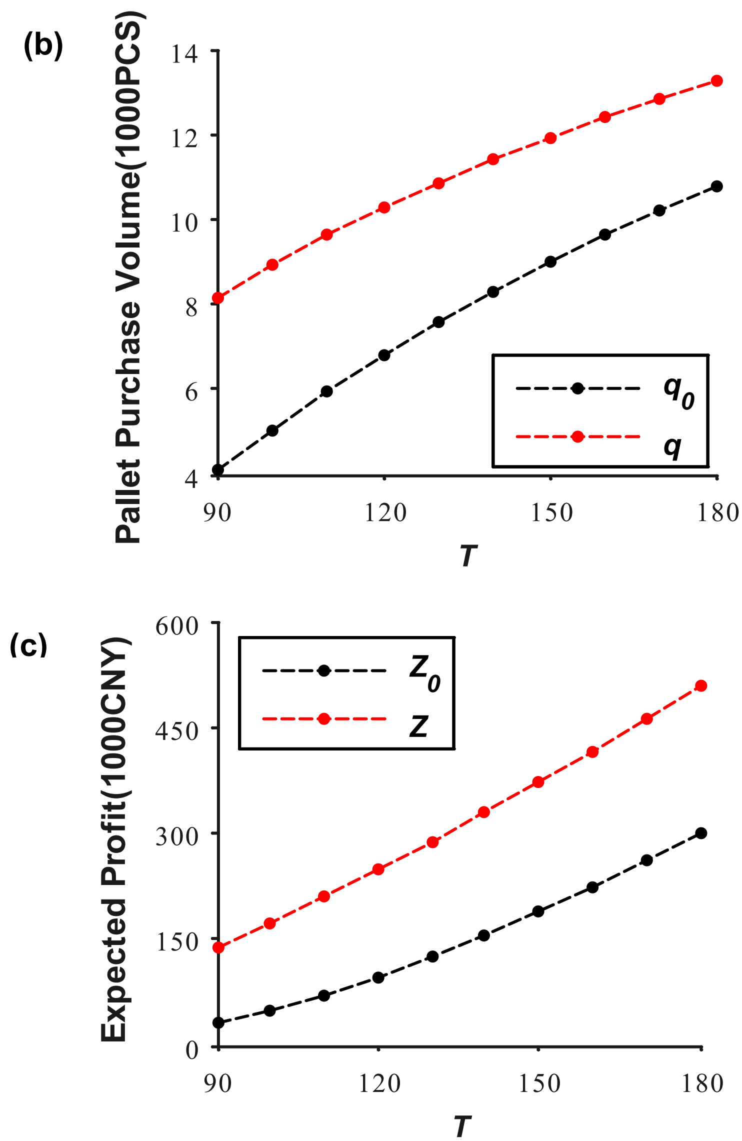



**Fig. 4.** Influence of pallet lease duration on optimal strategy; (a) Change in rental price ($p$, $p_0$), (b) Change in purchase volume ($q$, $q_0$), and (c) Change in expected profit ($Z$, $Z_0$)

It is known from the graph that when the lease duration was short, the expected profit was very small. In this case, the wooden pallet leasing supply chain cannot function properly. When the lease duration increased, the leasing price of wooden pallets decreased, and both of the purchase volume of the pallets and expected profit increased. In actual operation, pallet rental companies should focus on developing long-term customers and increasing the utilization rate of wooden pallets.

## Optimal Strategy Under Stochastic Market Demand

Under the stochastic market demand, the daily out-of-stock cost per 1,000 pallets $g = 100$, the daily holding cost per 1,000 pallets $h = 50$, and the market demand stochastic factor $\varepsilon$ follows a uniform distribution on (–5, 5) with mean 0. The other parameters are the same as the deterministic condition. Substitute the above parameters into Eqs. 23 and

27. The comparison results of optimal pricing, optimal purchase volume, and expected profit with and without maintenance are shown in Table 3.

**Table 3.** Comparison of Optimal Strategies Under Stochastic Market Demand

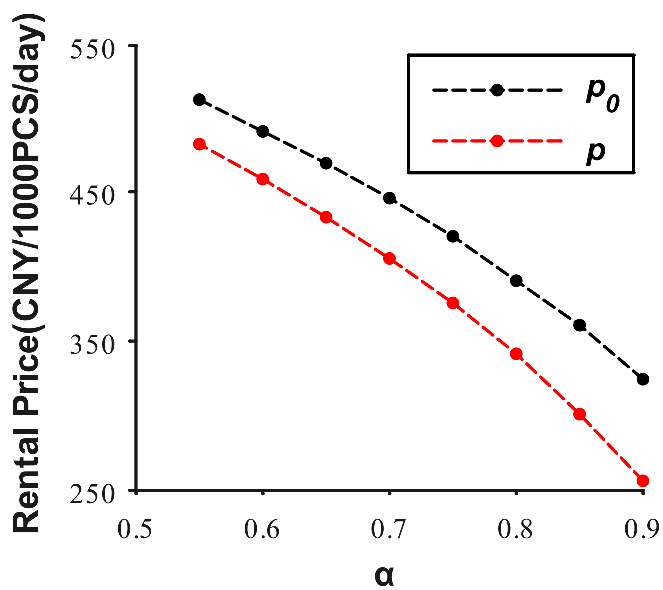
| Maintenance Strategy | Optimal Rental Price (CNY/1,000PCS/day) | Optimal Purchase Volume (1,000 PCS) | Optimal Profit (CNY) |
|---|---|---|---|
| **No maintenance Mode** | 391.28 | 12.461 | 229,019.5 |
| **Maintenance Mode** | 340.18 | 16.550 | 475,122.2 |

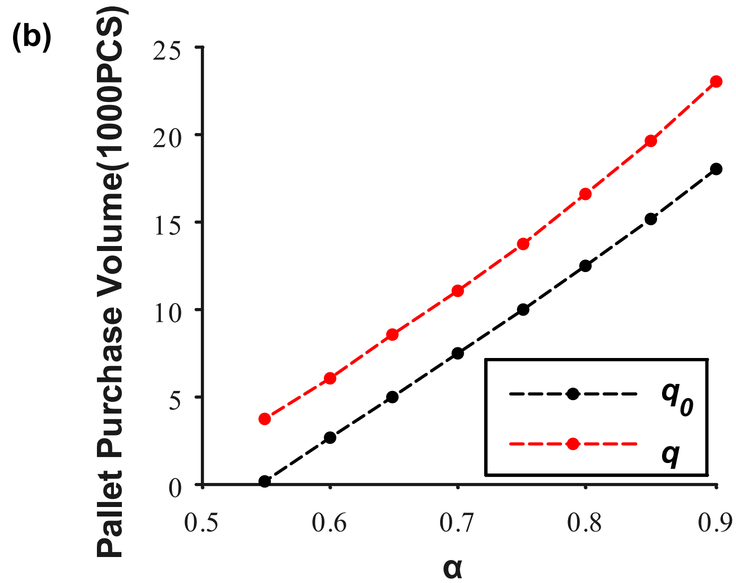
As shown in Table 3, the price decreased 13.06% and the profit was increased by 107.5% when adopting the rental strategy that considers repairing and reusing the recovered broken pallets. This result was similar to the analysis under deterministic market demand, indicating that the strategy is feasible while the market demand is random. To clarify the effects of pallet recycling volume, operating cost on the optimal strategy, the following sensitivity analyses were conducted for both strategies with and without considering repairing and reusing the recycled broken pallets.

*Impact of pallet recovery rate on optimal strategy*

When $\beta$ is constant and the pallet recovery integrity proportion $\alpha$ increased from 0.55 to 0.9, the results of the leasing price, purchase volume, and expected profit with and without considering maintenance are shown in Fig. 5.

(a)





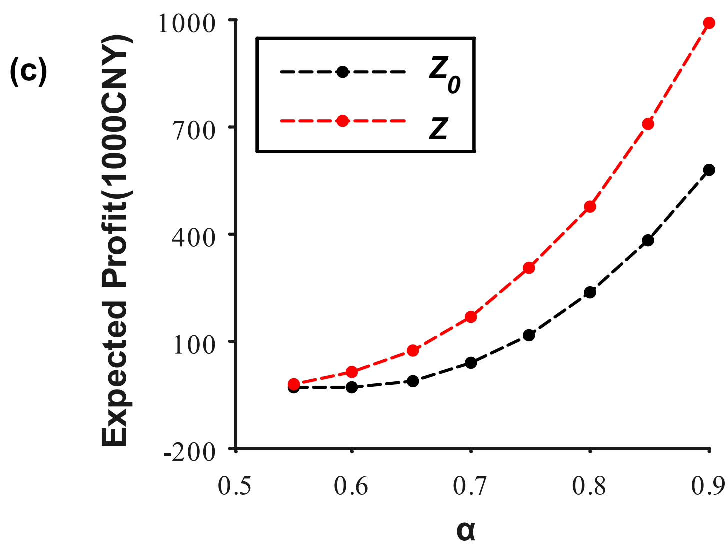
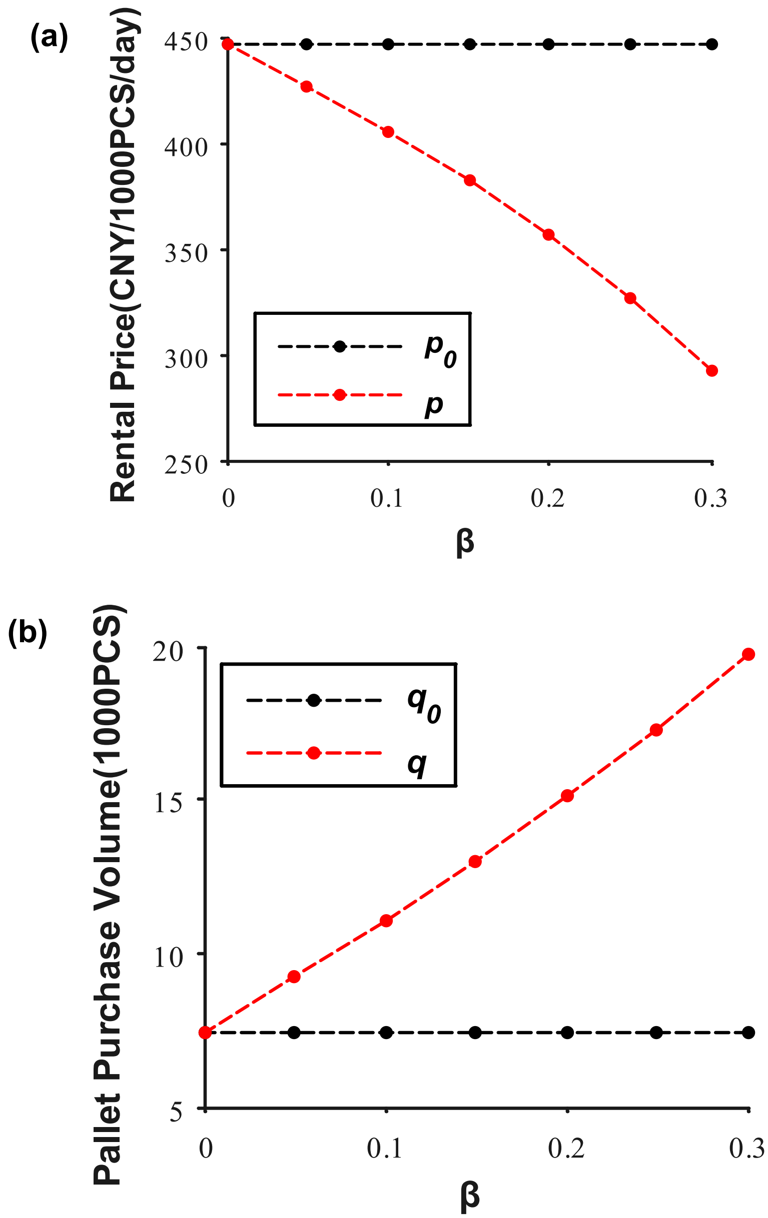




**Fig. 5.** Impact of recovery integrity on optimal strategy; (a) Change in rental price ($p$,$p_0$), (b) Change in purchase volume ($q$,$q_0$), and (c) Change in expected profit ($Z$,$Z_0$)

It can be seen that when the pallet recovery integrity proportion increased from 0.55 to 0.9, the leasing price decreased 88.6% and 58.0% for considering maintenance *versus* not considering maintenance, respectively. The expected profit also increases with the increase in the recovery integrity proportion. When the recovery integrity proportions were low, *i.e.*, $\alpha < 0.5655$ for maintenance strategy and $\alpha < 0.6812$ for non-maintenance strategy, the expected profit was negative. Therefore, ensuring the integrity proportion of pallet recovery is a necessary factor for the normal operation of wooden pallet leasing supply chain. The quality of recycled pallets can be effectively guaranteed by taking measures such as purchasing pallets with good quality and providing timely and correct operation guidance to customers in the daily leasing process.

The quality of pallets currently circulating in the pallet rental market varies, which is dependent on the utilization scenarios. According to the actual business, it is known that the range of the general pallet recovery integrity proportion is 0.4 to 0.8. Here α = 0.7 is used. When α was constant and the broken pallet repairability rate $\beta$ increased from 0 to 0.3, the comparison results of leasing price, pallet purchase volume, and expected profit with and without considering maintenance are shown in Fig. 6.



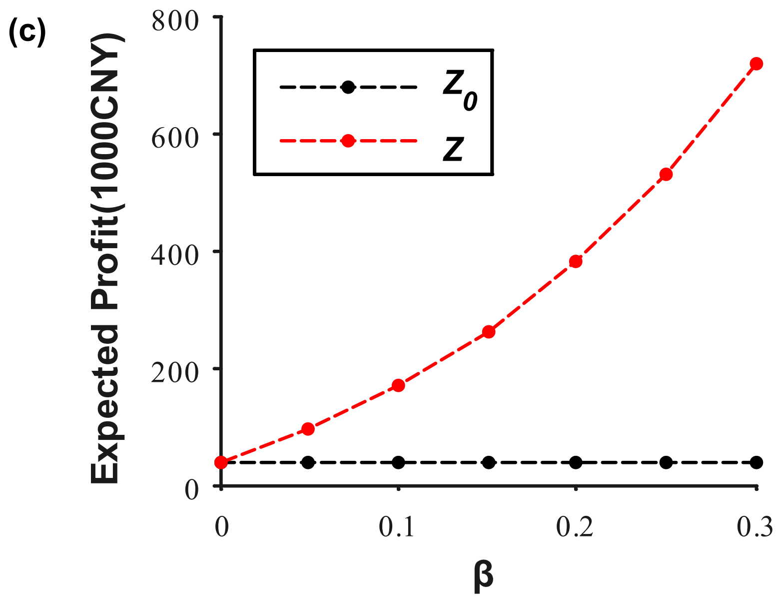



**Fig. 6.** Impact of broken pallet reparability proportion on optimal strategy; (a) Change in rental price ($p$,$p_0$), (b) Change in purchase volume ($q$,$q_0$), and (c) Change in expected profit ($Z$,$Z_0$)

As shown, the reparability rate of broken pallets had no effect when maintenance was not considered. For the strategy of considering maintenance, with the increase of the broken pallet repairability proportion, pallet rental price continued to decline, and pallet purchase volume and the expected profit continued to increase. The reason for this is that as the broken pallet repairability proportion increased, more wooden pallets can be put back into the pool for subsequent leases with simple maintenance at the end of each rental. While the utilization proportion of pallets increases, rental companies can expand their markets by lowering pallet rental prices, thus bringing an overall increase in profits.

*Impact of operating cost change on optimal strategy*

Under stochastic demand conditions, out-of-stock cost and inventory holding cost are the two essential operating costs. Their effects on the optimal strategy are shown in Figs. 7 and 8. It can be seen that the pallet rental price was positively proportional to these two costs and the expected profit was inversely proportional to these two costs, regardless of whether maintenance was considered or not. Therefore, pallet renters should purchase a reasonable number of pallets at the beginning of the period to reduce the expenses of out-of-stock and inventory holding costs.

## Discussion

The wooden pallet rental supply chain is more complicated than that of plastic pallets due to the reparability of the wooden pallets. Several studies have been conducted to analyze the rental pricing of pallets under different scenarios. For example, Xu (2011) presented a framework of pallets pooling supply chain system and established an optimal profit model based on the demand model of rental pallets and inventory control model by considering the dynamics of uncertain rental demand and return processes. The optimal

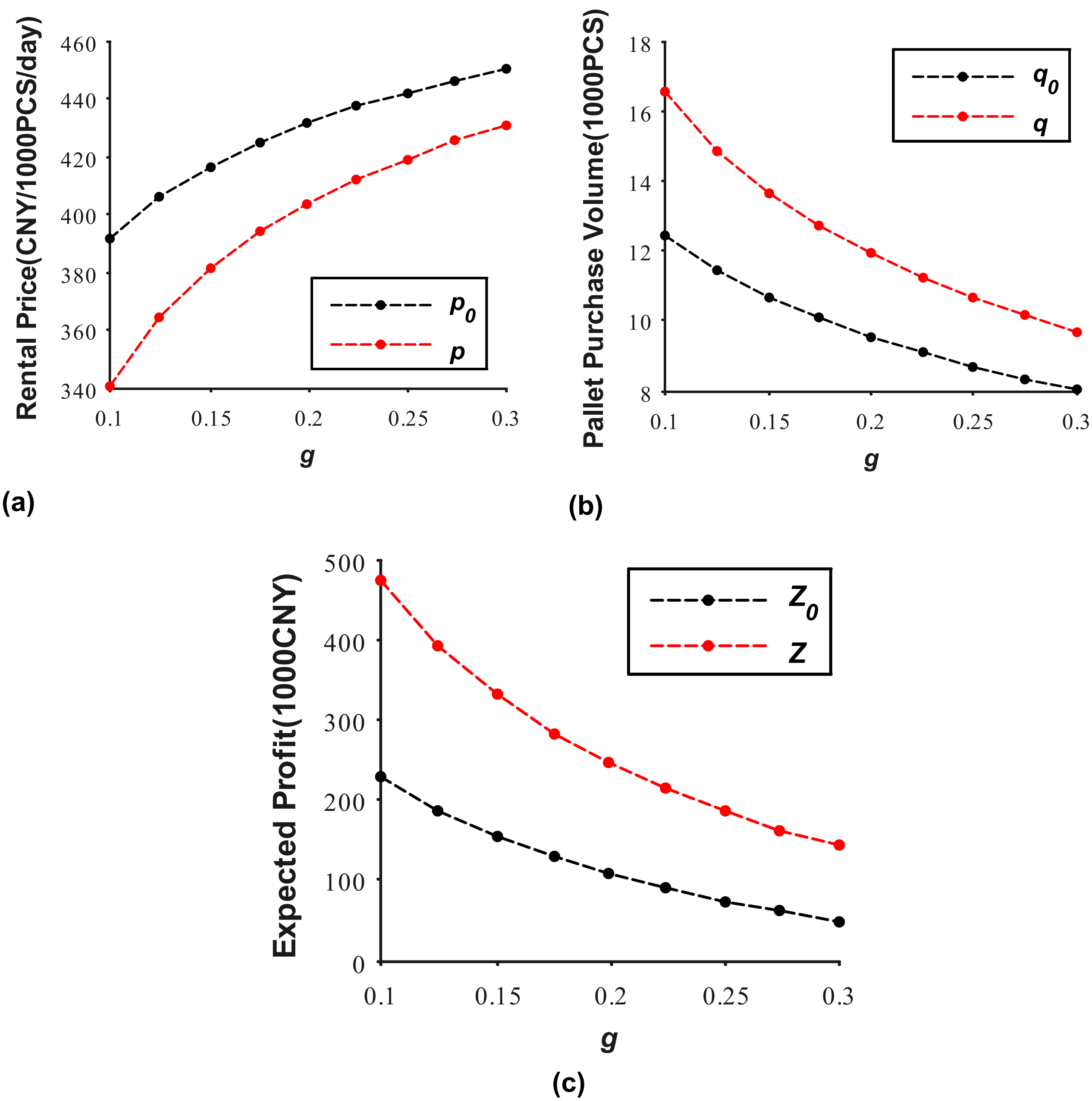



**Fig. 7.** Impact of out-of-stock cost on optimal strategy; (a) Change in rental price ($p$,$p_0$), (b) Change in purchase volume ($q$,$q_0$), and (c) Change in expected profit ($Z$,$Z_0$)

rental pricing strategy under different pallets rental duration and return rate were considered. Wu *et al.* (2015) proposed a pallet rental pricing method based on the concept of revenue management, including market execution price and key customer execution price, and established corresponding pricing models. By applying the new pricing method, the revenue of case enterprise can be improved by 5%. Previous studies focused on common pallets or plastic pallets, and the maintenance for the returned pallets were not considered in the pallet rental pricing. This study took the wooden pallet maintenance into consideration and established joint optimization models of purchase volume and rental pricing for wooden pallet leasing supply chain under deterministic and stochastic market demand, which can fill the gap of wooden pallet rental pricing.

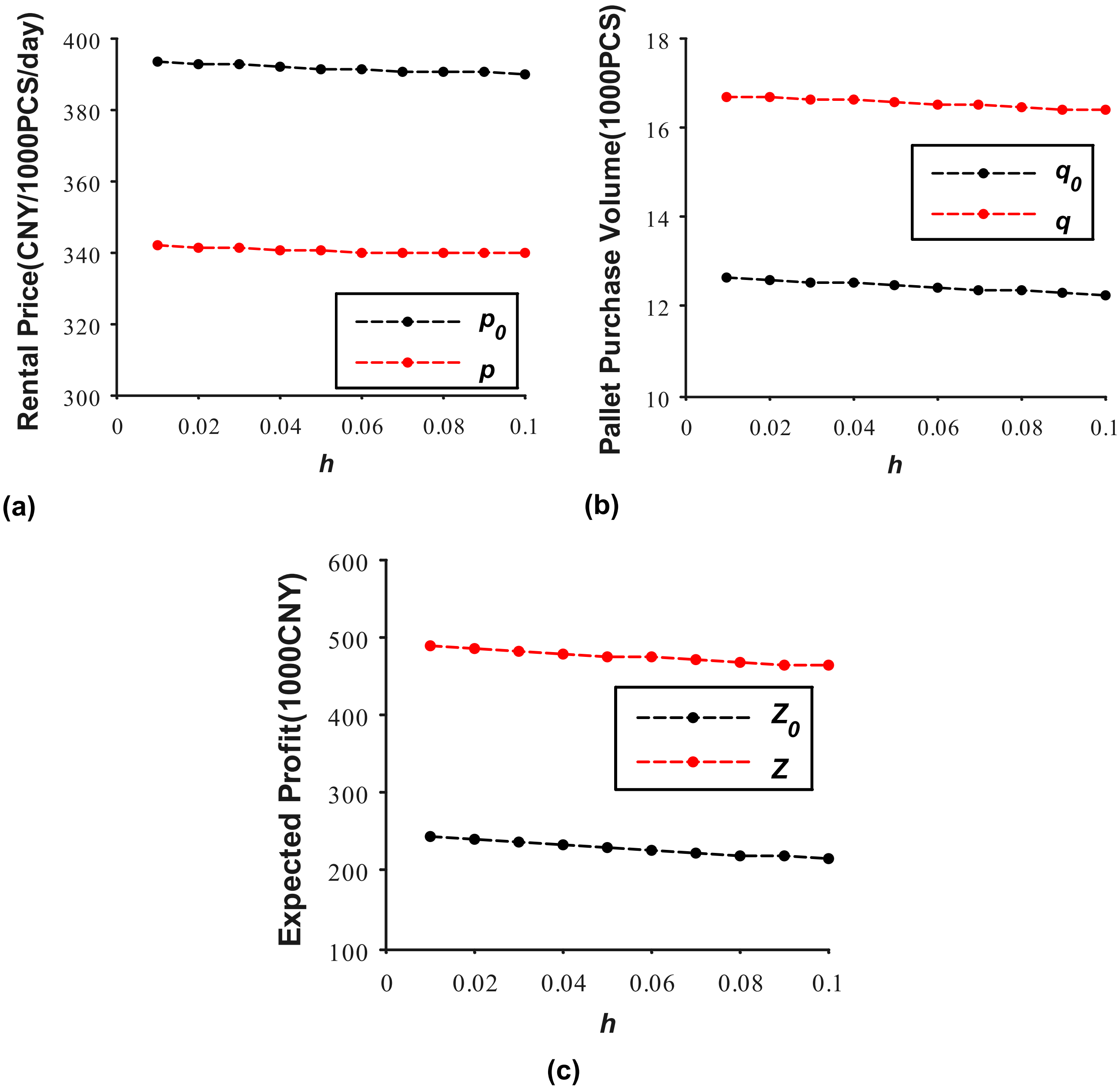



**Fig. 8.** Impact of inventory holding cost on optimal strategy; (a) Change in rental price ($p$,$p_0$), (b) Change in purchase volume ($q$,$q_0$), and (c) Change in expected profit ($Z$,$Z_0$)

According to the market survey results of wooden pallet leasing in China, the rental price per 1,000 wooden pallets ranged from CNY 200 to 500. In the present study, the optimal rental price of wooden pallets under deterministic market demand ranged from CNY 360.18 to CNY 386.30 per 1,000 pallets; the optimal rental price of wooden pallets under stochastic market demand ranged from CNY 340.18 to 391.28 per 1,000 pallets. Therefore, the study results are in line with the actual business, which indicated that the model proposed in this study is feasible. In addition, the study results also showed that more profits can be gained if the wooden pallet leasing company adopted the maintenance strategy.

There are also some limitations in this study. For example, only two-term leasing was considered, and the optimal strategy was derived based on the three-level wooden pallet rental supply chain system. Future research can be conducted by considering more leasing terms and more complicated wooden pallet rental supply chain systems in order to fulfill the requirements of business practice. In addition, the profit allocation among the members of the wooden pallet rental supply chain should be considered in order to promote the sustainable development of wooden pallet leasing industry.

## CONCLUSIONS

In this study, joint optimization models of wooden pallet rental pricing and purchase volume were established for two strategies of considering maintenance and not considering under deterministic and stochastic market demand with the objectives of maximizing the expected profit of the pallet rental company. Under the deterministic market demand, the existence and uniqueness of the optimal solution were demonstrated, and the analytical solutions for the optimal pricing, purchase volume, and expected profit were derived. Under the random market demand, the computational method to obtain the optimal solution was derived when the random factor followed uniform distribution. Finally, the optimal purchase volume, optimal leasing price, and expected profit under the given parameters were calculated by arithmetic examples and sensitivity analyses were performed. The following conclusions are drawn from the theoretical derivation and the example analysis:

1. For a wooden pallet rental supply chain, the strategy of considering pallet maintenance can result in higher profits and the pallet rental price can be decreased by 6.76% to 13.06% in the study case. This is a “win-win” situation for both the pallet leasing company and the customers.
2. Reducing the level of breakage and the number of lost pallets, while ensuring the integrity of the pallets, can increase the profitability of the pallet rental company. This can be achieved by purchasing better quality pallets and guiding customers to use them properly.
3. Pallet rental companies can optimize the pallet-related costs by developing business and forming strategic partners with more companies to enlarge the capacity of pallet pool and purchasing pallets and other maintenance services at lower prices, to realize sustainable development. For the costs incurred during the rental operation process, pallet rental enterprises can reduce the uncertainty of the pallet market by improving the accuracy of forecasts, so as to reduce the cost of out-of-stock and inventory holding costs.

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