Prediction of Flexural Stiffness of Wooden Beams with Cross-sectional Loss that Are Reinforced with Screwed Steel Plate Based on Numerical Simulation

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As a biodegradable material, wood is subject to deterioration if proper conservation techniques are not observed. Thus, several buildings, especially those of historical heritage, present pathological manifestations that can cause accidents. The interventions in these constructions must be planned to maintain the original elements and the aesthetics of the environment, with the indication of fixing additional elements in the degraded structure. The modification of the section of structural elements is commonly observed in the literature; however, few studies have been intended to analyze the effects of reinforcement in these geometrically discontinuous elements. Furthermore, the use of screwed steel plates guarantees greater ease of execution, even though it is not yet a method that has been well explored in the scientific literature. The objective of this study was to propose an equation for estimating the flexural stiffness of wooden beams with loss of cross-section that are then reinforced with screwed steel plates using a regression model. The considered variables correlated the elastic modulus of the wood and the reinforcement, the configuration of the defect, and the reinforcement. It was possible to identify that the properties of the wood and the position of the defect were variables with a significant impact on the stiffness of the reinforced beam.

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INTRODUCTION

Wood is commonly found in historic buildings across the planet, perpetuating the identity and culture of different societies. Preserving these structures becomes fundamental for maintaining the history of civilizations (Chen and Guo 2017).

The maintenance of historical heritage is imperative for the preservation of human culture and history. The recovery of the structures of these constructions should be considered a priority to ensure the expected performance of the building systems. The method adopted for the intervention must be defined considering the non-distortion of the environment or the structural element (Jasieńko and Nowak 2014).

The use of steel elements to reinforce wooden beams is present in several studies and approaches. The transmission of forces between elements is a topic with recurrent studies and propositions (Burdzik and Skorpen 2016; Waseem *et al.* 2022). However, applying this type of reinforcement in damaged elements with loss of cross-section is still insufficiently explored. Reinforced wood usually does not have the same initial geometric configuration, either due to attack by xylophagous agents or continuous usage (Verbist *et al.* 2019). Hence, the recovery analysis of these elements must take these imperfections into account in order to predict the behavior closest to reality.

The behavior of the wood-steel composite element is directly associated with the ability of mechanical fasteners to transmit forces. Waseem *et al.* (2022) verified that the stiffness of the wood-steel component increases when the spacing of the mechanical fasteners is smaller. Currently, there is a gap in the studies of the application of this solution in beams with loss of cross-section (discontinuous geometries), a situation commonly found in historic buildings, as well as the study of the participation of mechanical fastening elements in the structural recovery capacity of this type of element.

Currently, there is no analytical method for obtaining the flexural stiffness of wooden beams with loss of cross section that are then reinforced with a screwed steel plate. Obtaining this property is important to enabling the application of this easy-to-execute reinforcement method in real situations of degraded wooden structures with restrictions on replacing their elements.

The objective of this study was to obtain an equation to determine the flexural stiffness of wooden beams with loss of cross-section. Hence, a parametric study was carried out using finite element analysis with different wood species, defects, and reinforcement configurations. To obtain the stiffness equation, a multiple-variable regression model was performed with the results of the parametric study.

EXPERIMENTAL

Parametric Study

The parametric study was carried out using the Abaqus software, simulating wooden beams with a cross-section of 200 mm x 300 mm, with a total length of 6900 mm, supported with a distance between supports of 6300 mm, meeting the dimensions and criteria provided by the Brazilian standard ABNT NBR 7190 (2022). A more detailed presentation of the numerical model can be seen in Jardim *et al*. (2022).

The loading situation was defined to simulate a four-point bending test expected to meet the Serviceability Limit State (SLS), with a maximum displacement of 31.5 mm, according to ABNT NBR 7190 (2022). Therefore, the equivalent displacements of 27.39 mm were imposed at the force application points.

Five hardwood species characterized by other researchers were chosen to obtain the necessary data for numerical simulation. The species were chosen based on their differences in the modulus of elasticity, as follows: Paricá (*Schizolobium amazonicum* Herb), with 7.32 GPa (Almeida *et al.* 2013), Angelim Araroba (*Vataireopsis araroba*), with 12.17 GPa (Branco *et al.* 2014), Envira Branca (*Xylopia* sp), Canelão (*Ocotea* sp), and Breu Vermelho (*Protium* sp), with 14.97 GPa, 15.35 GPa, and 16.39 GPa, respectively (Christoforo *et al.* 2013).

The cross-section losses were considered on the upper and lower face of the critical section, with a prismatic configuration since this is the critical configuration for the distribution of internal stresses. The length of the defect was fixed at 10% of the beam (690 mm) and varied transversely between 15, 30, and 45% (45 mm, 90 mm, and 135 mm, respectively).

The adopted reinforcement consisted of steel plates, with a modulus of elasticity of 210 GPa, which is the common value for steel A36, located on the lower face of the critical section of the beam, aiming at the best use of the steel reinforcement capacity (Lukin *et al*. 2021). The adopted dimensions were 200 mm x 1530 mm x 9.5 mm and were not varied in this study.

The screws were provided with fixation aligned on the axis of the cross-section, following the model studied by Burdzik and Skorpen (2016). They were considered with commercial diameters of 12.7 and 15.9 mm, varying in quantity from 1, 2, and 3 screws on each side of the plate, equidistant by 95 mm, with sufficient length to pass through the beam section, with a modulus of elasticity similar to one of the plates.

Considering the simulation within the elastic regime of the material, linear analyzes were performed for each model (Kim and Harries 2010), with the resulting force at the displacement application point (F) and the maximum displacement obtained in the critical section (U) as the results of interest. With these results, the modulus of elasticity of both materials together (E) was determined utilizing Eq. 1,

$$E = \frac{\left(F_{50\%} - F_{10\%}\right) \cdot L^3}{\left(U_{50\%} - U_{10\%}\right) \cdot 4 \cdot b \cdot H^3} \tag{1}$$

where $F_{50\%}$ and $F_{10\%}$ are half, and a tenth of the total force obtained (N), $U_{50\%}$ and $U_{10\%}$ are half and a tenth of the maximum displacement of the beam (mm), and b, H and L are the measurements of the base and height of the cross-section and the distance between the supports of the beam (mm). Its product with the moment of inertia (I), determined by Eq. 2, resulted in the stiffness of the set (EI) simulated,

$$I = \left(\overline{I_{beam}} + A_{beam} \cdot d_{beam}{}^2\right) + \left(\overline{I_{plate}} + A_{plate} \cdot d_{plate}{}^2\right) \tag{2}$$

where I and $\bar{I}$ are the moments of inertia of the set and the isolated element (beam and plate) (mm^4), A is the cross-sectional area (mm^2), and d is the difference between the center of gravity (CG) of the isolated element for the CG of the set (mm).

Each element was modeled in Abaqus using solid elements, type C3D8R, except for screws, which were modeled using tetrahedral elements, type C3D10. The adopted convergence criterion was an automatic displacement increment, with a maximum limit of 10^4 increments of the initial size of 0.001, maximum of 0.01, and minimum of 10^{-15}.

The contact between the elements was configured by friction, with two native Abaqus contact properties: tangential and normal behavior. The tangential was configured with the Penalty-type friction formulation and a coefficient of friction of 0.4 (Fajdiga *et al*. 2019). The normal behavior of the interaction was configured with the Hard Contact property. In all, 180 simulations were performed.

Regression Model

A multiple-variable regression model was used to estimate the stiffness value (EI) of the defective wooden beams with the inclusion of the steel plate, highlighting the lack of analytical methodology for calculating the *EI*, which motivated the development of the present research.

In the regression model (Eq. 3), the variables considered were based on the ratio between the length of the beam (*L*) and the respective height (*H*) of the wood cross-section, disregarding the defect existing in the middle of the span (*L*/*h*), the ratio between the modulus of elasticity of the steel sheet (E_s) and of the wood (E_w) considered (E_s/E_w), the ratio between the height of the cross-section of the wood without (*H*) and with (*h*) the defect (*H*/*h*), the position of the defect (P_o), the number of screws (*N*), and the ratio between the diameter (*D*) of the screw and the thickness (*t*) of the steel plate,

$$EI = \beta_0 + \beta_1 \cdot L/H + \beta_2 \cdot E_s/E_w + \beta_3 \cdot H/h + \beta_4 \cdot P_o + \beta_5 \cdot N + \beta_6 \cdot D/t + \varepsilon \qquad (3)$$

where β_i consists of the coefficients adjusted by the least squares method and ε is the random error. It should be noted that the quality of the adjustment was obtained by the adjusted coefficient of determination (R^2_{adj}). The ANOVA of the regression model, at the 5% significance level, was used to investigate the terms' significance and order of significance (Pareto chart), which consists of a sensitivity analysis.

RESULTS AND DISCUSSION

With the results of the simulations, the stiffness values were obtained through the product of Eqs. 1 and 2, considering the defects and the reinforcements adopted. This result was compared with the stiffness of each wood species without the presence of the defect and reinforcement (reference model). These values are shown in Fig. 1.

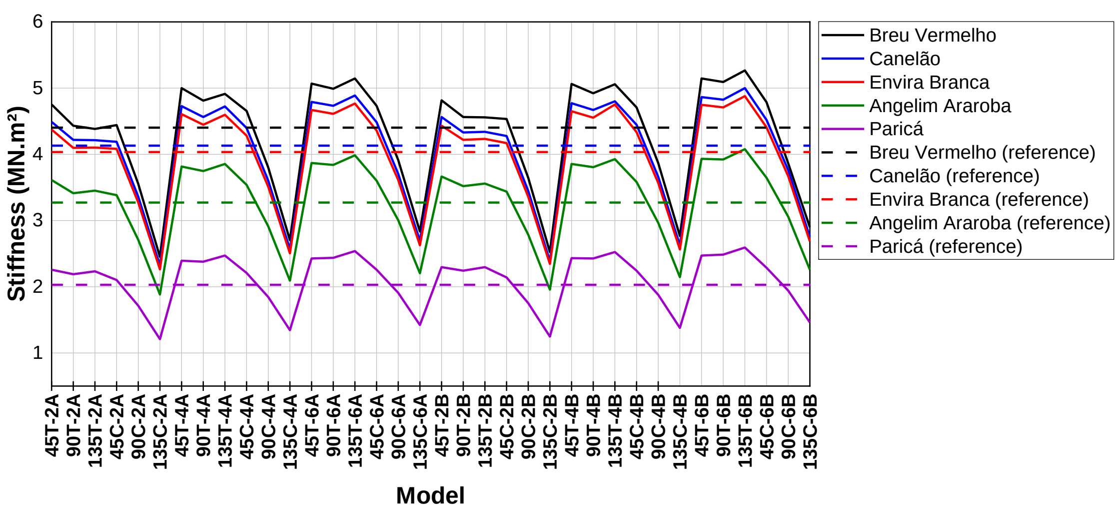

Fig. 1. Stiffness of the simulated models

In Fig. 1, alphanumeric codes were defined to identify each model's defect and reinforcement configuration. The first term consists of a number referring to the size of the defect considered (45, 90 and 135) and a letter indicating the region of the defect (T = lower region and C = upper region). The second term consists of a number indicating the total number of reinforcement screws (2, 4, and 6, considering both sides) and a letter indicating the diameter of the screws (A = 12.7 mm and B = 15.9 mm).

The participation of wood in the stiffness of the system is greater when its strength is greater, as observed by Kim and Harries (2010), Burdzik and Skorpen (2016), and Isleyen and Kesik (2021). Woods with a lower strength class or a reduction in their physical-mechanical properties tend to have greater recovery of flexural stiffness when reinforced. Thus, the best results obtained were with reinforcement in pieces of wood with lower strength values. No significant differences were found between the species Breu Vermelho, Canelão, and Envira Branca (species with greater strength).

For all species, only the models with 90 and 135 mm of defects in the upper region did not have their stiffness restored, evidencing the possibility of using partial reinforcement to recover wooden beams in defects in the tensioned region and minor defects in the compressed region, as also observed by Jesus *et al.* (2012). The models that recovered their initial stiffness, and had two screws in total, presented the results closest to the reference (approximately 10% higher), with better results being found in configurations with more screws (variation between 11 and 22% higher in relation to the reference).

In studies by Jasieńko and Nowak (2014), an increase of 58% was found in relation to the reference model when analyzing the fixation of steel sheets with epoxy on the lower or upper faces, *i.e.*, with the transmission of stresses throughout the glued region. In the analyzed simulations, the models that reached the results of Jasieńko and Nowak (2014) were those with 90 mm and 135 mm of section loss in the tensioned region, even using a screw on each side. The results show the possibility of using this reinforcement method in wooden beams with part of their cross-section compromised. The regression model obtained to estimate the stiffness of defective wooden beams reinforced with steel plates is expressed in Eq. 4. The values of 0 or 1 were adopted for P_o, representing the defect in the lower and upper region of the beam, respectively. In Fig. 2 (Pareto chart), the results of the ANOVA are presented, highlighting that for the use of Eq. 4 in estimating the EI, it is necessary to observe the limits of the variables of the parametric study.

$$\begin{aligned} EI(N\cdot mm^2) &= 6713131292681 - 40574319946\cdot L/H \\ &-133539003637\cdot E_s/E_w + 55426068873\cdot H/h - 987779794614\cdot P_o + \\ &177730560481\cdot N + 208988520442\cdot D/t \qquad \mathbf{R^2adj = 86{,}75\%} \end{aligned} \tag{4}$$

$$\begin{cases} 24.71 \le L/H \le 38.18 \\ 12.82 \le E_s/E_w \le 28.69 \\ 2.22 \le H/h \le 6.67 \\ P_o = 0 \text{ or } P_o = 1 \\ 1 \le N \le 3 \\ 1.34 \le D/t \le 1.67 \end{cases}$$

From Eq. 4, the good accuracy of the regression model in estimating flexural stiffness can be seen from the value obtained from the adjusted determination coefficient, which was close to 87%. From the sensitivity analysis, the most significant variable (≥ 1.97) consisted of the ratio between the modulus of elasticity (E_s/E_w) followed by the position of the defect (P_o), the number of screws, and the ratio between the length and height of the section (L/H). It should be noted that the ratio between the heights (H/h) and the ratio between the screw diameter and the thickness of the steel plate (D/t) were not considered significant by the ANOVA (5% significance).

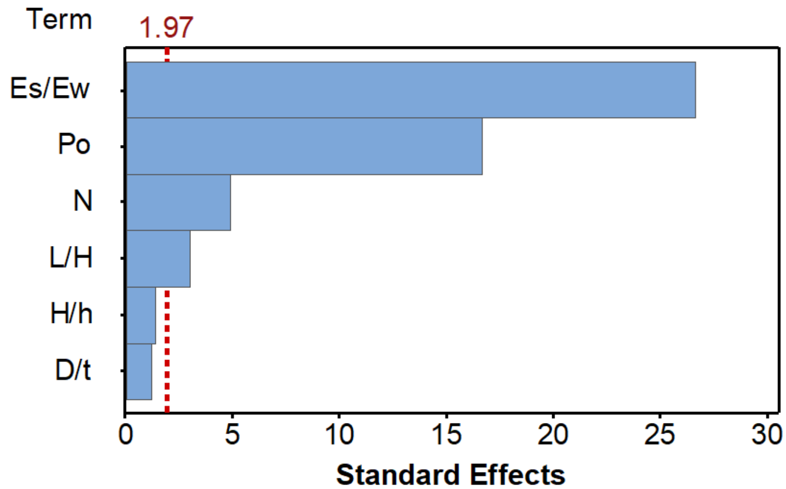

Fig. 2. Pareto chart

The high significance of the E_s/E_w ratio shows that this reinforcement technique is more efficient in woods with a low modulus of elasticity or with reduced properties due to biodeterioration. The location of the defect directly influences the moment of inertia of the beam, justifying the degree of importance of the parameter.

Of the less significant variables, the H/h ratio can be explained by not drastically altering the section's moment of inertia, maintaining the behavior of a composite section. This result was also observed in Fig. 1, where the models showed similar stiffness when only the defect size varied, except for the 90 mm and 135 mm defects in the upper region. The D/t ratio also showed little significance in the stiffness gain, being recommended to increase the number of screws in the detriment of changing the D for stiffness gain.

In order to demonstrate the use and to access the estimation capability of Eq. 4, one new simulation was made with the length of 3000 mm (common dimension found in structural design projects), with cross-section of 100 mm total height and 45 mm of section loss in the lower region. The wood species of Caixeta (*Simarouba amara* Aubl.) was considered with E_w equal to 9.7 GPa (Moritani et al. 2023). The reinforcement was set with a bolt with 22.22 mm in each side (7/8 in) and a steel plate with 16 mm thickness, both with Es equal to 206.8 GPa, representing the use of galvanized steel since it has higher resistance to humidity (Hassan et al. 2023). Hence, all the parameters are withing the limits presented in Eq. 4. Figure 3 presents the result of stiffness obtained by Eqs. 1 e 2, which was calculated with the results obtained from the additional numerical simulation and the estimated stiffness by Eq. 4 proposed in this work.

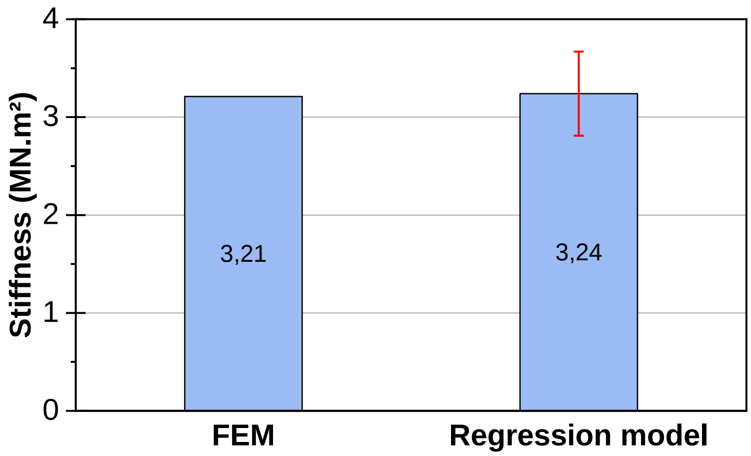

Fig. 3. Comparison of the stiffness obtained in the numerical simulation with that estimated by the regression model

As observed in Fig. 3, the numerical model resulted in a stiffness of 3.21 MN.m², while the regression model resulted in 3.24 MN.m². Thus, the Eq. 4 presented in this work was able to adequately estimate the stiffness of the model, with an error of 1.03% in the additional simulation performed by varying all parameters.

CONCLUSIONS

1. The modulus of elasticity of wood plays a significant role in the flexural stiffness of the wood-steel composite.
2. The use of a screwed steel plate is a more efficient reinforcement solution for wooden beams with a low modulus of elasticity, when compared with the models with no reinforcement.
3. The proposed equation presents good accuracy in predicting the flexural stiffness of wooden beams with loss of cross-section reinforced with screwed steel plate.

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Author Contributions

Conceptualization, P.I.L.G.J., E.A.M.M., T.H.P, D.H.A., A.L.C.; methodology, P.I.L.G.J., H.F.S., E.F., D.H.A., A.L.C.; software, P.I.L.G.J., A.L.C.; validation, P.I.L.G.J., A.L.C.; formal analysis, P.I.L.G.J., E.A.M.M., T.H.P, V.A.A., A.L.C.; investigation, P.I.L.G.J., A.L.C.; resources, P.I.L.G.J., D.H.A., A.L.C.; data curation, E.A.M.M., T.H.P, F.J.R.M., D.H.A., F.A.R.L., A.L.C.; writing—original draft preparation, P.I.L.G.J., F.J.R.M., V.A.A., H.F.S., E.F., A.L.C.; writing—review and editing, E.A.M.M., T.H.P,

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