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THE STRUCTURE, EVALUATION AND APPLICATION OF DYNAMIC MODELS FOR CONTROL

G. DONKIN, Automation Systems Division, Ferranti Ltd.

Synopsis The paper is intended as a tutorial introduction to some of the principles used in model building techniques, which place emphasis on modelling of the process behaviour as shown to the outside world by records of inputs to and outputs from the process, rather than by attempting to model details of the physics or chemistry internal to the process.

Introduction

IN A general sense, a model is a convenient summary of useful information about the system it represents. In a more detailed sense, the nature of a model depends on the purpose for which it is intended. For the purpose of this paper, a model of a process consists of a suitable mathematical representation of that process. It is a means to an end, the end being the design of an improved control system to minimise the effect of unwanted transient disturbances that upset that process. Process designers also use models to help them design improved processes. Research workers, however, postulate models to increase fundamental knowledge of cause and effect relationships between observed data. In their case, the model is virtually an end in itself, as the use to which this increased knowledge is eventually put is not necessarily known at the time of its development.

Because of the different purposes of modelling, different types and techniques have evolved. The process designer is naturally interested in modelling the basic physical or chemical processes taking place inside the hardware of the plant, as well as the dynamics of the plant hardware itself. To him, a model consists of physical and chemical equations involving heat balances and mass balances, usually in the form of steady state equations at first; but, where transient conditions have to be investigated, the steady state equations become differential equations as necessary. Solution of such equations on analog or digital simulators has become a standard technique. However much is known,

Under the chairmanship of N. C. Underwood there are usually some parameters of his model that cannot be measured directly like a temperature can be measured. Typical of such parameters are heat transfer coefficients, mass transfer coefficients, chemical reaction rates and imperfections of mixing. Indeed, some of the disturbances that upset a plant are changes in these parameters as the result of plant throughout changes. At best, these parameters can be calculated or inferrred only from other measurements. This type of model will usually be difficult, hence expensive to formulate; lack of complete measurements may make it impossible to evaluate all the parameters in the model. Nevertheless, an advantage of the above approach is that important non-linear effects and process interactions can be revealed.

The more the control engineers became aware of the limitations of the above modelling approach, the more effort they put into developing a 'black box' approach. In this, the basic aim is to deduce from process records of output responses to arbitrary input variations a set of simple equations that reproduce the important behaviour characteristics of the process to a certain degree of accuracy, ignoring the need for structural similarity between model and system. Techniques are still under development and some of them have involved a mountain of mathematical processing in the search for the molehill of the simple model. Within the scope of this paper, it is possible to give only glimpses of the underlying principles being used.

Basic principles of black box modelling

BEFORE we can deal with black box situations, we must first study results deducible from known situations. For illustrative purposes, consider the simple system shown in Fig. 1. The assumption made here is that the system may be separated into a static non-linear function to represent the control valve, a transport time delay and a linear dynamics component. The linear dynamics are represented by the equation—

Rate of change of output $= \frac{1}{T}$ input/output)

That is,
$$\frac{dX}{dt} = \frac{1}{T}\left[W(t) - X(t)\right]$$

or
$$T\frac{dX}{dt} + X(t) = W(t) \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

where T is the time constant,

$W(t)$ is any arbitrary input waveform,

$X(t)$ is the resultant output waveform.

An input step change of valve position $u(t)$ will produce the various responses shown. In the presence of process noise $n(t)$ and measurement noise

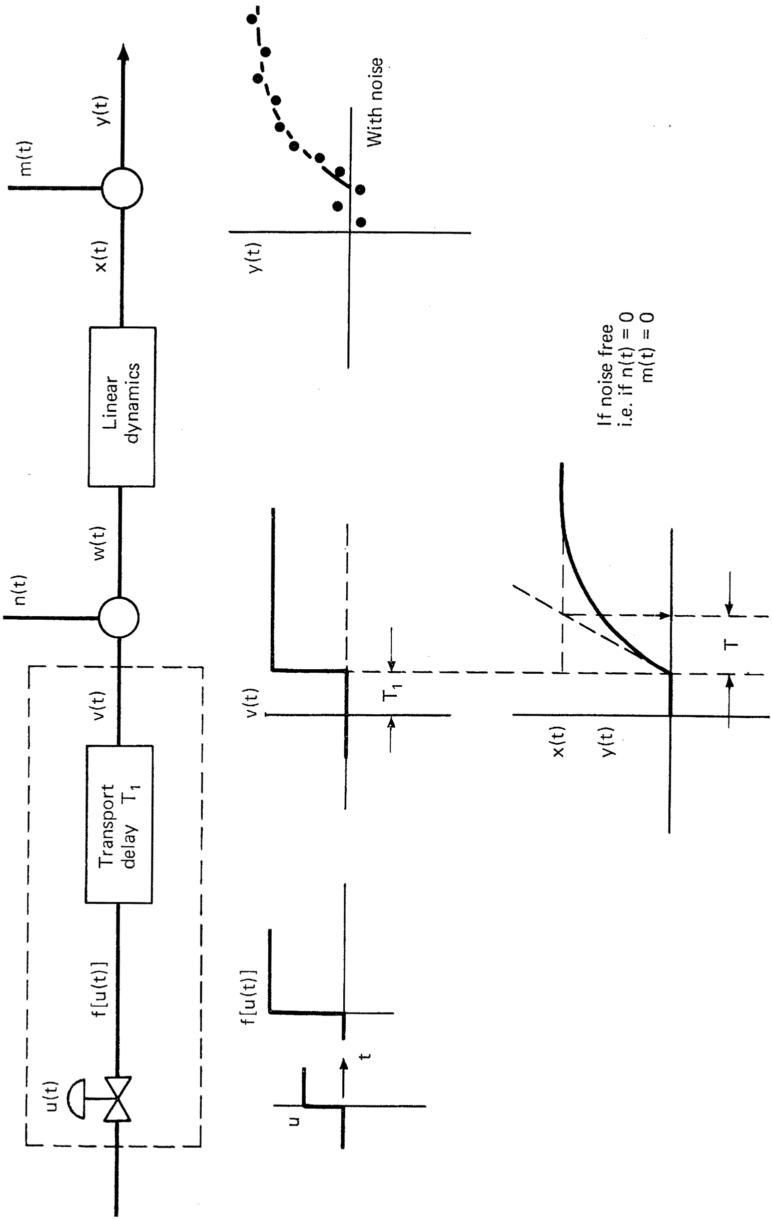

u(t)
f[u(t)]
Transport
delay T_1
v(t)
n(t)
w(t)
Linear
dynamics
x(t)
m(t)
y(t)
u
t
f[u(t)]
v(t)
T_1
y(t)
With noise
x(t)
y(t)
T
If noise free
i.e. if n(t) = 0
m(t) = 0

Fig. 1

$m(t)$, the measured output $y(t)$ is noisy somewhat as shown. If the noise is small (or, ideally, zero), tangents drawn to the initial rate of rise and the final steady state intersect to define the time constant T. In the presence of noise, this is not so clearly defined.

The pure time delay may be lumped together with either the static or dynamic part of the system. If the delay occurs essentially in the output dynamics, equation (*1*) would become—

$$T\frac{dX}{dt}+X(t-T_1) = W(t)$$

From a modelling point of view, it raises problems that are not easily dealt with in an introductory approach. Hence, we choose here to combine it with the static non-linear function as implied by the dotted rectangle and temporarily forget it. Thus—

$$v(t) = f\{u(t-T_1)\} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Continuing for the time being to assume that noise (nt) and $m(t)$ are zero, let us assume that the model equations have to be solved on a digital computer where the sampling interval between calculations is τ. For the purpose of this illustration, a simple difference approximation of $\frac{X_n-X_{n-1}}{\tau}$ can be used for $\frac{dX}{dt}$.

X_n can be regarded as the current sampled value,
X_{n-1} is then the previous sampled value and
τ is the sampling interval.

In similar vein, $\frac{X_n+X_{n-1}}{2}$ can be used for $X(t)$

and $\frac{W_n+W_{n-1}}{2}$ can be used for $W(t)$.

On substituting these values in equation (*1*) and rearranging to solve for X_n, we get—

$$X_n = \left(\frac{2T-\tau}{2T+\tau}\right) X_{n-1}+\left(\frac{\tau}{2T+\tau}\right)(W_n+W_{n-1}) \quad . \quad . \quad (3)$$

Assuming we know the value of the process time constant T and since we know the value of τ, this equation can be written more simply as—

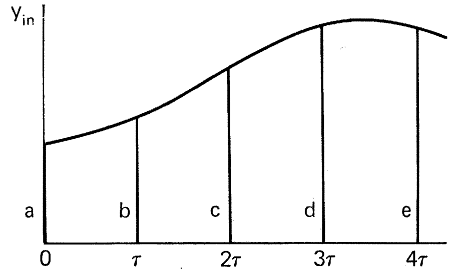
$$X_n = aX_{n-1}+b(W_n+W_{n-1}) \quad . \quad . \quad . \quad . \quad (4)$$

It is worth verbalising equation (*4*), because it represents a simple mathematical model of our dynamics in the example. It says that the current output X_n is a times the previous output plus b times some function of the current and previous inputs. The sampling interval should be chosen to be small compared with estimates of T so that typical operating disturbances can be well characterised. It follows that, in equation (*4*), a is much greater than b. This formulation emphasises the fact that uncontrolled dynamics react to earlier disturbances (that is, they 'remember' earlier disturbances) for considerable elapsed times. This and the responses shown in Fig. 1 indicate that such dynamics smooth or filter 'sharp corners' on input changes into smoother output changes.

If we now adopt the black box philosophy and pretend that we do not know the value of T, it appears that we can in principle work back from plant records using different sets of output data X_n and input data W_n, to back calculate parameters a and b. This is the basis of the black box modelling approach despite certain mathematical problems that will be mentioned later. Practical situations are usually more complicated than our simple example. More complicated dynamics require more complicated differential equations than are used in equation (*1*), hence the corresponding difference equations equivalent to equation (*4*) may have more than just two parameters to evaluate. They may also involve differences of current and previous data values further back in time than the previous sample. Such differences are called backward differences and there are many different mathematical shorthand notations that serve as time markers to help those expert in these matters to tag relative time shifts. Fig. 2 shows some of the alternative forms of *mathematical shift operators*.

Once it is realised that they mean move backwards (or forwards) in time and are associated with parameters that have to be evaluated in the search for a black box model, they tend to lose some of their mystery. Later, we shall use the shorthand notation $F(z)$ to mean 'some general form of dynamic filter based on the concepts embodied in equation (*4*), but of a more complicated form'.

This is not, however, the most fundamental problem associated with the black box approach. Let us return our attention to the back calculation of parameters a and b in our simple model equation (*4*). Usually, the techniques of least squares regression analysis have been applied to the determination of these coefficients. Yet, our development above, even assuming the lack of process or measurement noise, has shown that a and b depend on T and τ, hence are not independent of each other. An essential prerequisite for successful use of least squares analysis is that the parameters should be independent of each other. Some workers recognise this and go to great lengths to minimise



Define special symbol(s) as time markers to tag amplitudes at successive sampling times

e.g. Tustin	$y_{in} = ax^0 + bx^1 + cx^2 + dx^3 + ex^4 +$ etc.
Z – Transform	$y_{in} = az^0 + bz^{-1} + cz^{-2} + dz^{-3} +$ etc.
Laplace shift operator	$y_{in} = a\delta + b\delta e^{-S\tau} + c\delta e^{-S2\tau} + d\delta e^{-S3\tau} +$ etc.

Fig. 2—Waveforms as time/number series

estimation errors that may arise from this cause. Others seek to avoid this problem by using automated search techniques to try different parameter values, until they find acceptable values that provide the best match between the model and the process.

If we now take into account the measurement and process noise that we temporarily ignored after showing it on Fig. 1, the *back calculation analysis* may be further complicated, depending on whether the noise samples are correlated with each other or not. The autocorrelation function is the mathematical tool used to test whether data is correlated or not. Since much use is made of it in model building, we will describe what it does and what its advantages are.

The autocorrelation function

The mathematical formulae that define it rigorously are complicated to understand. Its advantage is that the essential information extracted from and averaged over a long record of data can be obtained in condensed form. The principles underlying autocorrelation are shown graphically in Fig. 3, assuming that a representative section of the sampled data is as drawn. By further imagining that the thumb and fingers of one's left hand represent the magnitude of the successive samples *a*, *b*, *c*, *d*, *e*, respectively, superimpose an identical copy of this, for example, the right hand, palms together, calculate

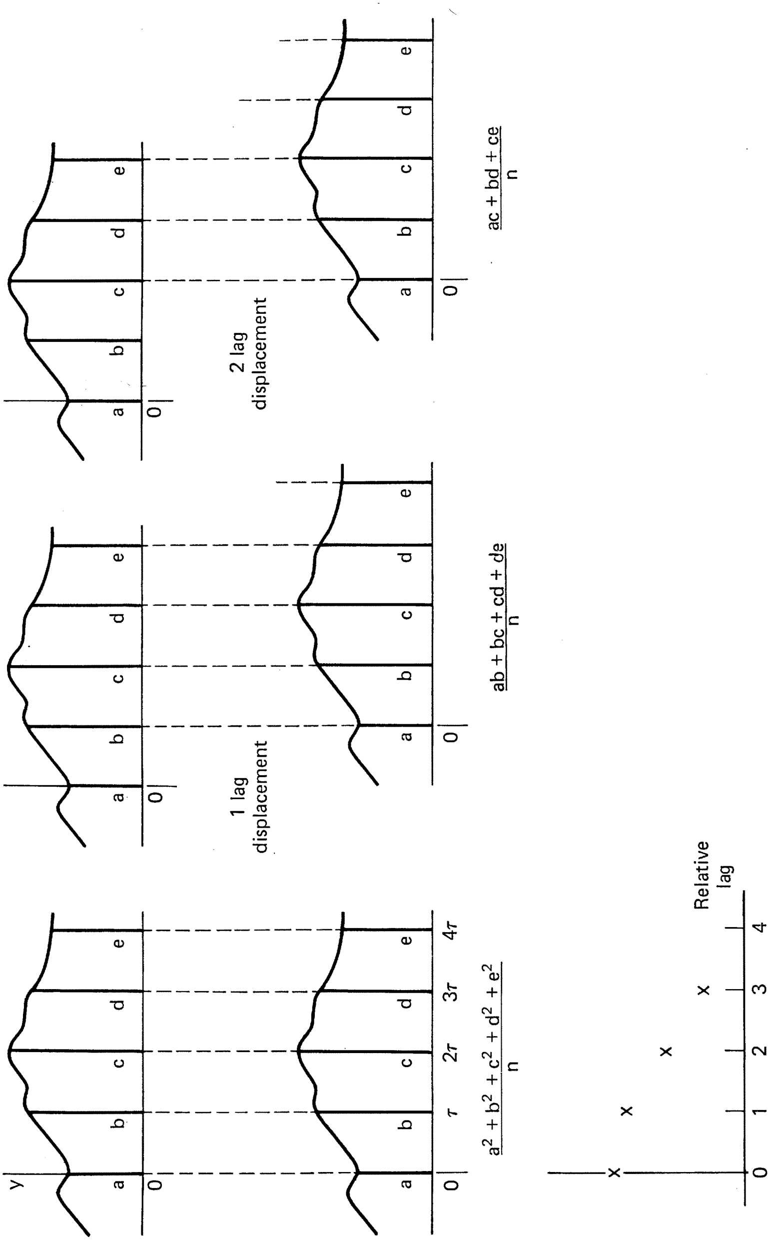


Fig. 3—Autocorrelation function

the products of the successive superimposed ordinates and average them. The magnitude of this average is called the autocorrelation at zero relative lag and for zero lag is the mean square of the amplitudes. This can be plotted on another graph of magnitude against relative time lag.

Now displace by one sample time the ordinates of one set of samples relative to the other.

Calculate the new superimposed successive products and average them. Plot this magnitude of the autocorrelation for a relative lag of one sample.

Displace the ordinates by another sample time, calculate the new products, average them and plot for a relative lag of two samples, etc. The resultant plot is the autocorrelation function or autocorrelogram. It clearly shows the extent to which, on the average, a data sample at one instant of time depends on data samples at previous instants of time. In the case of random or white noise, the autocorrelation plot in theory is zero everywhere, except at zero relative lag. In practice, the non-zero lag autocorrelations will be small and randomly scattered about zero. If the original data is not

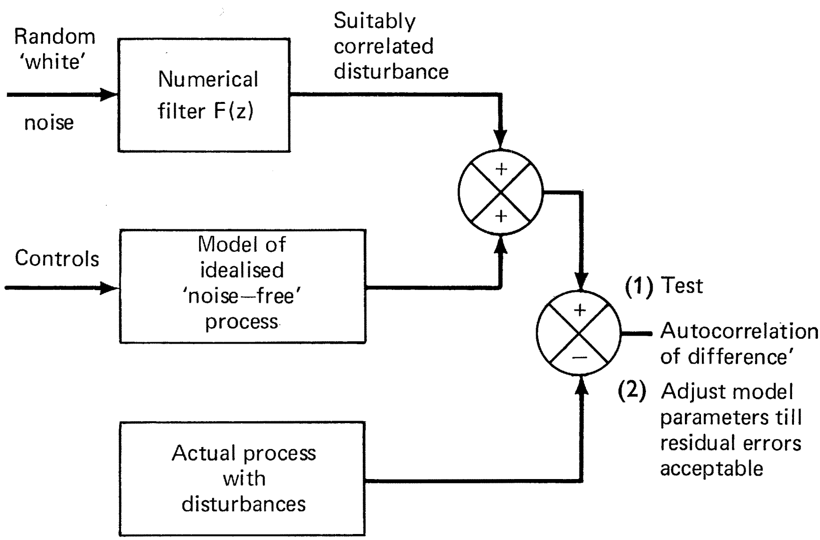


Fig. 4

random, the autocorrelation plot will have finite magnitudes at non-zero lags. The shape of the plot will be characteristic of the nature of the dependence of later samples on earlier ones, that is, characteristic of the memory inherent in the correlation.

With this understanding of autocorrelation, it is now possible to return to the concept of the modelling of process disturbances, presuming them to be correlated, as they often are in practice. First of all, the artifice is adopted of assuming that a sequence of uncorrelated noise samples is to be passed through and altered by an appropriate calculation filter $F(z)$ of the difference equation type hinted at in equation (*4*) and subsequently. The behaviour of $F(z)$ has to be chosen (that is, its parameters and mathematical structure have to be chosen) so that its output samples match the process disturbance samples in some sense. Because of the nature of noise sequences, we cannot expect that the output sequence of the calculation filter will be identically equal at each sample time to the process disturbance samples. What can be attempted, however, is to make their autocorrelation functions similar. This is what is attempted in noise modelling.

Conclusion

WHETHER the problem is to model the dynamic behaviour of a process or to model the disturbances entering or arising inside the process, at first we do not know the best structure for the model, nor the best values for the parameters in the model. Initial guesses have to be made and modified as necessary as the result of comparison of the models with the real situation being modelled. These features are common to all model building. The parameter adjustment method chosen depends on the use that is to be made of the model; the extent to which knowledge of the process exists; the extent to which process variables can be measured; the nature of disturbances affecting the process; and the complexity, storage requirements and time involved in solving the necessary computations. Some processes may possess unique properties that dictate the use of one particular modelling technique, rather than allowing of a choice.

In the black box approach, outlined in the paper, emphasis is placed on modelling of the process behaviour as shown to the outside world by records of inputs and outputs rather than by attempting to model details of the physics or chemistry internal to the process.

The paper has attempted to convey only some conceptual interpretation of what some of the mathematics of information extraction is trying to do. Hopefully, it has tried to take some of the 'black magic' out of the black box approach. It had deliberately avoided any attempt to relate analysis in the time domain of sampled data theory to spectral analysis in the frequency domain of continuous waveform theory. The statistical techniques used are those of dynamic statistics rather than those of the steady state quality control type. Details of how these various highly specialised branches of mathematics do their number crunching are obviously outside the scope of this paper.

Discussion

Mr B. W. Balls Mr Ward has given a fine paper, but perhaps he has treated a little lightly the formative years of process control. As one long connected with 'bellows, nozzles and levers' and who has met some of the first designers, I can say that they had a deep understanding of process control requirements and the operation of their hardware. First interest was in the oil industry, where the interest was predominantly flow, as it is in the paper industry. A study of the patent literature of the twenties and thirties would demonstrate these points.

Process control began in the time domain and has returned home. I believe it was Minorsky who around 1920 published a pioneer paper on feedback based upon observations of the wake of a ship. Papers by such as Ivanoff,* Callender, Hartree & Porter† and Mason‡ analysed process lags and still make interesting reading. The Cranfield conference in 1951§ brought together for the first time process control people and the backroom boys from the services, etc. and we moved rapidly into the frequency domain. I well remember the birth pains. Much useful work was done, but frequency response analysis of real processes was a disappointment in many cases, owing to extreme attentuation and noise. There was also understandable reluctance by production managers to let their plants be waggled.

Mr Ward has stated correctly that process computers have arrived in the paper industry. It is also true that a clear definition of aims is essential. Many early installations failed because of indigestion, whereas more modest aims might have produced some success. This requires total involvement of people who are well versed in process control applications and having a deep understanding of operating skills.

The Chairman My only comment to Mr Balls is that it was the war that made radio and electronics respectable in some of the universities and many of the graduates coming out at that time were well versed in the frequency domain. It is only more recently that we have moved back into a time domain.

* Ivanoff, A., *J. Inst. Fuel.*, 1934, **7**, 117
† Callender, A., Hartree, D. R. and Porter, A., *Phil. Trans. Royal Soc.*, 1936, **235** (A), 415
‡ Mason, C. E. and Philbrick, G. A., *Trans. A.S.M.E.*, 1941, **63**, 589
§ Tustin, A., *Automatic & Manual Control* (Butterworth, London, 1952)

Mr H. B. Carter Is there any significance in the continued use of a valve as an example when we talk of the single control loop? One of the earliest control systems that was applied in the papermaking process was a speed control and the earliest one of which I have any experience was patented in 1919, the Harland differential speed control.

Mr A. J. Ward As I pointed out, engine regulation was the first problem. It was really only the process industries that had an interest in the flow of material, hence the valves. If you search, you will find that there are many more valves in control situations than there are engines with speed controllers.

Dr I. B. Sanborn With regard to the modelling paper, let me say that, whenever such papers are given, I notice many people becoming discouraged by the mathematics involved. Please be assured that modelling is not as difficult as it appears. First, note that most of the processes in our industry can be modelled via a simple first order system with dead time. This means that only three parameters need to be estimated— a dead time, a gain and a time constant.

Dead time can usually be assessed by a direct inspection of the time series data available. The gain can usually best be estimated by noting the response in the output at steady state to a step of known size in the input. This can again be assessed from the time series record. This leaves only the time constant to be estimated, which is easily accomplished by any one of a number of one-dimensional search methods available.

Mr G. Donkin I agree that useful estimates of process time delays and time constants can be made without resort to prolonged mathematical analysis, especially for small systems. The techniques outlined represent some of the many tools in the model builder's tool kit. The paper was not intended as a survey of all possible modelling techniques, nor was it meant to imply that the methods outlined must be used. An appropriate mixture of engineering judgment and mathematical refinement is inherent in all modelling. The mix chosen by an engineer may well differ from that chosen by a mathematician. When the mathematicians have developed what they believe to be useful tools, it is sensible to explain them to the engineer. Engineers would welcome more 'talky-talky' explanations of recent developments such as this paper has tried to present.

The Chairman These contributions show that value is still left in simple methods than can be handled by a desk calculator.