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COMPUTER CONTROL OF A PAPERMACHINE USING A LINEAR STATE SPACE MODEL

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Synopsis Perturbation experiments on a Fourdrinier papermachine have confirmed that its dynamic behaviour can be represented adequately by a state space model in the form of a matrix difference equation. The basic equations involved have been treated in general terms, but the discussion on the model building and control system design is made more explicit by reference to a specific system.

Methods have been developed for investigating and describing the papermaking system as a process to which modern control theory can be applied. It has been shown how the model can be used to determine possible control strategies to change grade in such a way that the grade change time is at a minimum and certain papermaking criteria are obeyed. The control objectives have been stated by analytical performance criteria in the form of quadratic cost functions.

A simple grade change at constant machine speed was achieved by altering the thick stock flow according to a trajectory determined by the rate constraint of the flow valve. It was found necessary to manipulate and synchronise the thick stock and thin stock flows together with the machine speed in order to change grade at constant production rate.

Based on optimum control and filter theory, an on-line controller has been designed to manipulate the thick stock flow in order to minimise the variance of the measured basis weight. The developed formulation incorporates optimum estimation of inaccessible state variables as an implicit feature. The control action is given by a proportional term together with a memory term to account for past values of control and basis weight. The controller has been implemented on a machine and is shown to have stabilised the system considerably.

Introduction

A SERIOUS problem in the production of multi-grade paper is to make fast and smooth grade changes and to maintain uniform and reproducible properties of the paper.

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The process is characterised by transport delays and long-term transient behaviour, owing to the inherent capacitance of hold-up vessels and pipe network, which can cause the paper to drift off specification, especially immediately after a grade change.

A number of variables such as certain consistencies and flows are inaccessible to measurement, consequently information about the state of the process is incomplete and it may not be possible to compensate for disturbances.

There is a major economic incentive to optimise grade changes and thereby minimise production of off-grade paper. Furthermore, the system is generally not in a steady state immediately after a grade change and it is most important to ensure that the paper is on specification while the system settles down and that it is maintained steady for as long as it is required. Paper breaks caused by undesirable transients during grade changes can result in a considerable loss of production.

A basic prerequisite for the design of a control system to perform the tasks indicated above is to establish an adequate mathematical model of the process.

The papermaking process comprises a certain number of basic operations such as the transport of fibre and additives between tanks and chests where mixing takes place. Another important feature is the coming together at a point of several streams of stock with different consistencies, then emerging as one stream. Thus, four basic concepts are fundamental to the process—

1. Transport delays.
2. Mixing in chests and tanks.
3. Mixing at a point.
4. Flow dynamics.

Some of these are essentially non-linear in character. It is assumed in the present investigation, however, that we are interested only in the dynamic behaviour of the system in the neighbourhood of some specified operating level; therefore, it will be sufficiently accurate to consider a linear model. Further direct digital control is to be used and the attention can be focused on a linearised, discrete time model as follows—

$$x(n+1) = \sum_{i=0}^{h} F_i x(n-i) + Eu(n) + d \quad . \quad . \quad . \quad (1)$$

where n is the independent, discrete time variable,

x is the $m \times 1$ state vector of the papermachine,

u is the $s \times 1$ control vector,

d is the $s \times 1$ plant noise vector

and F_i and E are $m \times m$ and $m \times s$ transition and input matrices, respectively.

Present methods of automation in papermaking provide for some control of the basic process variables, but this control can be described as being partial.

Effective control, however, must provide automatic control of all the basic parts of the process and must take into account the interrelationships of the corresponding controlled quantities. If this is to be achieved, it is necessary to develop a method for investigating and describing the papermaking system as a process to which modern control theory can be applied.

This integrated approach of dealing with the interactions between variables and their effect on the finished product remain to be established.

The objectives of the present investigation were as follows—

1. To devise a manageable and realistic model of the process in the form of equation (*1*).
2. To use the model to determine possible control strategies to change grade in such a way that the grade change time is small and certain papermaking criteria are obeyed.
3. To design a computer controller that maintains constant grade with minimum deviation from the desired specifications.

System description and control objectives

THE actual process on which the controls is to be implemented will be used as an example to make the developed general methods of model building and control theory more explicit.

The papermaking system is shown in Fig. 1. From a constant head tank, thick stock of about 3·5 per cent consistency is continuously supplied at a rate of 600–800 gal/min to the system at the mixing pump in a suction pipe, which also supplies recycled backwater with a concentration of 0·4–0·5 per cent fibre.

Thin stock (0·9–1·0 per cent) is formed by mixing, then pumped by a constant head pump at a rate of 7 000 gal/min through the cleaning equipment.

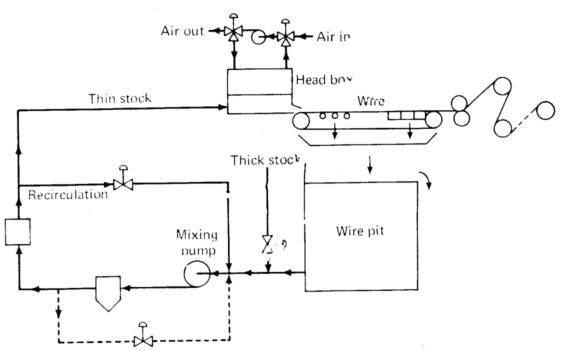

Fig. 1—Layout of process

Recirculation pipes are fitted to maintain a high flow through the cleaning equipment for efficient performance. The flow of thin stock into the head box is about 5 000 gal/min and is discharged through the slice opening on to the moving wire. The machine and wire speed ranges 650–950 ft/min.

A fraction of about 0·6 of fibre and additives filters through the wire together with about 98 per cent of the water into the wire pit. This is fitted with an overflow in order to supply the mixing pump from a constant head.

For papermaking reasons, it is necessary to maintain a constant low level of stock in the head box; the head box is therefore pressurised and air is continuously circulated through the head box airspace by a pump as indicated in Fig. 1.

The wire pit is the largest of the tanks and contains a constant volume of 2 300 ft^3 of backwater. This gives rise to a time constant of about 3 min to changes in consistency. The combined effect of consistency changes arising from recirculation dynamics associated with the cleaning, together with the head box and wire pit mixing dynamics, yields a system with a very long settling time of about 30 min. This makes manual control difficult, especially at grade changes, when it is necessary to manipulate several input variables simultaneously to force the system to go from one state to another.

The majority of grade changes for this particular machine are carried out at constant machine speed or constant production rate. The first method is mainly used for small changes in basis weight and when it is necessary to alter production rate for papermaking reasons. A major economic incentive is to maintain the highest possible production rate, usually constrained by drying capacity. Under these conditions, a grade change is best effected by keeping the production rate constant.

Sheet strength and formation depend to a great extent on the ratio of stock velocity in the slice to wire speed, called the efflux ratio. It is particularly important to keep the efflux ratio constant during a grade change.

A grade change subject to these papermaking criteria (constant production and efflux ratio) can be realised by simultaneously manipulating the three input variables— thick stock flow, thin stock flow and wire speed. If the papermaking criteria are violated, the result is that often the sheet breaks or the basis weight and moisture can be outside the tolerance limits for long periods of time. A general strategy for a constant production change can now be formulated. It is required to change the basis weight from one level to another as quickly as possible, subject to saturation and rate constraints of the input variables while production rate and efflux ratio remain constant.

Grade changes at constant speed are carried out by changing the thick stock flow, subject to constraints on the valve movements.

The third control objective is to design a direct digital computer controller that minimises the variance of basis weight by manipulating the thick stock flow based on measurements of the basis weight.

State space representation of the process

WATER is used as a vehicle to transport fibre in a network of pipes and tanks, hence lags will occur with respect to changes both in flow and in concentrations. It will be necessary to consider the past states of the system, as well as the present states, owing to transport delays that affect the concentration of fibre and additives. Finally, the interactions between flow and concentration dynamics must be considered, as these determine the distribution of fibre concentrations throughout the system.

The object of the modelling problem is to evaluate these functions and determine the resultant dynamic behaviour of the plant.

We are interested mainly in the dynamic behaviour in the neighbourhood of a fixed operating level and certain assumptions can therefore be made—

1. Simplified fluid flow equations in the form of a direct analog between fluid flow and electric current in networks is considered to be adequate.
2. Perfect mixing takes place in the head box and wire pit.
3. The delays are time invariant and independent of the state variables.
4. Changes in the fraction of fibre and additives that filter through the wire is taken to be proportional, but of opposite sign to the changes in the initial basis weight on the wire.

The last approximation has been verified and used by several investigators[1, 2] and provides the mathematical link between the flow and consistency dynamics of the head box and wire pit.

The total hydraulic pressure at the slice opening is of particular interest and this, together with the stock level, form the variables of the simultaneous differential equations that describe the head box flow dynamics—

$$C_1\frac{dP_1}{dt} = q_{ai}-q_{ao}+(C_1+C_2)\frac{dP_2}{dt} \qquad (2)$$

$$C_2\frac{dP_2}{dt} = q_i-q_o \qquad (3)$$

The input and output concentrations in the mixing tanks, head box and wire pit are assumed to be related by the first-order differential equations—

$$\frac{dV}{dt} = -q_o+\sum_{j=1}^{r} (q_i)_j \qquad (4)$$

$$\frac{d(c_oV)}{dt} -c_oq_o+\sum_{j=1}^{r} (c_i)_j(q_i)_j \qquad (5)$$

The thick stock flow, the recirculation flow and the wire pit flow converge immediately before the mixing pump. The output concentration from the pump can be derived as a special case of equations (*4*) and (*5*) by putting $V = 0$ and $r = 3$.

The head box and wire pit equations were derived with $r = 1$.

The basic equations (*2*)–(*5*) were linearised about a chosen operating level. In order to obtain a discrete model as indicated in the introduction, it was assumed that information of the state variables was available only at periodic intervals of time (every 10 s) and the forcing functions were to be held constant throughout the interval and changed in a step manner at the sampling instants.

In order to account for delays, a modified version of a method described by Tou[3], was used to form the difference equations of the linear continuous equations. This yields the complete deterministic model in the discrete form given by equation (*1*), where now h = 3, m = 5 and s = 4.

$$x(n+1) = \sum_{i=o}^{3} F_i x(n-i) + Eu(n) \qquad (6)$$

The system is characterised by five state variables and four control variables, thus the transition matrices F_i and control matrix E are 5×5 and 5×4, respectively.

The dynamic behaviour of the system is the n given by the discrete time history of it state variables, which constitute—

x_1 = Hydraulic pressure at the slice,
x_2 = Stock level in the head box,
x_3 = Consistency immediately after the mixing pump,
x_4 = Head box consistency,
x_5 = Wire pit consistency.

The dynamic behaviour can be controlled by the four inputs—

u_1 = Head box air flow,
u_2 = Thin stock flow,
u_3 = Thick stock flow,
u_4 = Machine speed.

It is difficult to determine a rational criteria for the adequacy of the model; nevertheless, it is of paramount important to establish that the model represents the plant behaviour sufficiently well in order to devise practical control schemes.

The thick stock flow on the machine was changed according to a predetermined pseudo-random binary sequence of thick stock valve positions and the response of basis weight was observed from beta-gauge measurements.

The model was then perturbed by an identical sequence of assumed thick stock flows u_3. The response was observed by augmenting the state space of equation (*6*) by a linear discrete output equation to represent changes in basis weight—

$$x_6(n+1) = Mx(n+1) \quad . \quad . \quad . \quad . \quad (7)$$

where M is a 1×5 matrix and x is the state vector of equation (*6*). The simulation thus represents a tie between analysis and experiment and, as can be seen in Fig. 2, the agreement between the predicted behaviour of basis weight and the actual behaviour seems quite satisfactory for our purpose.

It should be pointed out that the basis weight trace contained a small amount of noise and was smoothed by eye before it was plotted on Fig. 2.

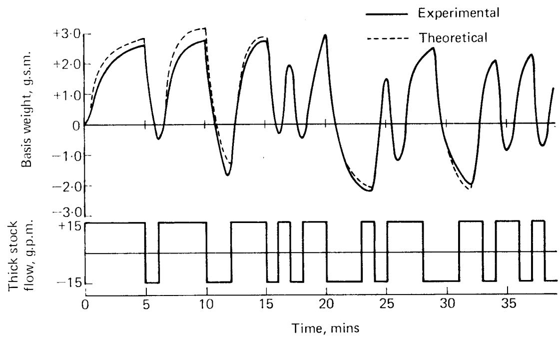

Fig. 2—Perturbation experiment (this refers to a plant and model with the additional recirculation indicated in Fig. 1)

Statement of control problems

WE WILL attempt to control a process that can be described by a linear, discrete matrix difference equation of the following form—

$$x(n+1) = \sum_{i=o}^{h} F_i x(n-i) + Eu(n) \quad . \quad . \quad . \quad (8)$$

For grade changes, it is required to find the input vector—

$$u(n),\ n = 0,1,---,p-1,$$

which minimises a performance criterion of the form—

$$J = \sum_{n=o}^{p-1} \tfrac{1}{2}||x_d(n+1) - x(n+1)||^2 \quad . \quad . \quad . \quad (9)$$

subject to equation (*8*) and the rate constraint—

$$|u(n) - u(n-1)| \leq v \quad . \quad . \quad . \quad . \quad (10)$$

A quadratic non-linear programming method developed by Kishi,[4] suitable for on-line computation, was extended to multi-variable systems with delays in order to solve the above problem.[5] With some modifications, the same method was used to solve the regulator problem by minimising a performance criterion of the form—

$$J = \sum_{n=o}^{p-1} \tfrac{1}{2}\|x(n+1)\|_Q^2 + \tfrac{1}{2}\|u(n)\|_R^2 \quad . \quad . \quad . \quad (11)$$

subject to equation (*8*). This leads to a feedback control law—that is, the controls are given as a function of the state variables, thus—

$$u(n) = -\sum_{i=o}^{h} D_i x(n-1) \quad . \quad . \quad . \quad . \quad (12)$$

where D_i are s×m matrices.

The discrete form of dynamic programming[6] can also be used to derive the feedback law, but it is then necessary to augment the state space of equation (*8*) to account for the delays.

Control strategies

Grade change at constant machine speed—The object was to determine the input trajectory of thick stock flow u_3 in order to raise the basis weight 10 g/m² from the assumed operating level. At the same time, it was required to minimise the variance about the target subject to the rate constraint—

$$|u_3(n) - u_3(n-1)| \leq 0{\cdot}15 \text{ ft}^3/\text{s per } 10 \text{ s}$$

Fig. 3 shows the determined optimum trajectories. It can be seen that it is required to increase u_3 as fast as possible for about 60 s, to slightly more than twice the change in flow necessary to achieve the new grade. The thick stock is then gradually brought down towards the demanded change in flow of 0·37 ft³/sec. As a comparison, the response of basis weight to a hypothetical step change of 0·37 ft³/sec in u_3 is also shown in Fig. 3.

The minimisation of quadratic performance criteria often results in oscillatory input trajectories being determined, as that exhibited by the thick stock flow. In this case, a less oscillatory grade change could be obtained by imposing a more severe rate constraint—for example, 0·05 ft³/sec per sampling interval (10 s). The time taken to achieve the new grade is then obviously longer, but compares very favourably with the faster oscillatory response.

The method used above to determine thick stock trajectories can be implemented on-line as a feedback control system, especially with a single input/single output system like the one considered here. A large grade change at constant machine speed is likely to cause the dryline to change position (the

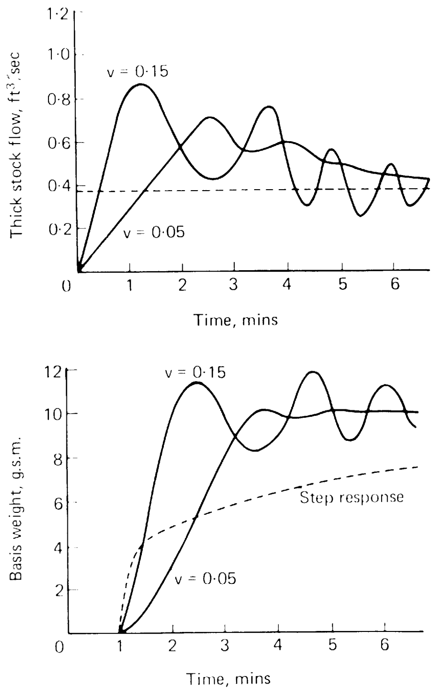

Fig. 3—Grade change trajectories at constant machine speed rate constraints of thick stock flow $V = 0{\cdot}05, 0{\cdot}15^3/\mathrm{s}/10\ \mathrm{s}$

position on the wire at which no stock remains above the sheet) with the result that the sheet is either too dry or too moist to be transferred from the wire without breaking. In the present investigation, the dryline position was assumed to be adequately represented by—

$$s_d = \frac{kWv_m}{\Delta P} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

which was derived from general filtration theory, as stated by Meadley.[7] A grade change at constant machine speed is therefore likely to require adjustment of ΔP, the suction box vacuum.

A more elaborate and complicated equation for the dryline position has been used by Sullivan & Schoeffler,[8] who considered grade changes subject to the constraint that the dryline position be invariant.

Grade change at constant production rate and efflux ratio—For the sake of ease in presentation and discussion of this topic, it is considered necessary first to deal briefly with the problem of head box control.

The formation of the sheet depends to a great extent on a constant stock level being maintained in the head box. This problem has previously been studied using conventional means.[9] In the present investigation, a time optimum control law was determined and used both on-line and in simulations to regulate the stock level by the air-flow valves, thus—

$$u_1(n) = a(x_1(n)_d - x_1(n)) + bx_2(n) \qquad (14)$$

where a and b are constants computed from the head box model and $x_{1,d}$ is the desired value of the total pressure at the slice opening. Equation (*14*) was then substituted for u_1 in equation (*6*) to reduce the number of control inputs to those of particular interest in the present grade change problem—namely, thin stock flow u_2, thick stock flow u_3 and machine speed u_4.

Before the grade change trajectories are determined for these three control inputs, it is necessary to examine the specific performance requirements of the present problem. It is assumed that the following linear output equation relates changes in total head, head box consistency and machine speed to basis weight, production rate and efflux ratio—

$$\begin{bmatrix} \Delta W \\ \Delta M \\ \Delta \phi \end{bmatrix} = M \begin{bmatrix} x_1 \\ x_4 \\ u_4 \end{bmatrix} \qquad (15)$$

For the grade change problem in question, $\Delta M = 0$ and $\Delta\varphi = 0$. In addition, the change in machine speed is severely limited by the rate constraint given by—

$$|u_4(n) - u_4(n-1)| \leq 0{\cdot}278 \text{ ft/ s/ 10 s}$$

This simplifies the problem, as the desired trajectories of x_1 and x_4 can now be determined explicitly assuming that it is required to change u_4 as fast as possible to the new value, governed by the choice of ΔW.

We then envisage using the available control inputs in an optimum sense such that the process outputs (x_1 and x_4, in particular) are kept close to the desired trajectories. This operation has then to be maintained throughout a predetermined optimisation interval so that the process is left in a state ready for the on-line regulator to maintain constant conditions.

The transport delay from the mixing pump to the head box makes it impossible to alter the head box consistency x_4 immediately by the thick stock flow u_3. No changes should therefore be made to either thin stock flow u_2 or machine speed u_4 until this delay has elapsed.

All the available control inputs affect the consistency distribution in the system and this is, of course, taken into account by the model when the input trajectories are determined. The control inputs must be synchronised to produce the desired trajectories of total head and head box consistency. Failing to do so often results in paper breaks, owing to insufficient strength, excessively high moisture content or low basis weight. Experience on the plant has shown that the dryline position remains constant for grade changes carried out at constant production rate; this can also be inferred from equation (*13*) as $M = Wv_m$.

In order to raise the basis weight by 12 g/m² at constant production rate and efflux ratio, it was required to manipulate the control variables as shown in

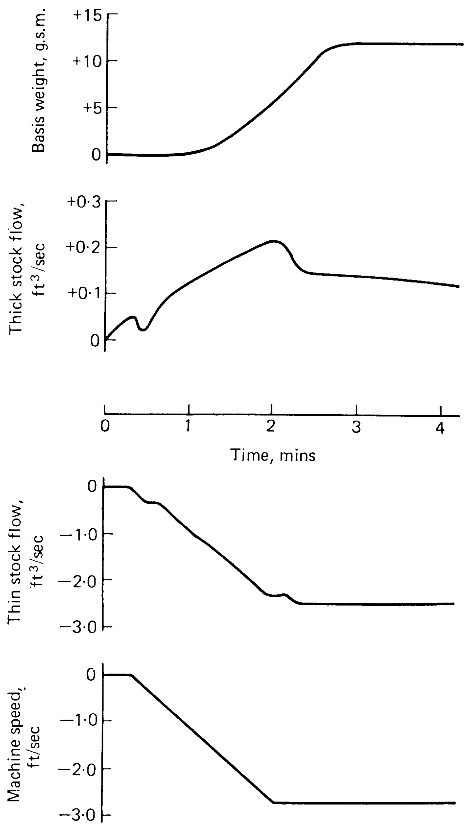

Fig. 4—Control trajectories for a grade change at constant production rate and efflux ratio

Fig. 4. The demand was to increase the head box consistency from 0·88 per cent to 0·99 per cent and decrease the total head by 42·8 lb/ft², according to

the predetermined trajectories. The machine speed was lowered as fast as possible by 2·78 ft/s.

The control strategy can be resolved into three distinct phases—

1. Initial recirculation dynamics and head box level control demand an abrupt change in thick stock flow and a slight relaxation in the thin stock flow in order to realise the initial path of the desired trajectories of x_1 and x_4.
2. The system is quickly stabilised and a period of almost uniform change in the input controls follows until just before the desired targets are reached.
3. During the grade change, the system has gained a considerable consistency 'momentum', which is dissipated by the gradual decrease of thick stock flow in the last phase. In fact, the thick stock flow will change until it is slightly below the reference value before the system has reached the new steady state.

The control strategies described above are at present being implemented on-line.

Basis weight control

THE specific task of maintaining the basis weight constant is particularly important, not only immediately after a grade change, but also in face of disturbances such as drifts and fluctuations in thick stock consistency and changes in the drainage properties of the fibre.

In order to realise a feedback control law as the one suggested in equation (*12*), it is required that all the papermachine state variables are known or can be measured. The object of the regulator problem must therefore be to design a controller that makes optimum use of information about the state variables contained in the basis weight measurement.

Kalman[10] has derived a linear estimator for the state variables that has the property that it minimises the discrepancy between observation and the state variable estimate according to a criterion based on the noise characteristics of the process and the measurements.

The present problem requires estimates of the state variables at the sampling instants as well as estimates of the past state to account for the transport delays in the system. In order to do this, it is best to think of the system in an augmented state space, thus—

$$x^*(n+1) = \begin{bmatrix} F_0 & F_1 & . & . & F_h \\ I & O & . & . & O \\ O & I & . & . & O \\ . & . & . & . & . \\ O & O & . & I & O \end{bmatrix} x^*(n) + \begin{bmatrix} E \\ O \\ O \\ . \\ O \end{bmatrix} u(n) \quad . \quad . \quad . \quad (16)$$

where the augmented state vector $x^*(n)$ includes all the vectors—

$$x(n), x(n-1), \ldots, x(n-h); \text{ as follows—}$$
$$x^*(n)^T = [x(n)^T x(n-1)^T \ldots x(n-h)^T]$$

The augmented plant state $x^*(n)$ and the measurement $y(n)$ can then be described in a stochastic environment by the following matrix equations—

$$x^*(n+1) = Ax^*(n)+Bu(n)+z \qquad (17)$$
$$y(n) = Mx^*(n)+w \qquad (18)$$

where M is an $1 \times mh$ output matrix.

It is assumed that z and w are Gaussian random independent variables representing the plant and measurement noise, respectively.

The linear estimator has the following structure—

$$\hat{x}^*(n,n) = \hat{x}^*(n,n-1)+K[y(n)-M\hat{x}^*(n,b-1)] \qquad (19)$$
$$\hat{x}^*(n+1,n) = A\hat{x}^*(n,n)+Bu(n) \qquad (20)$$

The initial estimate $\hat{x}^*(0,0) = 0$, that is, the best estimate of the state at time $n = 0$ given 0 measurements (the intial state has zero mean). After taking the measurements $y(n)$, the best estimate of the state at time n given all n measurements is equal to the best estimate before the nth measurement plus a weighting matrix times the difference between the actual measurement and the expected measurement.

The control law is given by—

$$u(n) = -C\dot{x}^*(n,n) \qquad (21)$$

The gain matrix K and control matrix C can both be computed by various iterative methods.[11]

The thick stock flow was chosen as the control variable in the present investigation and, for a particular set of machine conditions, it was found to be given by—

$$u_3(n) = 0{\cdot}544[3{\cdot}79x_3(n)+0{\cdot}02x_4(n)+4{\cdot}34x_5(n)+1{\cdot}17x_3(n-1)+1{\cdot}07x_3(n-2) +0{\cdot}08x_3(n-3)] \qquad (22)$$

with 10 s sampling interval. It can be seen that this is not immediately realisable, as u_3 is a function of inaccessible state variables. If equation (*22*) is to be used as a control law, it is first necessary to estimate the state variables involved from the measurement of basis weight.

A considerable computational load is placed on the computer control system if equations (*19*), (*20*) and (*21*) have to be evaluated on-line at every sampling instant to produce a control action. To meet this problem, the basis weight controller was here based on a canonical form of minimum arithmetic for the given state space representation, as developed by Lee.[12] For ease of on-line implementation, the control action was formulated as a z-transform of the basis weight. A general computational procedure was developed to derive the controller directly in the required z-transform—

$$u(z) = c\left[-d\times\frac{\sum_{n=1}^{mh} a_n z^{-n}}{1-\sum_{n=1}^{mh} b_n z^{-n}}\right]e(z). \quad . \quad . \quad . \quad (23)$$

which is based on the developed state space model of the process together with plant and measurement noise characteristics.

Åström[13] has derived similar control expressions based on experimentally identified processes and noise models. Equation (*23*) represents an optimum control transform relating the change in thick stock flow necessary to compensate for an observed deviation in basis weight. The control and estimation are both implicit features of this formulation, which consists of a proportional term and a memory term to account for past values of both control action and basis weight.

In place of equation (*22*), the new controller, which accounts for inaccessible state variables and noise, is given by—

$$u_3(n) = 0{\cdot}544\left(-0{\cdot}681\times\frac{0{\cdot}144z^{-1}-0{\cdot}294z^{-2}-0{\cdot}126z^{-3}-0{\cdot}06z^{-4}}{1+0{\cdot}058z^{-1}-0{\cdot}522z^{-2}-0{\cdot}322z^{-3}-0{\cdot}07z^{-4}}\right)x_6(n) \quad . \quad . \quad . \quad (24)$$

with a sampling interval of 10 s.

This controller has been successfully implemented and tried on the process itself, but has not been in operation long enough to have been fully evaluated quantitatively. There is, however, ample indication that the system has been considerably stabilised as can be seen from the plant records shown in Fig. 5.

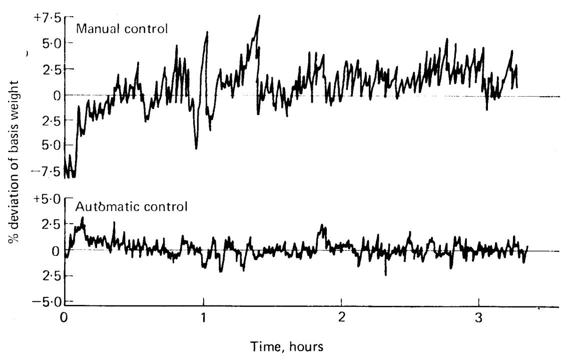

Fig. 5—Comparison between manual control and on-line computer control of basis weight

The above controller was derived under the assumption that the thick stock consistency could take on values with equal probability between certain limits. The developed methods are, however, not limited to problems with random noise disturbances. The following controller was derived by assuming the thick stock consistency to behave in a random walk manner—

$$u_3(n) = 0{\cdot}544\left(-2{\cdot}71+\frac{0{\cdot}58z^{-1}-1{\cdot}65z^{-2}+0{\cdot}182z^{-3}+0{\cdot}451z^{-4}+0{\cdot}36z^{-5}}{1-0{\cdot}87z^{-1}-0{\cdot}564z^{-2}+0{\cdot}01z^{-3}+0{\cdot}3z^{-4}+0{\cdot}12z^{-5}}\right)x_6(n) \quad . \; . \; . \quad (25)$$

This has not yet been implemented, but simulation tests indicate that more integral type action has been introduced, which will tend to eliminate offsets to a greater extent than with the first controller.

Conclusions

CONSISTENT with certain assumptions, a linear, discrete time, state space model was derived for the papermachine in the form of a matrix difference equation. Perturbation experiments on the plant were used to confirm the validity of the model. The system dynamics was found to be adequately predicted by the model. It comprises an independent flow model (which represents the flow of water as a vehicle for the transportation of fibre and additives) and a fibre model (which depends on the flow model and represents the concentration and distribution of fibre throughout the system).

Having formulated and established a mathematical model, the control objectives were stated by analytical performance criteria in the form of quadratic cost functionals, which were to be minimised, subject to the model equations and constraints on the control inputs.

The intent of this work has been to exploit the state space model and to develop a number of direct digital control systems compatible with available instrumentation and on-line digital computers, in order to improve the performance of a papermachine.

The optimum grade change strategies were formulated, one for grade changes at constant machine speed and the other for grade changes at constant production rate.

It has been shown that a systematic treatment of the problem of inaccessible variables is possible provided the state equations are known. This approach led to the synthesis of an on-line basis weight controller that was found to stabilise the system considerably when implemented on the papermachine.

It is thought that further studies of the treatment of inaccessible state variables would reveal the importance of being able to measure other state variables such as thin stock, head box and wire pit consistencies in order to approach an optimum feedback control system where all the state variables are used.

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Nomenclature

Physical representation

C_1,C_2 = Hydraulic capacitance, of head box air space and stock volume, respectively.

c_i,c_o = Consistencies before and after mixing.

P_1,P_2 = Hydraulic pressure at slice and pressure attributable to head box stock level alone.

M = Production rate, defined as $M = Wv_m$.

ΔP = Applied differential pressure at suction boxes.

q_{ai},q_{ao} = Airflows in and out of the head box.

q_i,q_o = Stock flows into and out of mixing tanks.

r = Number of converging streams before mixing.

s_d = Dryline position as measured from the point where suction is first applied.

t = Time, independent variable.
V = Volume of stock in mixing tanks.
v_m = Machine speed.
W = Basis weight.
φ = Efflux ratio.

State space representation

A = Augmented transition matrix (mh × mh).
B = Augmented input matrix (mh × s).
C = Augmented control matrix (s × mh).
D_i = Control matrices (s × m).
E = Input matrix (m × s).
$e(z)$ = Measured deviation of basis weight.
I = Identity matrix.
F_i = Transition matrices (m × m).
h = Number of delay matrices.
K = Filter gain matrix (mh × 1).
l = Number of accessible state variables.
m = Number of state variables.
n = Discrete time.
Q = Positive semi-definite weighting matrix (m × m).
R = Positive definite weighting matrix (s × s).
s = Number of input variables.
u = Input vector (s × 1).
v = Vector of input rate constraints (s × 1).
x = State vector (m × 1).
x_d = Desired state vector (m × 1).
$\hat{x}^*(i,j)$ = Best estimate of augmented state vector at time i, given j measurements.
y = Measurement vector (1 × 1).
z = Delay operator.
$||x||^2$ = Quadratic form (for example, $||x||_Q^2 = x^T Q x$).
T = Transpose of a vector or matrix.

Transcription of Discussion

Discussion

Mr A. J. Ward Could you please indicate whether you tried this automatic grade change on the actual mill. If so, did you find that the constant values of the parameter in the equations remained valid over the whole range tried?

Mr S. Hem No. Unfortunately, I have been unable to implement these control strategies in full, but the grade change at constant production rate and efflux ratio have been implemented practically by synchronously changing the thin stock flow and wire speed. There is no doubt that, by manipulating the thick stock flow as well, an even better response could be achieved.

Mr H. B. Carter Can you give some figures on the improvement made in basis weight variation with the implementation of this control?

Mr Hem This is an experimental project, not long enough in operation to have reliable figures, but the sort of *variance* that we hope for on basis weight is 0·3 per cent.

Mr W. T. Whight A certain amount of investigation on the results has been done by Mr Burrows of our research and development department. We have at the moment another controller of our own that gives the basis weight in the machine-direction to within $\pm 1{\cdot}25$ per cent. The information on Mr Hem's controller (in the experimental stage) is that it is as good as this and there is evidence to believe that it is suitable for our purposes. If more tuning effort were put into it, it would perform better.

Mr R. E. Johnston Would you like to guess at the major contributing effect to the difference between your controller and any other controllers that might be used? Is it the fact that a more complicated state space model than a linear estimator was used or is it that the amount of stock valve movement was included in the cost function?

Mr Hem There is a considerable difference between this controller and the one used by Åström, for instance. With this controller, we have the possibility of weighting the inputs in order to tune the controller. We can then prevent wear on valves, especially when, in this case, we were taking corrective action on the thick stock valve every 10 s. I think that the controller used here contains a larger memory than the Åström controller; in addition, it is a multi-variable approach applicable in a general sense.

Mr O. Alsholm I would add a few words that may be of interest. What I have encountered during this session is very much the same as Mr Johnston said earlier. It seems to me that everybody is trying to do the same thing, but using more and more complicated mathematics. I do not intend to discuss the differences between the Åström controller and other controllers presented here today or explain how much more efficient you could work with our DDC package than in CONRAD, but I would like to ask the authors to translate their nice mathematics into somewhat simpler terms. I enjoy listening to these excellent mathematicians and I really believe we need them for the future, but the majority of the problems that we implement today could be presented in a much simpler manner. If, instead of using the term Åström controller, for instance, one explains that there is a digital controller corresponding to PI plus dead time correction, people would not be so confused that they do not dare implement the strategy in practice. On the other hand, we should give credit to the mathematicians, because, if they do not continue with their advanced work, we will be left stranded.

Mr Hem May I say that, although the mathematics may sound awfully complicated, the actual process, once it has been done, can be performed on a fast computer in about 30 s and the implementation takes no longer than to implement the Åström controller.

Mr J. A. S. Newman Is it possible to use such mathematical models not only to predict how they can be controlled by the application of, say, DDC, but also how they can be made more inherently stable or controllable by modifications to their structural parameters such as pipework and tank sizes.

Mr Hem Yes, the structure of the plant is easily recognised in the formulation of the model. The parameters are very quickly changed if you want to investigate their effect on the plant behaviour.

Mr T. J. Boyle In 1959, dynamic programming was in vogue. I was in graduate school and applied this method on a chemical reactor problem similar to a grade change. I was successful in finding an optimum change on a simulated basis, but was very disappointed to find that some very simple strategies did quite as well as the optimum strategy. In this paper and that by Johnston & Kirk, we seem to have a similar situation with one being an optimum change, the other selecting the best version of a heuristically developed strategy. Has either author applied his technique to the other's model and thus developed a comparison?

Mr Hem Well, the grade change strategy at constant production rate and influx ratio was obtained purely and simply by solving the equations. There are no special optimisation procedures involved at all. This is mainly because the wire speed can be changed only at a certain rate. When one computes all the other manipulated variables, we find their trajectories never violate their constraints. It is therefore very easily obtained.