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A NEW THEORY OF THE TENSILE BEHAVIOUR OF PAPER

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Summary

Following considerations of previous work, the authors' investigations indicate that the first interfibre bonds to rupture under tensile strain do so by a peeling action with shear deformation of the paper in locally weakened regions. Progressive breakdown occurs along specific narrow bands within which strain is largely concentrated. The direction of these 'strain lines' has been predicted. Only for weak papers does final fracture occur along a single strain line, when considerable shear also results. For strong papers, only short lengths of strain lines form in a widely scattered criss-cross formation. In all cases, ultimate failure depends on the relationship between the tensile strength of fibres and the shear strength of bonds.

The mechanism of strain line formation results in paper thickness increases. Permanent deformation, frozen-in stresses and the effects of drying stresses are largely associated with strain line formation and frictional forces between fibre 'mats'. Changes in moisture content modify the frictional effects and stress distribution and some aspects of dimensional stability can be explained. The finer details of fibre orientation distribution in machine-made papers can be attributed to shear in strain lines.

Une nouvelle théorie sur le comportement du papier sous tension

Après un examen critique des travaux antérieures, les auteurs montrent par leur recherches que les premières liaisons interfibres qui se rompent sous tension le font par pelage accompagné d'une déformation de cisaillement des zones du papier localement affaibli. La rupture progressive se produit dans des bandes étroites privilégiées où

la déformation est concentrée. La direction de ces 'lignes de déformation' a été prevue d'avance. Seulement dans le cas des papiers legers la rupture totale se produit-elle selon une ligne de déformation unique et dans ce cas un cisaillement important se produit en même temps. Pour les papiers forts il ne se forme que des lignes courtes de déformation en treillis largement réparti. Dans tous les cas la rupture finale dépend de la relation entre la résistance à la rupture des fibres et la résistance au cisaillement des liaisons.

La formation des lignes de déformation entraîne une augmentation de l'épaisseur du papier. Déformations permanentes, tensions internes et effets de contrainte dus au séchage ont tous une influence sur la formation des lignes de déformation et sur les forces de frottement entre les couches de fibres.

Les variations de teneur en humidité modifient les effets des forces de frottement et la répartition des contraintes. Ainsi s'expliquent certains aspects de la stabilité dimensionelle. De plus, le détail plus fin de la distribution de l'orientation des fibres dans les papiers fabriqués sur machine peut être rattaché au cisaillement dans les lignes de déformation.

Eine neue Theorie über das Verhalten von Papier unter Zugbeanspruchung

Auf Grund früherer Arbeiten sind die Autoren der Ansicht, dass die Zerstörung der ersten Zwischenfaserbindungen bei Angriff einer Zugdehnung die Folge einer Schälwirkung bei einer Scherverformung des Papiers innerhalb örtlich geschwächter Bereiche ist. Die fortschreitende Zerstörung verläuft entlang spezifisch schmalen Bändern, innerhalb derer die Dehnung weitgehend konzentriert ist. Die Richtung dieser 'Dehnungslinien' kann vorausgesagt werden. Nur bei schwachen Papieren erfolgt der endgültige Bruch entlang einer einzigen Dehnungslinie bei gleichzeitiger beträchtlicher Scherwirkung. Bei festen Papieren bilden die Dehnungslinien bei geringen Längen eine weitgestreute, kreuzweise Formation. In jedem Fall hängt der endgültige Bruch von der Beziehung zwischen der Reissfestigkeit der Fasern und der Scherfestigkeit der Bindungen ab.

Der Mechanismus der Bildung von Dehnungslinien führt zu einer Erhöhung der Bahndicke. Permanente Verformung, eingefrorene Spannungen und die Wirkung von Trocknungsspannungen sind weitgehend mit der Bildung der Dehnungslinien und den Reibungskräften innerhalb des Faservlieses verbunden. Änderungen im Feuchtig-

keitsgehalt variieren die Reibungswirkung und die Spannungsverteilung, was in gewissem Mass die Dimensionsstabilität erklärt. Die Einzelheiten der Faserorientierung in Maschinenpapieren können auf die Scherwirkung in den Dehnungslinien zurückgeführt werden.

Introduction

EXPERIMENTALLY, the tensile behaviour of paper has been widely studied, especially during the last twenty years, but theoretical interpretation of the work has been more limited. Thus, Steenberg and his associates have greatly advanced our factual knowledge by excellent experimental techniques, but largely confined their interpretive work to describing tensile phenomena by the well-known rheological models using springs and dashpots.

This approach has produced results of great value and interest, but has several limitations. In particular, (*a*) it fails to connect the structural characteristics and constituent behaviour of the paper with those of the model and (*b*) the model lacks any characteristic to terminate the straining process. To overcome the first of these criticisms, Ivarsson and Steenberg(1) suggest that individual paper constituents (fibres, fibrils, etc.) possess mainly spring-like characteristics, whereas the bonded junctions between them supply most of the viscous element. Although this enables the required rheological model to be formed, it is unconvincing to suggest that the paper constituents exhibit such a demarcation of characteristics and that they are so conveniently combined.

The most valuable use for spring and dashpot models would appear to be the provision of a convenient and easily remembered analogy (not physical explanation) of the behaviour of a material under stress. As Steenberg(2) has said, 'I still think that there is no method superior to the mechanical model of paper, if I have the aim in the shortest possible time to give somebody a condensed qualitative knowledge about the behaviour of paper under stress and strain.'

Nissan's(3) proposed theory of hydrogen-bonded solids, as applied to paper, assumes that all extension under stress results from the hydrogen-bond strain in the amorphous regions of the fibres and between fibres. The stress/strain curve is derived from a modified Morse function chosen to represent the hydrogen bond energy/strain relationship, hence the theory fails to associate the latter part of the curve with permanent deformation of the paper. The stress/strain curve predicted rises to a maximum with a monotonically decreasing gradient and then decreases asymptotically to zero with continued strain. The calculated position of the maximum occurs at a strain of about 3 per cent (as Nissan himself points out). This shows a wide discrepancy between theory and behaviour for paper dried with freedom to shrink, which commonly exhibits an ultimate strain of the order of 10–15 per cent and, at fracture, shows an increasing gradient of the stress/strain curve. This and similar shortcomings would seem to be at least partially due to the exclusion from the theory of any considerations of the geometrical structure.

Detailed aspects of paper behaviour have been discussed theoretically by several authors, but a discussion of such work would not be appropriate in this context. The work of Rance,(4, 5) however, is in a different category, since he has had considerable success in relating the mechanical to the structural properties of paper. He postulates the fracture of interfibre bonds at a comparatively early stage of straining and so explains the general shape of the stress/strain curve. The further assumption of large frictional forces between unbound fibres allows extension of the theory to cover permanent deformation and time-dependent phenomena. Certain paper characteristics (for example, machine- and cross-direction differences) are explained on a basis of a spectrum of slackness in the network that is progressively taken up during the piecemeal rupture of bonds. Rance's theory is based essentially on the predominant weakness of interfibre bonds and on few fibres breaking when the sheet finally ruptures. This theory has radically different origins from the present one, but possesses, perhaps significantly, many similar features that are discussed as the occasion arises. The present theory and its basis are now presented in Part 1, whilst Part 2 discusses further implications and ramifications. Only brief accounts of our experimental work have been given and these occur mainly in Part II.

PART 1

The tensile strength of paper

INITIALLY, in attempting to investigate the complete tensile behaviour of paper, the ultimate strength was considered. For the majority of papers, the most commonly held view seemed to be that final fracture is predominantly due to interfibre bond failure, with very few fibres being broken. The evidence in favour of this belief has been analysed and found to lack conviction. Some of the points considered are adequately discussed in a paper by Van den Akker and his colleagues.(6) Their paper also describes an experimental determination of the number of fibres broken at ultimate fracture, using sheets incorporating a small percentage of dyed fibres. In one case, as many as 77 per cent of the fibres involved in the fracture were broken. Although noting that many fibres broke after the start of fracture (which could still have been initiated by bond failure), Van der Akker concludes that 'the strength of fibres of a papermaking pulp is of much greater importance than had hitherto been generally recognised'.

A weakness in this experimental work, which may detract from its significance, is the determination of the number of fibres (dyed) involved in the fracture. Broken fibres are easily counted, but bond failure is more difficult to assess. If from the fracture line of the paper there is a reasonable length of unbroken fibre, it is reasonable to assume that it was previously bonded to the rest of the paper. The more doubtful assumption of previous bonding between short fibre projections and the sheet from which they were pulled out is also required if the fibre is to be considered as involved in the fracture. A different situation is illustrated in Fig. 1 in that the dyed fibre is involved in the fracture, but would not be counted. It is probable that the latter situation outweighs the first, so that the total number of involved fibres is underestimated, hence the proportion of broken fibres is overestimated. Despite the influence of these disputable factors, however, the general conclusions are not likely to be altered.

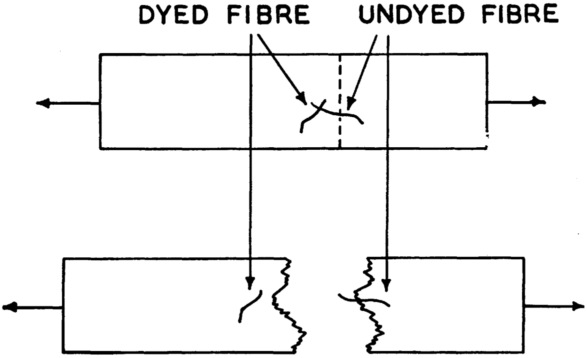

Fig. 1—Dyed fibre involved in fracture, but not counted

Zero-span tensile test results apparently support the 'importance of interfibre bond failure in tension' theory. In the test, it is considered that the majority of fibres undergoing strain are gripped by both jaws, hence the tensile strength will be a function of fibre strength and orientation only. Hoffman Jacobson,(7) who first suggested this test, called the ratio of the normal span to zero-span tensile strengths the *percentage adhesion*, with the implication that the lower value with normal span tests is due to bond failure. The zero-span strength changes very little with the degree of beating except in the initial stages and would seem to be a good indication of fibre strength,(6) although more recent work(8) has cast some doubt on this.

If the variation of *percentage adhesion* with beating is examined, it is found that an initial value of 10–20 per cent for unbeaten sheets increases on beating, but apparently asymptotes to a seldomly exceeded value in the region of 50–70 per cent (*see* Fig. 2). This apparent support for the fibre bond failure theory ignores two considerations.

Firstly, it was established that the mode of deformation for the two tests differ, since a normal test strip contracts significantly in width when stretched. Defining a Poisson's ratio for paper as the lateral contractile strain per unit longitudinal tensile strain, an average value was found for this ratio of 0·95 for beaten sulphite handsheets* and 0·64 for sulphate. For machine-made sack kraft, an average value of 0·78 for machine-direction strips was obtained and 0·40 for cross-direction samples. An average spread of results of about ± 40 per cent is typical of these measurements. Obviously, the fibre stress will depend on orientation ranging from maximum tension when parallel to the tension to maximum compression when lying across the contracted width of the strip. In the zero-span test, the paper is constrained by the jaws, lateral contraction is prevented and only tensile forces can exist in the fibres.

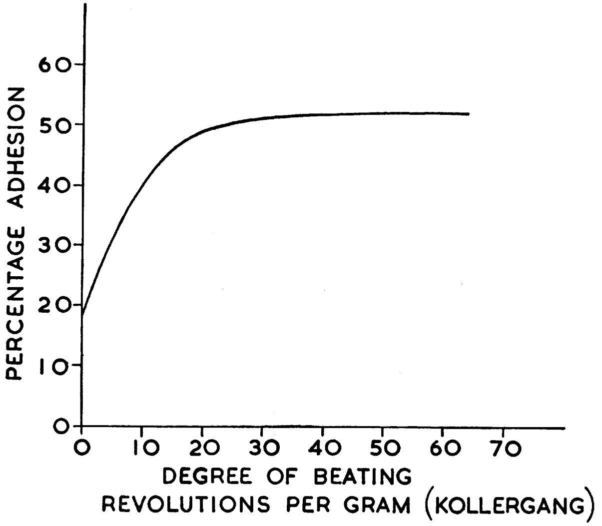

Fig. 2—Typical increase in percentage adhesion with beating

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* The handsheets used in these and the majority of experiments described in this paper were plate-dried. A separate discussion of the behaviour of paper dried free to shrink is included in Part 2.

Simplified models of the two situations are analysed in Appendix 1 and it is shown that, even if fracture is determined wholly by fibre failure, the zero-span test might still be expected to yield a value some 30 per cent higher than the normal figure.

The second important factor concerns the type of tester used. It is obvious that constant strain rate instruments are unsuitable and pendulum-type instruments have generally been used. In this case, the strain rate during the test varies and is partially determined by the stress/strain curve of the test specimen. Appendix 2 shows that, if the total time of test is the same in both cases (the recommended procedure), the strain rate at fracture in the zero-span test may be as much as ten times the corresponding rate in the normal test. This factor alone will result in the zero-span figure being larger than the normal strength by an amount depending on the type of paper, but probably of the order of 10–20 per cent.

When both effects described above are taken into account, it follows that, if normal tensile failure is controlled solely by fibre failure, then the zero-span tensile strength may still be in the region of 1·4 times as large as the normal figure corresponding to an apparent *percentage adhesion* of about 70 per cent. This, of course, places in a different light the previously mentioned figures of 50–70 per cent.

Both the effects noted above are continuous—that is, the tensile strength will vary steadily as the span is altered between the two extremes. On the other hand, if the tensile strength is controlled to any significant extent by interfibre bond failure, an additional change in strength would be expected when the span is reduced to the magnitude of the fibre length of the sheet. Cross-direction tensile strips were cut from sack kraft and tested at various spans with a pendulum-type instrument, adjusting the instrument speed so that a time of approximately 20 sec was needed for each test. Fig. 3 shows that there is a steady variation in strength, which is exponential in form, with perhaps a suggestion of an additional factor at spans below about 0·4 cm. The value at zero-span in this experiment was obtained, using the normal instrument jaws touching each other and not with special jaws. It must not therefore be regarded as a true zero-span figure, but this does not invalidate the conclusions.

The steady change in tensile strength when extrapolated gives a ratio of zero-span to normal strength of 1·30, that is, an apparent *percentage adhesion* of 77 per cent, which agrees well with the value of 80 per cent predicted from Appendixes 1 and 2, using a value of 0·40 for Poisson's ratio. Significantly, this estimate excludes any consideration of the probability of weak spots being found in the strip, an explanation of the variation of strength with length advocated by many workers(9, 10, 11) and the results, therefore, indicate that this effect is relatively unimportant when using strips of the normal width (15 mm). Measurements at various spans, made on papers of widely contrasting formation,(12) give further confirmation. On the 'weak spot' theory, such differences in formation would be expected to affect the relation between tensile strength and span, but the measurements referred to show no such effect.

In view of the preceding evidence, it was decided that it would be worthwhile to attempt to describe the tensile behaviour of paper as a process terminated predominantly by fibre failure and to investigate the extent to which known phenomena relating to this behaviour could be explained by such a process. One such phenomenon, extensively investigated by Nordman(13) and his colleagues, is the increase in optical opacity during tensile testing.

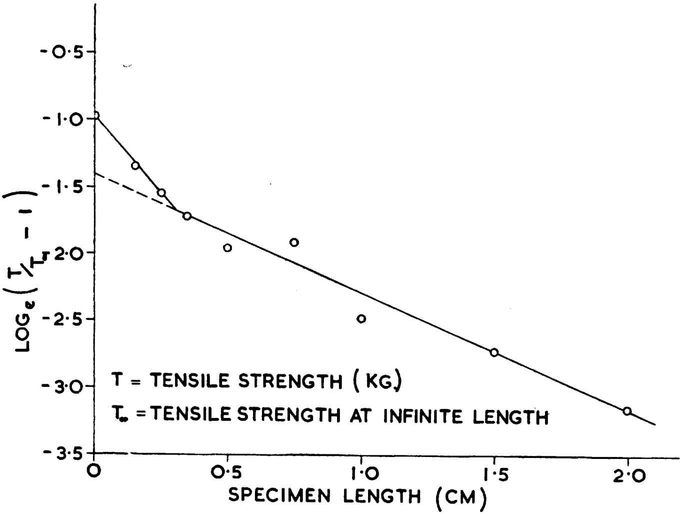

Fig. 3—Variation of tensile strength with specimen length

Variation in opacity during straining

NORDMAN has found that the opacity remains substantially constant during elastic deformation, but that it starts to increase when permanent or plastic deformation commences, continuing to do so until fracture occurs. It increases also if the paper extends by creep when under constant load.

Several possible causes of this opacity change have been suggested.(14)

Nordman, Rance and others believe that interfibre bonds are continually being broken during the latter part of the straining process, giving rise to fresh fibre/air interfaces. Two experiments were undertaken by us to investigate the alternative possibility of new fibre/air interfaces being formed by the fracture of fibres.

Firstly, we have shown theoretically that, if a proportion of the fibres in a sheet were broken, the fibre length distribution curves in strained and unstrained samples would be different and permanent extension of some of the fibres, if it occurred, could not counterbalance this change. Handsheets were prepared from a medium beaten, dry sulphate pulp, tensile strips cut from them and half the strips strained to breaking point. The strips were then thoroughly wetted, small areas teased from both strained and unstrained samples (excluding cut or fractured edges) and these were gently disintegrated. About 2 500 fibre lengths in each class were measured by optical projection of prepared slides. The differences between the two length distribution curves so obtained were extremely small and could not be ascribed to any combination of fracture and permanent extension of fibres; they were therefore classified as random errors because of the finite size of population sampled.

Secondly, handsheets were made from a medium beaten, dry sulphite pulp, about 2 per cent of the fibres being dyed black prior to sheetmaking. In a preliminary experiment, the effect of dyed fibres on the tensile strength was found to be negligible.

Tensile strips, strained to fracture, were examined microscopically for evidence of broken fibres in the body of the paper. Although about 1 000 fibres were examined, no evidence of fibre fracture was discovered.

It is felt that the results of both experiments, taken in conjunction, indicate that sufficient fibre rupture to explain the opacity increase does not occur in the body of the paper before final fracture, hence tend to confirm the belief that the breakage of interfibre bonds is responsible for the change.

Paradoxically, this implies that the interfibre bonds are sufficiently weak to allow a significant number to be ruptured during the straining process and yet, at the same time, can be sufficiently strong to result in a large proportion of fibres being broken at final fracture.

A satisfactory explanation of this paradox forms the basis of the current theory.

Mechanism of bond breaking during straining

BECAUSE paper approximates to a two-dimensional structure, tensile forces are transmitted between fibres via shear of the bonded area between them. For fibres in tension, only small distortions out of the plane of the paper are possible; however, when the fibres are in compression, owing to the lateral contraction of the specimen (and this applies to a large proportion of them), it appears that their slenderness will make them prone to buckle. Although such buckling will be hindered by the surrounding mat of fibres, in most papers (with densities below, say, 1·0 g/cm^3), the number of voids present must be sufficient to allow considerable distortion of this type to take place. A bonded area between a buckling fibre and one held in tension will be stressed in such a way that failure by 'peeling' can occur and, although the total energy requirement is unaltered, the activating stress level is considerably lower than for bond failure in shear. This mechanism is postulated to account for bond breakage and the opacity increase during plastic strain, but it is necessary to account also for the peculiar distribution of the opacity change observed with certain types of paper by Rance.(5)

Occurrence of strain lines

Rance found that when a cross-direction strip of tracing paper is strained, the regions of increased opacity form a network of lines at characteristic angles to the direction of tension. He further showed, by indirect methods, that these lines were exhibited by many different types of paper and it was merely the translucent nature of tracing paper that enabled them to be observed so easily in this case. By way of explanation of these strain lines, Rance drew attention to the phenomenon of Lüder's lines in the field of metals and said, 'there seems little doubt that there is a common rheological origin in the two cases.'

Lüder's lines are associated, however, exclusively with a sudden decrease in stress at the yield point as in the case of annealed mild steel and, although load/extension curves of paper have been reported to exhibit such a fall in load, such behaviour is exceptional. Moreover, Lüder's lines are formed only during the initial stage of plastic straining, before the lower yield point is reached, whereas strain lines in paper continue to be formed until final fracture occurs. Finally, whereas Lüder's lines are regions of locally decreased thickness or 'necks', evidence presented later shows that a local increase in thickness accompanies the formation of a strain line in paper.

Clearly, then, the two situations are far from analogous and the formation of strain lines in paper must be considered as a phenomenon in its own right.

The process of fibre buckling already outlined is compatible with the increase in thickness at a strain line, but the problem of accounting on this basis for the characteristic angles at which the lines appear is obviously a more critical test.

The strain line is assumed to be initiated by the failure (by peeling) of one bond or a small group of bonds, which will result in a small local extension. It cannot be imagined, however, that the weak bonds in the sheet are 'pre-distributed' in the diagonal strain line pattern and it therefore follows that bond failure must occur in the bulk of the line as a result of the local deformation itself—that is, the process is self-propagating. Now, whatever the angle at which the line is formed, if the direction of separation of the two sections of the paper is parallel to the tension, the situation will be analogous to that in the zero-span tensile test. There will be no further lateral contraction and very little tendency for additional bond failure (by peeling) as a consequence. The required property of self-propagation would therefore be absent.

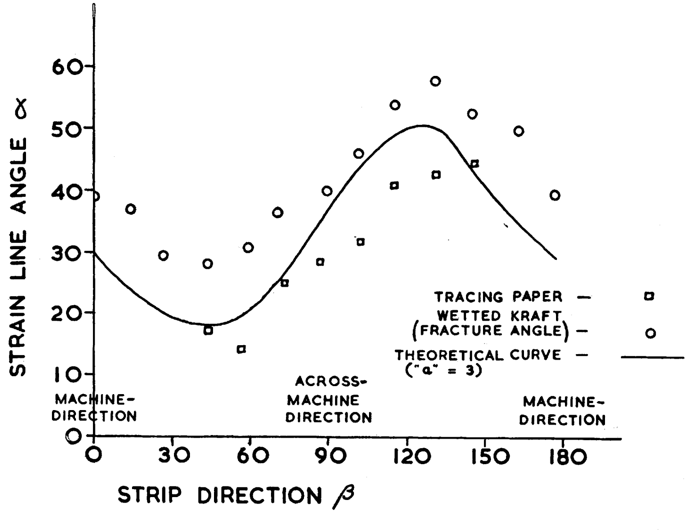

Fig. 4—Variation of strain line angle with strip direction
(β as defined in Fig. 14)

If shear or slip at a strain line is assumed, the sections of the paper moving parallel to the line itself, such a process would subject a large proportion of the fibres involved to compressive strain and provide an adequate self-propagation mechanism. In Appendix 3, a simplified model of the structure of paper is considered on this basis and the strain line angle predicted. Assumptions are discussed and the theoretical and some experimental results are given in Fig. 4.

It is of interest to recall at this stage that the suggested mechanism of bond failure during plastic straining, hence of strain line formation, had its origin in the difficulty of explaining the behaviour of strong papers in which a large proportion of fibres ruptured at final fracture. It is clear, however, that the process of strain line production arising from that approach is applicable to papers of any strength, since, however weak the interfibre bonds are compared with the fibres, they will always be weaker with respect to peeling than to shear.

Summary of theory

It has been shown that tensile extension of paper is accompanied by lateral contraction and it is suggested that this can induce buckling of some fibres, which can in turn promote bond breakage by a peeling action at relatively low stress levels. With the additional assumption of shear movement at these locally weakened positions, it is shown that the most probable distribution (that is minimum energy configuration) of bond rupture is within diagonal bands. In such a system, the bond breakage will be promoted along these bands (strain lines), until fibres in tension inhibit the process (for most papers) at some position, whilst the rising stress level initiates the process elsewhere. This leads to a network of intersecting strain lines and offers an explanation for many phenomena associated with tensile straining—such as opacity increase, thickness increase, the interaction of fibre and bond strength in determining ultimate tensile performance, irregular straining on a microscale and shear or slip effects.

PART 2

The effect of specimen length

The effect of specimen length on tensile strength has already been discussed in Part 1. The most significant additional effect in the present context is the variation of modulus of elasticity. Contrary to published experimental results,(10, 15, 16) Appendix 1 predicts a decreasing modulus with increasing strip length.

Most of the published results utilised the separation of the instrument jaws to measure the extention: the effect of any slipping of the paper from between the clamps during the test is proportionately greater the shorter the strip.

An investigation of the variation of the modulus was made using Ballotini particles (glass spheres of about 0·2 mm diameter) attached to the surface of the paper either by a spot of silicone grease (applied with a needle point) or by a thin layer of pressure-sensitive adhesive and the assembly was illuminated by a small lamp. Under these conditions, the spheres act as convex mirrors and form an image of the light source very much smaller than the sphere itself. A microscope with eyepiece micrometer was used and it was found that the precision of alignment of the eyepiece crosswire and the lamp image was at least an order of magnitude greater than when alignment on needle holes in the paper was attempted.

Various length strips from sulphite and sulphate handsheets were dead-loaded to well within the elastic limit. Using the technique described above, it proved possible to measure relative moduli of elasticity with strip lengths down to 0·5 cm and typical results are plotted in Fig. 5, together with the results of Setterholm and Kuenzi for comparison. It was concluded that the usually reported variation of modulus of elasticity with strip length is severely in error, possibly owing to paper slippage within the jaws as previously suggested.

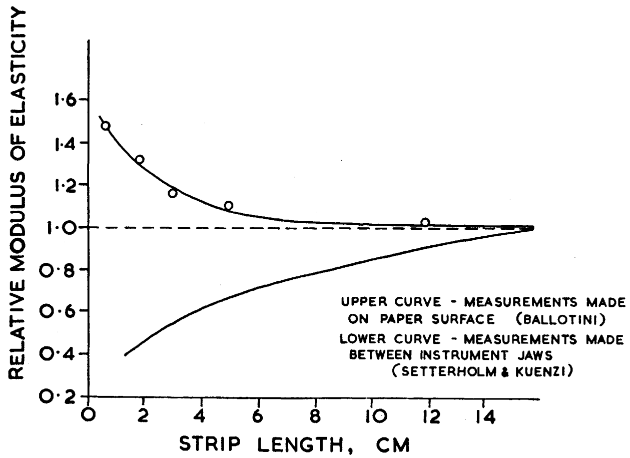

Fig. 5—Variation of Young's modulus with strip length

Shear deformation

UNEQUIVOCAL evidence that shear deformation takes place during plastic strain has not yet been obtained, although several experiments strongly suggest that this is so. In fact, the marked shear movement that occurs at fracture with weaker papers is clearly a closely allied phenomenon that can be interpreted on the basis of the present theory.

The suggested process of strain line formation and propagation have been given in Part 1. The halting of strain lines by the build-up of tensile forces in fibres was also discussed and, in the case of strong papers, a widespread pattern of strain lines is initiated, but some of the tension fibres in a weak paper may be pulled loose before equilibrium is attained. Consequently, a still higher stress is required in the remaining fibres and, before this is reached, more fibres may break away from the surrounding paper and so on. Clearly, the possibility of an unstable process exists, terminating in fracture along a strain line and exhibiting the marked shear that is commonly observed.

This tentative identification of fracture line with strain line in weak papers was utilised for a further experiment. When investigating the variation of strain line angle with strip orientation (using tracing paper), it was found that the lines in strips cut near to the machine-direction were too indistinct to measure. A machine-direction strip of sack kraft, on the other hand, when weakened locally by wetting, tends to fracture along a diagonal line by shear. The angle of this fracture line was measured for strips cut at various angles to the machine-direction and the results are plotted in Fig. 4. It will be seen that, over the appropriate part of the range, the trend for these results closely follows that for tracing paper and the theoretical curve deduced from Appendix 3, although the magnitudes are significantly different.

Thickness changes during straining

THE apparatus for this set of experiments consisted of a fixed and a moving anvil on opposite sides of the paper, the moving anvil being mounted on a compound balanced parallel spring movement as described by Jones.(17) The movement of this anvil was measured and recorded continuously. The anvil pressure was of the order of 0·5 kg/cm^2.

Typical gross thickness changes as measured with $\frac{1}{2}$ in diameter anvils are shown in Fig. 6. It will be seen that, in general, a marked increase in thickness occurs and this increase for unbeaten and free-beaten papers commences at the start of the straining process. The theory suggests therefore, in this case, that the rupture of bonds and consequent strain line formation commences at a very early stage. This is consistent with the known effects of beating and explains also the almost negligible initial straightline section of the load/strain curves in such cases.

In the hope that the effect of single strain lines could be detected, $\frac{1}{8}$ in diameter anvils were used and a typical result is shown in Fig. 7. Such records are characterised by a number of instances of fairly rapid thickness increase during the course of straining followed by a slower decrease. Presumably, these indicate the initial formation of a strain line followed by further extension and/or relaxation within the anvil area. Two consecutive humps could indicate two distinct lines or possibly successive deformations of the same line.

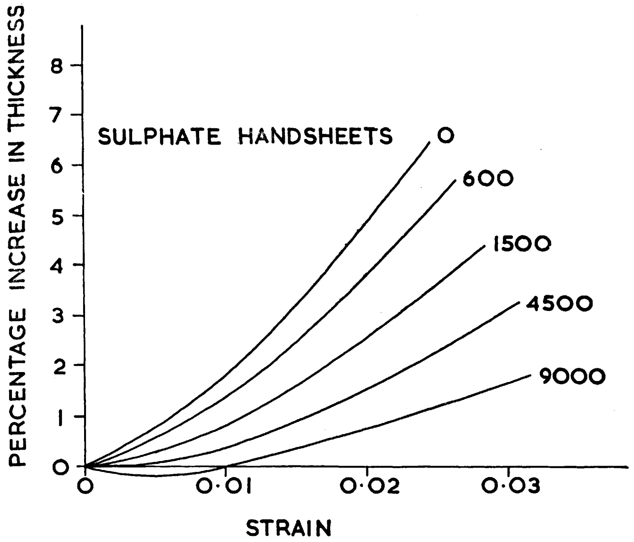

Fig. 6—Thickness variation with straining (½ in anvil): figures by curves indicate revolutions of Lampén mill

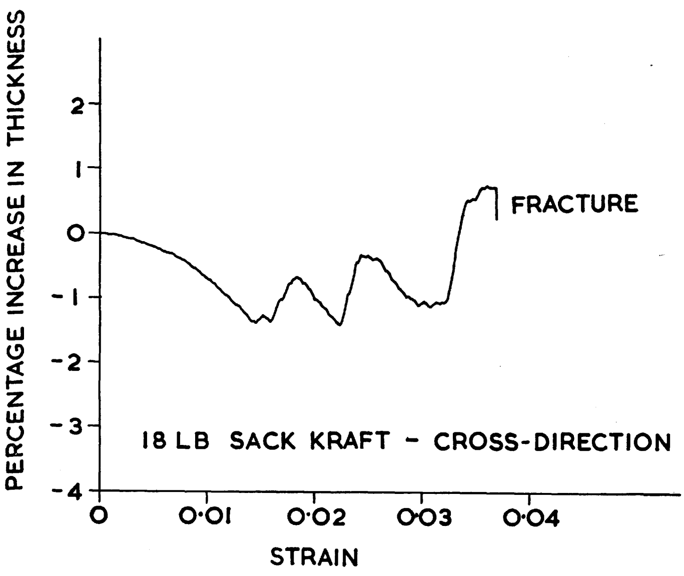

Fig. 7—Thickness variation with straining (⅛ in anvil)

Unsuccessful attempts were made to obtain direct evidence of the increase in thickness at strain lines by surface profiling. It would appear that the background of surface roughness already present is too great to permit a simple detection of local thickness change in this manner.

In the special case of tracing paper, evidence on this point was obtained by a printing technique. After prestraining, a strip of the paper was printed with the IGT apparatus using such a small quantity of ink that only the high spots were contacted. Distinct diagonal ink lines were noted, which, by pricking through with a needle and viewing from the other side, were observed to coincide with the normal high opacity strain lines.

The fracture of interfibre bonds during straining

A PROCESS in which the rupture of interfibre bonds resulted in immediate fracture of the paper would be easy to understand, but the available evidence indicates that this is far from being the case. Bonds are apparently ruptured at quite low strains with little obvious effect. The argument that the local weakening caused by bond failure is compensated for by stress redistribution throughout the network seems satisfactory at first sight. It must be emphasised, however, that this picture is essentially one of 'spot' weakening. When it is remembered that strain lines sometimes extend across the whole width of a test strip without marking the position of final fracture, the explanation becomes less convincing. It was similarly puzzling to find that, when tensile strips were creased (quite sharply) at rightangles to the direction of tension before testing, the tensile break did not always occur at the crease and the tensile strength was not significantly reduced despite the obvious damage inflicted. This suggests that the bonds ruptured during creasing would not normally have exerted a great influence on the tensile behaviour.

It follows that, in addition to two types of bond failure—by shear and 'peeling'—certain bonds must be more prone to rupture by one mechanism than by the other. This can be predicted from the present theory. Fig. 8 illustrates a fibre undergoing longitudinal strain (either compressive or tensile). The stress is induced by the movement of the other fibres to which it is bonded and the type of stress distribution to which this will give rise is also shown. Therefore, the bonds that are most likely to fail by shear are those at the extremities of the fibre, where the maximum bond stress (maximum difference between stresses in adjacent segments) is developed. Those most likely to fail by peeling, on the other hand, are situated near the mid-point of the fibre, since this is the region of maximum fibre stress, hence of maximum tendency for fibre buckling. It is clear from this model that the failure of a bond near the middle of a fibre by peeling will not greatly alter the stress distribution in that fibre and the effect will be distributed in the surrounding network. In contrast, the rupture of a bond at the end of the fibre by shear will immediately cause a much higher stress to be applied to the penultimate bond and this type of failure could well produce the type of instability that results in complete fracture. The effect of failure of the fibre itself would be equally severe.

In the case of the precreased tensile strips, one surface of the paper is stretched and the other compressed during creasing, no lateral contraction being allowed to take place. From the original argument on the relative strengths of bonds in tension and in compression regions, it follows that bond failure will occur preferentially in the compressed part of the paper leading to the delamination type of internal rupture that occurs on a larger scale in the folding of board. Furthermore, since the bonds affected by this deformation are not those that would play a predominant part in subsequent tensile testing, a marked decrease in tensile strength would not be anticipated.

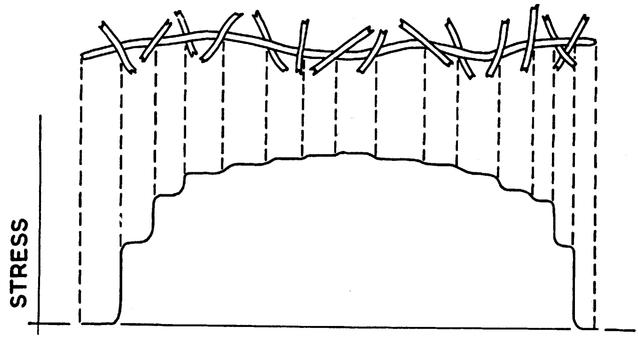

Fig. 8—Fibre bonded to other fibres undergoing longitudinal strain: graph indicates stress distribution

Because of the essentially two-dimensional structure of paper, any interfibre bond breaking can be regarded as 'micro-delamination'. Consequently, a pronounced effect would be expected on the results of tests designed to measure this type of strength—that is, 'surface' or 'transverse tensile' strength tests.

To investigate this, a method was used in which a tapered strip of transparent pressure sensitive adhesive tape is stuck to the paper. The wider end of the tape is loaded with a weight and the strip is peeled from the paper at a speed that increases as the width of the 'peeling region' decreases. The method has the advantage that the tape can be subsequently stuck to a black surface and the different types of damage occurring at different stages examined in retrospect. Fig. 9 shows typical results obtained with tracing paper before and after straining and the weakening effect due to straining is obvious. Similar, but less marked results were obtained with coated papers.

An allied phenomenon described by Rance(18) is the increased pliability of paper strained to near breaking point. Clearly, the rupture of interfibre bonds is consistent with this effect (as Rance points out), especially when it is regarded as a delamination process.

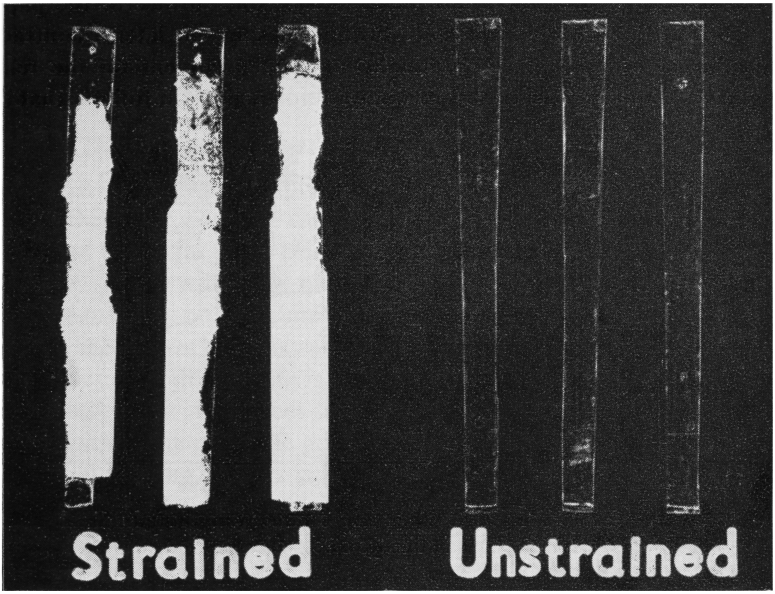

Fig. 9—Surface strength tests on tracing paper

Permanent deformation

Rance has shown that many of the major characteristics of the load/strain curve of paper can be explained on a basis of interfibre bond breaking during straining, but he introduces the additional idea of interfibre frictional forces(19) to account for permanent deformation. Bond breakage is followed by fibres sliding over one another to new positions and the frictional forces maintain these positions when the load is removed from the paper. This poses difficulties, however—for instance, the frictional forces must possess directional properties. They must allow sliding after bond rupture and yet resist it when the load is removed, despite the fact that the internal forces operating in the second case are comparable in magnitude with those in the first.

Secondly, whilst the coefficient of friction between fibres may be adequate, it seems doubtful if sufficiently large normal forces between fibres could exist, bearing in mind that (*a*) most of the permanent set occurs at strain lines (discussed subsequently) and (*b*) the strain lines are regions of increased paper thickness.

In addition, such a friction-maintained permanent deformation would seem to be susceptible to removal (or at least significant reduction) by vibration. Accordingly, several strips of different types of paper were prepared and strained, then the permanent set induced was measured. Some were set aside so that the effect of creep recovery could be ascertained; the rest were subjected to a wide variety of vibrational treatments for periods up to 15 min at frequencies ranging 20–20 000 c/s, including resonant conditions. Both longitudinal and transverse oscillations were employed, but with no detectable influence on the permanent set.

Applying the present theory, after a strain line has been formed, a state of equilibrium is reached inasmuch as, although further deformation at the line is possible, the net force acting on each part of the paper is parallel to the direction of tension. Consequently, when the load is removed, the two parts of the paper move together in this direction and not along the strain line. It is suggested that the fibre mats on the two sides of the line become enmeshed at an early stage in this recovery and that frictional forces prevent further recovery and therefore induce permanent set.

In this situation, the required normal force mentioned earlier is supplied by the tensioned fibres crossing the line and the two parts of the paper are held together by these members. Such a frictional effect is unidirectional, of course, since it is not called into play until the paper is unloaded.

Frozen-in stresses

THE existence in strained, but not loaded, paper of a considerable number of fibres held in tension provides an explanation of the frozen-in stresses first postulated by Ivarsson and Steenberg(1) as a result of the analysis of load/strain curves. It is a point of great interest that these authors found virtually no such residual stress in plate-dried handsheets. This is to be expected on the basis of the present theory, since, as a result of the drying stress in this case, all the fibres are subjected to tension; consequently, the proposed mechanism of strain line formation cannot come into play. There remains, of course, the problem of accounting for permanent set under these conditions and this, together with several other features of such straining, is still under consideration.

The drying of paper under uniaxial tension

THE machine-direction tension normally existing during the manufacture of paper is generally accepted as being a major cause of anisotropy in the finished product. Several investigators have examined this phenomenon, both on papermachines and by means of simulated experiments in the laboratory. According to Sapp and Gillespie,(20) the most marked effects of tension applied during drying are a decrease in the machine-direction stretch and a decrease in the cross-direction tensile strength. That the former should occur is a natural result of removing some of the stretch during the drying process. The decrease in cross-direction strength is also to be anticipated, providing it is assumed that the permanent deformation (either actual or effective) occurring during drying involves strain line formation and the previous discussion on frozen-in stresses suggests that such is the case. During the subsequent cross-direction tensile test, strain lines will be formed in the paper that will intersect those formed during drying and there will be an additional, wide distribution of fibre stresses superimposed on those already present. This will in turn lead to failure either of fibres or of interfibre bonds at a lower load than normal owing to the piecemeal nature of such failure.

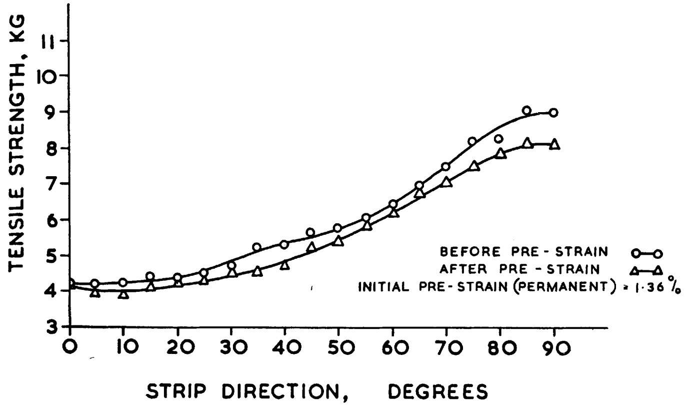

Fig. 10—Tensile strength of sack kraft before and after prestraining in the cross-direction

This reasoning should apply equally well to dry paper subjected to straining in two directions and this has been tested experimentally. Large samples (12 in × 6 in) of various papers were strained in the long direction so that appreciable permanent set was induced and then 15 mm wide strips were cut from them at various angles and tensile tested. Fig. 10 shows a typical result obtained with a sack kraft strained originally in the cross-direction. It will be seen that this resulted in a reduction in the subsequent machine-direction tensile strength of some 9 per cent.

The effect of strain line formation on ultimate strain

THE existence of strain lines automatically implies a non-uniform distribution of strain and, apart from the effect of local variations of strength, the final break will be determined in position and time by the maximum local strain attaining a critical value. If this local value is that of an isolated strain line (in which the strain greatly exceeds that of the rest of the paper), the stretch measured over the complete specimen will be low; whereas, if the strain is almost uniform throughout the specimen, the total stretch at fracture will be high. It follows that any reduction in the severity of strain line formation (or ideally elimination of such formation) should increase the ultimate strain.

Such a reduction occurs in the case of a machine-direction sample compared with a cross-direction one of the same paper. Firstly, because of the fibre orientation, fibres in the cross-direction have a shorter length between adjacent bonds with other fibres than those in the machine-direction. Consequently, at any given strain there will be fewer buckled fibres for a machine-direction strip than for one cut in the cross-direction. Secondly, for a machine-direction strip, consideration of the distribution of strain within a strain line shows that the sector of fibres undergoing tension contains the machine-direction and the compression sector includes the cross-direction. Therefore, compared with an isotropic sheet (with respect to fibre orientation), there will be fewer fibres contributing to the weakening process by buckling and more to halt the propagation of the line once it has been initiated.

According to the present theory then, a machine-direction specimen should have a greater ultimate strain than one cut in the cross-direction. Direct verification of this is not possible with machine-made paper because of the disturbing effect of the machine-direction tensions applied during drying, but an interesting confirmation of this view is provided by the work of Toroi.(21) With the aid of a specially constructed machine, he made handsheets in which the fibres were oriented in a similar manner to those in normal machine-made paper and he found on subsequent testing that the machine-direction strips had both a higher ultimate strain and a higher tensile strength than the cross direction strips.

Fibre orientation in machine-made paper

A COMPREHENSIVE investigation of fibre orientation in paper has been carried out by Steenberg.(22) One puzzling feature of the results is that the distribution curves do not, in general, show a smooth variation from machine- to cross-direction, but exhibit instead 'horns' at certain preferred angles. Fig. 11 shows two such curves, although others can be more complex than this.

The formation of strain lines by shear during the manufacture of the paper offers a possible explanation of this phenomenon, since the orientation of fibres involved in such deformation will be affected.

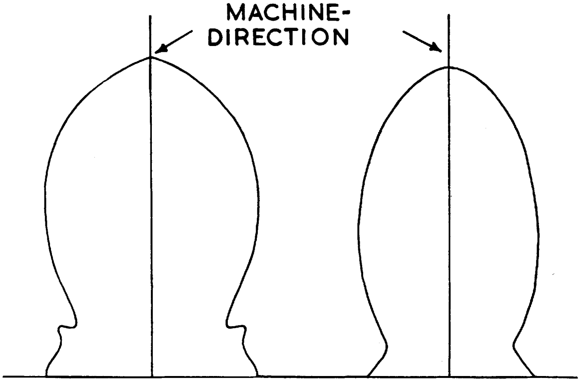

Fig. 11—Typical polar diagrams of experimentally determined fibre orientation (Steenberg)

Several initially elliptical distributions were formulated, including a circle as a special case and the total effect on such distributions of varying proportions of shear deformation (degrees of strain line 'coverage') was calculated. The total distribution was divided in five degree segments and the proportion of fibres in each segment modified according to the effect of each of the two strain line directions (assumed for this calculation to be at 60° to the machine-direction). Fig. 12 shows the results of two sets of calculations and the similarity between these and Steenberg's distribution curves is clearly apparent.

The dimensional stability of paper

THAT the dimensional stability of paper with moisture variations may justifiably be considered in the context of tensile behaviour is obvious in view of the marked interrelation between these properties. Rance,⁽⁴⁾ for instance, has noted that permanent tensile deformation of a cross-direction strip results in a diminished wet expansion in that direction. On complete wetting and redrying, however, the original length and hydrosensitivity are regained.

On the basis of the present theory, the strain lines are held in a 'bound' condition by the tensioned fibres that cross them and their contribution to hygro- or hydrosensitivity will be small, since moisture will not appreciably affect the length of these fibres. The hygro- and hydroexpansion will therefore be reduced in proportion to the extent to which the strain lines cover the strip, which will in turn be related to the degree of permanent deformation previously induced. Complete immersion in water will cause a reduction in the coefficient of friction between fibres and the consequent recovery of strainline deformation under the action of the tensioned fibres.

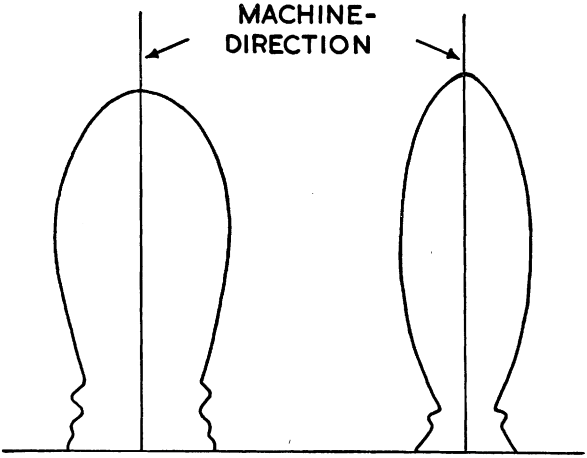

Fig. 12—Calculated orientation diagrams: initial distribution (both cases) ellipse of axis ratio 2

The hygro- and hydroexpansion anisotropy of machine-made paper can be explained in a similar fashion, although, in addition, there is probably an anisotropic effect from the fibre orientation itself. Complete wetting and redrying, however, has a more complex effect on machine-made paper, the most common result being a partial (sometimes negligible) decrease in length and the retention of at least a large part of the original anisotropy. This can be explained partly by the effect of fibre orientation, which will of course remain, partly by supposing that some of the strain line formation on the machine occurs at such an early stage (say, before and between the presses) that subsequent interfibre bonding can be effected, also a more thorough enmeshing of the fibre mats adjacent to strain lines. The mere reduction of the coefficient of friction on wetting may then be insufficient to allow the permanent deformation to be recovered, although the tensioned fibres will still hinder moisture expansion in the same way as before.

Direct measurement of local deformation

An attempt has been made to investigate directly the permanent local deformation in paper. The Ballotini technique was employed, a total of 196 particles being placed in a 14 × 14 square mesh pattern at 1 mm spacing on the surface of a tensile strip. Several different types of paper were used and the particle spacing in both directions was measured in each case before and after stretching the sample.

The results clearly demonstrated the local nature of the permanent deformation—for example, a sample of 26 lb sack kraft with a total permanent extension of 1·3 per cent showed variations from millimetre to millimetre from 0·2 per cent to 4·7 per cent. Moreover, the regions of large deformation formed lines at the characteristic strain line angle and for tracing paper, they could be identified visually with lines of increased opacity. Unequivocal evidence of shear movement, however, could not be obtained. It would appear that the length of the majority of strain lines is comparable only with the particle spacing used in these experiments and the inhibition of the shear movement at the ends of such short lines, results on a millimetre scale, in a complex pattern of distortion.

The behaviour of handsheets dried free to shrink

Most of the experimental work described in this paper has been carried out on machine-made paper or plate-dried handsheets, but paper dried with complete freedom to shrink is in many respects of more fundamental interest. Unfortunately, its behaviour is not easy to account for in terms of the present theory.

If a chemical pulp is wet beaten so that the final freely dried sheet tends to be translucent, then, on tensile straining, an increase in opacity can be visually observed, though the familiar strain line pattern is absent.

In addition, for the same pulp beaten to the same extent, a freely dried tensile strip decreases in thickness during straining, whereas the corresponding plate-dried strip shows an increase. Less reliance can be placed on these observations, because of the cockled nature of the freely dried paper. Nevertheless, it seems that the nature of freely dried paper is such as to inhibit the formation of strain lines.

This behaviour might arise from the structure of the sheet or from the load/strain characteristics of the fibres dried under these conditions. One effect of the cockling of a freely dried sheet, for instance, must be that certain areas of the paper are initially slack during tensile testing, the load being borne by the remainder as is evident from the lower modulus of elasticity obtained compared with a plate-dried sheet. These regions of slackness will automatically tend to inhibit strain line formation, since fibres under compression will tend to peel away from other fibres only if the latter are held taut.

According to the argument outlined in a previous section, an absence of strain lines should result in a higher ultimate strain. It is interesting to note that, in the many experiments we have carried out, it has invariably been the case that the ultimate strain for a freely dried sheet has been greater than the sum of its linear shrinkage and the ultimate strain of the corresponding plate-dried sheet.

These views are still largely speculative; even if they are substantially correct, the major problem still remains of accounting for the permanent deformation of such paper.

Conclusion

A CONSIDERABLE amount of evidence, both direct and circumstantial, has been accumulated in support of the main arguments of the present theory. It is not suggested, however, that the proposed mechanisms involved in tensile straining are exclusive. The experiments described in the penultimate section, for example, show that, whilst shear movement may play a large part in permanent deformation, local distortion may tend to mask the effect. Similarly, the rupture of interfibre bonds during straining is believed to be caused predominantly by fibre buckling, but other processes may well play their part.

In short, it is obvious that the mechanical behaviour of paper is a subject of challenging complexity and, if the present paper succeeds in throwing a little light on this problem, it is largely due to the wholehearted assistance offered to the authors by other members (too numerous to name individually) of the Bowater Research Division.

REFERENCES

1. Ivarsson, B. and Steenberg, B., *Svensk Papperstidn.*, 1947, **50** (18), 419–432
2. Steenberg, B., Contribution to discussion at Third Mechanical Pulping Conference, *Tappi*, 1960, **43** (1), 188A–252A
3. Nissan, A. H., *Trans. Faraday Soc.*, 1957, **53** (5), 700–721
4. Rance, H. F., *Proc. Tech. Sect. B.P. & B.M.A.*, 1948, **29** (2), 449–476
5. Rance, H. F., The Rheological Behaviour of Paper, in *Some Recent Developments in Rheology*, edited by V. G. W. Harrison (United Trade Press Ltd., London, 1950), 90
6. Van den Akker, J. A., Lathrop, A. L., Voelker, M. H. and Dearth, L. R., *Tappi*, 1958, **41** (8), 416–425
7. Hoffman Jacobsen, P. M., *Paper Trade J.*, 1925, **81** (22), 51–52
8. Stone, J. E. and Clayton, D. W., *Pulp and Paper Mag. Can.*, 1960, **61** (10), T475–T484
9. Nissan, A. H., *Tappi*, 1959, **42** (12), 928–933
10. Andersson, O. and Sjöberg, L., *Svensk Papperstidn.*, 1953, **56** (16), 615–624
11. Graham, D., *Tappi*, 1956, **39** (3), 147–148
12. The Institute of Paper Chemistry, 'Instrumentation Studies—XL': *Paper Trade J.*, 1942, **115** (5), 12–14, 15, 18; (7) 14, 16–18, 21–22 and (19), 18–20, 22
13. Nordman, L. S., *Fundamentals of Papermaking Fibres*, Ed. F. Bolam (Technical Section, B.P. and B.M.A., Kenley, 1958), 333
14. *Ibid.*, Discussion by several contributors
15. Rance, H. F., *Tappi*, 1956, **39** (2), 104–115
16. Setterholm, V. C., and Kuenzi, E. W., *Tappi*, 1957, **40** (6), 197A–204A
17. Jones, R. V., *J. Sci. Instr.*, 1951, **28** (2), 38–41
18. Rance, H. F., in *Mechanical Properties of Wood and Paper*, edited by R. Meredith (North Holland Publishing Co., Amsterdam, 1953), 209
19. *Ibid.*, page 195
20. Sapp, J. E., and Gillespie, W. F., *Paper Trade J.*, 1947, **124** (9), 120–122, 124, 126
21. Toroi, M., *Paper and Timber (Finland)*, 1959, **41** (5), 271, 273, 275–279
22. Steenberg, B., *Svensk Papperstidn.*, 1947, **50** (13), 301–305
23. Ranger, A. E., *Paper Tech.*, 1961, **2** (2), T71–T76

Appendix 1

Variation of mode of deformation with specimen length

A SIMPLIFIED two-dimensional model of the sheet is considered in this and subsequent appendixes, the major approximations being (*a*) the fibres are assumed to be Hookean in their behaviour and (*b*) the structure is assumed to be 'hinge-jointed'—that is, fibre flexure is ignored. The latter is not as serious as might at first appear, since those fibres to play the major part in determining tensile behaviour (oriented approximately parallel and perpendicular to the tension) are also those least affected by such flexing distortion.

It can be shown(23) that, in an isotropic two-dimensional array of fibres, the number of fibres with angles between θ and $(\theta + d\theta)$ to a line perpendicular to the tension, which intersect unit length of this line is N_θ, where—

$$N_\theta = \frac{L}{\pi} \sin \theta \, d\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which L is the total fibre length per unit area of the sheet.

In the central region of a long strip, that is, where the full lateral contraction is allowed, the fibre strain e_2 is given by—

$$e_2 = e_1\{1-(\sigma+1)\cos^2\theta\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where e_1 is the longitudinal paper strain and σ the Poisson's ratio as defined in the text.

Combining equations (*1*) and (*2*), the force contribution dF in the tension direction of this group of fibres is given by—

$$dF = \frac{L\alpha e_1}{\pi}\{1-(\sigma+1)\cos^2\theta\}\sin\theta \, d\theta$$

where α is the effective mean force/strain constant of proportionality of the fibres.

The total force per unit width F is therefore given by—

$$F = \frac{L\alpha e_1}{\pi}\int_0^\pi \{1-(\sigma+1)\cos^2\theta\}\sin^2\theta \, d\theta$$

$$= \frac{L\alpha e_1}{\theta}(3-\sigma);$$

therefore,

$$\frac{F}{e_1} = \frac{L\alpha(3-\sigma)}{8} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and this is a measure of the modulus of elasticity.

In the zero-span situation, no lateral contraction is allowed and therefore the corresponding expression is obtained from equation (*3*) by putting $\sigma = 0$. Thus—

$$\left[\frac{F}{e_1}\right]_z = \frac{3L\alpha}{8}$$

The ratio of the two moduli of elasticity R is therefore—

$$R = 3/(3-\sigma)$$

The experimental values of σ reported in the text indicate that R can therefore vary from about 1·15 to 1·46, with a mean value of about 1·3.

If the fibre behaviour is Hookean up to fracture (or even approximately so, since departures from such behaviour will occur in both cases and the ratio of strengths will tend to be unaltered), then this value of R will give a lower limit for the ratio of tensile strengths in the two cases. Taking R as 1·3, this will correspond to a maximum possible *percentage adhesion* of 77 per cent.

Appendix 2

Variation of strain rate with specimen length (pendulum tester)

IF the crosshead of the instrument moves a distance δS in time δt, its velocity V is given by—

$$V = \delta S/\delta t$$

If, during this time, the top clamp moves a distance δs, the specimen extension δL is given by—

$$\delta L = \delta S - \delta s$$

Therefore, change in strain δe is—

$$\delta e = \delta L/L = (\delta S - \delta s)/L$$

and the strain rate e' is—

$$e' = \delta e/\delta t = (\delta S - \delta s)/L\,\delta t.$$

Assuming that the change in load δF is proportional to the movement of the top clamp—

$$\delta F = k\,\delta s$$

where k is a constant

If G is the instantaneous gradient of the load/strain graph

$$G = \delta F/\delta e = k\delta s/e'\delta t$$

But $$e'L = \delta S/\delta t - \delta s/\delta t = V - \delta s/\delta t$$

Therefore, $$\delta s/\delta t = V - e'L$$

and $$G = k/e' \cdot \delta s/\delta t$$

$$= \frac{k}{e'}(V - e'L)$$

Thus, $$e' = kV/(G + kL)$$

During a tensile test therefore (k, V and L approximately constant), the strain rate continually increases owing to the decrease in G. Moreover, the shorter the specimen, the more marked this effect becomes.

If e'_N is the strain rate at fracture in the normal span case and e'_z that for zero span and G_F the load/strain curve gradient at fracture, then—

$$\frac{e'_z}{e'_N} = \frac{V_z}{G_F} \cdot \frac{G_F + kL}{V_N}$$

(NOTE—the two values of G_F will not be exactly equal, but this difference will produce only a second order error.)

Some experiments were carried out with a pendulum tester to evaluate this ratio. V_z was found to be $1{\cdot}32\ V_N$ (in order to maintain constant the time to fracture—the standard zero-span procedure). k was 4·42 kg/cm and L was 15·0 cm; G_F varied 10–20 kg, hence—

$$e'_z/e'_N = 5{\cdot}7 \rightarrow 10.$$

Such an increase in strain rate would be expected to produce about a 10 per cent increase in tensile strength for most papers.

Appendix 3

The angle of strain lines in uniaxial tension

SUPPOSE that, owing to the mechanism outlined in the text, a band of local deformation occurs at an angle α as shown in Fig. 13. The movement of the two parts of the strip relative to each other is assumed to be 'shear-like' along the deformation band. Firstly, the relationship between local paper strain e_1 in the direction of the applied force F and the fibre strain e_2 within the band is derived.

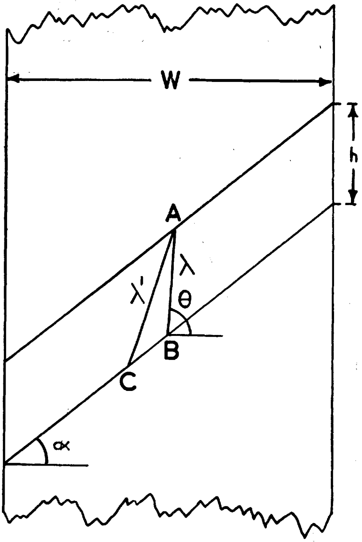

Fig. 13—Deformation of fibre within strain line

Suppose the original fibre length within the band (not necessarily the complete fibre length) is λ and the corresponding strained length λ'—

$$BC = he_1/\sin \alpha$$

Therefore, $(\lambda')^2 = (\lambda \sin \theta + he_1)^2 + (\lambda \cos \theta + he_1 \cot \alpha)^2$

Then $$\lambda' \doteqdot \lambda\left\{1 + \frac{2he_1(\sin \theta + \cos \theta \cot \alpha)}{\lambda}\right\}^{1/2}$$

and, since e_1 is small—

$$e_2 \doteqdot \frac{he_1}{\lambda}(\sin \theta + \cos \theta \cot \alpha) = \frac{he_1}{\lambda} \cdot \frac{\cos(\theta - \alpha)}{\sin \alpha}$$

but
$$\frac{h}{\sin(\theta - \mathrm{a})} = \frac{\lambda}{\sin\left(\frac{\pi}{2} + \mathrm{a}\right)}$$

Therefore
$$\frac{h}{\lambda} = \frac{\sin(\theta - \alpha)}{\cos\alpha}$$

Consequently,
$$e_2 = e_1 \frac{\sin(2\theta - 2\alpha)}{\sin 2\alpha}$$

Since the behaviour of the strain lines in strips cut in various directions from machine-made papers is of interest, a fibre orientation function is introduced. Suppose a strip is cut as shown in Fig. 14 with the cross-direction at an angle β to the reference line.

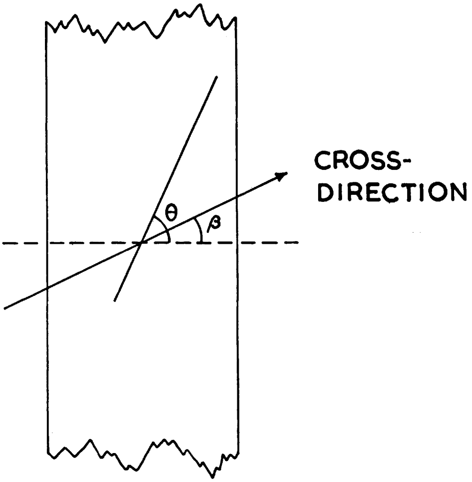

Fig. 14—Orientation of strip with respect to machine-direction

The type of orientation chosen is defined by the following equation. The number of fibres per cm^2 with angle between θ and $(\theta + d\theta)$ is given by—

$$N_\theta = k\{1 + a\sin^2(\theta - \beta)\}\,d\theta$$

where k is a constant and a defines the degree of orientation.

If N is the total number of fibres/cm^2—

$$N = k\int_{\beta}^{\pi+\beta}\{1 + a\sin^2(\theta - \beta)\}\,d\theta$$
$$= \pi k\left(1 + \frac{a}{2}\right)$$

Therefore,
$$k = \frac{2N}{\pi(2+a)}$$

and
$$N_\theta = \frac{2N}{\pi(2+a)}\{1+a\sin^2(\theta-\beta)\}\,d\theta$$

Adapting equation (*1*) of Appendix 1, the number of fibres with angles between θ and $(\theta+d\theta)$ that cut the strain line is given by—

$$dN_\theta = \frac{2L}{\pi(2+a)}\{1+a\sin^2(\theta-\beta)\}\,d\theta\cdot\frac{W\sin(\theta-\alpha)}{\cos\alpha}$$

where W is the width of the strip.

Therefore,
$$dN_\theta = \frac{2LW\sin(\theta-\alpha)}{\pi(2+\alpha)\cos\alpha}\{1+\alpha\sin^2(\theta-\beta)\}\,d\theta$$

Now, the relation between force and strain for an individual fibre is assumed to be of the form—

$$f = \gamma e_2$$

where γ is a constant.

The total force contribution opposing the tensile force, due to the group of fibres N_θ, is therefore—

$$dF = \frac{2LW\sin(\theta-\alpha)}{\pi(2+a)\cos\alpha}\{1+a\sin^2(\theta-\beta)\}\left\{\frac{\gamma e_1\sin(2\theta-2\alpha)}{\sin 2\alpha}\right\}\sin\theta\,d\theta$$

This is now integrated, but all that is required is the component that varies with the paper strain e_1. Additionally, it is assumed that the fibres in compression are buckled and the force f does not therefore vary with e_1. In consequence, the integral is evaluated only over the fibres in tension. This assumption can obviously produce errors in two ways. Firstly, the majority of the fibres could be in a buckled or near-buckled state and the subsequent fracture of bonds and formation of longer fibre 'struts' could result in an average fibre force that decreased with strain. Secondly, fibres already buckled and not subject to bond breaking could predominate. In this case, the surrounding fibres could partially restrict further buckling and give rise to an average force that increased with strain. The fact that these effects work in opposite directions and that the present assumption is a sort of 'average' situation decided its adoption. The disagreement in magnitude between experiment and the mathematical treatment can be ascribed therefore to the predominance of one or other of these factors and this approach is in good agreement with qualitative observations.

If the component of total force that varies with strain is F_1, then according to the above assumption—

$$F_1 = \frac{2LW\gamma e_1}{\pi(2+a)\sin\alpha\cos^2\alpha}\int_{\alpha}^{(\pi/2)+\alpha}\sin^2(\theta-\alpha)\cos(\theta-\alpha)\sin\theta\{1+a\sin^2(\theta-\beta)\}\,d\theta$$

This can be split into two integrals and each can be evaluated by expansion in multiple angles leading to—

$$\begin{aligned}F_1 = \frac{2LW\gamma e_1}{\pi(2+a)\sin\alpha\cos^2\alpha}\Big[&\frac{1}{16}(4\cos\alpha+\pi\sin\alpha)\\ &+\frac{a}{96}\{\pi[3\sin a+3\sin\alpha\cos(2\alpha-2\beta)+3\sin(\alpha-2\beta)]\\ &+12\cos\alpha+6\cos(\alpha-2\beta)-2\cos(3\alpha-2\beta)\}\Big]\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{dF_1}{de_1} = \frac{B}{(2+a)\sin\alpha\cos^2\alpha}\Big[&4\cos\alpha+\pi\sin\alpha\\ &+\frac{a}{6}\Big\{\pi[3\sin\alpha+3\sin\alpha\cos(2\alpha-2\beta)+3\sin(\alpha-2\beta)]\\ &+12\cos\alpha+6\cos(\alpha-2\beta)-2\cos(3\alpha-2\beta)\Big\}\Big]\end{aligned}$$

where

$$B = LW\gamma/8\pi$$

It is now assumed that the angle α will automatically be such as to produce a minimum increase in F_1 for a given value of e_1—that is, the increase in potential energy will be a minimum. This is determined by differentiating with respect to α and equating to zero, which leads, after simplification, to a cubic equation of the form—

$$A\tan^3\alpha+C\tan^2\alpha+D = 0.$$

where

$$A = \pi+\frac{a}{6}(6\pi+16\sin 2\beta)$$

$$C = 2+\frac{a}{6}(3\pi\sin 2\beta+12\cos 2\beta+12)$$

$$D = \frac{a}{6}(3\pi\sin 2\beta-4\cos 2\beta-12)-2$$

The solution for an isotropic paper is obtained by putting $a=0$. With all values of a tried so far, the solution has always been of the same general form, that is, one real and two complex solutions.

The variation of α with β is shown in Fig. 4 and the qualitative agreement with experiment is good, although quantitatively there is considerable spread between the different results as might be expected from the considerations previously discussed.

Transcription of Discussion

DISCUSSION

DR. P. LUNER: Both yesterday and today, we have heard some highly interesting papers relating to a number of aspects of interfibre bonding. All the results discussing interfibre bonding have been obtained using either optical or electron microscopy. While these results are highly significant and give us an insight into the structure and structural changes occurring in paper, the data give very little information about the adhesive forces holding these structures together.

At the Cambridge symposium, Nordman described a method for evaluating the rupture of bonds when a fibrous network is stressed. He found that, for a given pulp, beating or wet pressing does not affect the linear relationship between specific scattering coefficient and energy loss in a straining/destraining cycle. This was interpreted to indicate that the bond strength (specific energy) for the rupture of hydrogen bonds remained constant during beating and wet pressing.

We would like to report some preliminary data using the same basic idea as Nordman, but in which, instead of measuring the free surface optically, we have determined the surface areas before and after straining by nitrogen adsorption measurements. These results are summarised in Table 1. It can be seen that, for relatively weak sheets, a low specific energy for the rupture of hydrogen bonds is obtained and that beating and wet pressing results in a marked increase in the specific energy for straining. The weakest sheets have straining energies very close to the theoretical values for the surface energy of a hydrogen-bonded system. The higher values obtained in the

TABLE 1—SPECIFIC ENERGY OF STRAINING

Pulp and freeness, CSF	*Wet pressure, lb/in^2*	*Specific energy, erg/cm^2*
Sulphite		
560	50	80
560	100	86
Sulphite		
290	0	250
290	50	700
290	100	1 670

stronger sheets most likely reflect increased energy losses, owing to deformation of the fibres. These results differ from Nordman's in two respects. Firstly, the specific energies are much lower and, secondly, beating and wet pressure results in an increase in the specific energy. Although this may partially be due to the type of pulp employed and extent of bonding achieved, part of the difference is no doubt a result of the method used in measuring the new surface created during straining. The absolute increase in areas is large, indicating extensive intrafibre rupture. It seems reasonable to associate the irreversible work in straining with all types of bond failure rather than only fibre-to-fibre bonds as assumed by Nordman.

Another method that we are currently employing to establish whether or not differences in bonding capacity exist between fibres is to determine the specific energy for the separation of two weakly bonded sheets. Pulps beaten and wet pressed to different extents are bonded together in the wet state, then dried. The bonding conditions are chosen so that transfer of fibres from one side to another does not occur during the delamination. In this way, the sheets separate cleanly with no fibre rupture involved. The surface area before and after delamination was measured by nitrogen adsorption and the specific energy calculated.

TABLE 2—SPECIFIC ENERGY OF PEELING

Pulp and freeness, CSF	*Wet pressure, lb/in²*	*Specific energy, erg/cm²*
Sulphite		
560	0	73
560	50	110
560	100	103
560	150	177
Sulphite		
290	0	284
290	100	550
290	150	1 030
Sulphate		
500	50	267
500	100	248

The results for a sulphite and sulphate woodpulp are shown in Table 2. It is seen that, for slightly bonded sheets, the specific energy for peeling is rather low, but—again as with the straining experiments—wet pressing and beating results in higher bond energies. A sulphate woodpulp gave higher energies than an equivalent sulphite pulp. Whether or not this is significant and indicates a real difference in bond quality between sulphite and sulphate pulps will be made clearer, it is hoped, by further investigations of this nature.

DR. F. L. HUDSON: If the theory developed from the behaviour of dry paper under straining is to be applied to the formation and structure of ordinary paper, it must be shown to be applicable to the straining of paper under wet conditions, because the anisotropy of paper is mainly caused by strains applied in the later stages (falling rate period) of drying.

Now, it can easily be demonstrated that natural tracing paper strained under wet conditions becomes opaque and shows a pattern of lines roughly similar to those formed under dry conditions. There are, however, important differences. The straining can be carried on much further and, with a tracing paper that ruptures under a 14 per cent dry strain in the cross-direction, it is possible to get an elongation of as much as 35 per cent and to stabilise this by hot drying under tension. (This was demonstrated by a slide of two pieces of tracing paper. One, treated, carried a circle; the other, stretched wet to 25 per cent and dried, showed a similar circle strained permanently to an ellipse. The paper itself showed a noticeable increase in opacity and wildness.) From the arguments on page 296, one would expect this to break far more bonds than would dry straining and to lower the tensile strength of the paper. Actually, although the work of rupture is decreased (as shown in Fig. D6), the tensile strength is increased and this result does not agree, therefore, with the typical dry straining results in Fig. 10 of the paper.

Although wet straining in this way can give papers that approach being isotropic in tensile strength and humidity expansion, it seems that there are still differences between this and paper stretched on the machine during manufacture.

It seems risky to explain machine effects by the existence of strain lines that cannot be seen, when the tensile test behaviour either wet or dry is explained by lines that are definitely visible.

Possibly some rebonding occurs on the machine as may be inferred from Corte, Schaschek or Broens' work,[1] possibly invisible hornification as suggested yesterday by Jayme[2] is producing irreversible bonding effects. It is likely that the drying temperature itself is important,[3] too, in its effects

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[1] *Tappi*, 1957, **40** (6), 441–447

[2] this vol., 135

[3] Hudson, F. L. and Heinsius, H. J., *Proc. Tech. Sect. B.P. & B.M.A.*, 1959, **40** (1), 31–47; Coles, D. R. and Hudson, F. L., *Paper Tech.*, 1961, **2** (5), T183–T189

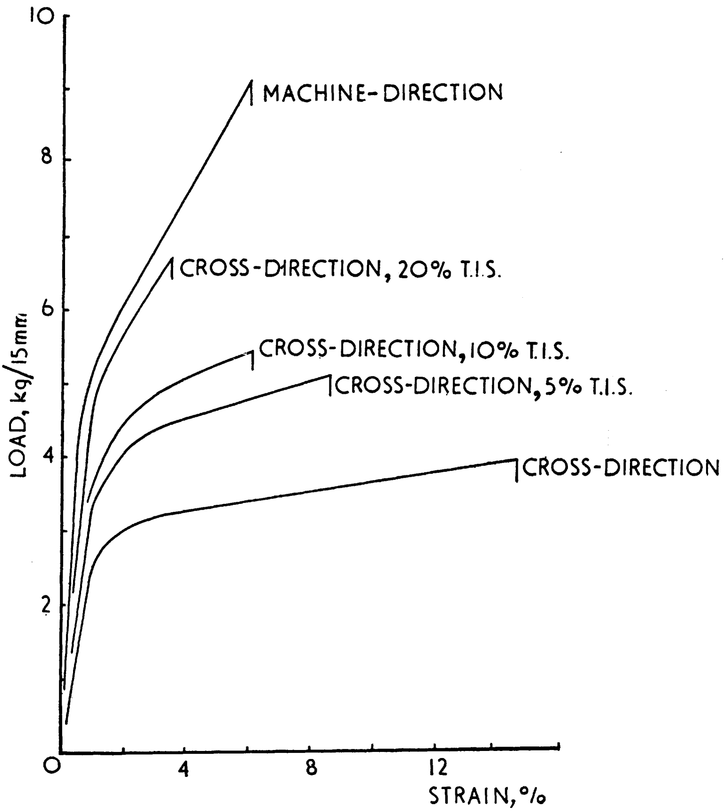

Fig. D6

through the structure on anisotropic properties, but there is no time to develop this today.

MR. A. E. RANGER: In the case of wet straining discussed by Dr. Hudson, the argument outlined by us suggests that a reduction in tensile strength perpendicular to the direction of wet straining should be observed. He does not state the direction of subsequent tensile testing and this, of course, is of prime importance.

It should be stressed that the strain lines are visible only in certain special instances of tensile behaviour (with tracing paper, for example), whereas it is our contention that the basic mechanism is of widespread occurrence. Dr. Hudson does not appear to care for an explanation of machine

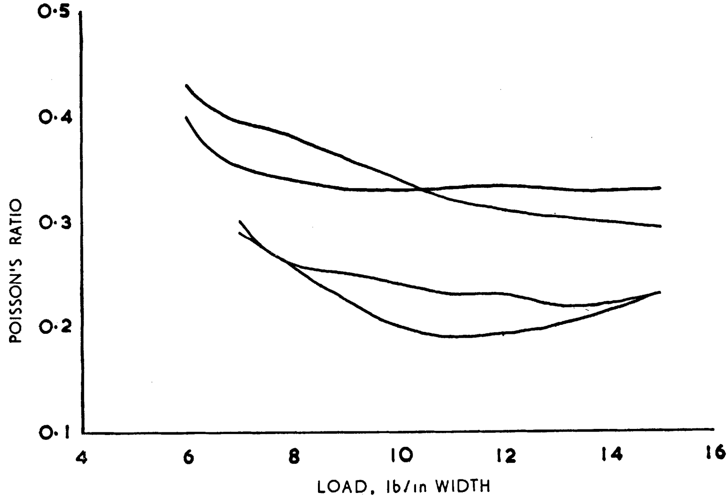

Fig. D7—Variation of Poisson's ratio for 36 lb kraft paper with tensile load— cross-direction

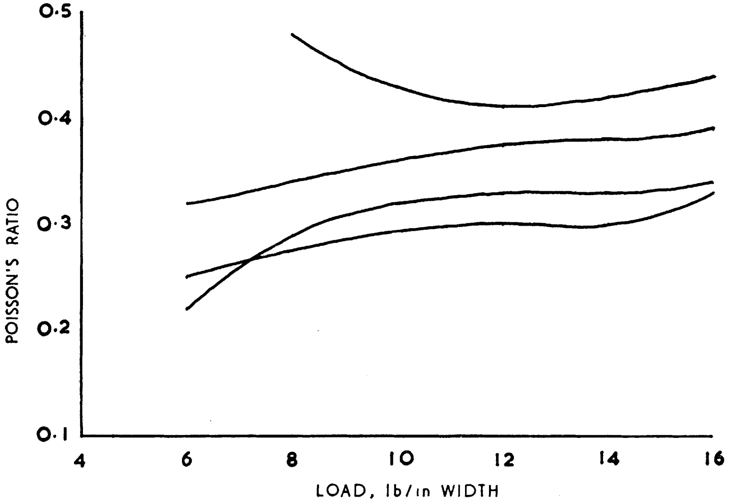

Fig. D8—Variation of Poisson's ratio for 36 lb kraft paper with tensile load— machine direction effects based on lines that cannot be seen, but apparently has no objection to the idea of 'invisible hornification producing invisible bonding effects'.

DR. M. ROTHMAN: The values of Poisson's ratio quoted in the paper did not agree with our fairly extensive experiments and were unrealistic if paper was assumed to be anywhere near elastic (which it was over a certain range).

Our results showed Poisson's ratio to have different values for different values of loading, somewhat asymptotically approaching a value of near 0·5 for loads approaching the breaking load of the paper (Fig. D7 and D8).

MR. RANGER: It is not clear what Dr. Rothman means by 'unrealistic' in this question. It must be remembered that Poisson's ratio cannot have its normal meaning when applied to such a grossly anisotropic material as paper, therefore comparisons between values obtained with paper and those typical of isotropic materials can be misleading.

There does not appear to be a great deal of disagreement between our measurements and those given by Dr. Rothman. The values that we quote are, on the whole, the larger of the two sets, but they were obtained with tensile loads very much less than 6 lb/inch width and, in this region, five of Dr. Rothman's eight curves are rising fairly rapidly.

PROF. R. H. PETERS: The zero span experiments show a discontinuity in gradient when the tensile properties are plotted against jaw span—does this effect vary with the kind of paper? If so, I would anticipate the discontinuity would be less with the stronger papers.

MR. RANGER: We would agree with Prof. Peters' expectations, but unfortunately we have no experimental evidence on this question.

MR. B. RADVAN: The shape of the strength orientation diagram shown in this paper may be explained more simply by fibre orientation—I should like to return to this point on Friday.

Secondly, would you explain again how the compression necessary for initiating the shear movement arises when the sample is in pure tension?

I believe that similar strain lines have been observed in stretching dilatant materials such as a layer of wet sand on a rubber band. The thickening of paper at the strain lines points the same way.

MR. RANGER: The compression necessary for initiating the shear movement arises from the lateral contraction that accompanies tensile extension.

Written contribution

DR. R. I. C. MICHIE: In Appendix 1, the authors calculate that, if a paper specimen is allowed to contract laterally during extension, the modulus will be lowered by the divisor R. It is then suggested that the ultimate strength will also be lowered in (at least) the same proportion. This assumption presupposes that the breaking strain will be similar in the two cases, which is not likely in practice. The lower modulus in the case of lateral contraction results from lower strain in individual fibres for a given paper extension: if the fibres are to be strained to break, it is clear that the paper extension will be greater than in the zero jaw span case (with zero lateral contraction). No quantitative relationship can be evaluated without taking this effect into account, but it seems quite possible that the breaking load might not be very different in the two cases.

As an alternative explanation, it should be emphasised that, if tensile strength results are subject to any spread not attributable to the method of test, the strength *must* decrease with increasing jaw span, owing to the greater probability of including in the test strip an area of lower strength.[4] Thus, the strength decrease in Fig. 3 of the paper over the range 0·25–2·0 cm jaw span would be accounted for on the basis of the Peirce equation(4) by a coefficient of variation of 8 per cent at a jaw span of 1·0 cm. It does not seem necessary to invoke any other mechanism to explain the effect of jaw span on strength within this range.

MR. A. E. RANGER: Dr. Michie states, 'The lower modulus in the case of lateral contraction results from lower strain in individual fibres for a given paper extension'. This is a misleading statement, since, although the average fibre strain is less, the maximum strain, occurring in fibres parallel to the direction of tension, will be the same in both cases. The results of our experiments on fibre failure in paper suggest that final fracture coincides with the failures of the first fibres. It is reasonable to assume that these fibres lie approximately parallel to the tension direction and are therefore strained to the maximum extent and that they fail by reaching their breaking strain. On this basis, the breaking strain should not depend on span. This simplified argument does, of course, ignore the effect of local variations of strain in the long span case, a factor whose importance is difficult to assess.

The suggestion that the 'span effect' might be due to local strength variations is not new, as pointed out in the paper. It has been known for

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[4] Peirce, F. T., *J. Textile Inst.*, 1926, **17**, T355–T368

some time, however, that this explanation leads to anomalies that can be satisfactorily resolved, if there is an additional mechanism present such as the one suggested by us. As a matter of interest, the coefficient of variation of the ten results at 1·0 cm jaw span in our experiment was 2·8 per cent, which would seem to support our contention.