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INTRODUCTION TO SYMPOSIUM

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DESPITE considerable advance during the last twenty years in the technology of paper and papermaking, there has been scarcely any explicit consideration of the subject that underlies all of our knowledge and all of our problems in these fields—the structure of paper. A brief survey of papermaking literature brings home forcibly our state of ignorance. None of the classic reference or textbooks includes even a reference to structure in its contents list or index. Even the few papers explicitly referring in their titles to structural considerations are indexed under alternative headings. An enquirer from another industry or technology might be forgiven for inferring that paper is a structureless body defying the normal laws of physical analysis and description!

This is an exaggeration, of course, for ideas about structure are implicit in a wide range of theories, discussions and practical operations on paper and papermaking, but the absence of explicit reference to structure does clearly indicate the superficial and empirical nature of our current knowledge and of the technology of paper and papermaking. At the same time, there are now many indications of a revolutionary change in this situation. On all sides, there is a mounting demand for a more fundamental understanding of the structural factors underlying the physical properties of the fibrous network of paper: a demand deriving from the technological pressures of a rapidly expanding industry. So long as acceptable paper could be made at acceptable rates of production without such understanding, there was no economic incentive for deeper probing; but now, when our ignorance is beginning to set limitations to achievement of quality or of economic rates of output, there is a strengthening demand to break down these limitations through knowledge and understanding of underlying factors.

The last few years have seen only the beginnings of this new approach; consequently, our proceedings will inevitably reflect the partial and exploratory nature of our knowledge of the subject—in sharp distinction to the rounded and comprehensive quality of our 1957 Cambridge symposium on fibres. Whereas many of the 1957 papers tended to be—at least in part—critical reviews of the state of current knowledge, we have in 1961 a much greater proportion of papers that report original work; this is inevitable in those sectors where there is so little previous work to review. Indeed, we have reason to be gratified that the timing of this symposium has coincided with the emergence of so much new and original material; otherwise, our proceedings might have been lean and relatively unproductive.

Among the objectives of this introductory paper is the need to indicate the reasoning that lay behind the planning of the symposium and to explain the pattern into which the contributions and sessions have been arranged.

The term *structure* of paper is self-explanatory: it refers to the pattern into which the component elements of the sheet are arranged to form the new artifact paper. It includes such descriptive factors as shape, size, isotropy and appearance; it includes reference to the relative placing of separable components and to the bonding together of these components; it includes reference to the interaction of the artifact with adjacent media, since no body can exist unrelated to surrounding or permeating bodies.

The term *formation*, which accompanies structure in the symposium title, requires more explicit definition, since it is used in connection with an industry that sometimes uses this word in a specialist and restricted sense. In this symposium, we are using the word formation to describe the act or acts of forming the artifact from its components, not merely to describe the optical appearance of the paper sheet by transmitted light, which is a traditional papermaking usage of the word. Formation in this latter sense is indeed one of the physical properties consequent upon the formation of the sheet in the general sense of an act or succession of acts of forming and it will certainly be referred to at some stage in this symposium. It is therefore important to clarify the distinction of meaning.

Structure is the link between the technology of papermaking and the technology of paper properties and usage. There are five basic elements relevant to our subject matter. Listed analytically, they are—

1. The usage behaviour of paper.
2. The physical properties that determine that behaviour.
3. The structure that underlies the physical properties.
4. The processes of formation that produce the structure.
5. The components that are joined together by the processes of formation.

In deciding the pattern of the conference, the Fundamental Research Committee was much exercised to present the subject matter in a way that would be both logical and practical. Although the 1957 conference dealt so fully with the subject of papermaking fibres (the components—element 5 above), it was decided that this element should be recapitulated in a series of papers oriented towards structural considerations: two papers dealing with the nature and selection of papermaking fibres—one dealing with the effect of pulping processes upon papermaking properties of the fibres, one dealing with the beating process. This series of papers was visualised as a precursor to the series on the papermaking processes as such and, taken together, these two series were seen to cover the practical aspect of the symposium, the range of physical processes of formation, ranging from the original selection and preparation of the fibres through stock preparation to formation on the making machine.

At the other end of the subject, much thought was given to the question of usage behaviour. Although this is seen as the ultimate objective of the whole complex of paper technology, it was realised at an early stage that it would be impracticable in this symposium to make more than cursory reference to the mass of empirical knowledge relating to the hundreds of different types of paper usage. Consequently, it was decided to concentrate attention upon the physical properties that determine usage behaviour, while recognising the importance of the distinction between these two elements.

Thus, the main elements of the subject were reduced to—

A. The physical properties of paper (with incidental reference to resultant usage behaviour).

B. The structure underlying these properties.

C. The processes of formation (including fibre selection and preparation) that produce the structure.

Viewed thus, structure is once again seen as *the* central element, produced by the processes of formation, itself *determining* the physical properties.

After much thought, it was decided to emphasise the central (or pivotal) character of structure by making this the opening theme of the proceedings: a decision to go straight to 'the heart of the matter'. The next consideration was that the symposium should culminate in a natural and practical climax, which determined the decision to place the session on the processes of formation at the *end* of the proceedings, with the sessions on physical properties on the intermediate days.

The result is a natural sequence moving from fundamental theory, through analytical and empirical studies, into practical operations.

The mathematics of paper structure

ONE square metre of paper (of, say, 40 g/m^2 substance) contains about 10^8 fibres— 10^8 structural components arranged within the main plane of the sheet according to a random process, with certain superimposed biases. Added to this is the fact that the components themselves cover a wide range of shape and size. It is therefore evident at once that an integral description of the structure of a sheet in terms of the arrangement of its components must involve probability theory and is bound to be highly complex in a mathematical sense. A brick wall can be quite simply described in terms of an invariable non-random arrangement and fixing together of identical components. A sheet of paper is immeasurably more complicated, yet our understanding of the structural reasons for physical properties and our hopes of a more deliberate control of the papermaking processes to produce desired structures and consequent properties depend upon solving these complexities and in some way reducing them to simple expressions.

The pioneers of the mathematical studies required to approach a solution were two men whose names are virtually unknown in the paper industry: H. L. Cox and J. E. Gordon. Cox and Gordon were concerned during the 1939–1945 war with the adaptation of paper base laminates to aircraft construction. The anisotropy of these laminates was of vital importance in that work; consequently, their attention was turned to the fundamental structure of paper itself. The resultant studies rest in the archives of the National Physical Laboratory and the Chemical Research Laboratory at Teddington, as well as the Royal Aircraft Establishment at Farnborough (1942 *et seq.*). Cox eventually published some of his work in 1952 (*Brit. J. appl. Phys.*, **3**, 72). Gordon's work, which concerned the practical application of Cox's theories and which involved intensive experimental observations on thin polished sections of laminates, was unfortunately never published.

Cox and Gordon's work was necessarily limited by incomplete premises on the nature of the paper and by inadequate mathematical tools. The mathematics of statistical geometry were yet to emerge; consequently, their studies did not succeed in solving the basic problem of describing mathematically the structure of paper in terms of the shape and position of its component elements. Nevertheless, this work represented a valuable advance, also a challenge to the technology of the paper industry.

That challenge was not to be taken up for many years. Rheological studies of paper properties pioneered by Gibbon (1944) and Farebrother (1944) and strikingly developed on a mathematical basis by Steenberg *et al.* (1947 *et seq.*) stimulated a renewed interest in the structure of paper, but did not advance far beyond empirical description of certain aspects of behaviour before fracture. The need for a new mathematical approach to rheological problems based explicitly upon structural considerations was discussed at length by Rance (1956), but the requisite effort has not been forthcoming until the last year or two. In 1959, Corte and Kallmes published an introductory paper *The statistical geometry of an ideal two-dimensional network* and their studies are further developed in the first paper of the present conference on general structural considerations and in a later paper dealing with the relation between structure and physical properties.

Looking at structure

WHILE the mathematics of the structure of paper have thus been tentatively probed during the last ten to fifteen years, other workers have been taking the direct approach of examining structure visually, recording the observations both photographically and by systematic description. The cellulose fibre normally used for making paper is on the borderline between microscopic and macroscopic. Hence, the structural arrangements of the fibres are directly and readily amenable to light microscopy—a fact that is sometimes overlooked in favour of the more esoteric attractions of the electron microscope.

The last few years have seen substantial advances in techniques of light microscopy adapted in various ways to the study of paper surfaces and paper structure. Metal shadowing, especially in connection with replica technique, has come into its own, together with examination under polarised light and allied techniques. The British Paper & Board Industry Research Association at Kenley has pioneered this work with striking success and two papers in the opening sessions of our symposium, given by teams led by Emerton and Page, testify to the advances made. These deal with structure as seen in the surface of paper and the holding together of the structural elements in the sheet.

At lower orders of dimension, the light microscope has to give way to the electron microscope. The finer resolution achieved by this instrument opens up a new potential for observation and facilitates the dovetailing of the structure of paper with the structure of its fibrous components. In this respect, Jayme's paper is an important link with the body of knowledge on the structure of fibres already recorded in the proceedings of our last symposium.

Finally, below the visible level—below even the resolution of the electron microscope—is the level of molecular events that must be dealt with by the mathematical, thermodynamic approach. The programme of this conference rightly emphasises the central significance of the near macroscopic fibre, but it would be unbalanced without reference to the molecular events that determine the way in which the fibrous elements are bonded together. Nissan's paper discusses the cement that sticks together the observed components and here perforce we return to a method of study based upon mathematical considerations, akin in principle to that employed in the work of Corte and Kallmes.

Structure, physical properties and usage quality

STRUCTURE is the prime determinant of the physical properties of the paper sheet, whether mechanical or optical or relating to fluid permeability. In turn, the physical properties of the sheet largely determine its usage quality, which is the ultimate objective of the papermaker.

The distinction between physical properties and the consequent usage quality is sometimes confused, but it is none the less real. Usage quality relates to a particular *event* or complex of events of usage, in which the paper sheet is subjected to a sequence or complex of physical operations. The behaviour of the sheet may depend upon the interactions of a number of different physical properties and it is this behaviour that defines the usage quality—for example, a paper subjected to aqueous coating, followed by drying, may show an objectionable curl. The physical properties determining this curl will include the rate of liquid penetration through the porous structure of the sheet, the hygroexpansivity behaviour of the sheet and the rigidity of the sheet. Each of these properties in turn is largely determined by the underlying structure of the sheet, which is itself determined by the nature of the component fibres and the manner in which they are assembled and bonded together.

This is the long and complicated sequence of cause and effect that the paper scientist tries to unravel and it is vital that at every stage he should distinguish between the successive causal steps. In the past, he has often failed to distinguish clearly between simple physical measurements and usage quality—and this has been a source of some misunderstanding between the commercial and technical sides of the industry. A single elementary physical property seldom determines alone the usage behaviour of a paper, yet there is a perennial tendency to try to use an elementary measurement as a direct index of ultimate behaviour.

Nowhere has this tendency been more evident than in the field of mechanical properties. These have always been a focus of attention for paper technologists, partly because they have central importance for certain uses of paper, but also because strength testing—in crude form—is so easy to achieve. Complete description of mechanical properties is, however, quite another matter: it calls for a complex of expressions involving stress, strain and time and this complex can be simplified only by making limiting and arbitrary assumptions. Furthermore, mechanical usage of a sheet of paper is itself almost invariably a complex event or series of events. Only rarely is it possible to accept an elementary mechanical measurement as a reliable index of usage behaviour.

Realisation of these facts stimulated a considerable amount of investigation into mechanical properties in the 'fifties and these properties are still a focus of attention for a substantial research effort. Hence the fact that a whole day of our present conference is devoted to various independent views on the interrelations between structure and strength. Five teams of workers are contributing, two from the U.S.A. and three from the U.K. Research associations are strongly represented here, including a most welcome paper from the Institute of Paper Chemistry presented by Van den Akker; but private industry also is active in this field, as exemplified by the paper by Ranger and Hopkins. With such a concentration of attention, this is one of the few subjects in which we may hope to reach a measure of real understanding before our discussions close.

It is perhaps a weakness of our programme that, relatively speaking, so little attention is being paid to other physical properties. For many of the main uses of paper—for printing, writing, copying, wrapping, etc.—the interaction of the sheet with permeating fluids—air, water, oil—is quite as important as mechanical strength. Furthermore, there are some clear and relatively simple relationships between structural parameters and fluid permeability. It is to be hoped, therefore, that Brecht's treatment of this subject will be well supplemented by discussion contributions, perhaps by associated references arising from some of the other papers concerning properties and structure.

Optical properties do not have the same central significance in the usage of paper, apart from certain specialities; but, as Harrison's contribution will show, they have particular interest as indices of structural arrangements. Over twenty years ago, the application of the Kubelka and Munk theory in studies on paper brightness and opacity indicated the structural basis of optical properties. The relationship between light transmission and bonded area between fibres was one of the first structural concepts to be established and accepted in our scientific thinking about paper properties. It has borne fruit in more recent studies by Nordman *et al.*

and by Kenley workers, relating optical to mechanical properties through their common structural basis.

Here, indeed, is one of the most important features of recent work in the field of paper properties: a growing realisation of their interdependence arising from their common dependence upon the underlying structure. This is so important as to justify a paper, given by Gallay, concerned not with a particular set of properties, but with the interdependence of all paper properties. This is intended to stress the behaviour of paper as a phenomenal whole, as a set of related events determined by its basic physical properties and, in turn, by its underlying structure. In this way, by tracing out the links between various environmental factors and patterns of behaviour, we may be able to see more clearly the central significance of structure in the usage of paper.

The formation of structure

Having observed and described the structure of the paper sheet and having related that structure to the physical properties and usage behaviour of the sheet, it remains to control the actual procedure of structural formation in the course of the papermaking process.

I have indicated that the selection of fibrous raw materials and their treatment rank as preliminary processes in this sense. In fact, the selection and treatment of the fibres together are the main determinants of the structural *type* of the paper. A few simple examples will demonstrate this. A very free beaten stock of, say, a hemicellulose-free woodpulp or cotton linters inevitably gives a bulky, permeable, opaque, absorbent type of paper; nothing the papermaker can do will change this structural type. At the other extreme, a very wet beaten stock of, say, a high hemicellulose woodpulp inevitably gives a dense, non-permeable, non-absorbent, semi-transparent paper, however the papermachine is operated.

In this sense, it is only too true that 'paper is made in the beater'—even before the beater—consequently, we have devoted a whole session to a survey of these preliminary but vital processes that do so much to determine the character of the final product. The first two papers, one by Dadswell and his colleagues, the other by Grant, deal with the influence of intrinsic fibre properties and types upon the structure and properties of paper; Giertz' and Higgins' papers deal respectively with the effects of the pulping process and the beating process, still—this must be repeated—within the context of the structure of the paper sheet. Here is the field that links our 1957 Cambridge conference and the 1961 Oxford conference. It must again be emphasised that we have a good working knowledge about the fibres that are the component elements for our sheets of paper. It ought to be possible to relate that knowledge to the properties and behaviour of the paper sheet, if only we can get a clearer understanding of the structural arrangements of the components within the sheet. Even qualitative relationships can be of considerable practical value in helping to determine choice, blending and treatment of the fibres before they are assembled together on the papermachine.

To conclude this session, a rather special paper has been added, somewhat obliquely related to the central theme. This is Groen's paper concerning the distribution of loading materials within the paper sheet. Despite the original decision to confine attention to structure of the sheet as an assemblage of fibres, it was felt that loading materials constitute a component of the sheet of significance too great to be ignored.

Groen's paper serves also as a link with the final session, dealing with the focus of attention for the practical papermaker, the formation of the paper web at the wet end of the papermachine. The papermaking process on the machine, which deals with the beaten stock presented to it, cannot change the *type* of product that is predetermined by the nature of the stock; but it does exert a fine control over a significant range of variability. Of even more importance, it determines the degrees of homogeneity—both small scale and large scale—of the final product and determines the anisotropies that can have such a profound effect upon the usage of the paper.

I have indicated earlier the difficulties inherent in mathematical description of structure and the complexities of the involved relationships between structure and behaviour. The control of structure in the papermaking process is no less difficult and complex. It is probably much more difficult, involving as it does economic and technological compromises at every stage.

The process of making a sheet on the machine—the deposition and filtration of a felted mass of fibres from a water suspension—is one of the most elementary methods of construction known to human history. Intrinsically, the method has not changed over two thousand years and only very slowly has the papermaker progressively learned to control the deposition and filtration of his fibres to give the desired final product. His degree of control is still relatively primitive. It is commensurate with the status of paper as one of the cheapest artifacts of the modern world, but, in many cases, it does not match the demands for precision that arise from the increasing complexity of paper-using equipment.

A property required above all in a web of paper is that of regularity. Modern, high-speed printing or converting machinery may be variable to allow for start-up changes in the properties of the fed-in web of paper, but it cannot tolerate serious variations in strength, porosity, etc., once the machine has started. Hence, the primary aim of the papermaker is to maintain a given structural pattern and to avoid both small scale heterogeneity and large scale variations.

In this last session of the conference are assembled a number of papers dealing with various aspects of the central events of fibre deposition and filtration, in which the fibres are separated from their carrier medium and are deposited together to form the structure that will determine the final product. Many physical factors are involved. The hydrodynamics of stock flow, fibre flocculation, speed of deposition, rate, direction and method of water removal; all these combine to determine the pattern of the ultimate sheet and all are dealt with in papers presented by contributors from four countries and three continents: Majewski from Australia, Robertson and Mason from Canada, Wrist from the U.S.A., Andersson and Steen from Sweden. It is fitting that such an assembly of international talent should concentrate upon this closing theme, for, despite all the technological advances of the last decade, we are still ignorant of many of the elementary factors in this area. The most we can hope for is some clarification of the problems, a slight lifting of the cloud of ignorance that covers the practical theme of paper formation.

Finally, what of the events that come after? Couching, pressing, drying, sizing, calendering: each of these processes leaves its mark upon the structure and character of the paper sheet and in due time each must be considered. For the present, we have had to limit our field of interest, have had to curtail our terms of reference. To deal reasonably and coherently with the main factors of our subject, we have decided to stop at the wet end of the papermachine. What I have already said indicates that vast scope of the subject matter, even with such limitation. Perhaps a future conference will carry the theme further, through the after-treatments and ancillary processes, to complete the story.

There are many other omissions that necessarily render incomplete our treatment of the chosen theme. There are many specialities with their special problems of structure and property that might well shed light upon the wider field, but of necessity the programme deals almost exclusively with conventional papers made from conventional raw materials. There is the colloidal aspect of papermaking, the whole physico-chemical complex that surrounds such subjects as deflocculation, sizing and dispersion of loading. There are the many specialised end uses of paper that introduce their own special problems and special knowledge, related to yet far more complex circumstances than the simple cases considered in our programme. All these aspects are of great importance, but they are in a sense marginal to the main theme and can be touched on only incidentally.

Even with these omissions, the programme is extensive and the theme complex, if not comprehensive. Indeed, viewed superficially, the programme could be criticised not so much on grounds of incompleteness as on grounds of diffuseness: it could be argued that it is too widely spread, not focused sharply enough upon the central theme of structure. Whether or not this criticism is justified will depend upon those who take part in the conference.

All participants, speakers, chairmen and contributors to discussion should keep in mind the central theme towards which all arguments must be pointed, towards which all evidence must converge. The theme is structure: everything we discuss must be related to structure; if we adhere strictly and consistently to this theme, knowledge and understanding will emerge. Prof. Rånby will have the arduous, but (I hope) rewarding task of trying to crystallise this when he sums up at the end of our conference.

DISCUSSION

PROF. R. H. PETERS: Could you comment on how easy it is to make a random sheet and, secondly, can you possibly explain at this stage how it is that the Poisson distribution has come into the picture as distinct from a normal distribution?

DR. H. CORTE: On the first question, when you make a $2\frac{1}{2}$ g/m^2 handsheet, you suspend 50 mg of fibres in 7 litres of water and you will see in the suspension (or you have at least the impression) that the fibres are moving quite independently of one another. We do not know whether they do that. When fibres move independently of one another, they should form a random network: that is the definition of it. Whether or not they do this we do not know beforehand, but we can check it afterwards.

Once this sheet is made, we can perform a number of quite simple experiments. These sheets have a diameter of 16 cm and, for instance, you draw a straight line and count the number of intersections between the straight line and the fibres—of the order of 1 200. Then you divide the straight line into a number of equal sections and compare the number of intersections per section with the Poisson distribution. This has to fit and that answers your second question. When you travel along a straight line and place events at random, the number of events at uniform intervals has a Poisson distribution. Thus, the number of telephone calls in a busy telephone exchange per minute has a Poisson distribution.

The existence of such a Poisson distribution is not a complete proof, but it is very strong support that the system we have produced is random.

MR. P. H. PRIOR: If you perform exactly the same operation with one of your flocculated sheets, how will that differ from the Poisson distribution? Will it necessarily be distinguishable?

DR. CORTE: Yes, it would be distinguishable and it will not be a Poisson distribution. You may have a bimodal distribution, for instance. If not, you may have a wider distribution and that is what normally happens.

DR. A. B. TRUMAN: As stated in your paper, that the fibres in a two-dimensional sheet are randomly distributed does not imply that they are uniformly distributed. Does it follow that flocculation of the fibres may be observed in a random sheet?

DR. CORTE: The term flocculation is, so to speak, a human expression and not a mathematical one. Papermakers use this word for a visual effect: when we use the word here, we mean the amount of non-uniformity that is beyond that inherent in a Poisson process. You can think of making the non-uniformity of a random sheet the standard (unit flocculation). The flocculation scale would then start at this point (the non-uniformity of a random sheet) and the term *flocculated* would apply to sheets with a formation worse than random.

MR. P. E. WRIST: I wish to refer to a point to be discussed in my paper at the end of the week. For reasons outlined there, we have been forced to the conclusion that the fibres in a sheet of paper are more uniformly distributed than would occur by a random distribution alone.

DR. CORTE: I am glad to hear this. I was under the impression that, in order to render a sheet, say, completely uniform, you have to control the deposition of every fibre. Only then can you control the position of the fibre centres and make a completely ordered and uniform structure like a woven fabric.

MR. WRIST: This is the ultimate in control. While we cannot control fibre deposition to any extent approximating to this ideal, we do have a small measure of control on the papermachine through the use of such means as wire shake, velocity differential between the jet and wire speed and the agitation that occurs over table rolls at high speeds—all of which produce relative motion between the fibre suspension and the forming web. Combined with the local variations of drainage resistance produced by any local variations in the concentration of fibre deposition, this relative motion promotes a more uniform distribution than would be produced by a randomising process alone.

DR. CORTE: Relative motion of fibres leading to a more uniform distribution may exist, but I have never seen them and they have not been considered.

MR. W. H. HALE: Have these statistics been compared with actual papers made on standard production machines?

DR. CORTE: It has never been done, because no two-dimensional sheet can be made on the papermachine and this statistical treatment refers to two-dimensional papers only. The extension of statistical geometry to three dimensions is possible and has been made, but direct observation of the geometric quantities is impossible; instead, one has to use them to predict physical behaviour. That will be the subject of our next contribution.

PROF. A. H. NISSAN: I am interested in comparing this work with other work, not in the paper industry at all, but where statistical geometry is called for. Bernal published about two years ago an article in *Nature*, in which he tried to explain (very tentatively as admitted by him) the structure of liquids and he calls for statistical geometry as a new science that does not exist as yet, for a higher form of mathematics. He has done some empirical experiments like yours to find out the number of sides of polygons that are produced randomly and he gives a table of results. I was interested to find that the maximum occurs at five in his work, whereas in yours I notice four sides.

DR. CORTE: The number of sides of the polygons are derivable.

PROF. NISSAN: His were not and I was wondering whether you have compared your work with Bernal's.

DR. CORTE: I know Bernal's publication on crystal lattices, but as a matter of fact the number of sides of the polygons in two dimensions is four: there is strict proof of it, very easy and quoted in our paper. We have in fact made enlarged photographs of such a piece of paper, cut these polygons out with a pair of scissors and written down the number of sides. We found to our surprise that 88 per cent of them had three or four sides, with an average of four. Unfortunately, we did not isolate the pentagons. (Bernal's polygons are the faces of polyhedra, which he finds on the average to be pentagons.)

PROF. NISSAN: So really your results differ from Bernal's.

DR. O. J. KALLMES: I think he is working only in three dimensions, not in two.

DR. C. W. CARROLL: Arising from Wrist's comments, if by *uniformity* he means *symmetry*, I would recall to you that the Poisson distribution can be approximated by the normal distribution under certain conditions. In the case of random fibre deposition, these conditions correspond to the requirement that in spite of the low probability of an event (that is, small chance of a fibre centre landing on a particular small sub-area), the number of fibres involved is so large that the product of the small probability of an event and this very large number of randomly deposited fibres (the parameter of the Poisson distribution) is itself relatively large. Thus, the more concentrated the fibre suspension from which a sheet is randomly formed, the greater the tendency that the sheet will be characterised by a normal distribution of fibre centres. In this sense, the Poisson and the normal distributions are not two discrete distributions, but represent a continuum in the realm of distributions, merging one into the other.

MR. WRIST: My comment did not refer to the choice of distribution function, but that the distributions we obtained on practical sheets of paper are more regular than would result from a random distribution alone. If we could do no better than achieve a random distribution as you seem to suggest and that flocculation worsens the situation still further, we would be unable to make a saleable sheet of paper.

DR. KALLMES: In any case, if you say basically the process gives random flocculation of unity by definition, to a more uniform sheet we could give that a flocculation figure of 0·9 or 0·8. The important point is that you have a reference mark.

A DELEGATE: I am getting a bit confused. Don't you get uniformity from averaging a large number of random distributions?

DR. CORTE: When the mean goes up, the variance goes up accordingly and the standard deviation (the square root) goes up too. The relative standard deviation (standard deviation divided by the mean) goes down with increasing thickness of the pile of random sheets.

MR. WRIST: If we restrict our discussion to two-dimensional sheets alone, a random distribution is acceptable; but, if you then go on to build up a three-dimensional sheet by stacking two-dimensional sheets, you are assuming that the relative position of the fibre in a given layer is completely independent of the positions of fibres in the layers beneath. This is where my interpretation of the papermaking process disagrees with yours. Once you have the first two-dimensional sheet laid down, the deposition of the subsequent fibres is not a completely random event, there is a strong tendency for it to be drawn to a place in the sheet where there is deficiency in fibres and the result is a tendency to build up a much more uniform sheet. It is fortunate that this can occur; otherwise, thin sheets of paper would be completely unacceptable in formation.

DR. CORTE: Nevertheless, although this may be so, it could be checked whether randomness occurs. For machine-made papers, it may not be the case, because the hydrodynamic effect could upset the whole picture. We do not seem to know exactly how the fibres are deposited on the wire. For a purely formal description, it would not really matter, because you could slice a paper and describe each layer no matter how it was formed. We want only to describe, we do not want to refer to the forming process at all. If it is not random, we have experimental means to find out how non-random it is and express this. The problem of how this state of affairs was produced is an entirely different thing and is not the subject matter of this paper.

MR. P. A. TYDEMAN: Could you clarify a point on nomenclature? You have defined g as mean free fibre length and I notice you call it also the distance between two intersections.

DR. CORTE: I mean the distance between centres of fibre intersections.

MR. TYDEMAN: That is surely not free fibre length?

DR. CORTE: We call it free fibre length. The distribution is, by the way, independent of the width of the fibres. When you take wider fibres—we assume that we have a large number of them with statistical or random distribution—then, of course, a number of these gaps would disappear, would be blocked, but those remaining are still an infinitely large number and their distribution would still be exactly the same.

MR. P. G. SUSSMAN: Have you ever used a scanning area smaller than 1 mm and so determined the statistical distribution of the mass of these small areas? If the scanning area is small enough, there is a definite probability of finding very dense spots in a sheet of purely random structure.

I once made sheets of 70 g/m^2 substance from highly dilute stock (0·002 per cent consistency) and they showed quite a few very dense spots, though they were very even over larger areas.

An ordinary handsheet, made from the same pulp at 0·02 per cent consistency, showed more general variation in look-through, but none of these dense spots.

DR. CORTE: We have never scanned areas smaller than 1 mm^2, but we have scanned larger areas by taking, for instance, four of them together to give a square. There is a certain rule about this, how the parameters of the distribution vary from size to size and, whenever they do not correspond to this, then you have a non-random structure. This is revealed by comparing the results of two adjacent squares or of two squares separated by one or more squares. The autocorrelation between squares that are a certain distance apart would indicate whether the distribution is random.

A random distribution is independent of the size of the squares. Only the parameters of the distribution vary with the size of the squares in a well-defined manner.

A DELEGATE: In practice, when one makes a random sheet does it work out according to the equation—and when one scans a sheet made from a high dilution, do you in fact find this so, even when you are scanning very small areas?

DR. CORTE: Yes, this is part of the equation. Take, for instance, the one-dimensional case analogy. The parameter of the negative exponential distribution would be one over the mean number of intersections per unit length, say, 1 mm: this gives the spread of the distribution. When the intervals have only half the width, say, 0·5 mm, then of course the mean is smaller, the parameter is larger and the spread is automatically larger.