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MEASUREMENT FOR SAMPLED DATA CONTROL

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Synopsis Sampled data can arise in several ways—for example, from manual samples taken from the process at reel change, from discontinuous instruments such as scanning basis weight gauges, also from digital computer control.

Sampled data in contrast with continuous data have the problem of how sampling should be carried out. This is discussed in relation to the spectral characteristics of the variable. The superiority of sampled data control for dead time processes and the relation between sampled data control and statistical quality control are mentioned.

Sampled data control system analysis and synthesis is introduced. The *z*-transform and modified *z*-transform are included.

Direct digital control is introduced, using sampled data forms as well as PID analog controller replacement. The relative features of pulse amplitude, pulse width and of velocity and positional algorithms are reviewed, also the selection of the sampling interval. A few of the applications to pulp and paper are reviewed.

Introduction

THIS paper is intended to provide a survey for the technically oriented person who is interested in broadening his knowledge into the sampled data control field and its application to pulp and paper process control.

The introduction defines and describes the general features and problems of sampled data control, as well as its applications in the industry. This is followed by a brief tutorial on sampled data control theory. The final section is concerned with reviewing some of the reported applications in the pulp and paper industry.

Although the theory of sampled data control is known to a relative few, the practice is in very common use, both in the pulp and paper and other industries and in our everyday lives.

We generally receive our bank statement only once per month. Those

Under the chairmanship of Prof. J. A. Van den Akker people who merrily write cheques and look only at their monthly balance usually find themselves in debit on occasions. This is an example of a sampling interval that can be too short unless another measurement is taken—for example, keeping a running total of all cheques written and deposits made.

Business accounting systems usually allow fixed time periods to elapse between display of income and expenditure. For example, a report of quarterly or annual gross income and profit is quite common. These figures hide or attenuate the fact that any single month was good or bad. Obviously, the frequency at which these data are available has a bearing on their possible use. We tend to take these sampled data measurement systems for granted, even though they are capable of seriously misleading us at times.

In pulp and paper making, many of the variables of interest, particularly those relating to quality, are available only on a sampled data basis, often with a long sampling period. Long sampling periods (low sampling frequency) are usually attributable to the cost of sampling and testing being considered too high; alternatively, they may result from the fact that a good use for more frequently sampled data has not been determined.

The sampling and testing costs are generally reduced considerably if an instrument can be found that can measure the property of interest on-line. Basis weight and moisture are good examples of measurements that have yielded to this approach. Previously, these variables were measured by destructive tests on an infrequent basis, say, once per reel. With on-line scanning gauges, strip charts are now filled up with a large amount of information, which is usually of little value in its raw state. In order to produce a useful measure of the basis weight for control purposes, the readings during a scan are usually averaged, with this scan average being used for control during the following scan. This is an example of filtering—that is, removal or attenuation of unwanted information in order to leave the useful information for control purposes. It is also an example of a *zero-order hold.* These will be discussed later.

The human operator who is manually controlling a plant equipped with on-line instruments employs sampled data control. He divides his attention among several instruments, as well as among other non-control tasks such as inspecting equipment. Each time he takes an instrument reading (that is, he samples it), he makes a mental decision about whether the variable has moved off target significantly—he filters the value, then estimates the control action required.

If the response of the measured variable to his control action takes a long time, he probably learns to reduce his sampling frequency. If the variable is fluctuating quite a lot, he examines it more carefully in determining whether it has deviated from target—that is, he filters it more heavily.

Sooner or later, he would probably persuade an engineer to install an automatic control loop to carry out the control. A skilled engineer, however, would recognise that there is a choice whether to make the control action continuous or discrete (that is, sampled data control). The fact that the measurement is continuous does not necessarily mean that continuous control is going to give the best control performance. Indeed, there are situations for which sampled data control can be superior to continuous control.

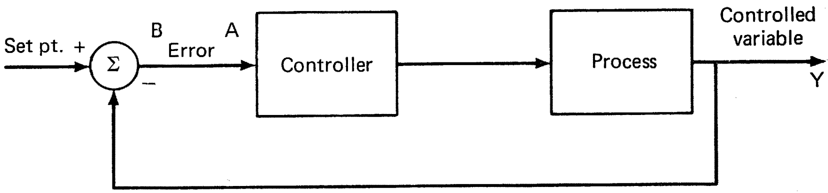


Fig. 1—Schematic of feedback control system

Soliman & Al-Shaikh[1] have made this comparison by determining the increase in loop gain that can be achieved in a sampled data system compared with that of the equivalent continuous system. The meaning of loop gain can be seen in Fig. 1, where for a proportional controller it would be the value of a signal increase as it proceeds clockwise round the loop. Suppose we start with a certain error at point A, multiply it by the controller gain, then multiply this product by the process gain, the signal would arrive at point B with the loop gain times its original value at A. For most processes, particularly those with dead time, there is a limit to which this gain can be increased (by, say, reduction of the controller proportional band), because instability usually occurs. The results[1] showed that for processes in which a dead time (transport lag) is the dominant feature, stable sampled data control can be achieved at loop gains of as much as three times greater than those possible for a stable continuous system. This improvement can be achieved only by proper selection of the sampling interval relative to the dead time.

Processes with dominant dead times are common in the pulp and paper industry. Buckley[2] gives an example of a simple sampled data controller for use with a dead time process.

The digital computer is a sampled data device. When used as a controller by necessity, it operates as a sampled data controller. It is of course well known that the digital computer has many advantages for control purposes[3] over more traditional methods.

For these reasons, sampled data control can be advantageous even when the controlled variable is measured continuously.

Variables that are measured discretely create another set of problems, to the extent that several different approaches have been developed to deal with them. As mentioned previously, these occur when there is no instrument available, when the instrument is too costly or when the instrument operates on a discrete basis.

It seems to have been the tradition that control of continuously measured variables was the realm of the control engineer, whereas control of discrete variables was handled by the statistical quality control engineer. In recent years, some common ground has been established by the statisticians moving into control engineering through adaptive quality control[4] and the control engineers moving into statistics using sampled data stochastic control.

Unfortunately, many people are still firmly entrenched in each camp and many communication problems arise as a result, because of the jargon used by each group.

When a control engineer uses the word *interaction*, for example, he is normally referring to a process that is multi-variable and for which a single input will produce a response in more than one output. The control engineer will most likely assume that this process can be approximated by a set of linear differential or differential-difference equations.

When the statistician talks about *interactions* in a process, he is generally referring to the necessity (in his opinion) of assuming that the process is multi-variable and that it should be represented by a non-linear quadratic algebraic model, which contains terms in the cross-products of the input variables.

These two uses of the word are therefore quite different and it is consequently well worth while for anybody working in this field to be able to cross the interdisciplinary boundary.

The word *frequency* is another good example of this semantic problem. To the statistician, it means the number of occurrences of an event—for example, the number of times that a sampled value lies between two limits. To the control engineer, it is equated with sinusoidal frequency or whether changes in a variable are occurring slowly or rapidly. The control engineer's concept of frequency can be helpful in discussing some factors relating to sampled data measurement and control.

Fig. 2 shows four different time series of a variable together with the frequency distribution and power spectrum for each time series.

Comparison of signal (*a*) with signal (*b*) indicates that, while (*a*) is made up primarily of slowly changing or low frequency values, signal (*b*) also contains some rapidly changing or high frequency components.

These differences are not reflected in the statisticians' frequency distribution of signals (*a*) and (*b*)—that is, they are both normal and have the same mean and standard deviation. The signal differences show up, however, in the respective power spectra. The power spectrum[5] gives the distribution of signal variance in terms of the (sinusoidal) frequency. The areas under the power spectra for (*a*) and (*b*) are the same. This is because the area under the power spectrum is equal to the variance of the signal. The area under the power spectrum between any two frequencies is that part of the variance contained in that frequency band. It can be seen that signal (*b*) contains proportionately more variance at the higher frequencies that does signal (*a*). Signals (*c*) and (*d*) may be compared with each other in a similar manner.

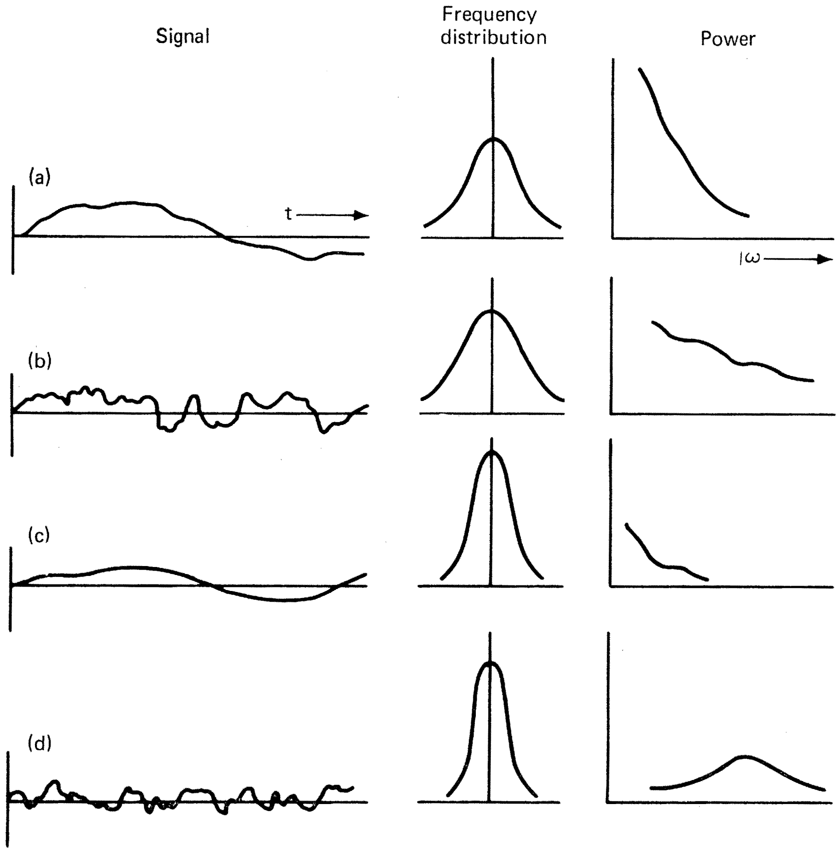


Fig. 2—Signal analysis

When making measurements, it is important to know the spectral characteristics of the variables and the use that will be made of the measurements.

This is particularly so for sampled data, because of the misleading results that can be obtained if the sampling frequency is improperly chosen. When the sampling frequency is too low, *beating* between the sampling frequency and the higher frequencies in the signal occurs, leading to the production of beat frequencies in the measurement.

A good example of how misleading this can be occurs when a movie film shows spoked wheels apparently rotating slowly in the wrong direction.

This phenomenon is known as *aliasing*[5-7] and can be avoided by making the sampling frequency at least twice as high as the maximum frequency present in the signal. This criterion presents certain practical difficulties when this maximum frequency is unknown. In such a case, the sampling frequency may be increased until there is no more change in the power spectrum. At this point, it may be assumed that the sampling frequency is high enough to eliminate aliasing. The use of too low a sampling frequency is a method of producing a time series with spectral characteristics approaching that of *white noise* —that is, the power spectrum is essentially constant at all frequencies.[5]

The power spectrum of a variable can be used to determine—

1. How much variance reduction we might expect to obtain by improving control.[8,9]
2. To make hypotheses about the causes of variations. This can be helpful as a source of solutions for reducing the high frequency variations (process noise) that cannot be reduced by feedback control of the output variable under consideration.

Sampled data may be used for process identification leading to controller design.[10, 11] To obtain values for, say, a process time constant and dead time, the sampling period would need to be several times less than these two parameters. Further decrease of the sampling period can be advantageous to allow filtering of both the input and output variables before identification.

Many variables in the pulp and paper industry that are at present sampled are important measures of product quality. The control objective is generally to minimise the variation of the variable so that it can be aimed as close to the relevant specification as possible.[8, 9, 12] Feedback control for this purpose has an inherent limitation, namely, that it becomes increasingly ineffective as the (sinusoidal) frequency of the disturbance increases.[8, 9, 12, 13] There is an upper frequency above which control is quite useless. The value of this limiting cut-off frequency depends on the response characteristics (time constant and dead time) between the manipulated input variable and the controlled (output) variable and to a lesser extent on the maximum permissible sampling frequency. All disturbance frequencies above the point that control becomes ineffective can be considered as *noise* produced either by the process or by the measurement. It is advisable to filter these higher frequencies from the measured signal before using it in any control calculations. Failure to do this adequately can lead to aliasing [7] or to noise amplification[14] in the control loop. The characteristics of the measuring system have a considerable bearing on the types of filtering that may be employed. When the measurement is from a continuous instrument, it is advantageous to use analog filtering, which can remove frequencies from infinity down to the order of one cycle/minute. For the removal of lower frequencies, digital filtering may be used. A moving average or the exponentially weighted average (exponential smoothing) are examples of digital filtering techniques.

Goff[7] describes the use of both analog and digital filtering systems and their relation to the process control cut-off frequency in the design of direct digital control systems. Filter systems, besides having frequency attenuation characteristics, also have lag characteristics.

Goff[7] gives the attenuation and phase lag characteristics for two types of digital filter. These lags are detrimental to control. It is therefore wise to eliminate the sources of process and measurement noise rather than to depend on filtering. If the loop in question is basis weight, for example, it is advisable to pay every attention to improving consistency control, to providing well agitated mixing chests, to reducing head box head variations and to reducing forming table instabilities and drive speed non-uniformities, etc.

The discussion to this point has dealt with sampling from a continuous measurement, in which the characteristics of the analog filter, the sampling frequency and the digital filter characteristics can be chosen with regard to the disturbance frequency spectrum and the process response dynamics. When continuous measuring instruments are not available and samples for testing must be taken from the process stream, the situation becomes more difficult to deal with.

When sampling from fluid streams, it is possible to perform some mixing during sample collection, which has a similar effect to analog filtering. Coulman[15] discusses the theory of designing a special mixing system for this purpose. Sampling from sheet systems, by its very nature, precludes this type of mixing. Additionally, sampling from a sheet is destructive, so that there is strong economic motivation for maintaining a low sampling frequency, the latter usually corresponding to reel changing.

The literature is rather sparse on the application of the design of control systems to utilise this kind of data and the situation leads to the often-heard comment that the sample results come too late to take any action. This statement is usually not true. It is true, however, that the cut-off frequency of the controlled system will be quite low, but this is a relative matter.

The authors recently examined the hourly test results from a process that has a delay of 3–4 h between changes in the manipulated variable showing in the result of the hourly laboratory test. The data showed that drifts in the frequency range of one cycle per week accounted for more than half of the standard deviation of this variable. The application of digital filtering and sampled data feedback control performed manually would be quite effective in reducing this variation. It is often a good assumption, because of the existence of mixing chests, etc., that the variations in the sampled variable can be separated into a low frequency component and a high frequency or uncorrelated (white noise) component. The latter component may be filtered by performing a number of tests on each sample and averaging the result. The sample average is then a better estimate of the low frequency variation or what Loeb(14) calls the *true process average.*

Two publications of interest in this connection concern the control of a process for making glass bulbs.(14, 16) In this case, some filtering was accomplished by averaging the test results from several bulbs at each sampling interval. The time delay introduced by testing was directly proportional to the number of samples averaged, so there was an optimum in the number of samples that would produce the minimum product variance. This sort of averaging is effective when the variations are due to very high frequency or uncorrelated causes such as measurement errors or, in the case of a sheet, local non-uniformities.

As mentioned earlier, a knowledge of the power spectrum of the variable to be controlled is valuable in designing a system such as the above. Although its determination in the case of a sheet may be expensive and tedious, it can help provide the correct perspective in designing the optimum control system. Because of aliasing, the major problem is determination of the proper sampling scheme.

Astrom(17) reported on the use of Kalman filtering for the estimation of cross-direction (CD) stretch from reel samples. He claimed that the variations could effectively be grouped into those below about 5 cycles per shift and into high frequency uncorrelated variations, which he called measurement error. The latter can be detected as a spike on the autocovariance function at zero lag—roughly equivalent to white noise. Astrom used his method to supply the operators with a prediction of CD stretch for feedback control, as well as for estimating the interpolated value of CD stretch for product acceptance purposes.

Both Loeb(14) and Astrom(17) comment on the statistical quality control engineer's use of control charts. These charts are a means of revealing extreme variations in a process output. From these, it is concluded that a variation has occurred from an assignable cause (low frequency disturbance), but no rules are provided to determine the sequence of adjustments that must be made to the manipulated input variable to minimise the variance of the controlled variable.

Cumulative sum (*Cusum*) charts suffer from the same disadvantage as control charts. They perform the function of a digital arithmetic filter. Another disadvantage of control charts is that they do not take into account the fact that much of the disturbance is low frequency (that is, correlated).

Adaptive quality control, proposed by Box & Jenkins,[4] was probably the first attempt to incorporate a knowledge of the process dynamics into design of a quality control system. This method, although differing in terminology and some detail, is basically the same as linear stochastic control. Not only are the dynamics of the process utilised in the controller design, but a model of the low frequency disturbances is used for prediction purposes. The method of Box & Jenkins has been utilised by several papermachine computer control systems for basis weight and moisture control.

Linear stochastic control theory has been applied by Astrom[11] to computer control of a papermachine. Like Box & Jenkins, this makes use of prediction of low frequency disturbances and at the same time filters the noise. A reduction of variance by a factor of 7 was obtained for basis weight.

These and other applications will be discussed in more detail after the following discussion on sampled data theory.

SAMPLED DATA CONTROL ANALYSIS AND DESIGN

Introduction

IN CONTRAST to a continuous system (which has data available at all times), a sampled data (or discrete) system has data available only at specific instants called sampling instants. When dealing with process control systems in which the process is continuous, the sampled nature of the system arises either from measurements that are taken and/or are available only at certain times (say, laboratory analyses or hand samples) or from controllers that take action only at specific times. An example of the latter is a digital computer, which is necessarily discrete, since it is constrained to perform only one calculation at a time. (For general tests on sampled data control, see two papers.[18, 19])

Sampling process

BASIC to the concept of a sampled data system is the process of sampling the value of a continuous variable. Although in many cases the fact that continuous knowledge of a variable is not available is incidental and continuous analysis techniques such as those afforded by the Laplace transform may be used without significant error, many instances do arise in which the sampled nature is a critical factor and sampled data analysis must be used.[19]

Given the continuous signal $f(t)$, it can be seen in Fig. 3 that sampling or 'reading' it every T s results in the sampled signal $f^*(t)$, where—

$$f^*(t) = \sum_{n=o}^{\infty} f(nT)U'(t-nT)$$

and where $U'(t-nT)$ is the unit impulse* train shown in Fig. 3. Clearly, in a real system, a sample is not obtained instantaneously and the impulse train should be replaced by a pulse train (finite height and width); but, as this actual sampling time becomes small compared with T, the impulse assumption is very little in error and is much easier to deal with analytically.

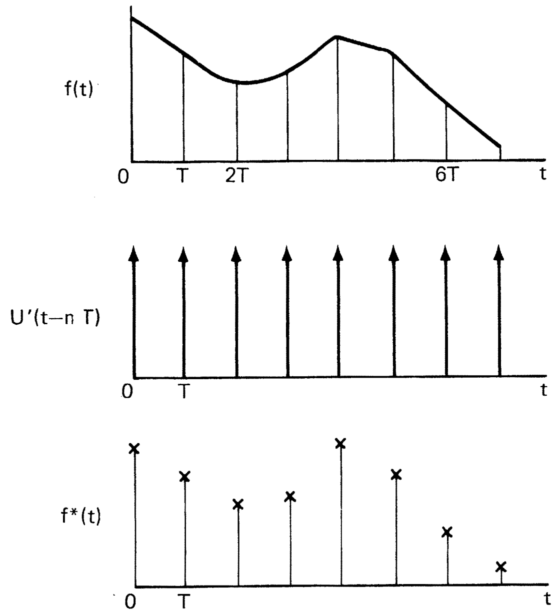


Fig. 3—Sampling a continuous system

When dealing with $f^*(t)$, it should be remembered that it is a representation of a continuous signal and that information has been lost in the sampling process. Although Shannon's sampling theorem (that is, by making T less than one half the period of the highest frequency component of $f(t)$, complete reconstruction of $f(t)$ from $f^*(t)$ is possible) tells us how to sample in order to gain full information from the signal, in control applications this approach is not practicable and reconstruction of the original signal is seldom attempted. Instead, the signal is assumed to be of a certain form between sampling intervals, as outlined in the following section.

* A unit impulse (the Dirac function) is a pulse of infinite height and zero duration occurring at time zero. The integral of the function (area under the curve) equals unity.

Holds

A zero-order hold maintains (or holds) a signal value over a sampling interval (from one sampling instant to the next). If a zero-order hold is applied to the output of a perfect sampler (which is an impulse modulator), the result is a series of steps as illustrated in Fig. 4. This type of hold is a reasonable assumption for the behaviour of the signal during the sampling intervals, if T is small and the signal smooth.

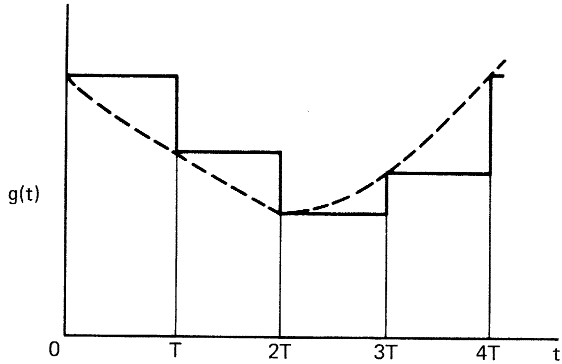


Fig. 4—Zero-order hold output

Yet other holds are possible. A first-order hold, for instance, provides a signal between sampling instants that is a linear extrapolation based on the previous two measurements (Fig. 5).

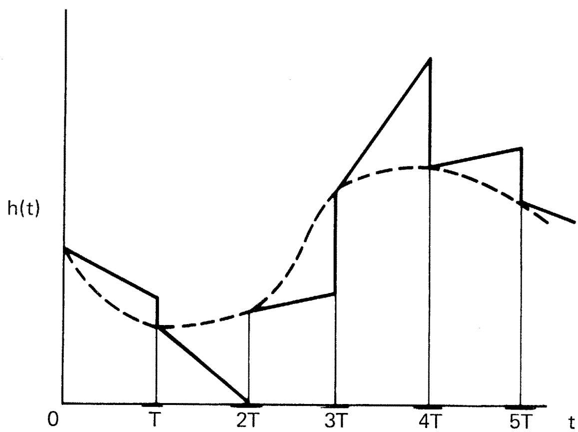


Fig. 5—First-order hold output

The z-transform

PROBABLY, the most useful single tool in the analysis of differential equations describing continuous systems is the Laplace transformation, which transforms differential equations into an algebraic form. Analogous to this in the study of difference equations and discontinuous system is the z-transform. The sampled signal is represented by—

$$f^*(t) = \sum_{n=o}^{\infty} f(nT)U'(t-nT)$$

Taking the Laplace transform of both sides yields—

$$F^*(S) = \sum_{n=o}^{\infty} f(nT)e^{-nTs}$$

The form of this equation is not particularly convenient for the study of sampled data control systems, so by introducing the variable—

$$z = e^{Ts}$$

the equation becomes—

$$F^*(S) = F(z) = \sum_{n=o}^{\infty} f(nT)z^{-n}$$

Simply stated, this means that the z-transform of a continuous signal is an infinite series of negative powers of z with the coefficient of the z^{-n} term being given by the value of the function at the nth sampling instant. This definition can often lead directly to z-transforms of functions through direct summing of the infinite series. For example, the unit step has $f(nT) = 1, n \geqslant 0$, thus yielding—

$$F(z) = \sum_{n=o}^{\infty} z^{-n} = \frac{z}{z-1} \,;\, |z| > 1$$

The restriction $|z| > 1$ for the unit step transform to be valid is equivalent to—

$$|z| = e^{Ts} = |e^{\sigma T}| > 1$$

$$\text{or } \sigma = \text{Real part of } s \geqslant 0,$$

which is the same condition that must apply in order to assure the existence of the Laplace transform. This illustrates the following point. Since the z-transform is mathematically the Laplace transform with a change of variable, no new restrictions are imposed for its existence.

TABLE 1—z-TRANSFORMS

	Time function	*Laplace transform*	*z-transform*
Description	$f(t)$	$F(s)$	$F(z)$
Unit impulse	$u,(t)$	1	1
Unit step	$u(t)$	$1/s$	$z/(z-1)$
Ramp	t	$1/s^2$	$T/z(z-1)^2$
Exponential	e^{-at}	$1/(s+a)$	$z/(z-e^{-at})$
Sinusoidal	$\sin \omega t$	$\omega/(s^2+\omega^2)$	$\dfrac{z \sin \omega t}{z^2-2z \cos \omega t+1}$
Multiply by e^{-aT}	$e^{-aT}f(t)$	$F(s+a)$	$F(z+e^{aT})$
Delay by time nT	$f(t-nT)$	$e^{-nTs}F(s)$	$z^{-n}F(z)$

Table 1 lists some of the more common z-transforms. As with the Laplace transform and continuous systems, the z-transform becomes most useful when characterising the dynamic properties of sampled data systems. It should be appreciated, however, that, owing to the hybrid nature of a sampled data system (that is, some components are continuous, Fig. 6), it may not always be possible to derive a sampled data transfer function, which is usually the desired end result in Laplace analysis. The following is an example of a system for which a transfer function can be obtained.

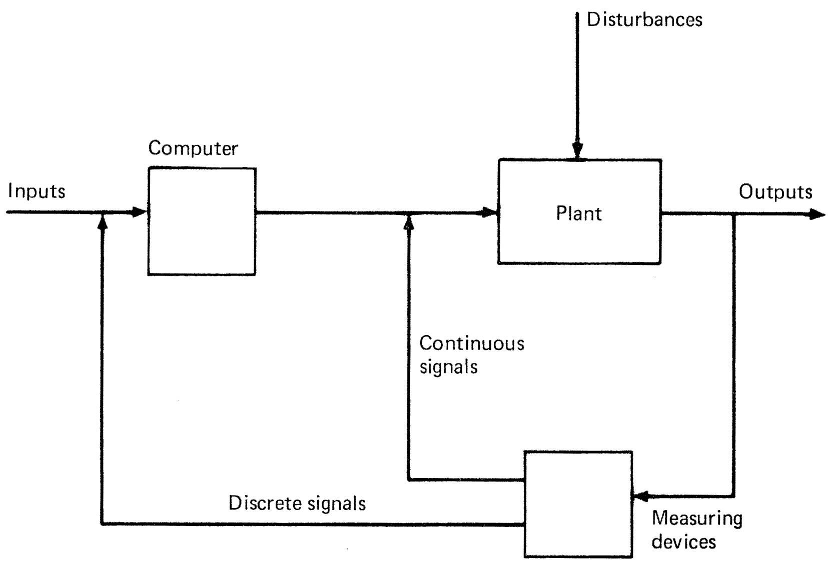


Fig. 6—Typical 'hybrid' sampled data system

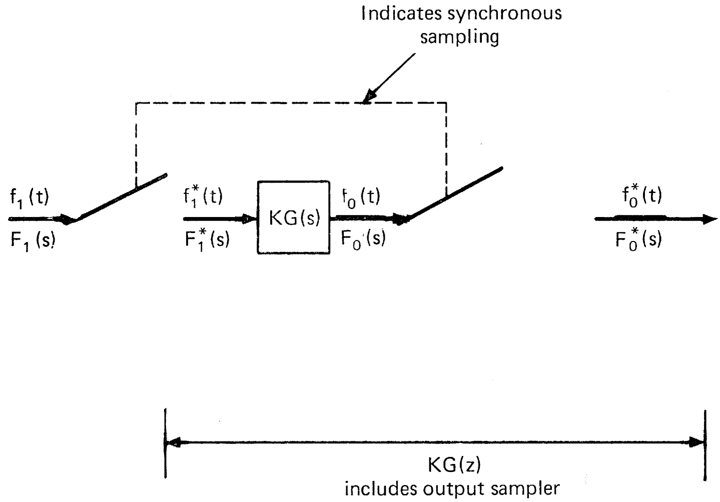


Fig. 7—Continuous and pulse transfer functions

Given the continuous system $KG(s)$ as in Fig. 7, with sampled input and output (synchronously), the sampled Laplace output is—

$$F_0{}^*(s) = KG^*(s)F_1{}^*(s)$$

which yields the transfer function—

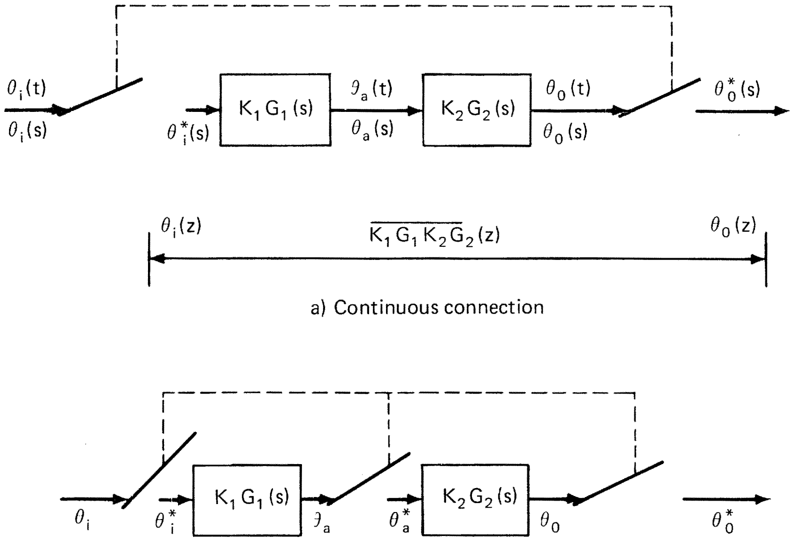
$$KG(z) = \frac{F_0{}^*(s)}{F_1{}^*(s)}$$

For cascaded continuous networks as in Fig. 8*a*, however, the final relationship is—

$$\frac{\theta_0(z)}{\theta_i(z)} = \overline{K_1G_1K_2G_2(z)}$$

where the superscript bar indicates that the z-transform is taken of the product of the terms and there can be no intermediate transfer function—for example, $\frac{\theta_a(z)}{\theta i(z)}$. This is because the output of the first continuous system is continuously supplied to the second, which clearly is different from only sampling the output of the first system as in Fig. 8*b*. To illustrate this point consider—

$$\begin{aligned} K_1 &= 1 \\ K_2 &= 1 \\ G_1 &= 1/(s+a) \\ G_2 &= 1/(s+b) \end{aligned}$$



a) Continuous connection

b) Sampled connection

Fig. 8—Cascaded continuous systems

Then
$$\overline{K_1K_2G_1G_2(z)} = \frac{(A-B)z^{-1}}{(1-Az^{-1})\ (1-Bz^{-1})}$$

where $A = e^{aT}$ and $B = e^{bT}$, although—

$$K_1G_1(z)K_2G_2(z) = \frac{1}{(1-Az^{-1})}\ \frac{1}{(1-Bz^{-1})}$$

Clearly, the two functions are not the same and it can be appreciated that z-transform analysis is not simply substituting z-transforms for Laplace transforms wherever they arise.

Before proceeding to the analysis of closed loop sampled data systems, it would be worthwhile to summarise a few properties of z-transforms. A full theoretical treatment is not possible without making extensive use of complex variable theory, so it would be out of place in this paper.

1. The inverse z-transformation may be carried out by methods analogous to those of inverse Laplace transformation. Another technique, power series expansion (simply dividing numerator by denominator), is also applicable and is in many instances simpler and easier to use.

2. The z-transform contains information about the corresponding time function at the sampling instants only.
3. The initial value theorem states—

$$\lim_{t \to o} f^*(t) = \lim_{z \to \infty} F(z)$$

4. The final value theorem states—

$$\lim_{t \to \infty} f^*(t) = \lim_{z \to 1} \frac{z-1}{z} F(z)$$

5. When a system function $G(S)$ contains a term e^{-as} (that is, a delay), the sampling period should be chosen so that $a = kT$, where k is an integer, otherwise the standard z-transform is not applicable and the modified or advanced z-transformation must be used. This technique allows the calculation of the response at times other than the sampling instants. Fig. 9 illustrates the derivation of the modified z-transform. A fictitious delay element is placed between

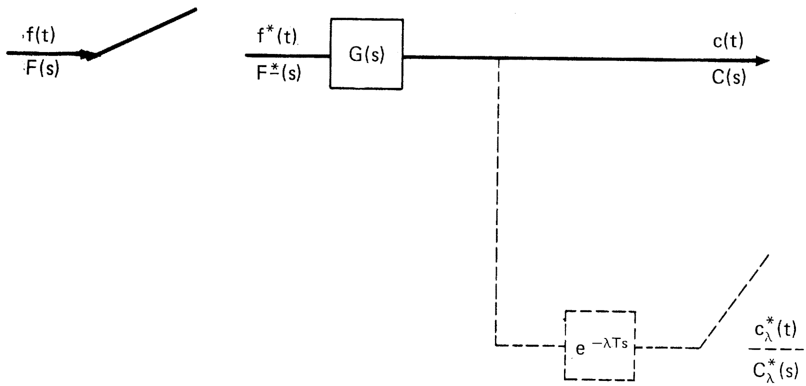


Fig. 9—The modified z-transform

the system and the fictitious output sampler (that is, the output is really continuous). Define—

$$G_\lambda(s) = G(s)e^{-\lambda Ts}$$

$$\text{then } C_\lambda(z) = G_\lambda(z)F(z)$$

$$\text{where } G_\lambda(z) = \sum_{n=1}^{\infty} g(nT-\lambda T)z^{-n}$$

$$C_\lambda(z) = \sum_{n=1}^{\infty} c(nT-\lambda T)z^{-n} \quad . \quad . \quad . \quad (1)$$

Since $g_\lambda(nT) = g(nT-\lambda T)$ and $C_\lambda(nT) = c(nT-\lambda T)$, introducing the parameter $m = 1-\lambda$, we find equation (*1*) may be written—

$$C_\lambda(z) = \sum_{n=o}^{\infty} c[(n-1)T+mT]z^{-n}$$

and, by redefining the dummy index on the summation, we have—

$$C(z,m) = z^{-1} \sum_{n=o}^{\infty} c[(n+m)T]z^{-n}$$

which is the definition of the modified z-transformer. Through the use of the fictitious delay element, the value of the response has been obtained at times other than the sampling instant and can then be used in calculations involving dead times that are not integral multiples of the sampling interval.

Closed loop control systems

THERE is no generalised sampled data system as there is for continuous systems, since the number and location of samplers greatly influences system performance. Two common situations are often encountered, however, especially when dealing with digital computer control—

1. Continuous controller with sampled measurement (Fig. 10).
2. Sampled data control system with a sampled data controller (Fig. 11).

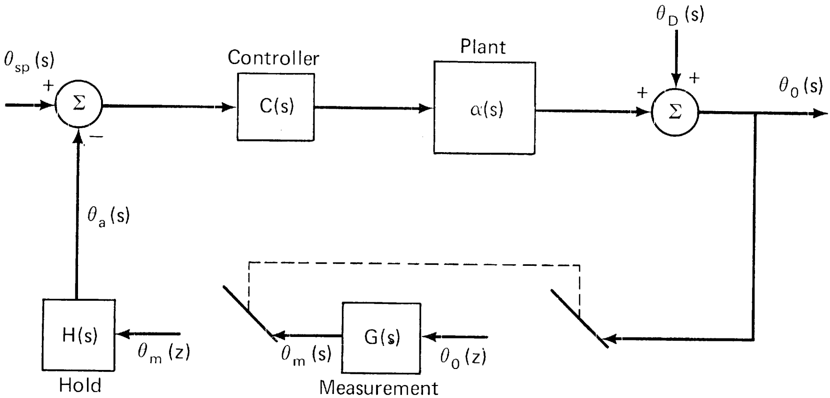


Fig. 10—Continuous controller (sampled measurement)

In the first type, no total transfer function between set point change and response can be obtained, but the closed loop response equation is found to be—

$$\theta_0(z) = \frac{\overline{C\alpha\theta_{sp}}(z)}{1+\overline{C\alpha H}(z)G(z)}$$

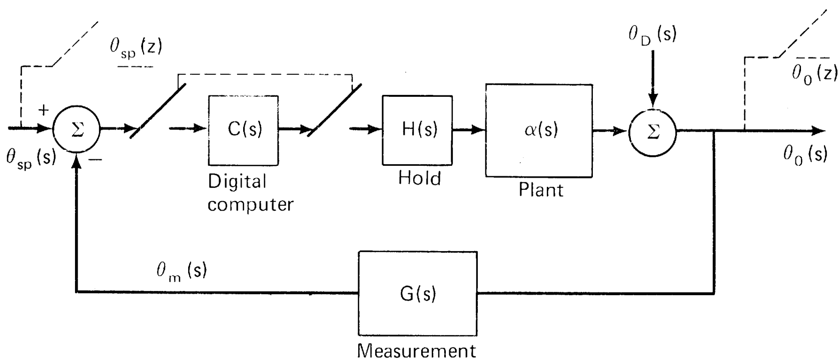


Fig. 11—Sampled data controller

In response to a disturbance, the transfer function is—

$$\frac{\theta_0(z)}{\theta_D(z)} = \frac{1}{1+\overline{C\alpha H}(z)G(z)}$$

For systems of the second type (such as direct digital control), a setpoint change transfer function is found to be—

$$\frac{\theta_0(z)}{\theta_{sp}(z)} = \frac{C(z)\overline{H\alpha}(z)}{1+C(z)\overline{GH\alpha}(z)}$$

While the disturbance transfer function is not obtainable in an explicit form from the equation—

$$\frac{\overline{G\theta_0}(z)}{\overline{G\theta_D}(z)} = \frac{1}{1+\overline{GH\alpha}(z)C(z)}$$

1. Sampled measurements

As an example of the first type of sampled data system described above with the sampling taking place in the measurement channel, consider the first-order system and conventional two-term controller of Fig. 12. The usual function of such a control system is to regulate against load disturbances rather than to follow setpoint changes, so this is what will be considered.

Standard continuous control system design techniques such as the transient response, frequency response, root locus and Nyquist plots are all equally applicable to the design of sampled data systems, but in many cases their application is more difficult. Therefore, the use of sampled data control theory for the selection of controller settings for satisfactory control is often a simple trial and error approach[2]—

1. Start with the system pulse transfer function (or system output equation), choose a value for gain (and/or integral time constant) and system disturbance (say, a unit step) and solve for the system output as a function of z.
2. Expand this equation as a power series in z.
3. Read off the sampled time response as the coefficients of the z terms.
4. Repeat steps 1, 2 and 3 with different controller settings until satisfactory response is obtained.

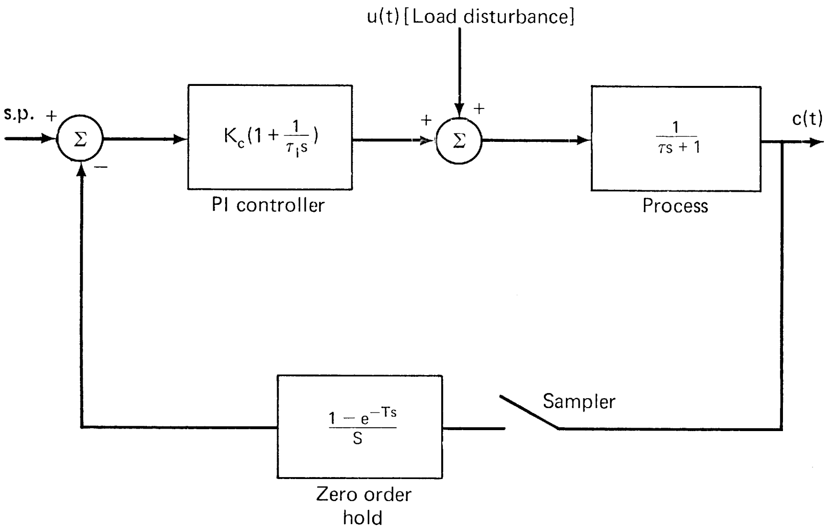


Fig. 12—Typical system

Treating the system in Fig. 12 in this way, we have—

$$\alpha(s) = 1/(\tau s+1) \qquad \text{(first-order system)}$$

$$C(s) = K_c(1+\frac{1}{\tau_I s}) \qquad \text{(PI controller)}$$

$$H(s) = (1-e^{-Ts})/s \qquad \text{(zero-order hold)}$$

Following the same approach as in the preceding section, we arrive at the equation—

$$C(z) = \frac{\overline{\alpha U}(z)}{1+\overline{C\alpha H}(z)}$$

The characteristic equation—

$$1+\overline{C\alpha H}(z) = 0$$

the roots of which are the poles of the closed loop system, may now be analysed as a function of Z. Substitution of the given Laplace transforms for α, C and H, then taking the transform of the product $\overline{C\alpha H}$, the following equation results—

$$\overline{C\alpha H}(z) = \frac{K_c}{\tau_I}\left[\frac{T}{z-1}+\frac{(\tau_I-\tau)(1-A)}{(z-A)}\right]$$

$$\text{where A} = e^{-T/\tau}$$

Although any value τ_I may be chosen, it is often practical to choose it equal to the first-order lag τ, as is the case in continuous systems. This means a process pole is being cancelled by the controller zero,* which is a reasonable design criterion.

The characteristic equation reduces to—

$$z-(1-K_cT/\tau) = 0$$

which has a root—

$$z_r = (1-K_cT/\tau)$$

The response of the system may be characterised as a function of K_c in the following way—

$K_cT/\tau < 0$	Unstable
$0 < K_cT/\tau < 1$	Overdamped
$1 < K_cT/\tau < 2$	Underdamped
$2 < K_cT/\tau$	Unstable

Usually, the underdamped response is desired. Specification of the decay ratio (ratio of adjacent peaks in the response) desired (say, $\frac{1}{4}$) will then totally specify the system (assuming T is fixed) and the system response may be calculated.

For this case, it turns out to be—

$$c(nT) = (1-A)\frac{A^n-(1K_cT/\tau)^n}{A-(1-K_cT/\tau)}$$

which, it must be emphasised, is valid only at sampling instants.[21]

2. *Discrete control*

THE second type of sampled data system is one in which the measurement may be continuous, but the controller has been replaced by a discrete system. Two types of discrete control are possible, analog or digital. A discrete analog

* Root of the transfer function numerator

controller may be of the pulse width modulation (PWM) variety (see below), in which the controller drives a valve motor forward or reverse (for a certain length of time) at discrete intervals only. This latter restriction is required, otherwise the motor might easily wear out. The other kind of discrete control has been labelled direct digital control (DDC).

With a digital controller, one is not constrained to choose only three-term algorithms such as proportional plus integral plus derivative (PID). This may be seen in the following generalisation.

The two-term PI (proportional plus integral) analog controller algorithm may be written—

$$m(t) = K_c\left[e(t)+\frac{1}{\tau_I}\int_o^t e(\theta)d\theta\right]$$

where K_c = the proportional gain,
$e(t)$ = the error signal,
τ_1 = the reset rate constant,
θ = a dummy integration parameter,
$m(t)$ = the controller output signal.

In discrete form, this algorithm becomes—

$$m(nT) = K_c\left\{e(nT)+\frac{T}{\tau_I}\sum_{j=0}^{n-1} e\left[(n-1-j)T\right]\right\} \quad . \quad . \quad (2)$$

where the continuous integral has been replaced by the sum of all previous sampling instant errors. This discrete two-term algorithm may be rewritten—

$$m(nT) = k_1 e(nT)+k_2\sum_{j=0}^{n-1} e[(n-1-j)T]$$

and it can be seen that the integral and proportional modes may be tuned independently. In practice, the full benefit of this advantage may not be felt immediately, as instrument engineers are used to tuning controllers that have mode interaction and some training will be necessary. Equation (*2*) may be simplified by substituting the previous controller output $m(nT-T)$—

$$m(nT) = Kc\left\{e(nT)-\left(1-\frac{T}{\tau_I}\right)e(nT-T)\right\}+m(nT-T) \quad . \quad . \quad (3)$$

which is the operational digital equivalent to the two-term controller. It requires the storage of only three signals, the latest two error signals and the previous controller output. The obvious generalisation of this algorithm is—

$$m(nT) = \sum_{j=0}^{n} q_j e(nT-jT)+\sum_{j=1}^{n} p_j m(nT-jT)$$

which requires the storage of all previous errors and all previous outputs and the specification of the $(2n+1)$ coefficients q_j and p_j. In a practical algorithm, many of these will be zero, but even so the tuning of so many parameters on-line is a difficult task. In practice, some coefficients will be fixed in value as a result of the controller design, whereas others will be fixed using numbers derived from process identification experiments, leaving only one or two to be tuned on-line.

It should be noted that it would be possible to start with this form of the sampled data controller algorithm and, by proper choice of the coefficients, derive a standard two-term algorithm—in other words, the reverse of what has just been done.

Replacement of conventional analog controllers with PID digital algorithms is only one DDC application. Such functions as compensation, cascade control, feedforward control, dead time compensation, digital filtering and sequence control may also be implemented using a digital computer. In many instances, the use of such advanced control techniques would otherwise be impossible.

Positional and velocity algorithms—In the conventional PI control algorithm derived above, the calculated value $m(nT)$ is related to the new valve or other control actuator position. Actually to impress this value on the actuator, a digital to analog (D to A) converter is required. Furthermore, the computer must recalculate the required valve position at every sampling instant. This type of algorithm is called a *position algorithm.*

Looking at equation (*3*), however, it is possible to see that—

$$m(nT)-m(nT-T) = \nabla m(nT) = K_c\left\{e(nT)-\left(1-\frac{T}{\tau_I}\right)e(nT-T)\right\} \quad . \qquad (4)$$

is the change in controller signal required as a result of a change in error signal during the sampling interval and in essence represents the velocity of the control actuator, since the time interval is fixed. Hence, the term *velocity algorithm.* If the computer output goes to a stepping motor or integrating amplifier, no *D* to *A* converter is required and the output may be a digital pulse train.

Two unique properties of the velocity algorithm should be noted—

1. Some small amount of integral action is always required. Examining equation (*4*) and rearranging the terms, we see—

$$\nabla m(nT) = Kc\left\{e(nT)-e(nT-T)\right\} + \frac{KcT}{\tau_I}e(nT-T)$$

If we substitute—

$$e(nS) \times V_n - S$$

where T = set point and Vn = measurement at $t = nT$, this becomes—

$$\nabla m(nT) = K_c(V_n - V_{n-1}) + \frac{K_c T}{\tau_I}(S - V_{n-1})$$

and it can be seen that only the integral term contains any reference to the set point and that the elimination of this term would result in severe controller drift.

2. At any given sampling instant, proportional and integral contributions to the controller output could be of opposite sign (in this case, if V is moving away from S, the sign is opposite; if towards S, the signs are the same). This may be utilised by including only the proportional term for both increasing and decreasing errors when V is within a certain band around S and excluding it for decreasing errors outside this band. This limited proportional action may be accentuated by having different proportional gains in the two regions with a higher value outside the band. This would allow for much faster response to process upsets.[23]

Pulse amplitude and pulse duration control—The final step in a digital control system is the application of the calculated control signal to the final control actuator. This can be accomplished by a zero-order hold, which maintains the calculated signal as input to the actuator over the next sampling intervals. This type of actuation is called *pulse amplitude modulation* (PAM) and results in a controller signal as in Fig. 13.

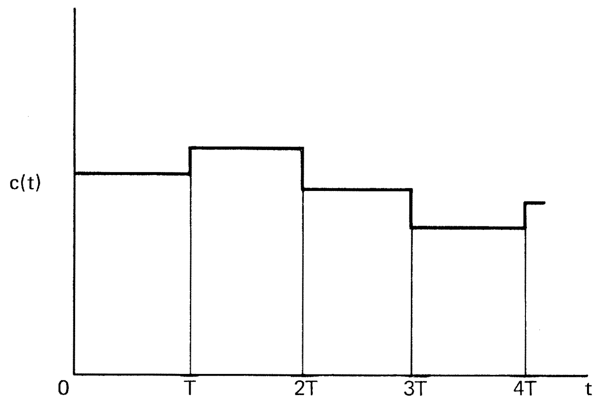


Fig. 13—PAM control signal

An alternative to PAM is pulse duration control, that is, pulse width modulation (PWM). There are several advantages in using PWM, as Emery & Lin summarised[25]—

'*1.* PWM systems are less susceptible to the influence of signal noise, since the information contained by an individual pulse resides in the duration of the pulse and not in its amplitude.
2. The output stage of a PWM system can be a simple relay circuit. Thus, very high gains can be realised with little equipment complexity and relatively low cost.
3. The torque limiting problems that are encountered in pulse amplitude modulated (PAM) and continuous systems are easily handled in a PWM system, since only one value of torque (pulse amplitude) is ever applied in the latter.
4. No hold circuit is required in PWM systems.
5. Loss or interruption of the input signal in a PWM system yields a fail safe condition.
6. PWM or pulse duration actuators are commonly employed in industrial plants for push-button adjustment of process variables. It is advantageous to employ PWM control when installing computer control systems so that the existing actuators can be left intact. This preserves normal operating procedures throughout the installation and thereafter, during computer shutdowns. In addition, it reduces the complexity in switching from manual to automatic modes of control[(26)] and the overall installation costs are considerably less.'

A velocity type of algorithm is almost always used with PWM systems, as can be appreciated by considering the PWM signal as a sequence of instructions to the control valve— 'open a bit', 'open a bit more', 'close a bit', etc., with the duration of the pulse indicating how much and the polarity indicating open or close. PWM control can be implemented using a digital computer, which would allow complete flexibility in the design of the control law. Emery & Lin[(25)] describe the use of Lyapunov's second method in the design of an approximately optimum PWM controller.

The algorithm they arrive at is equivalent to a PI controller, with both the sign of the control signal and its duration being functions of the present and past errors as below—

$$sgn\{c(t)\} = sgn\{K_1 e(nT) + K_2[e(nT) - e(nT-T)]\}$$

$$\text{and } d(n) = |K_1 e(nT) + K_2[e(nT) - e(nT-T)]|$$

$$\text{for } nT < t < (nT+T)$$

where $d(n)$ = duration of the pulse starting at $t = nT$,
K_1, K_2 = tuning parameters.

They showed that near optimum K_1 and K_2 could be chosen as functions of process parameters. Note that, although the algorithm may appear to be proportional plus derivative, the process used in the derivation contained an integrating term $1/S$—for example, an electric motor driven valve, which would convert this to proportional plus integral. An example of PWM signal is seen in Fig. 14.

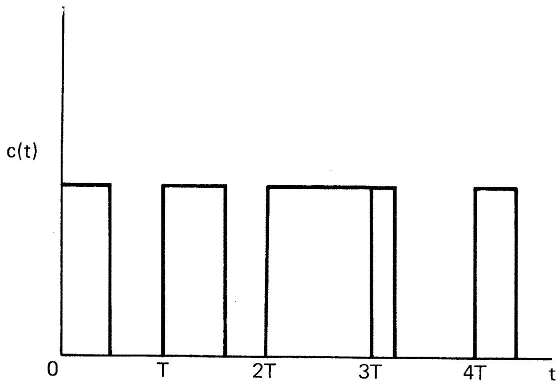


Fig. 14—PWM control signal

Stability of sampled data control systems—No discussion of sampled data control systems would be complete without a mention of the stability problem. In linear control theory, the stability of a closed loop system is determined by the location of the transfer function poles on the S-plane (Fig. 15). If all the

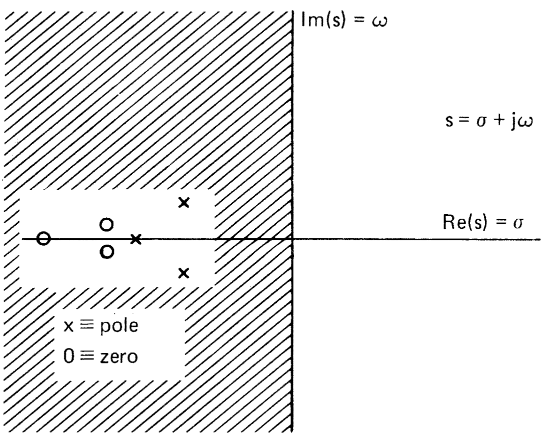


Fig. 15—The S-plane

poles of the system (roots of the characteristic equation) are in the left half plane (that is, they have negative real parts) for a particular gain, then the system is stable for that choice of gain. The poles should be far enough from the axis so that the system remains stable when process parameters change.

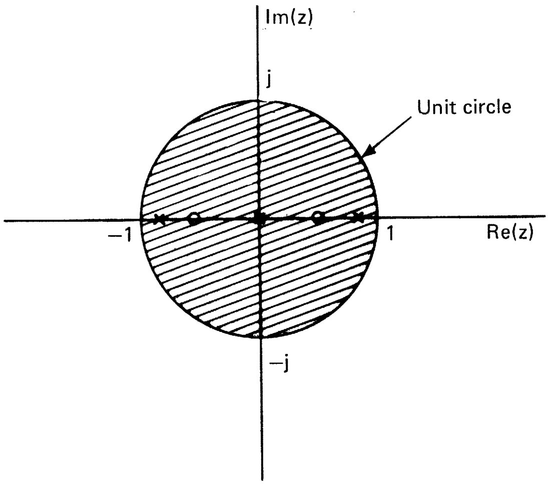


Fig. 16—The z-plane

The equivalent condition for a sampled data system is found by mapping the left half of the S-plane to the z-plane (Fig. 16) by using the transformation—

$$z = e^{Ts}$$

This is a multi-valued transformation and the results in the mapping of each strip $\sigma > 0$, $nT < \omega < (nT+T)$ on the S-plane into the unit circle of the z-plane. The Routh stability criterion described above for continuous systems then requires that the poles of the sampled data transfer function lie within the unit circle. The root locus technique may thus be applied to determine the motion of the poles as the controller settings are varied and the closed loop gain changes.

Using this Routh criterion, Soliman & Al-Shaikh[1] have investigated the stability of a first-order system containing a finite delay and have shown that discrete control gives stability region improvement over continuous control for various delay time to sampling interval ratios. The largest improvement cited was a trebling of the maximum allowable controller gain. The delay time to sampling interval ratio was found to be a critical stability parameter as may be anticipated from discrete system theory.

Selection of the control interval—One question that has not yet been answered is 'How often should a control action be taken?' Note that this control sampling interval is not necessarily the same as the measurement sampling interval, as in many cases a number of measurements are taken for each control action. The choosing of the control rate is still very much a matter of engineering judgment, as it represents a compromise between poor control (which results from too low a rate) and increased computer loading and equipment wear (which result from too high a rate). Guidelines can be set down, however, as has been done by the DDC Users Workshop [(33)] and discussed by Goff.[(7)] The guidelines proposed are control sampling rates of—

1. Once per second for flow loops.
2. Once every 5 s for level and pressure loops.
3. Once every 20 s for temperature and composition loops.

In practice, the dynamics of the specific loop must be taken into account. The control algorithm used and the type of disturbance expected also have a bearing on the selection of the sampling rate, as do such considerations as the speed of response to set point change requests from the operator, who is used to having immediate response and may not like having to wait until the next control sample instant. This latter problem may be handled by having special routines to act on set point changes immediately.

Some representative values of control sampling rates are—

Head box control	2–8 s
Basis weight and moisture	
Scan average	30–270 s
Single point	10– 30 s
Refiners	5– 60 s

APPLICATIONS OF SAMPLED DATA THEORY TO THE CONTROL OF PULP AND PAPER PROCESSES

THE literature abounds with articles describing the application of computer control to pulp and paper processes. For information on the literature available, attention is drawn to Brewster & Bjerring's paper,[(32)] which has an extensive bibliography as well as a review of pulp and paper application areas and a summary of major computer control installations in U.S.A.

Applications of computers to control of pulp and paper making are described also in papers given at this symposium. For example, three papers,[(30, 34, 35)] all describe some application of computer control.

The application areas and the techniques used in these installations vary widely, but almost all involve more than simple DDC replacement of PID analog controllers. This latter application is still very controversial, owing to the questionable economic justification of installing a computer to perform simple PID DDC when analog controllers are often required for back-up anyway. For this reason, most installations have been performing such non-PID functions as cascade control, non-interacting control and feedforward control. Since dead time processes are very common in the pulp and paper industry, a few of the major papers dealing with the design of digital controllers for such systems will now be reviewed.

Dahlin(13) describes a controller synthesis method based on cancellation of process poles by controller zeroes, which gives good results once the open loop process transfer function has been determined. He assumes a first-order plus dead time process in his design of basis weight and moisture control. The identification of the model parameters is accomplished by an on-line perturbation technique.

The controller synthesis method also requires the specification of the desired closed loop response to a step change in set point. Dahlin specified a damped exponential, which resulted in a controller transfer function—

$$D(z) = \frac{\Omega(1-e^{-AT}z^{-1})}{K(1-e^{-AT})}$$

where $\Omega = (1-e^{-\lambda T})/[1-e^{-\lambda T}z^{-1}-(1-e^{-\lambda T})z^{-N-1}]$
NT = the process transport lag,
$-A$ = the process pole,
K = the process gain

and λ is the time constant of the desired closed loop transfer function. Dahlin uses λ as the on-line tuning parameter, while N, A and K are determined from the on-line identification experiments.

Although this algorithm looks fairly complex, it can be reduced to a form that shows the ease with which it can be implemented on a digital computer. The discrete form of the algorithm is—

$$c(n) = a_1e(n)+a_2e(n-1)+a_3c(n-1)+a_4c(n-N-1) \quad . \quad . \quad (5)$$

where $c(n)$ = the controller output at $t = nt$,
$e(n)$ = the error at $t = nt$,
$a_1 = (1-e^{-\lambda T})/K(1-e^{-AT})$
$a_2 = -a_1e^{-AT}$
$a_3 = e^{-\lambda T}$
$a_4 = 1-a_3$

It can be seen that the coefficients are functions of λ (the tuning parameter), hence all control modes are affected as λ is varied. It should also be noted that as the delay (N) goes to zero, the algorithm becomes identical in structure to the discrete PI controller—equation (*3*). Algorithms developed in this manner have been used successfully providing stable head box, basis weight, moisture and colour control.[27–30]

In a series of articles dealing with computer control at Billeruds, Astrom in one of them[11] describes the development of control laws based on linear optimum stochastic control theory, specifically[11] for basis weight and moisture control. Sampled data theory is used extensively in modelling and process dynamics, modelling the dynamics of the disturbances (which are assumed to be stationary random processes) and eventually in formulating the control law. He uses a minimum variance design criterion, but shows that, if there is a non-minimum phase process singularity (that is, a zero outside the unit circle on the z plane), then this design strategy will give unstable control as process parameters change. A modified criterion can be used partially to eliminate this stability problem.

A typical control algorithm derived by Astrom is the following for a first-order model relating wet basis weight to thick stock flow with a dead time of four sampling intervals[36]—

$$\nabla u(t) = -K\frac{1-b_1z^{-1}}{1+a_1z^{-1}+a_2z^{-2}+az_3^{-3}}\,y(t)$$

which can be converted to the easily programmed form—

$$\nabla u(t) = Ky(t)-Kb_1y(t-1)-a_1\nabla u(t-1)-a_2\nabla u(t-2)-a_3u\nabla(t-3)$$

where $\nabla u(t) = u(t)-u(t-1)$,
$u(t) =$ the controller output (thick stock flow),
$y(t) =$ the process output (wet basis weight),
K, b_1, a_1, a_2, a_3 are all positive constants.

Although Astrom did not do so, this velocity algorithm can be converted to the equivalent positional algorithm so that it may be compared with Dahlin's—

$$u(t) = \alpha_1y(t)+\alpha_2y(t-1)+\alpha_3u(t-1)+\alpha_4u(t-2)+\alpha_5u(t-3)+\alpha_6u(t-4)$$

Since a_1, a_2 and a_3 are all very nearly the same, α_4 and α_5 are small compared with α_3 and α_6 and the algorithm is approximately—

$$u(t) = \alpha_1y(t)+\alpha_2y(t-1)+\alpha_3u(t-1)+\alpha_6u(t-4)$$

Comparing this with Dahlin's equation (5) and noting that in Astrom's example the delay $N = 4$, it can be seen that the only structural difference lies in the fact that Astrom uses the control action taken one delay ago in his prediction, whereas Dahlin uses the control action taken a delay plus a sampling interval ago.

This same sort of difference exists between Astrom's algorithm and one derived by Buhr *et al.*[31] from a model reference feedback structure. In this latter approach, Buhr used the identified process delay time in a process model to predict what the output (as a result of the control action taken one delay time previously) should now be. Comparing this to the actual measured output gives a measure of the input disturbance that existed at that point in time. The prediction of the present input disturbance on the basis of this and past disturbances is where Buhr deviates slightly from Astrom's approach. Firstly, the prediction is a weighted linear extrapolation of the just-measured disturbance and one a fixed number of intervals ago. Secondly, the extrapolation is made to a point more than just a dead time in the future as with Astrom's, the advantage being that this reduces the stability problems introduced if the original plant model is in error.

In comparing these various alternatives, personal preference is probably the most reasonable excuse for choosing one over the other. Part of this, of course, will depend on the ease with which the algorithm can be implemented and, in particular, tuned. Neither Astrom nor Buhr discusses the practical difficulties in tuning their algorithms, whereas Dahlin describes a simple method that can be programmed and implemented on-line.

Many other DDC installations have been reported in the literature. For example, Bockstanz & Keyes,[37] writing about an Eastex installation, mention that sampled data control synthesis methods were used to do preliminary control algorithm design and that pulse duration control was used. Again, attention is drawn to[32] and its extensive bibliography for other references to computer control applications in the pulp and paper industry.

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Discussion

Mr R. E. Johnston I would like to confirm the results presented by Mr Bjerring on the equivalence between the two design methods. The proof can be made more rigorous than this presentation and, in fact, leads to the time constant that you should choose for the closed loop response being uniquely described by the disturbance characteristics that may be determined during the process of identification.

My question is that, since 'statistic' control systems are usually designed to cope with a disturbance that has a zero mean, how do you finish up with a controller that does not give you any offset after drifts or stop changes?

Mr A. K. Bjerring I have no personal experience with that, but I am sure that there are people here who have. Perhaps, they would like to comment.

Mr R. E. Jones The terms in the numerator of the dead time control algorithm can be shown to be equivalent to proportional plus integral control. Hence, there is reset action against long-term offset.

Mr Bjerring Yes, but, in Astrom's case, I believe that the reset terms are not there. This might have been what has been referred to by Mr Johnston. If you remove the dead time, you do not take into account the controller output a sampling interval ago. That does not enter into it.

Mr Johnston May I just add to that. The problem appears to be that in the first part of Astrom's paper he has assumed a model that in effect has a zero mean and a variance that is finite; but, in the last part, he assumes a model of the disturbance that has an infinite variance, yet eventually leads to a system that gives no offset. How you identify the constant parameters I do not know.

Dr A. P. Wardle When statistical methods are used for system identification, sampling is carried out over a finite time and the infinite limits of the summation terms involved become finite. What error does this introduce and how will the length of sample affect the accuracy of the results?

Dr D. B. Brewster Unfortunately, I did not have time to discuss this point. The estimation of the autocorrelation and power spectrum and the theory of stochastic control depend on the assumption of stationarity. This states that the statistics such as the mean, standard deviation, autocorrelation, etc. do not change with time. With a sample of finite length, very low frequency variations (which have a period greater than some submultiple of the sample length) will in effect show up as drifts in the mean. These drifts should be removed from the data before starting the calculations, otherwise a spurious result will occur. Jenkins & Watts' paper[5] goes into more detail on this, in particular on the relationship between length of sample and the lowest frequency that can be estimated. At the other end of the frequency range is the problem of aliasing, which is dealt with in the paper. When the sampling interval is restricted to reel changes, for example, the power spectrum almost inevitably will contain aliasing. The autocorrelation (or autocovariance), although normally more difficult to interpret, is more useful in this case.

Mr D. L. Cooper The procedure that we normally follow is this. If we have a very long series of results, we will see whether there is any long-term drift. In interpreting the spectrum, long-term drifts show up as frequencies; because you have only one or two peaks perhaps in the whole series. The errors associated with the estimate of those frequencies are very large. We normally do some very simple smoothing of the curve, taking out as far as we can what appears as long-term trends before we apply the autocorrelation function and obtain spectral estimates.

Dr J. N. Chubb The impression I receive from these two papers is that it would be wise to put more effort into improving the hardware side of data sampling techniques instead of concentrating so much on mathematical analysis. I suggest that, for example, instead of sampling at a fixed repetition rate, it would be better to sample only when there has been a significant deviation of a signal from the previous sampled value. This requires some signal storage and comparison on each input signal line and the ability to call the attention of the main processor when information transfer is required. This system would minimise the amount of computer attention required to ensure fully detailed tracing of a number of input signal variations.

Mr Bjerring To your first point, one comment is that this would be all right so long as we maintain the same control interval, which is the interval at which the controller output is sampled. A technique called modified z-transform analysis may be used here and must be used when using a sampling interval somewhere in your system that is not an integral multiple of the sampling measurement interval.

A Speaker There has been some work done with so-called sensitivity models. This sensitivity approach has been used for more generally adaptive schemes where you adjust your parameters on-line as in your Dahlin control, on a model reference approach. You can also extend that to the case of adjustable sampling parameters, for example, the sampling period. The basic work in these areas was done by Prof. George Beckie of the University of Southern California.

Dr Brewster I would like to ask a question that may sound a little heretical for a fundamental research symposium. Has this actually been used in industry?

A Speaker It is my understanding that the basis of the sensitivity model approach has been applied to the steel industry by Beckie and his associates.

Dr I. B. Sanborn One should remember, with the idea of sampling on the basis of a change of signals, that there will probably be a change of signal from the beta-gauge, owing to basis weight profile variations. One of the large problems in getting basis weight samples at high frequencies on papermachines is that one should be taking control actions on the average basis weight. In addition, the papermaker likes to see his profile so that appropriate corrective action can be taken at the slice.

There is a possibility that one could mount a second beta-gauge in a fixed position, but that costs 60 000 dollars or more. There are other approaches as well. For example, one could sample consecutively while traversing the web and estimate the deviation from the average profile (obtained via a 10 scan average) as a best estimate for basis weight change. How well this would work, however, is a difficult question.

One final comment about some practical information on Dahlin's approach. We have been using this approach at CPE for approximately two years and we have found that the basis weight stability of our papermachine has improved significantly. I think that the most important factor in the improvement, however, is that mass flow rate of stock to the machine is controlled by a tight loop with a 4–8 s sampling intervals. As a result, it would be foolish to use an algorithm such as Dahlin's to adjust the stock valve directly. Instead one should adjust the set point of a tight DDC loop on the stock valve.

Mr Johnston We have all been referring to Dahlin's approach and we should realise that this all stems from Smith's original work on designing controllers for systems with dead time. In that respect, the controllers designed by Dahlin, Smedhurst and Ramaz *et al.* and by others will all be identical, no matter how they are dressed up and in what terms they are expressed.