Performance Analysis of Bimodulus Frame Structures Based on Deformation Energy Decomposition Method

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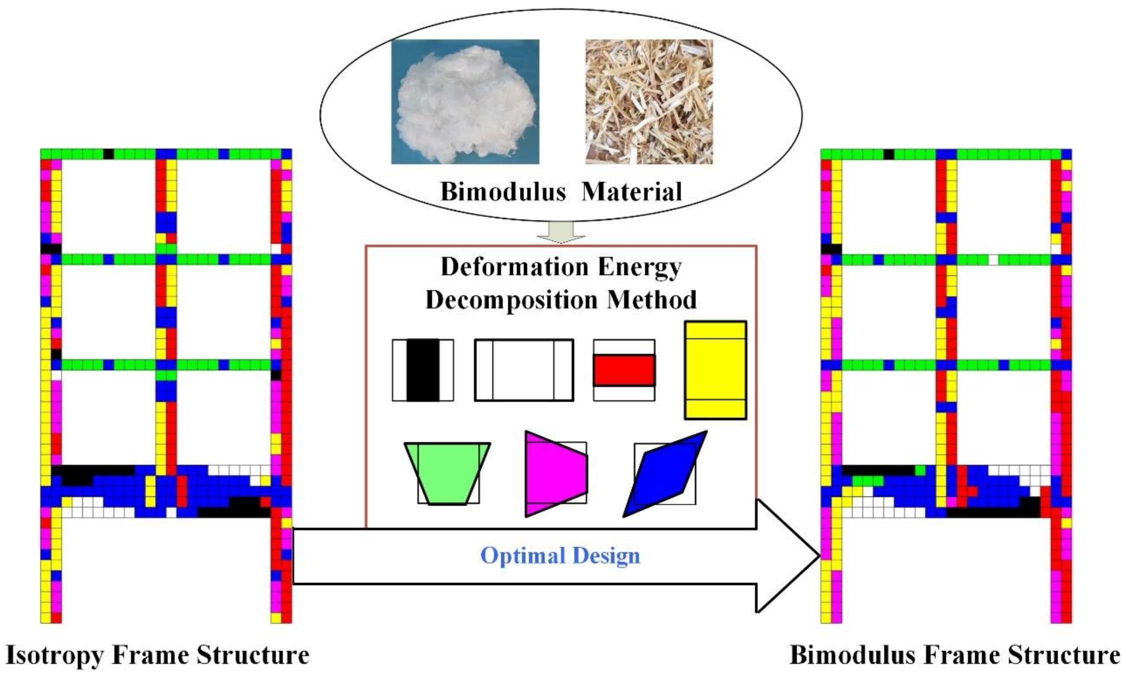
GRAPHICAL ABSTRACT



Performance Analysis of Bimodulus Frame Structures Based on Deformation Energy Decomposition Method

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Some biological materials have bimodulus properties. The elastic modulus in the tensile state is different from its value in the compressive state. The deformation energy decomposition method for bimodulus material can be obtained, and then the deformation energy decompositions of the isotropic and bimodulus frame structure are further realized. On the basis of the quantitative results of the basic deformation energy, the proportions of the areas dominated by shear deformation energy were proposed, which can characterize the ductility of the frame structures. The cases showed that the ratio of the elastic modulus in tensile state to the elastic modulus in compressive state is the important index of bimodulus material, which affects the deformation energy distribution of the bimodulus structure. When the ratio of bimodulus material for the deep beam was 0.2, the proportions of the regions dominated by shear deformation energy for the deep beams located on the 1st to 3rd floors were reduced by 10.00%, 7.77%, and 11.11%, respectively. The bimodulus material improved the ductility performance of the frame structure.

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Keywords: Bimodulus material; Shear deformation energy; Ductility performance; Deformation decomposition

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INTRODUCTION

For some biological materials, the elastic modulus in tensile state is different from the elastic modulus in compressive state. These materials, which include some stalk fibers, are referred to as bimodulus materials (Li *et al.* 2013; Tavakoli *et al.* 2019), and such behavior has been reported for fiber-reinforced composites (Khan and Patel 2014; Boon *et al.* 2019). Some scholars have begun to study the mechanical properties of bimodulus structures. The semi-inverse method (Yang *et al.* 2014) and the power series method (He *et al.* 2019) are derived the elastic solution of functionally graded cantilever beams with different tensile and compressive elastic modulus. Shah *et al*. (2017) derived the displacement equation and analytical solution of a simply supported beam with bimodulus material. Beskopylny *et al*. (2019) derived the maximum tensile and compressive stresses of bimodulus beams. However, the calculation processes of these methods are complex. The deformation energy decomposition method is different from the analytic method, which is a quantitatively numerical method and the calculation process is simple (Wang *et al.* 2023). Therefore, it is an interesting work that studies mechanical analysis of bimodulus structures based on the deformation energy decomposition method.

The frame structure has many advantages, such as clear transmission paths, good integrity, and easy spatial division. It is the preferred structural system for multi-story public buildings with important functions, such as schools and hospitals. The large space frame structures with deep beams are important frame structures (Li *et al.* 2021). The deep beams, which can significantly increase the initial stiffness and the corresponding ductility, should be analyzed (Xu *et al.* 2016). Therefore, it is of great significance to adopt bimodulus materials to improve the performance of frame structures. In this paper, the deformation energy decomposition method of the bimodulus element is introduced. The deformation energy of bimodulus structures can be decomposed into basic deformation energies, such as tension, compression, bending, and shear. Then the quantitative results of basic deformation energies are realized for bimodulus structures. A quantitative index is proposed to evaluate the ductility of the structure. Therefore, suitable double modulus materials are selected to optimize the design of the frame structure.

EXPERIMENTAL

Deformation Energy Decomposition Method of Bimodulus Structure

Based on mathematical complete orthogonality, the deformation of a 4-node square element in a plane can be decomposed into 8 mutually orthogonal basic deformation or displacement modes. The corresponding displacement vectors can form matrix, namely

$$\boldsymbol{d} = \left[\boldsymbol{d_1} \quad \boldsymbol{d_2} \quad \boldsymbol{d_3} \quad \boldsymbol{d_4} \quad \boldsymbol{d_5} \quad \boldsymbol{d_6} \quad \boldsymbol{d_7} \quad \boldsymbol{d_8}\right] \tag{1}$$

where d_1 is X-axis tensile (compressive) deformation, d_2 is Y-axis tensile (compressive) deformation, d_3 is X-axis bending deformation, d_4 is Y-axis bending deformation, d_5 is shear deformation, d_6 is X-axis rigid body linear displacement, d_7 is Y-axis rigid body linear displacement, and d_8 is rigid body rotational displacement.

The specific form of the displacement mode matrix can be expressed as

$$\boldsymbol{d} = \begin{bmatrix} b & 0 & 1 & 0 & 1 & 1 & 0 & -1 \\ 0 & b & 0 & 1 & 1 & 0 & 1 & 1 \\ -b & 0 & -1 & 0 & 1 & 1 & 0 & -1 \\ 0 & b & 0 & -1 & -1 & 0 & 1 & -1 \\ -b & 0 & 1 & 0 & -1 & 1 & 0 & 1 \\ 0 & -b & 0 & 1 & -1 & 0 & 1 & -1 \\ b & 0 & -1 & 0 & -1 & 1 & 0 & 1 \\ 0 & -b & 0 & -1 & 1 & 0 & 1 & 1 \end{bmatrix} \tag{2}$$

where b is 1 or -1. When b is 1, d_1 is X-axis tensile deformation, and d_2 is Y-axis tensile deformation. When b is -1, d_1 is X-axis compressive deformation, and d_2 is Y-axis compressive deformation.

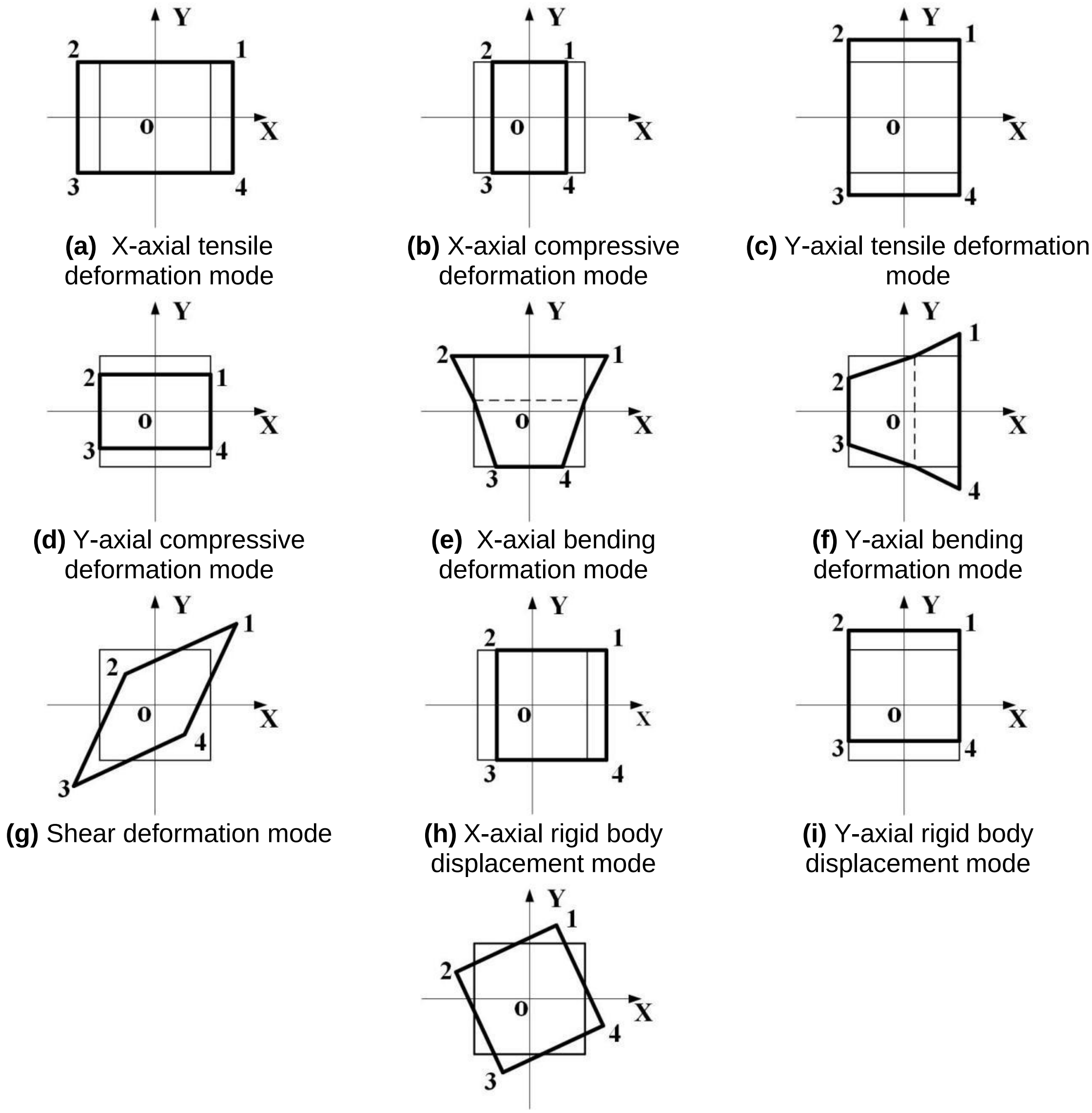
In addition, the displacement mode matrix satisfies mathematical orthogonality, namely,

$$\boldsymbol{d}_p^{\mathrm{T}}\boldsymbol{d}_q = 0 \quad p \neq q \ (p, q = 1, 2, \ldots, 8) \tag{3}$$

The nodal displacement vector of the element can be linearly represented by the basic deformation (displacement) mode vectors, namely,

$$\boldsymbol{u} = \sum_{k=1}^{8} \beta_k \boldsymbol{d_k} \tag{4}$$

where β_k is the projection coefficient of $\boldsymbol{d_k}$ $(k = 1, 2, \ldots, 8)$.



(a) X-axial tensile deformation mode
(b) X-axial compressive deformation mode
(c) Y-axial tensile deformation mode
(d) Y-axial compressive deformation mode
(e) X-axial bending deformation mode
(f) Y-axial bending deformation mode
(g) Shear deformation mode
(h) X-axial rigid body displacement mode
(i) Y-axial rigid body displacement mode
(j) Rigid body rotation displacement mode

Fig. 1. Basic deformation and rigid body displacement mode of planar square bimodulus element

The tensile and compressive states of the bimodulus material have to be considered, respectively. Ten basic deformation and displacement modes are constructed, as shown in Fig. 1. The constitutive relation of the bimodulus theory is different from that of the isotropy theory (Ambartsumyan 1982). For the bimodulus material, the stress-strain relationship can be analyzed in tensile and compressive states, respectively. The simplified processing has sufficient accuracy. The three rigid body displacements do not generate deformation energy. The five basic deformation energies can be calculated for a bimodulus square element in both tensile and compressive states, as shown in Eqs. 5 and 6,

$$W_k^{\,t} = \frac{1}{2}\int_x\int_y \boldsymbol{d}_k^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{D_t}\boldsymbol{B}\boldsymbol{d}_k t dxdy \quad (k = 1, 2, 3, 4, 5) \tag{5}$$

$$W_k^{\,c} = \frac{1}{2}\int_x\int_y \boldsymbol{d}_k^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{D_c}\boldsymbol{B}\boldsymbol{d}_k t dxdy \quad (k = 1, 2, 3, 4, 5) \tag{6}$$

where B is geometry matrix, D_t is elastic matrix in tensile state, and D_c is elastic matrix in compressive state.

$$\boldsymbol{D_t} = \frac{E_t}{(1-\mu_t^{\,2})}\begin{bmatrix} 1 & \mu_t & 0 \\ \mu_t & 1 & 0 \\ 0 & 0 & \dfrac{1-\mu_t}{2} \end{bmatrix} \tag{7}$$

$$\boldsymbol{D_c} = \frac{E_c}{(1-\mu_c^{\,2})}\begin{bmatrix} 1 & \mu_c & 0 \\ \mu_c & 1 & 0 \\ 0 & 0 & \dfrac{1-\mu_c}{2} \end{bmatrix} \tag{8}$$

where E_t is elastic modulus in tensile state, μ_t is Poisson's ratio in tensile state, E_c is elastic modulus in compressive state, and μ_c is Poisson's ratio in the compressive state.

The rigid body displacement does not provide energy. Then five basic deformation energies of the bimodulus element are obtained by Eq. 9,

$$W = \sum_{k=1}^{5} W_k \tag{9}$$

where W_k is the deformation energy of the k^{th} basic deformation mode.

Based on the quantitative results, the values of various basic deformation energies were compared. The maximum value represents the main deformation type of the element, which corresponds to a color. Based on the method, the deformation energy decomposition results of the bimodulus structure were obtained.

RESULTS AND DISCUSSION

Analysis of Deformation Performance for Bimodulus Frame

Deep beams are used as transfer beams of large open centers, such as hospitals and high-rise buildings (Liu *et al.* 2019). When overloading occurs, deep beams often determine the performance of the entire structure. Taking the 2011 Christchurch earthquake as an example (Goldsworthy 2012; Elwood 2013), the earthquake generated unforeseeable vertical ground acceleration and caused shear failure of multiple deep transfer beams. Within a few months after the earthquake, some buildings were on the brink of collapse and had to be demolished. Therefore, using dual-mode materials to improve the ductility performance of frame structures and deep beams in the structure is of great significance.

Taking a four-storey frame structure with a large conference room as an example, and the large conference room is set on the 1st to 3rd floors. The length direction of the frame beam is specified as the X-axis direction and the height direction as the Y-axis direction. Among them, the height of the large space floor is 4.5 m, the height of the deep beam is 1.5 m, and the height of the other floors is 3.0 m. The elastic modulus of the frame structure is 30 GPa, and the Poisson's ratio is 0.20. According to the bottom shear force method, a seismic fortification intensity of 8 degrees is selected for Class II sites, and the design earthquake group is divided into the second group. The horizontal load conditions of each frame structure are calculated by Code for Seismic Design of Buildings GB 50011 (2010). The structural dimensions and load conditions are shown in Fig. 2. The mass and stiffness of deep beams are significantly greater than those of shallow beams. The bottom shear force method depends on the mass and stiffness distributions. Therefore, the load conditions of frame F1, F2, and F3 are different.

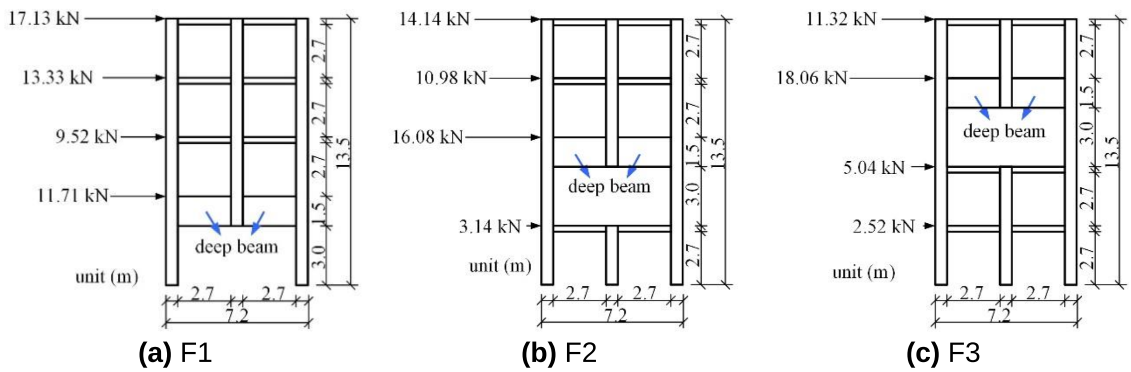


Fig. 2. The dimensions of frame structures and the corresponding load conditions

The isotropy frames F1 to F3 were divided into square elements. For the isotropy element, the elastic modulus in tensile state is equal to the one in tensile state. The deformation energy decomposition method of bimodulus element can be degenerated into the one of isotropy element. The decomposition diagrams were performed on three isotropic frames, as shown in Fig. 3. On this basis, the bimodulus material was adopted for frame beams. The tensile elastic modulus was 6 GPa, and the compressive elastic modulus was 30 GPa. The tensile-compression elastic modulus ratio of bimodulus materials for the deep beam was 0.2. Based on the deformation energy decomposition method of bimodulus element, the decomposition diagrams of three bimodulus frames BF1 through BF3 are shown in Fig. 3.

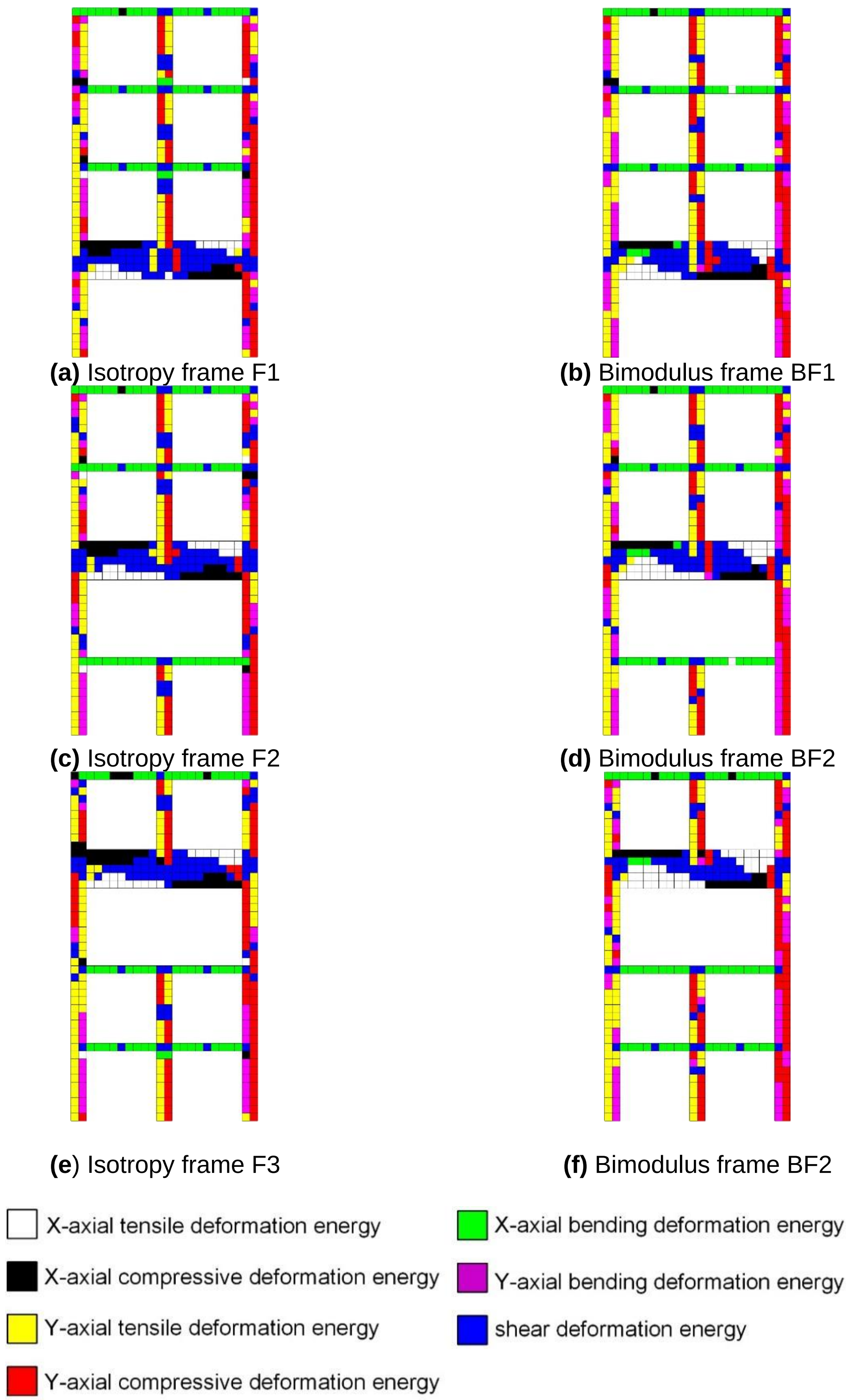
The proportion of the area dominated by shear deformation energy is calculated from the decomposition diagrams, which can characterize the ductility of the deep beam for frame structures. It is expressed as follows,

$$\psi = \frac{S_\tau}{S} \times 100\% \tag{10}$$

where S_τ is the regional area of shear deformation energy, and S is the total area of the frame structure. Based on Eq. 10, the proportions of the regions dominated by shear deformation energy for the isotropic and bimodulus frames can be obtained, as shown in Table 1. The deep beam is the important component and the shear deformation energy is dominant in the deep beam. In addition, as the number of floors in the large space increases, the area of the deep beam dominated by shear deformation energy gradually decreases.

Table 1. Shear Deformation Energy Proportions of Deep Beams

Frame Number	F1	BF1	F2	BF2	F3	BF3
ψ (%)	48.89	38.89	43.33	35.56	40.00	28.89
Reduced proportion (%)	—	10.00	—	7.77	—	11.11



(a) Isotropy frame F1 **(b)** Bimodulus frame BF1

(c) Isotropy frame F2 **(d)** Bimodulus frame BF2

(e) Isotropy frame F3 **(f)** Bimodulus frame BF2

Fig. 3. Deformation energy decomposition diagrams of Isotropy and bimodulus frames

According to Table 1, as the number of floors in the large space increases, the shear deformation energy proportion decreases, and the ductility increases for isotropy and bimodulus frame structures. Compared with F1, the shear deformation energy proportion of F3 is reduced by 8.89%. Compared with BF1, the shear deformation energy proportion of BF3 is reduced by 10.00%. Therefore, the large space should be placed on the upper floors of the frame structure, which can ensure its ductility.

Compared with isotropic frames, the bimodulus materials can reduce the proportion of shear deformation energy for bimodulus frames. According to Table 1, the proportions of the area dominated by shear deformation energy for deep beams are decreased by 10.00%, 7.77%, and 11.11%, respectively for the three bimodulus frames, BF1, BF2, and BF3. Figure 3 shows that the proportions of the areas dominated by shear deformation energy for shallow beams also slightly decreased. Compared with F1, the shear deformation energy proportions of shallow beams for BF1 on the 3rd and 4th floors slightly decreased. Compared with F2, the shear deformation energy proportions of shallow beams for BF2 on the 1st and 4th floors slightly decreased. Compared with F3, the shear deformation energy proportions of shallow beams for BF3 on the 2nd and 4th floors slightly decreased.

The results show that by reducing the ratio of tensile and compressive elastic moduli of beam components, the proportion of the area dominated by shear deformation energy dominates the structure under seismic loads can be reduced. Thus, the ductility performance of the frame structure is improved. The bimodulus material can improve the performance of the frame structure.

CONCLUSIONS

1. The basic deformation (displacement) modes are constructed based on mathematical complete orthogonality. Based on elastic modulus in tensile or compressive state, the basic deformation energies of the bimodulus element can be obtained.
2. The ratio of the elastic modulus in tensile state to the elastic modulus in compressive state is the important index of bimodulus material, which can affect the deformation energy distribution of bimodulus structures.
3. Based on the quantitative results of basic deformation energies for bimodulus frame structures, optimization analysis of multi-layer frame structures with large spaces has been achieved. When the ratio of the elastic modulus in tensile state to the elastic modulus in compressive state is 0.2, the proportions of the regions dominated by shear deformation energy in the deep beams located on the 1st to 3rd floors are reduced by 10.00%, 7.77%, and 11.11%, respectively. Then the ductility performance of the deep beams in the frame structure is improved.

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