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MECHANICAL PROPERTIES OF PAPER

PART I: ELASTICITY OF HANDSHEETS

O. J. KALLMES AND MISS G. A. BERNIER
ST. REGIS PAPER CO., WEST NYACK, N.Y., U.S.A.

Synopsis

A theory of the elasticity of handsheets has been formulated, based on the hypothesis that elastic deformation is caused by bending and, to a small extent, shearing and stretching of the unbonded parts of the fibres within the plane of the sheet. Experiments have shown that the theory holds for dense, well-bonded sheets, but deviates for light, poorly bonded sheets by a factor directly related to a geometric property—the mean free fibre length. It is concluded that the arrangement of the fibres in handsheets is the connecting link between the fibre properties and sheet elasticity.

Les propriétés mécaniques du papier: L'élasticité des feuilles de formette

Les auteurs proposent une théorie d'élasticité des feuilles de formette. On part de l'hypothèse que la déformation élastique provient de la flexion et, jusqu'à un certain point, du cisaillement et de l'élongation des parties des fibres non liées. La théorie est justifiée par les résultats d'expériences sur les feuilles denses et dont les fibres sont bien liées. Dans le cas des feuilles légères composées de fibres faiblement liées, on établit un rapport entre la déviation de leurs propriétés de celles exigées par la théorie et la longueur moyenne des parties non liées des fibres.

On tire la conclusion que le rapport entre l'élasticité des feuilles et les propriétés des fibres individuelles dépend de la disposition de celles-ci dans la feuille.

Die mechanischen Eigenschaften des Papiers: Die Elastizität von Versuchsblättern

Eine Theorie der Elastizität von Blättern ist entwickelt worden auf der Basis der Hypothese, dass die elastische Deformierung durch

Biegen und, zu einem kleinen Grad, durch die Verdrehung und die Dehnung der nicht gebundenen Anteilen der Fasern innerhalb der Blattebene verursacht wird. Experimente haben gezeigt, dass die Theorie für dichte gut gebundene Blätter Gültigkeit hat, aber dass sich Abweichungen einstellen wo es sich um dünne nur wenig gebundene Blätter handelt mit Bezug auf einen Faktoren der in direkter Beziehung steht mit einer geometrischen Eigenschaft—der mittleren freien Faserlänge. Man kommt zu der Schlussfolgerung, dass die Anordnung der Fasern in den Blättern die Brücke darstellt wodurch die Fasereigenschaften und die Elastizität der Blätter in Beziehung gebracht werden.

Introduction

IN the past, studies of paper behaviour have been based on artificial models, because the internal structure of paper was unknown. Although these studies have measured certain paper properties as quantities, they have not led to an understanding of the phenomena taking place within a sheet during its use.

The link between the mechanical properties of a structure such as a fibre network and the properties of its elements, the fibres, is the arrangement of the elements making up the structure. Little was known of the arrangement of fibres in paper until 1960, when the geometry of the fibres in a randomly formed, ultra-thin (2-D) network was described.(1) This theory has been extended to a sheet consisting of a pile of 2-D sheets, to a multi-planar (MP) sheet and to handsheets formed in the usual manner.(2) By applying concepts of the strength of materials to the elements of a random fibre network, it has been possible to work out a theory of the elasticity of handsheets: that theory is the subject of this article.

The theory combines the concepts of the deflection of beams, which in paper are the unbonded portions of the fibres between fibre-fibre contacts (crossings) and the geometry of a random fibre network.(2) Because the mathematics for solving this problem with complete rigour is not yet available, the following assumptions and approximations had to be made—

1. Only the unbonded portions of the fibres are deflected. The bonded parts of the crossings are rigid and undergo only linear displacement—that is, no rotation.
2. The same force, $F^{\star}$, parallel to the direction of the external force, is applied to every free fibre length or gap (these two terms are used interchangeably for the unbonded portions of the fibres).

We realise that both of these assumptions are not strictly valid and the experimental data have indicated that, without qualifications, they appear to hold only over a limited range of conditions. We know of no technique, however, for resolving into components the static forces in equilibrium at the extremely large number of crossings in a random fibre network.

3. The network initially contains no built-up stresses, but every free fibre length is deflected by the first application of an infinitesimal force to the system.[4]
4. There is no movement of the fibres out of the plane of the sheet.
5. The length and width of the sheet are orders-of-magnitude larger than the mean free fibre length.
6. No crossings fail during the elastic distortion of the sheet; the failure of the first crossing, no matter how soon it occurs after the initiation of strain, marks the end of the elastic zone of the stress/strain curve.

Average deflection of a free fibre length

To find the average deflection of a free fibre length due to the force F^*, the free fibre length is considered to be a double cantilever beam deflected

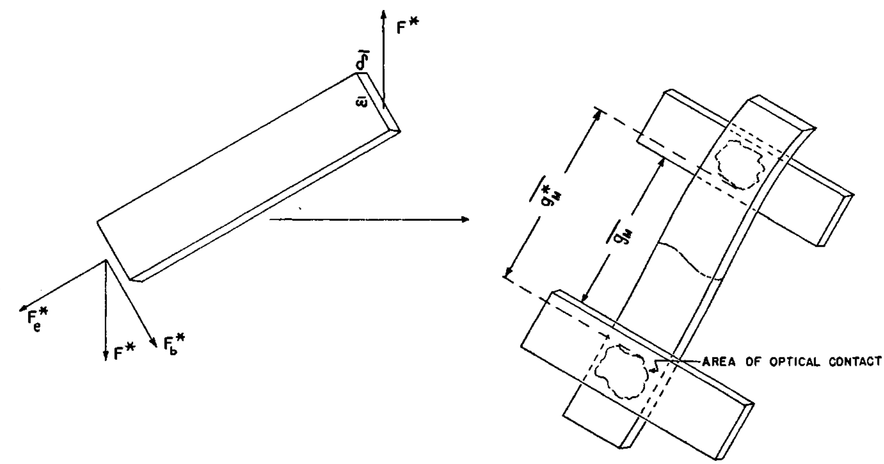

Fig. 1—Distortion of free fibre length during straining (exaggerated for demonstration purposes)

by forces F^* acting on the crossings at its ends (Fig. 1). The forces are resolved into two mutually perpendicular components, F_b^* and F_e^*, the first of which causes bending and shearing, the second, elongation.

The amount of deflection caused by $F_b^\star$ is a function of the fibre's cross-section shape, which in paper is highly irregular (Fig. 2). It varies from species to species and is affected by every phase of pulp and paper-making. Even within a single sheet, it varies from fibre to fibre and along every fibre. In papers made from softwoods, the fibres are generally long, flat ribbons with a cross-sectional shape not grossly different from rectangular. Because the nature of the cross-sectional shape (circular, rectangular, etc.) must be specified in considering shearing and stretching, we have assigned

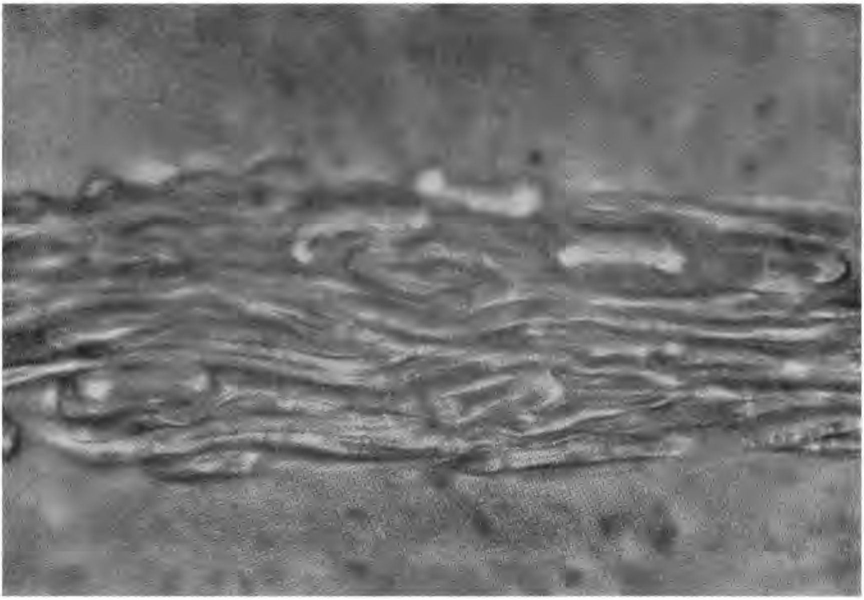

Fig. 2

a rectangular one to the fibres in considering these mechanisms; however, deflection due to *bending* can be calculated for *any* cross-sectional shape with a general moment of inertia I. By using I in considering bending, we hope eventually to determine the fibres' average cross-sectional shape more specifically. The calculation of I of ribbon-like fibres should be possible, because it turns out that bending is generally the dominating mechanism.

The deflection caused by bending in the direction of $F^\star$ is—

$$\frac{F^\star (g_m^\star)^3 \cos^2 \theta}{12 E I_Z} \qquad \qquad (1)$$

and that due to shearing in the same direction is—

$$\frac{F^\star}{E}\left[\frac{2\cdot8(g_m{}^\star)}{\omega\delta}-\frac{0\cdot8}{\delta}\right]\cos^2\theta \qquad . \; . \; . \; . \; . \; . \quad (2)$$

where $(g_m{}^\star)$ is the distance between the bonded zones of the crossings; E, ω and δ are, respectively, the elastic modulus, width and thickness of the fibre; I_Z is the moment of inertia of the fibre cross-section about its vertical neutral axis; $(I_Z)_{\max}=\delta\omega^3/12$ (*see* Timoschenko,[5] pp. 170–175, particularly equation k on p. 175).

The deflection caused by $F_e{}^\star$ in the direction of $F^\star$ is—

$$\frac{F^\star(g_m{}^\star)\sin^2\theta}{E\omega\delta}. \qquad . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

The average deflection of a free fibre length in the direction of $F^\star$ for a random fibre orientation is—

$$\frac{2}{\pi}\int_0^{\pi/2}\frac{F^\star}{E}\left\{\left[\frac{(g_m{}^\star)^3}{12I_Z}+\frac{2\cdot8(g_m{}^\star)}{\omega\delta}-\frac{0\cdot8}{\delta}\right]\cos^2\theta+\frac{(g_m{}^\star)\sin^2\theta}{\omega\delta}\right\}d\theta$$

$$=\frac{F^\star}{2E}\left\{\left[\frac{(g_m{}^\star)^3}{12I_Z}+\frac{2\cdot8(g_m{}^\star)}{\omega\delta}-\frac{0\cdot8}{\delta}\right]+\frac{(g_m{}^\star)}{\omega\delta}\right\} \qquad . \; . \quad (4)$$

It has been shown[1] that the gap size distribution is negatively exponential and independent of fibre width. Taking into account the length distribution of $(g_m{}^\star)$[1] and, using mean values of I_Z, E, ω and δ, the average deflection of a free fibre length S_{11} in the direction of $F^\star$ is—

$$S_{11}=\frac{F^\star}{\bar{E}}\left[\frac{(\bar{g}_m{}^\star)^3}{4\bar{I}_Z}+\frac{1\cdot9(\bar{g}_m{}^\star)}{\bar{\omega}\bar{\delta}}-\frac{0\cdot4}{\bar{\delta}}\right] \qquad . \; . \; . \; . \; . \quad (5)$$

Equation (*5*) gives the average elastic deflection of a free fibre length in the direction of the applied force—provided, of course, the assumptions are valid. This equation, combined with one for the number of free fibre lengths deflected, will lead to an expression for the elasticity of handsheets.

Total elastic deformation and modulus of elasticity of handsheets

It has been shown[2] that the elasticity of MP sheets is not significantly different from that of equivalent, normally formed handsheets of the same basis weight and that equations for the structure of MP sheets can be applied to handsheets. By the same token, the following considerations, which are based on MP sheets, are reasonable for normal handsheets.

Our assumptions predicate that the elastic deformation of paper is equal to that of a single 'chain' of end-to-end free fibre lengths between the same points of the ends of a sheet (Fig. 3). We have made the approximation that every chain contains the same number of free fibre lengths and that all of the chains of a sheet act parallel to each other. It follows that every fibre held in the clamps is the end of exactly one chain.

Previous work[2] has shown that the mean distance between the bonded zones of crossings $(\bar{g}_m^{\star})$ is at a maximum in the two edge layers of an MP sheet and falls to a minimum in the central layer. Variations in $(\bar{g}_m^{\star})$ are negligible from the second layer inward, however. We also recognise that the number of free fibre lengths per chain varies from chain to chain within

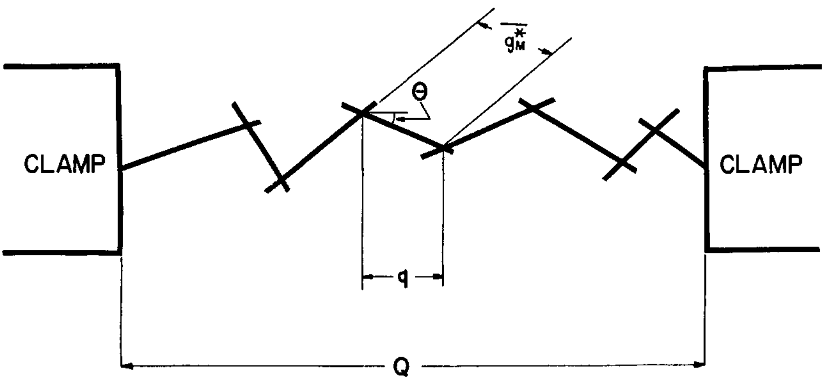

Fig. 3—Chain of free fibre lengths

any given layer, but these variations become small enough to be neglected when the sample length $Q \gg (\bar{g}_m^{\star})$. Accordingly, to assume that every chain of a sheet contains the same number of free fibre lengths is reasonable, except in the case of very small samples and extremely thin sheets such as those composed of one, two and three 2-D layers.

Consider a chain of length Q of fibres of no width (Fig. 3). For any one free fibre length, $q=(\bar{g}_m^{\star})\cos\theta$, where $(\bar{g}_m^{\star})$ has the distribution $(1/\bar{g}_m^{\star})\exp(-g_m^{\star}/\bar{g}_m^{\star})$. The angle θ is random in the range $-\pi/2$ to $\pi/2$ and $(\bar{g}_m^{\star})$ and θ are independent variables. Thus, $q=(\bar{g}_m^{\star})\overline{\cos\theta}=2(\bar{g}_m^{\star})/\pi$.

Kallmes and Corte[1, 6] showed that a fraction $[1-(\pi\bar{\omega}\bar{r}\sqrt{\beta}/2\lambda)]^{\bar{c}_m}$ of the external surface of fibres of width $\bar{\omega}$ is not bonded to other fibres; λ is the mean fibre length, $\bar{r}$ the mean curl factor,[1] $\bar{c}_m$ the mean number of crossings per fibre, and β the fraction of the area of crossings bonded. The mean number of free fibre lengths in a chain of real fibres is—

$$\frac{Q\pi}{2(\bar{g}_m{}^\star)}\left[1-\frac{\pi\bar{\omega}\bar{\tau}\sqrt{\beta}}{2\bar{\lambda}}\right]^{\bar{c}_m} \qquad (6)$$

The unit stretch per chain is the product of equations (5) and (6) and $(1/Q)$—

$$\epsilon_{11}=\frac{F^\star\pi}{2\bar{E}(\bar{g}_m{}^\star)}\left[\frac{(\bar{g}_m{}^\star)^3}{4\bar{I}_Z}+\frac{1{\cdot}9(\bar{g}_m{}^\star)}{\bar{\omega}\bar{\delta}}-\frac{0{\cdot}4}{\bar{\delta}}\right]\left[1-\frac{\pi\bar{\omega}\bar{\tau}\sqrt{\beta}}{2\bar{\lambda}}\right]^{\bar{c}_m} \qquad (7)$$

The number of chains per layer of an MP sheet of dimensions QR is found by dividing expression (6) into the total number of free fibre lengths in the layer—

$$N_f\bar{c}_m\frac{QR}{A}\left[1-\frac{\pi\bar{\omega}\bar{\tau}\sqrt{\beta}}{2\bar{\lambda}}\right]^{\bar{c}_m} \qquad (8)$$

a quotient equal to NR/L. The term N_f is the number of fibres in one 2-D layer of an MP sheet of area A, $(g_m{}^\star)\approx g_m$ and $\bar{c}_m \gg 1$. From this, $\bar{g}_m$ is the mean free fibre length between completely bonded crossings and N is the number of fibres in one 2-D layer intersecting a scanning line of length L; NR/L is also the number of fibres per layer clamped at each end of a sheet of width R so that *each fibre held in the clamp is, on the average, the end of exactly one chain.*

Since every layer of an MP sheet contains an average of NR/L chains and since we have made the approximation that the same force $F^\star$ is applied to every free fibre length, the relation between $F^\star$ and the externally applied force F is—

$$F=F^\star\left(\frac{NR}{L}\right)N_L \qquad (9)$$

in which N_L is the number of layers in an MP sheet or the equivalent number of layers in normally formed handsheets.

Substituting equation (9) into (8) and solving for the elastic modulus of the sheet $Y=F/R\epsilon_{11}$, then—

$$Y=\frac{\bar{E}N_L(N/L)}{\pi\left[\frac{(\bar{g}_m{}^\star)^2}{8\bar{I}_Z}+\frac{0{\cdot}95}{\bar{\omega}\bar{\delta}}-\frac{0{\cdot}2}{\bar{\delta}(\bar{g}_m{}^\star)}\right]\left[1-\frac{\pi\bar{\omega}\bar{\tau}\sqrt{\beta}}{2\bar{\lambda}}\right]^{\bar{c}_m}} \qquad (10)$$

The same equation, solved for $\bar{I}_Z$, becomes—

$$\bar{I}_Z = \frac{(\bar{g}_m{}^{\star})^2}{8\left\{\dfrac{\bar{E}N_L(N/L)}{Y\pi\left[1-\dfrac{\pi\bar{\omega}\bar{\tau}\sqrt{\beta}}{2\bar{\lambda}}\right]\bar{c}_m} + \dfrac{0{\cdot}2}{\delta(\bar{g}_m{}^{\star})} - \dfrac{0{\cdot}95}{\bar{\omega}\delta}\right\}} \quad . \; . \; . \; (10a)$$

Expressions for $(\bar{g}_m{}^{\star})$, $\bar{c}_m$ and β were derived in earlier publications[1,2,6] and are as follows—

$$(\bar{g}_m{}^{\star}) = \bar{g}_m + \bar{\omega}\sqrt{(\pi/2)(1-\beta)} \quad . \; . \; . \; (11)$$

$$\bar{c}_m = \bar{\lambda}/\bar{g}_m\bar{\tau} \quad . \; . \; . \; (12)$$

$$\bar{g}_m = \frac{\bar{\lambda}}{\bar{c}\bar{\tau}B\left\{\sigma_1 + 2\left[\left(1-\dfrac{1}{N_L}\right)\left(\sigma_2 + \dfrac{\sigma_3 p(0)}{1-p(0)^2}\right) + \dfrac{\sigma_3 p(0)[1-p(0)^{2(N_L-1)}]}{N_L[1-p(0)^2]^2}\right]\right\}} \quad (13)$$

$$B = \tfrac{1}{2}[1+p(0)] \quad . \; . \; . \; (14)$$

$$\beta = \frac{\sigma_1\beta_1 + 2\left(1-\dfrac{1}{N_L}\right)\sigma_2\beta_2 + \dfrac{2\sigma_3\beta_3 p(0)}{1-p(0)^3}\left[1-\dfrac{1}{N_L}-\dfrac{1}{N_L[1-p(0)^3]^2}\right]}{\sigma_1 + 2\left(1-\dfrac{1}{N_L}\right)\sigma_2 + \dfrac{2\sigma_3 p(0)}{1-p(0)^2}\left[1-\dfrac{1}{N_L}-\dfrac{1}{N_L[1-p(0)^2]^2}\right]} \quad . \; (15)$$

Equation (*10*) expresses the slope of the initial straightline portion of the stress/strain curve of handsheets in terms of the properties of the fibres and their arrangment in the sheet. All of its terms, except $\bar{I}_Z$, can be measured or calculated. Since $\bar{I}_Z$ is itself a direct measure of the rigidity or flexibility of fibres in paper, we have in equation (*10a*) a possible means of evaluating the influence of the pulp and papermaking variables on the basic fibre properties of rigidity and flexibility.

Experimental programme

AN experimental programme was carried out on four different softwood pulps. Three of these were unbeaten and fines-free: a sulphite and a kraft, whose preparations have been described[2], and an experimental pulp referred to here as pulp S. The same kraft pulp was also treated in a pulp evaluation disintegrator at 1 per cent consistency for 2 h and fractionated to remove fines:[2] this was beaten kraft. From each of the pulps, four sets of handsheets were prepared, one of MP sheets and one of equivalent, normally formed sheets pressed at 1 000 lb/in^2 and similar sets pressed at 50 lb/in^2, but otherwise according to the standard procedure. In this way, the effects of wet pressing could also be studied.[2] Each set of sheets contained samples with widely different basis weights, so that equation (*10*) could be tested over a large range of conditions.

The thickness of the fibre walls was determined from the polarisation colours produced when single fibres were viewed in transmitted light between crossed nicols using a gypsum retardation plate.[7] Here, the optical path difference of two polarised beams passing through the cellulose of two fibre walls is measured. The height of the lumen has no effect on the measurement so long as the cross-sectional shape of the fibre does not differ greatly from that of a flat ribbon.

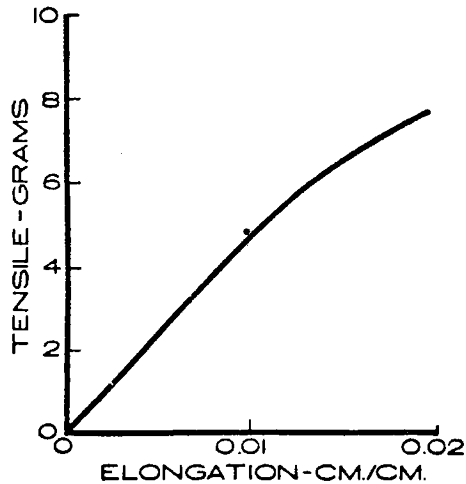

Fig. 4—Stress/strain curve of a single fibre (spruce sulphite pulp–1 000 lb/in^2 pressure)

The modulus of elasticity E of single fibres was measured on the Instron tensile testing instrument by the technique of Van den Akker *et al.*,[8] with a constant elongation rate of 0·0508 cm/min. The only change made in the procedure was that the single fibres, which were obtained from ferroplates after about 100 fibres were put through the standard sheetforming process, were treated identically with the sheets. A typical stress/strain curve is shown in Fig. 4.

Our procedure for measuring the average bonded area per crossing, based on the technique of Page and Tydeman,[9, 10] was described earlier.[2] The weighted degree of optical contact of each type of crossing, where at least some degree of optical contact exists, is β_K (*see* equation (*15*)[6]).

TABLE 1—FIBRE PROPERTIES

Pulp	*Pressure of formation, lb/in²*	*Fibre dimensions, cm*				*Fibre flexibilities*			*Relative bonded area*			*Fibre elasticity, $E \times 10^{-7}$ g/cm²*	*Specific fibre mass, $\bar{w} \times 10^6$ g/cm*
		$\bar{\lambda}$	$\bar{\omega} \times 10^3$	$\bar{\tau}$	$\bar{\delta} \times 10^4$	σ_1	σ_2	$C^{(a)}$	β_1	β_2	$D^{(b)}$		
Unbeaten sulphite	1 000	0·22	3·7	1·07	2·73	0·95	0·89	1·49	0·91	0·80	1·63	35	1·3
	50	0·22	3·7	1·07	2·73	0·92	0·71	1·00	0·86	0·77	1·54		1·3
Unbeaten kraft	1 000	0·22	3·5	1·13	2·64	0·84	0·65	0·78	0·94	0·85	1·48	28	1·5
	50	0·22	3·5	1·13	2·64	0·74	0·51	0·54	0·88	0·73	1·06		1·5
Beaten kraft	1 000	0·22	3·5	1·06	2·64	0·92	0·84	1·35	0·98	0·82	1·54	32	1·4
	50	0·22	3·5	1·06	2·64	0·73	0·61	1·04	0·93	0·64	1·37		1·4
Pulp S	1 000	0·22	3·7	1·10	2·44	0·89	0·83	1·28	0·93	0·85	1·30	31	1·5
	50	0·22	3·7	1·10	2·44	0·80	0·80	1·38	0·96	0·84	1·27		1·5

(a) $C=\sigma_3/p(0)$; (b) $D=\beta_3/p(0)$

Fibre and sheet properties comprise Tables 1 and 2, respectively. Unless otherwise indicated, Y values are the average of six measurements of $3 \times 1{\cdot}5$ cm samples run on the Instron at the straining rate used for single fibres, at 72°F and 50 ± 2 per cent r h (*see* original curves[2]).

Discussion and conclusions

THE experimental programme was designed to test the validity of some of our assumptions and approximations and to apply equation (*10*). Although the assumption that the chains of free fibre lengths act in parallel could not be tested directly, it was found that approximately the same elastic modulus per unit mass per crossing was developed—that is, $Y/W\bar{c}_m$ is constant in sheets of widely varying basis weights and structures (*see* Table 2, last column).

Relative theoretical contributions of bending, shearing and elongation to the total deflection of typical sheets are listed in Table 3. Clearly, bending predominates among the three terms. Shearing becomes important only when $(\bar{g}_m{}^\star)$ approaches $\bar{\omega}$ in magnitude. When softwood fibres are involved, this degree of bonding occurs only in glassine-type papers.

We had hoped that equation (*10*), besides being able to describe the elastic deformation of handsheets, would permit us to follow changes in $\bar{I}_Z$ caused by variations in the pulpmaking and sheetmaking conditions. To this end, $\bar{I}_Z$ was calculated from equation (*10a*) and the results plotted as a function of $(\bar{g}_m{}^\star)$ in Fig. 5. The term $\bar{I}_Z = 0{\cdot}6 \times 10^{-12}$ cm^4 for the densest sheet structure attainable—that is, the one having complete interfibre bonding and minimum $(\bar{g}_m{}^\star)$. [$(\bar{g}_m{}^\star)_{\min}$ is calculated by setting the σ_k and β_k values in equations (*13*) and (*15*) equal to unity.] The cross-sections of softwood fibres in sheets pressed to this extent tend towards the rectangular, for which shape $\bar{I}_Z$ approaches its maximum value, $\delta\bar{\omega}^3/12$. In the case of the four types of fibre examined, $(\bar{I}_Z)_{\max} \approx 1 \times 10^{-12}$ cm^4, a value in close agreement with that corresponding to $(\bar{g}_m{}^\star)_{\min}$ in Fig. 5.

The $\bar{I}_Z$ of Fig. 5 rises above $(\bar{I}_Z)_{\max}$ as $(\bar{g}_m{}^\star)$ increases, however—that is, as the sheet becomes less dense. The deviations of the calculated $\bar{I}_Z$ values from $(\bar{I}_Z)_{\max}$ are not serious in the case of the heavier well-bonded sheets (four or more layer sulphite, beaten kraft and pulp S), but they are quite marked for the poorly bonded kraft pulp and all of the extremely thin two- and three-layer sheets. It was obvious that one or more effects not considered in the theory had caused the rigidity of the fibres to appear to increase rather than decrease with increasing $(\bar{g}_m{}^\star)$.

We realised that all our assumptions introduce errors and that it is

TABLE 2—SHEET PROPERTIES

Pulp	*Pressure of formation, lb/in²*	*No. of layers (MP) or equivalent no. of layers (E)*	*Basis weight, g/m²*	*Mean no. of crossings per fibre, $\bar{c}_m$*	*Mean free fibre length, $\bar{g}_m$*, cm × 10³*	*Measured modulus of elasticity, g/cm × 10⁻⁴*	*Stiffness correction factor, K*	*Elastic modulus per unit mass, cm × 10⁻⁷*	*Elastic modulus per unit mass per mean no. of crossings per fibre, cm × 10⁻⁶*
Unbeaten sulphite	1 000	(MP) 2	5·7	35·6	7·52	3·1	2·86	5·5	1·5
		3	8·9	46·6	6·31	4·9	1·66	5·5	1·2
		4	14·2	57·8	5·46	8·6	1·12	5·9	1·0
		8	25·3	62·0	5·28	14·0	0·86	5·8	0·9
		12	31·8	58·8	5·51	20·8	1·18	6·5	1·1
	50	(MP) 2	6·0	32·7	8·25	3·2	3·60	5·4	1·6
		3	8·7	39·2	7·29	5·0	2·76	5·8	1·5
		4	13·2	46·9	6·46	8·4	2·03	6·4	1·4
		8	24·1	50·3	6·21	14·7	1·69	6·1	1·2
		12	35·1	51·5	6·12	21·9	1·63	6·2	1·2
	1 000	(E) 2	6·6	39·8	6·92	3·4	2·07	5·2	1·3
		3	9·3	47·7	6·17	5·6	1·71	6·0	1·3
		4	11·7	51·2	5·93	6·8	1·41	5·9	1·2
		8	23·7	59·8	5·41	13·7	0·98	5·8	1·0
	50	(E) 3	10·6	44·8	6·62	6·2	2·06	5·9	1·3
		4	12·5	45·2	6·61	6·7	1·87	5·9	1·3
		8	22·6	48·4	6·37	13·6	1·87	6·0	1·2
Unbeaten kraft	1 000	(MP) 2	6·2	25·1	9·14	2·2	6·36	3·6	1·4
		3	9·5	31·2	7·75	4·3	5·16	4·6	1·5
		4	11·7	32·6	7·53	5·7	5·08	4·9	1·5
		8	24·5	38·6	6·67	12·9	3·76	5·2	1·4

Beaten kraft	50	(MP)	2	6·0	20·5	11·35	2·0	10·60	3·3	1·6
			3	8·4	23·5	10·26	3·5	10·18	4·1	1·7
			4	11·2	25·9	9·78	5·4	10·17	4·8	1·8
			8	22·5	29·7	8·69	11·2	7·64	5·0	1·7
	1 000	(E)	2	5·6	23·2	9·78	2·1	8·06	3·8	1·6
			3	7·4	26·0	9·01	3·7	8·64	5·0	1·9
			4	10·2	29·6	8·14	3·9	4·98	3·8	1·3
			8	20·0	34·0	7·37	11·7	5·74	5·6	1·6
	50	(E)	2	3·9	14·6	15·23	1·5	24·95	3·8	2·6
			3	7·1	20·8	11·38	2·9	12·98	4·1	2·0
			4	8·9	21·6	10·94	3·1	10·07	3·5	1·6
			8	24·3	31·2	8·36	11·4	5·28	4·7	1·5
	1 000	(MP)	2	6·1	35·1	7·28	2·2	2·74	3·6	1·0
			3	9·3	44·9	6·15	4·7	2·19	5·1	1·1
			4	10·2	44·4	6·31	5·0	2·28	4·9	1·1
			8	26·7	61·0	5·08	14·3	1·15	5·4	1·9
			12	47·6	70·1	4·64	30·2	0·94	6·3	1·9
	50	(MP)	2	7·1	29·5	8·91	2·6	4·71	3·6	1·2
			3	10·8	37·3	7·70	4·0	3·08	3·7	1·0
			4	13·0	38·7	7·58				
			8	32·6	50·6	6·36	14·9	2·04	4·5	1·9
			12	45·4	50·6	6·40	23·3	2·40	5·1	1·0
	1 000	(E)	2	6·3	36·0	7·14	2·3	2·54	3·7	1·0
			3	7·0	37·4	7·21	3·3	3·25	4·6	1·2
			4	11·1	46·9	6·05	6·2	2·26	5·7	1·2
			8	24·6	58·4	5·25	11·5	1·12	4·7	0·8
			12	37·3	61.5	5·09	23·4	1·34	6·2	1·0
			20	60·4	62·8	5·04	31·2	1·06	5·2	0·8
	50	(E)	2	5·7	25·4	10·16	2·7	8·57	2·7	1·1
			3	9·2	33·5	8·35	3·6	4·12	3·9	1·2
			4	12·6	40·6	7·42	5·8	3·86	4·6	1·1
			8	22·2	48·0	6·63	11·6	2·16	5·2	1·1
			12	41·3	45·1	6·94	18·7	3·23	4·5	1·0
			20	57·8			29·1		5·0	

TABLE 2—SHEET PROPERTIES (*continued*)

Pulp	*Pressure of formation, lb/in²*	*No. of layers (MP) or equivalent no. of layers (E)*	*Basis weight, g/m²*	*Mean no. of crossings per fibre, $\bar{c}_m$*	*Mean free fibre length, $\bar{g}_m$*, cm × 10³*	*Measured modulus of elasticity, g/cm × 10⁻⁴*	*Stiffness correction factor, K*	*Elastic modulus per unit mass, cm × 10⁻⁷*	*Elastic modulus per unit mass per mean no. of crossings per fibre, cm × 10⁻⁶*
Pulps S	1 000	(MP) 3	10·1	41·0	6·56	4·9	2·52	4·86	1·2
		6	20·7	51·8	5·65	12·0	1·76	5·81	1·1
		9	31·2	55·4	5·43	21·5	1·71	6·83	1·2
		12	41·6	57·1	5·33	31·4	1·81	7·59	1·3
	50	(MP) 3	9·4	37·1	7·07	4·1	2·87	4·32	1 2
		6	18·6	47·1	6·06	8·4	1·77	4·42	0·9
		9	30·1	52·6	5·63	16·0	1·58	5·35	1·0
		12	42·6	54·5	5·41	24·7	1·55	5·79	1·1
	1 000	(E) 3	8·7	37·0	7·10	4·4	3·38	5·14	1·4
		6	18·0	47·9	5·99	9·9	2·06	5·52	1·2
		9	25·8	50·0	5·85	15·2	2·02	5·88	1·2
		12	30·6	48·8	5·99	18·6	2·24	6·08	1·2
		20	57·6	54·0	5·59	37·1	1·84	6·46	1·2
		Standard handsheet	65·0	53·9	5·60	40·5	1·71	6·24	1·2
	50	(E) 3	9·4	37·1	7·07	4·2	2·94	4·19	1·1
		6	17·9	46·1	6·16	8·6	1·97	4·82	1·0
		9	25·4	48·1	6·01	14·5	2·16	5·69	1·2
		12	32·3	48·9	5·98	19·3	2·20	5·98	1·2
		20	58·2	52·9	5·67	34·0	1·78	5·85	1·1
		Standard handsheet	60·3	52·8	5·69	31·7	1·61	5·23	1·0

1 000	(MP)	3	9·3	38·7	6·85	5·7	3·86	6·15	1·6*
		6	18·9	49·1	5·87	15·0	2·77	7·97	1·6
		9	29·2	53·5	5·57	20·9	2·02	7·16	1·3
50	(MP)	3	9·3	36·8	7·10	4·3	3·10	4·62	1·3*
		6							
		9	27·6	50·4	5·82	19·1	2·34	6·92	1·4
		12							
1 000	(E)	3	8·1	35·2	7·34	4·1	2·62	5·10	1·4*
		6	17·0	46·3	6·14	12·9	3·09	7·34	1·6
		9	26·0	50·0	5·85	17·8	2·43	6·84	1·4
		12	33·7	51·3	5·77	23·9	2·30	7·10	1·4
		20	55·7	53·1	5·68	42·4	2·29	7·61	1·4
		Standard handsheet	66·2	53·3	5·64	48·3	2·16	7·27	1·4
50	(E)	3	8·4	34·4	7·50	3·5	2·42	4·17	1·2*
		6	16·8	44·6	6·32	10·6	2·83	6·31	1·4
		9	24·4	47·3	6·10	17·3	2·82	7·09	1·5
		12	32·6	49·1	5·96	23·3	2·61	7·15	1·5
		20	56·3	52·2	5·74	37·6	2·12	6·67	1·3

* The size of the tested sheets in these sets was 10 cm × 1·5 cm

TABLE 3—EXAMPLES OF THEORETICAL RELATIVE CONTRIBUTIONS OF BENDING, SHEARING AND ELONGATION TO DEFLECTION OF FREE FIBRE LENGTHS

Pulp	*Pressing pressure, lb/in²*	N_L	*Fractional deflection due to*		
			Pure bending	*Pure shear*	*Fibre elongation*
Unbeaten kraft	50 MP	2	0·831	0·049	0·119
		3	0·760	0·055	0·185
		4	0·745	0·059	0·196
		8	0·736	0·073	0·191
	E	2	0·761	0·026	0·213
		3	0·761	0·045	0·195
		4	0·787	0·050	0·163
		8	0·746	0·080	0·174
Beaten kraft	1 000 MP	2	0·797	0·111	0·092
		3	0·755	0·144	0·101
		4	0·767	0·132	0·101
		8	0·728	0·198	0·074
		12	0·703	0·208	0·069
	E	2	0·795	0·115	0·090
		3	0·774	0·110	0·116
		4	0·745	0·147	0·108
		8	0·742	0·190	0·068
		12	0·720	0·195	0·085
		20	0·729	0·203	0·068
Sulphite	1 000 MP	8	0·738	0·208	0·054
		12	0·738	0·192	0·070
	E	8	0·753	0·172	0·075
	50 MP	8	0·760	0·159	0·082
		12	0·755	0·162	0·082
	E	8	0·763	0·152	0·085
Pulp S	1 000 MP	12	0·694	0·200	0·106
	E	12	0·729	0·163	0·108
		24	0·721	0·182	0·097
	50 MP	12	0·722	0·194	0·084
	E	20	0·735	0·182	0·083
		24	0·735	0·181	0·084

only a question of how far the conditions of straining may deviate from those stipulated before the theory breaks down. The fibres of 2-D sheets are visibly distorted out of the plane of the sheet during straining, perhaps because of a non-uniform stress distribution. Thus, the assumptions are inadequate for 2-D sheets, probably to a lesser extent for other sheets with large free fibre lengths. *It may be* the case in low-density sheets that fibres essentially parallel to the direction of external loading support more than their share of the stress. That would assure such sheets a stiffness somewhat greater than expected.

Whatever their nature, the unaccounted for phenomena appear from Fig. 5 to be functions of sheet structure. In equation (*10*), we replaced the variable $\bar{I}_Z$ with $(\bar{I}_Z)_{max}$ and multiplied the right side by a factor K to provide a measure of the difference between theory and experiment. Then, substituting the experimental data in the same equation, we computed K for each sheet.

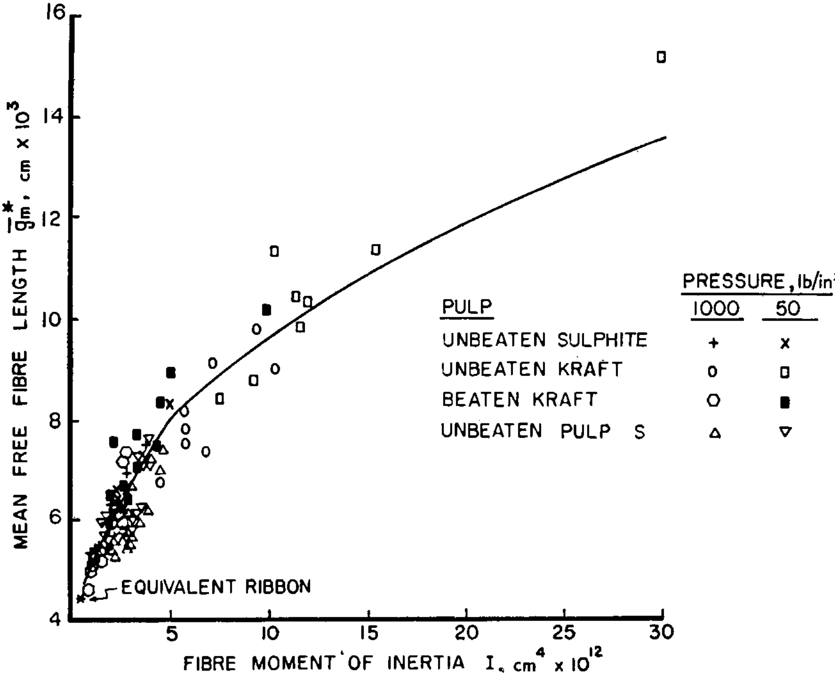

Fig. 5—Relationship between 'fibre moment of inertia' and mean free fibre length

When calculated and measured values are in perfect agreement, K is unity. Its regression equation—

$$K = 0{\cdot}0138[(\bar{g}_m{}^*)^3 10^3]^{2.76} \quad . \quad . \quad . \quad . \quad (16)$$

is plotted in Fig. 6 on log-log paper and has a correlation coefficient of 0·931. The range of K for heavy, well-bonded sheets (see above) is 1–3; for all two- and three-layer sheets except unbeaten kraft, 2–5; for the unbeaten kraft, 4–25. It should be noted that the true deviation of the present theory from experiment is slightly less than that indicated by K, because $(\bar{I}_Z)_{max}$ was used in the calculations.

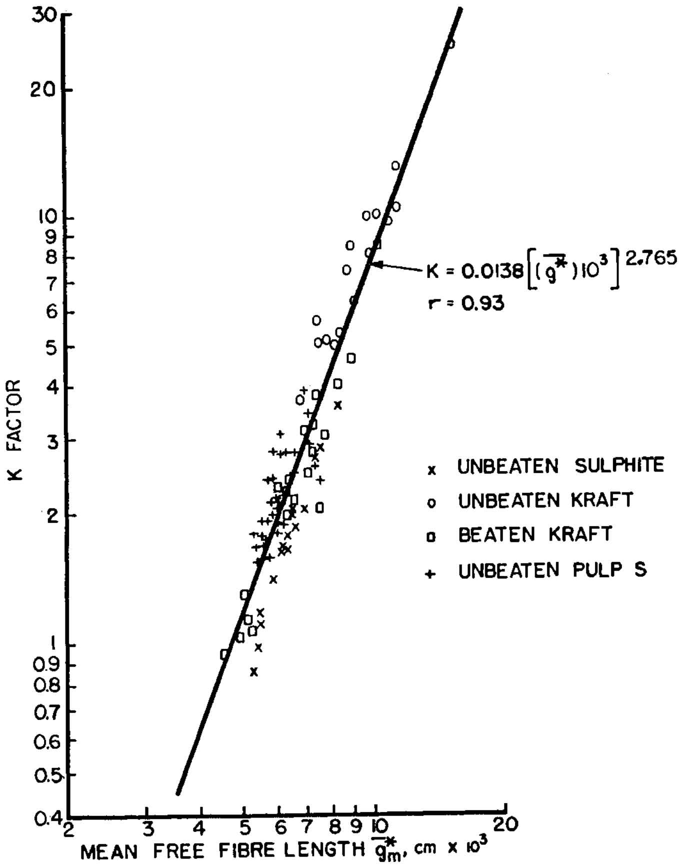

Fig. 6—Relationship between K factor and mean free fibre length

With equation (*17*) and $(\bar{I}_Z)_{max}$ in equation (*10*), the latter becomes—

$$Y = \frac{0{\cdot}0138\bar{E}N_L(N/L)[(\bar{g}_m{}^\star)^3 10^3]^{2.76}}{\pi\left[\frac{1{\cdot}5(\bar{g}_m{}^\star)^2}{\bar{\omega}^3\bar{\delta}}+\frac{0{\cdot}95}{\bar{\omega}\bar{\delta}}-\frac{0{\cdot}2}{\bar{\delta}(\bar{g}_m{}^\star)}\right]\left[1-\frac{\pi\bar{\omega}\bar{\tau}\sqrt{\beta}}{2\lambda}\right]\bar{c}_m} \quad . \quad (17)$$

The elastic modulus per unit mass, Y/W or Y/N_L, is seen to be primarily a function of the sheet density as expressed by $(g_m{}^\star)$ and the factors responsible for fibre stiffness, $\bar{E}$ and $\bar{I}_Z$. In the range of $(\bar{g}_m{}^\star)$ values germane to paper, Y/W is inversely proportional to $(\bar{g}_m{}^\star)$ (*see* Fig. 7) and directly proportional to $\bar{E}$ and $\bar{\omega}$ and $\bar{\delta}$, hence to $\bar{I}_Z$ in equation (*10*). It is interesting to note that equation (*17*) asserts Y/W to be independent of the fibre length. This result substantiates the work of Arlov, who found no variation in elastic modulus among handsheets containing fibres of very different length distributions.[11]

In Fig. 7 the correlation coefficient between the data points and the curve is sufficiently high to indicate the points would be along a single curve,

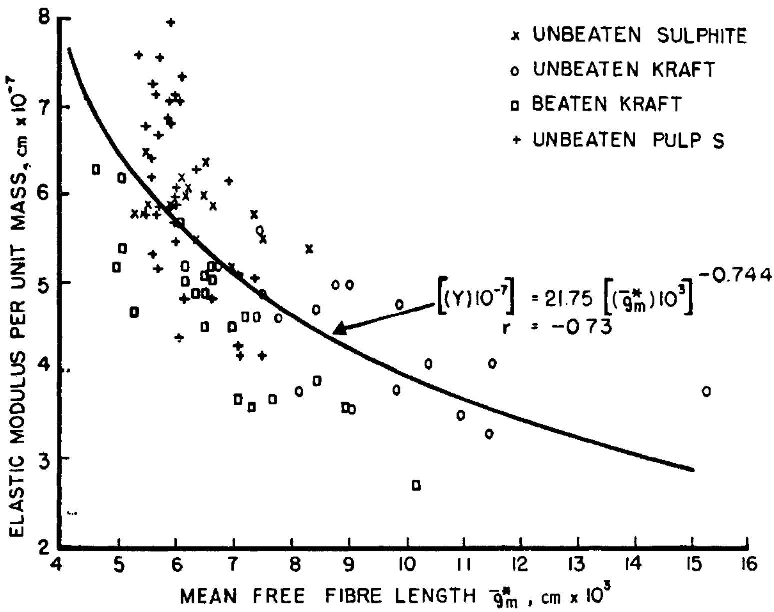

Fig. 7—Relationship between elastic modulus per unit mass and mean free fibre length

if the experimental error in Y and particularly in $(\bar{g}_m{}^\star)$ could be eliminated. Then, for the range of softwood pulps, pressing pressures and basis weights used, Y would be defined by a single expression involving only geometric laws and fibre properties. Such an expression could properly be called an equation of state, because of its ability to predict paper properties without knowledge of the pulp and sheetforming processes (provided fibre deposition on the wire is random). Equations of this nature, once derived and proven generally valid for all types of fibre network, may yet make scientists of us all.

Principally, the results of this investigation lend substance to the main premise on which our studies are based: that success in relating sheet properties to those of fibres is primarily contingent on a knowledge of the structure of the sheet.

Acknowledgements

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REFERENCES

1. Kallmes, O. J. and Corte, H., *Tappi*, 1960, **43** (9), 737–752
2. Kallmes, O. J., Corte, H. and Bernier, G. A., *Tappi*, 1961, **44** (7), 519–528
3. Litt, M., *J. Colloid Sci.*, 1961, **16** (3), 297–310
4. Van den Akker, J. A., this vol., 205–241
5. Timoschenko, S., *Strength of Materials: Part I, Elementary Theory and Problems* (Third edition, D. Van Nostrand Co. Inc., Princeton, N.J., 1955)
6. Kallmes, O. J., St. Regis Paper Co., *unpublished results*
7. Kallmes, O. J., *Tappi*, 1960, **43** (2), 143–153
8. Van den Akker, J. A., Lathrop, A. L., Voelker, M. H., and Dearth, L. R., *Tappi*, 1958, **41** (8), 416–424
9. Page, D. H., *Paper Tech.*, 1960, **1** (4), T165–T169
10. Page, D. H., and Tydeman, P. A., *Paper Tech.*, 1960, **1** (5), T207–T218
11. Arlov, A. P., *Norsk Skodind.*, 1959, **13** (10), 324–351

Transcription of Discussion

DISCUSSION

MR. D. H. PAGE: In the multi-planar theory presented in your earlier paper, you have considered the distribution of bonding that arises when two 2-D sheets are separated by a third (Fig. D9). I believe I am right in saying that you have defined a factor for the probability that crossings between the lower and upper layer are bonded and that you have used this factor to obtain the distribution of distances between bonded crossings in a multi-planar sheet. There are sound physical reasons that this treatment is unsatisfactory. By defining a probability factor that is constant over the whole sheet, it is implied

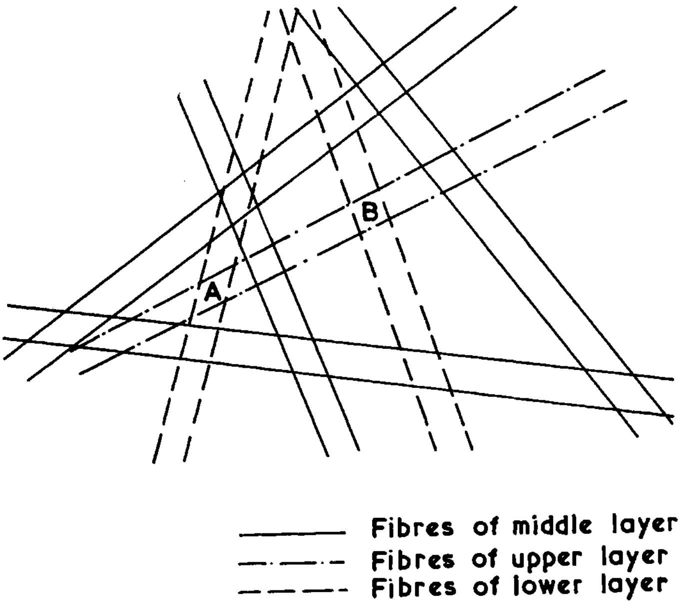

Fig. D9

that the two fibres of the first and third layer crossing over a small opening of the second layer have the same chance of bonding to one another as have two fibres crossing over a large opening. This is not a physically justifiable assumption. The factor controlling whether or not bonding occurs between the first and third layer is not only the flexibility or plasticity of the fibres, but also the length of the span over which the fibres are free to deflect.

Thus, it is clear that crossing *B* (Fig. D9) has a much greater chance of being bonded than crossing *A*. This effect will control to a large extent the shape of the distribution of distances between centres of bonded crossings.

While the negative exponential curve for gap length is correct for the distances between *all* crossings, it will be radically modified when the distances between *bonded* crossings are considered. The effect will be to decrease the number of short distances between bonded crossings, also to decrease the number of very large distances. Our own experimental values, obtained on one side of the fibre only in a sheet (the side facing inwards towards the body of the sheet), show this effect clearly. Fig. D10 gives the experimental values of

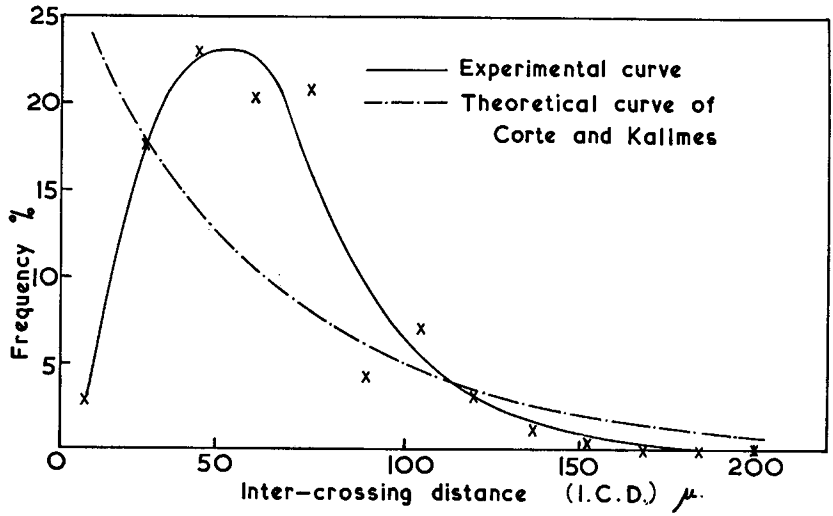

Fig. D10

intercrossing distance, together with the theoretical negative exponential curve fitted to give the same mean I.C.D. Whereas the negative exponential predicts that about 8 per cent of the distances will be greater than 150—in fact, none are observed in 188 readings. This sharp cut-off of high intercrossing distances is of importance in considering the theory of Kallmes for the elasticity of paper, since it is necessary for his theory that high intercrossing distances with their high potential extension in bending should exist.

While I realise that your model of paper had to be simplified to permit a mathematical treatment, I think it is a dangerous position when the mathematics enforce assumptions that are not physically justifiable and particularly so when attention is not drawn to them.

Secondly, I would like to ask how many clicks you hear, when a sheet of normal basis weight is extended to failure, compared with the number of fibre-to-fibre bonds in the sheet. We find that, in the surface, of the order of 5 per cent of fibre-to-fibre bonds separate completely. It would be interesting to have evidence of total fibre separation within the body of the sheet. In such a tightly bonded state, we believe there is the possibility that total bond breakage is very infrequent.

DR. O. J. KALLMES: The term B is not a probability factor: it is the experimentally determined fraction of the crossings bonded and, as such, has to give the correct mean free fibre length. The distribution of (g_m) *inside* the sheet can only be assumed at present.

Your description of our experiment is correct. I consider your experiment measuring the free fibre length *on the surface* is at least as suspect as our mathematical approximations. It is presumably impossible to measure the mean free fibre length in a sheet; the only test possible is to use the calculated value and apply it in a physical experiment. We have done this in our second contribution in two entirely different experiments, a porous and an optical. The good checks obtained between theory and experiment lend considerable support to the mean free fibre length of the entire sheet.

DR. H. CORTE: All we do is to measure the fraction of those bonds that see each other and are bonded. We do not call this fraction probability (because it has nothing to do with mathematics), but the bonded fraction of those bonds that see each other (symbol σ) and the free fibre length does not come into play at all. It turned out that the factor σ is roughly proportional to the open area. We have no idea why this is so—in fact, we know that it is not so, but it is good enough as an approximation. No mathematical assumptions have been made here, only experiments. The values are independent variables that have to be measured and cannot be computed. Once they have been determined, however, we are not satisfied by merely tabulating or plotting them. Instead, we use them for further quantitative evaluation of sheet properties.

The number of clicks depends, of course, on the size of the sample and on the basis weight—that is, the number of fibres present (you said 10^7). The number of clicks you hear depends not only on the number of bonds present, but on the shape of the sample. The film shown allowed the actual counting of the clicks, because they could be recorded on a tape recorder and played back. For thicker sheets, the clicks are too numerous and too fast to be recorded reliably on any mechanical instrument, but we have fed the output into an electronic counter that counts up to 2 000 pulses per second and there is a programme on at the moment. I am not quite sure, but in the last experiment I saw myself (with a sheet of about 25 g/m^2, say, 8 mm wide and 10 mm long) the number of clicks was about 1 500. We will refer to that in due course. In a 2-D sheet, the total number of bonds in a sample of that kind was about 4 000 and the number of clicks averaged about 180—that is, the number of broken bonds averaged 180—and the application of extreme value theory gives at best only the number of bonds that break along the breaking zone. In other words, the value should always be lower than the observed value, because no theory so far produced permits calculation of the extra bonds that break at random all over the sample.

PROF. A. H. NISSAN: I think Corte is absolutely correct that a fracture is discontinuous in paper. Where I take issue with him is his suggestion that a fracture is continuous anywhere else. It is a matter of time scale, also of energy amplifications. At the moment, we are studying at Rensselaer the actual straining of minute crystals under the electron microscope at very high resolution by observing the motion of dislocations. This, to all outward appearances, would give a clear case of continuity as your click on the cellulose film indicated, because it is crystal. It is not so. If you amplify enough to get the machine and apparatus commensurate with the phenomena being measured, it is found to be discontinuous. There is danger in assuming that we can use only one system of experimentation in the study of phenomena of a wide range. The flow of air at atmospheric pressure appears perfectly continous and the theory of viscous flow applies. The flow of air at 10^{-6} atm in wide tubes is discontinuous. You can 'count' the units. Now, supposing the enthusiastic physicist shows that the flow of air at 10^{-6} atm is discontinuous: he is perfectly correct, but I maintain that to study only that is to miss the opportunity of discovering viscous flow. This is very important, because flow at atmospheric pressure is continuous by ordinary scale. I suggest that the click heard when a fibre bond is breaking is comparable to the click heard when cellulose film breaks. Here, you have extended it commensurate with fibre length and fibre area, but with cellulose film you are not dealing with the same scale. There is some danger if we get too enthusiastic for one method: we might miss another aspect of truth by saying, 'Only this will work'.

DR. CORTE (*written contribution*): In my report yesterday of our first contribution, I stressed the importance of the structural scale. The scale reaches from the foil-like apparent continuity of larger samples of paper under certain experimental conditions to the molecular discontinuity of hydrogen bonds.

Each level is equally important. When we consider the structure to consist of undistinguishable fibres as elements, we do so because this viewing angle has been neglected. We think that such a treatment yields a new and useful insight into a number of interesting and important paper properties. We do not claim, however, that our method, though very powerful, is the only one to tell the whole truth.

MR. P. G. SUSSMAN: Referring to Fig. 3 and Fig. 4, Kallmes has confirmed that the ends of the curves show the actual breaking loads of the MP sheets and the normal handsheets? This being so, the tensile behaviour of the MP and 'normal' sheets is surprisingly similar.

I, too, have made sheets from a fibre suspension of very low consistency, 0·002 per cent, but 60 g/m^2 in substance, draining all the water through the sheetmachine in one go; one sheet took about 5 min to make. The 'random structure' sheets that I made had a slightly lower burst factor than had corresponding normal handsheets formed from 0·02 per cent stock. I had expected to make a better sheet than a normal handsheet.

In my 'random sheet' experiments, the amount of fibres lost through the wire was measured and found to be about 5 per cent of the total sheet weight, similar to the losses occurring with normal handsheets. If any fractionating took place in Kallmes' sheetmaking, I do not know what effect that would have.

DR. KALLMES: My only comment is that we made a large number of tests and the results were generally quite close. For heavier sheets, the differences became smaller, essentially alike within 10 per cent; on the two-layer sheets, there were some appreciable differences (indicated in the figures, especially Fig. 3).

In all our work to date, we have screened the pulp beforehand to deal only with the long fibre fraction: thus, no fines effect was added.

DR. J. KUBÁT: I have three comments on points discussed during the lectures today.

1. Bond breaking during straining—The concept of bond breaking must always be seen against the background of the relation between the number of bonds in the sheet and its modulus of elasticity. Irrespective of the actual form of this relationship, it can easily be demonstrated that the modulus is nearly constant along the whole stress/strain curve. Furthermore, when variations occur, there is always an increase in the modulus. The main part of the stress/strain curve is due to plastic flow, which takes place only during loading.

When deloading, the deformation is a superposition of an elastic and visco-elastic component. The slope of the stress/strain curve after the reversal of the straining procedure is thus identical with the modulus at the actual rate of straining. Another method of studying the magnitude of the modulus along the stress/strain curve is a combination of a constant rate of straining with a sinusoidal deformation of suitable frequency and amplitude. Experiments of this kind have been done at the Swedish Forest Products Research Laboratory in Stockholm. Different types of paper were investigated for modulus changes following straining, but no decrease could ever be observed.

2. *Inner stresses in the sheet*—Theories treating the sheet as a statistical assembly of interconnected fibres cannot (mainly for mathematical reasons) include the inner stresses, which are usually present in all types of paper. The magnitude of these stresses is by no means a matter of academic interest only. We could in fact demonstrate, mainly by measuring stress relaxation at different stress levels, that the inner stresses can reach values of 30–50 per cent of the tensile strength.

3. *Fluctuations, particularly of the stress during deformation (audible clicks, observed by Corte)*—The audible clicks heard during the straining of a strip of paper represent one of several different effects following from the discontinuity of the straining process on the microscopical scale. Apart from the fact pointed out by the Chairman that Corte's experiment much more reflected the conditions of a rupture process in tear in the sheet, there is no doubt that observations of the discontinuous nature of the deformation of the sheet represent a powerful means of getting direct insight into the kinetics of paper rheology. There are, of course, several methods of doing that. At our laboratory, we have chosen the measurement of electrical fluctuations. Such fluctuations can be measured with or without a direct current flowing through the strip. In the latter case, we found intense noise phenomena within the plastic region of the stress/strain process. The noise was not present in the elastic region and, furthermore, it disappeared when the contraction of the strip from the state of plastic flow began. We consider this another demonstration of the existence of at least two different flow mechanisms in paper—delayed elastic (visco-elastic) and plastic flow. There are still more methods for following the discontinuities in straining solid materials. One of them is the direct measurement of the microscopical stress increments by sensitive piezoelectric devices. In that way, it has been possible to show that, in the straining of metal crystals, the number of clicks is identical with the number of strain lines formed during the actual deformation, revealed and counted under electron microscope magnification.

All these examples demonstrate clearly that problems in connection with the deformation and flow of paper are amenable to direct observation on the microscopical scale. The coarse structure of a paper sheet, compared with the structure of a metal, makes such measurements still easier to carry out.

DR. KALLMES: My one comment is that the mechanical model used in this elasticity equation is very simple. The object of the exercise was to see if the structure theory could be applied at all—that is, if you can relate the properties of fibres and paper through arrangement of the fibres in the sheets. The mechanical model used is extremely simple, though we realise it has shortcomings.