

# Numerical Method of Time-domain Dynamic Responses for Timber Frame Structures with Viscoelastic Damper

Zhe Song,<sup>a</sup> Chongyang He,<sup>b</sup> and Shuxia Wang <sup>c,d,e,\*</sup>

The seismic performance analysis of timber frame structures with viscoelastic damper is important. Combined with Rayleigh damping model and stiffness proportional damping model, the damping matrix of the timber frame structures with viscoelastic damper is constructed. Based on the fine calculation method, the numerical method of time-domain dynamic responses for timber frame structures with viscoelastic damper is proposed, which can be used to calculate the time-domain dynamic responses. Then the peak top displacement and peak bottom shear force are selected as the indicators, which are used to analyze the arrangement of viscoelastic dampers in wooden frame structures. Numerical results show that compared to viscoelastic dampers at the top and middle, viscoelastic damper at the bottom is the most advantageous for improving the seismic performance of the structure, which can be given priority consideration in design.

DOI: 10.15376/biores.21.4.10269-10275

*Keywords:* Dynamic responses; Timber frame structures; Viscoelastic damper

*Contact information:* a: Henan Technical College of Construction, Zhengzhou 450064, China; b: Henan Earthquake Agency, Zhengzhou 450018, China; c: School of Materials Science and Engineering, Zhengzhou University of Aeronautics, Zhengzhou 450046, China; d: Lanshen Group Co. Ltd., Nanjing 211500, China; e: School of Materials Science and Engineering, Zhengzhou University, Zhengzhou 450001, China;

\* Corresponding author: shuxiawang1990@yeah.net

## INTRODUCTION

Timber frame structures have advantages such as light weight and good toughness (Kharrazi *et al.* 2008; Akan *et al.* 2021). However, traditional timber structures have disadvantages such as insufficient node stiffness and weak energy dissipation capacity (Matteoni *et al.* 2024; Pei *et al.* 2025). The lateral bearing capacity of the pure wood frame structure is insufficient, and the displacement between layers under strong earthquakes is prone to exceed the specified limit (Xu *et al.* 2025; Gholamizoj *et al.* 2026).

As an efficient passive energy dissipation component, viscoelastic damping devices can dissipate seismic energy through deformation, providing an effective way for seismic reinforcement of timber structures. Relevant research has become a hot topic in the field of seismic resistance of timber structures. The viscoelastic damping device can effectively improve the seismic performance of timber frame structures, and related research provides theoretical and technical support for seismic reinforcement of timber structures. Nakamura and Matsumura (2022) compared the installation methods of cross-story and inter-story in multi-story timber frames. It was found that dampers can significantly reduce the extension of the basic period and amplitude of the structure and thereby suppress the accumulation of residual deformation. Yi *et al.* (2024) pointed out that viscoelastic dampers can effectively control the residual deformation of damaged wooden frames on slopes and

improve the deformation control ability in weak areas based on the full-scale vibration table tests. However, a simple and effective time-domain calculation method of timber structures with viscoelastic dampers still needs to be further studied.

This paper conducts seismic performance analysis on timber frame structures with viscoelastic dampers. The damping matrix is established based on Rayleigh damping and stiffness-proportional damping models. A refined numerical approach is proposed to calculate the time-domain dynamic responses of the timber structures with viscoelastic dampers. Compared with the common complex modal superposition method based on state space, the calculation process of the proposed method is simpler. The peak top displacement, peak bottom shear force and peak story drift are selected as the indicators, the damper layout effect is quantitatively investigated. The seismic performances of timber frame structures with viscoelastic dampers at the top, middle and bottom are analyzed, respectively.

## TIME-DOMAIN METHOD OF DYNAMIC RESPONSES FOR TIMBER FRAME STRUCTURES WITH VISCOELASTIC DAMPER

The time-domain motion equation of the timber frame structure is as follows,

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = -\mathbf{M}\mathbf{I}g(t) \quad (1)$$

where  $\mathbf{M}$  is mass matrix of the timber frame structure;  $\mathbf{C}$  is damping matrix of the timber frame structure;  $\mathbf{K}$  is stiffness matrix of the timber frame structure;  $\mathbf{x}(t)$  is structural displacement vector;  $\mathbf{I}$  is the vector of load distribution vector;  $g(t)$  is the acceleration of seismic wave.

$\mathbf{C}$  is expressed by Eq. 2,

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K} \quad (2)$$

where  $\alpha$  and  $\beta$  are Rayleigh damping coefficients, defined as follows,

$$\begin{cases} \alpha = \frac{\omega_1\omega_2(\eta\omega_1 - \eta\omega_2)}{\omega_1^2 - \omega_2^2} \\ \beta = \frac{\eta\omega_1 - \eta\omega_2}{\omega_1^2 - \omega_2^2} \end{cases} \quad (3)$$

where  $\eta$  is loss factor of the timber frame structure,  $\omega_1$  is natural frequency of the first vibration mode,  $\omega_2$  is natural frequency of the second vibration mode. Based on Rayleigh damping model, the loss factors of the two modes are identical (Ryan *et al.* 2008), which is loss factor of the timber frame structure.

When the viscoelastic damper is added to the timber frame structure, Eq. 4 expresses the time-domain motion,

$$\mathbf{M}\ddot{\mathbf{x}}(t) + (\mathbf{C} + \mathbf{C}_d)\dot{\mathbf{x}}(t) + (\mathbf{K} + \mathbf{K}_d)\mathbf{x}(t) = -\mathbf{M}\mathbf{I}g(t) \quad (4)$$

where  $\mathbf{C}_d$  is damping matrix of the viscoelastic damper;  $\mathbf{K}_d$  is stiffness matrix of the viscoelastic damper.

The viscoelastic damper can adopt a stiffness proportional damping model (Sun *et*

al. 2024).  $C_d$  is expressed as follows,

$$C_d = \frac{1}{\omega_l} \eta_d K_d \quad (5)$$

where  $\eta_d$  is loss factor of the viscoelastic damper.

The damping system of the structure has become a non-proportional damping system (Sun *et al.* 2026). The equation of state is adopted. Equation 4 is rewritten as follows,

$$\dot{\mathbf{W}}(t) = \mathbf{H}\mathbf{W}(t) + \mathbf{R} \quad (6)$$

where

$$\mathbf{W}(t) = \begin{bmatrix} \mathbf{x}(t) \\ \dot{\mathbf{x}}(t) \end{bmatrix} \quad (7)$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{0} & \mathbf{E} \\ -\mathbf{M}^{-1}(\mathbf{K} + \mathbf{K}_d) & -\mathbf{M}^{-1}(\mathbf{C} + \mathbf{C}_d) \end{bmatrix} \quad (8)$$

$$\mathbf{R} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{I}g(t) \end{bmatrix} \quad (9)$$

The general solution of the equation can be expressed as Eq. 10.

$$\mathbf{W}(t) = e^{\mathbf{H}t} \mathbf{W}(t_0) + \int_0^t e^{\mathbf{H}(t-\tau)} \mathbf{R}(\tau) d\tau \quad (10)$$

$\mathbf{W}_0$  is initial value, which is calculated by Eq. 11.

$$\mathbf{W}_0(t) = \begin{bmatrix} \mathbf{x}(t_0) \\ \dot{\mathbf{x}}(t_0) \end{bmatrix} \quad (11)$$

The time step  $\Delta t$  is used to discretize. Any time can be represented as follows.

$$t_k = k\Delta t \quad (k = 0, 1, 2, \dots) \quad (12)$$

At the time  $t_k$ , the dynamic response is determined by Eq. 13.

$$\mathbf{W}(t_k) = e^{\mathbf{H}t_k} \mathbf{W}(t_0) + \int_0^{t_k} e^{\mathbf{H}(t_k-\tau)} \mathbf{R}(\tau) d\tau \quad (13)$$

At the time  $t_{k+1}$ , the dynamic response is determined by Eq. 14.

$$\mathbf{W}(t_{k+1}) = e^{\mathbf{H}t_{k+1}} \mathbf{W}(t_0) + \int_0^{t_{k+1}} e^{\mathbf{H}(t_{k+1}-\tau)} \mathbf{R}(\tau) d\tau = e^{\mathbf{H}\Delta t} \mathbf{W}(t_k) + \int_{t_k}^{t_{k+1}} e^{\mathbf{H}(t_{k+1}-\tau)} \mathbf{R}(\tau) d\tau \quad (14)$$

The integration term at the right end of Eq. 13 can achieve high accuracy by using a three-point Gaussian integration format, which is shown in Eq. 15,

$$\int_{t_k}^{t_{k+1}} e^{\mathbf{H}(t_{k+1}-\tau)} \mathbf{R}(\tau) d\tau = \sum_{i=1}^3 A_i T_i \mathbf{R}[t_k + 0.5\Delta t(1 + y_i)] 0.5\Delta t \quad (15)$$

where

$$\begin{cases} A_1 = \frac{5}{9}, & T_1 = e^{0.5\Delta t(1+\sqrt{0.6})\mathbf{H}}, & y_1 = -\sqrt{0.6} \\ A_2 = \frac{8}{9}, & T_2 = e^{0.5\Delta t\mathbf{H}}, & y_2 = 0 \\ A_3 = \frac{5}{9}, & T_3 = e^{0.5\Delta t(1-\sqrt{0.6})\mathbf{H}}, & y_3 = \sqrt{0.6} \end{cases} \quad (16)$$

and where

$e^{A\mathbf{H}}$ ,  $e^{0.5A(1+\sqrt{0.6})\mathbf{H}}$ ,  $e^{0.5A\mathbf{H}}$  and  $e^{0.5A(1-\sqrt{0.6})\mathbf{H}}$  are index matrices.

The fine calculation method is used to calculate the numerical results (Zhong and Cai 2000). Thus, the stepwise integration calculation of Eq. (4) can be carried out. The numerical method of time-domain dynamic responses for timber frame structures with viscoelastic damper is realized.

## NUMERICAL EXAMPLES

A three-layer shear type timber frame model is established, which is defined as Model A, as shown in Fig. 1. Here the lumped-mass shear-layer model is adopted. The mass and stiffness parameters are assumed, which is only for verifying the method. According to BS EN 1995-1-1: 2004+A1: 2008, the damping ratio of timber frame structures is 0.02, and the corresponding loss factor of timber frame structures is 0.04.

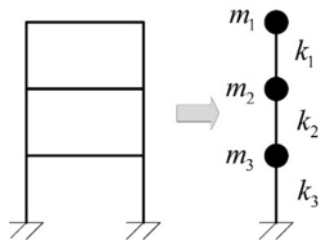


Fig. 1. Schematic diagram of timber frame structures

Based on Model A, the timber frame models with different viscoelastic dampers are established, respectively, as shown in Fig. 2. The damper of Model B is arranged at the bottom. The damper of Model C is arranged at the middle. The damper of Model D is arranged at the top. The parameters of numerical models are shown in Table 1.

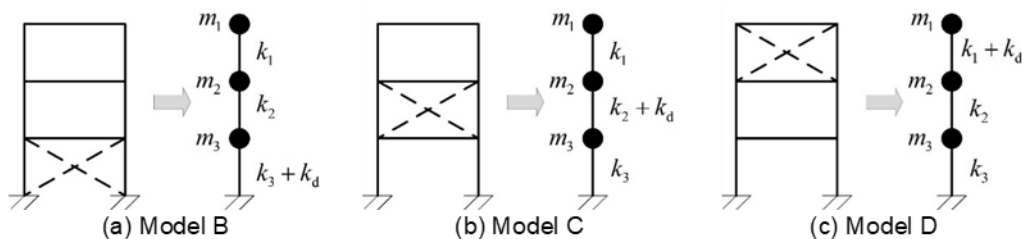


Fig. 2. Schematic diagrams of timber frame structures with different viscoelastic dampers

Table 1. The Parameters of Numerical Models

|                          | $m_1$             | $m_2$             | $m_3$             |                   |
|--------------------------|-------------------|-------------------|-------------------|-------------------|
| Value of mass (kg)       | 1000 kg           | 2000 kg           | 3000 kg           |                   |
|                          | $k_1$             | $k_2$             | $k_3$             | $k_d$             |
| Value of stiffness (N/m) | $1.0 \times 10^5$ | $2.0 \times 10^5$ | $3.0 \times 10^5$ | $0.5 \times 10^5$ |
|                          | $\eta$            | $\eta_d$          |                   |                   |
| Value of loss factor     | 0.04              | 0.20              |                   |                   |

Taking Model B as an example, El Centro wave and Tianjin wave are selected. The top time-history displacements are calculated by the proposed method and the complex modal superposition method (Chen and Peng 2022), respectively. The corresponding calculation results are shown in Fig. 3. Under El Centro wave, the peak top time-history displacements by the proposed method and the complex modal superposition method are 14.31 mm and 14.25 mm, respectively. Under Tianjin wave, the peak top time-history displacements by the proposed method and the complex modal superposition method are 15.88 mm and 15.86 mm, respectively. The results show that the calculation results of the two methods are consistent. The correctness of the proposed method has been verified. Compared with complex modal superposition method, the calculation process of the proposed method is simpler.

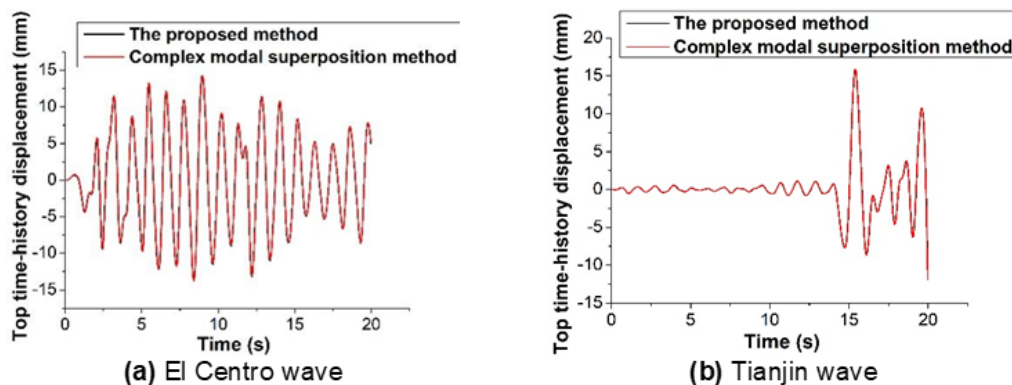


**Fig. 3.** Top time history displacements due to seismic waves

The undamped natural frequencies of the numerical model are 0.87 Hz, 1.82 Hz and 2.55 Hz, respectively. The proposed method is used to calculate the time-domain responses of the timber frame models due to El Centro wave. The peak top displacement, peak bottom shear force and peak story drift are selected as the indicators for seismic performance analysis. The comparative analysis of different models is shown in Table 2. The results show that compared with Model A, the peak bottom shear force and peak story drift of Model B are the smallest. The viscoelastic damper at the bottom can enhance resistance to seismic forces and improve local stiffness. Therefore, the viscoelastic dampers at the top and middle are not the most advantageous choices. The viscoelastic damper should be priority installed at the bottom of timber frame structures. However, the peak top displacement of Model C is the smallest. The peak top displacement of Model B is greater than the one of Model C. The improvement of the overall lateral stiffness of the timber frame structures with the bottom viscoelastic damper is limited.

**Table 2.** Comparative Analysis of Different Models

|                             | Model A | Model B | Model C | Model D |
|-----------------------------|---------|---------|---------|---------|
| Peak top displacement (mm)  | 16.32   | 14.31   | 13.86   | 15.21   |
| Relative error (%)          | —       | -12.32  | -15.07  | -6.80   |
| Peak bottom shear force (N) | 4701    | 4254    | 4840    | 4438    |
| Relative error (%)          | —       | -9.51   | 2.96    | -5.59   |
| Peak story drift            | 7.29    | 5.91    | 6.66    | 6.12    |
| Relative error (%)          | —       | -18.93  | -8.64   | -16.05  |

## CONCLUSIONS

1. A damping model was developed for timber frame structures, adopting the Rayleigh damping model. An additional viscoelastic damper adopted stiffness proportional damping model. Then the damping matrix of the timber frame structures with viscoelastic damper was constructed. Based on the fine calculation method, a numerical method of time-domain dynamic responses for timber frame structures with viscoelastic damper is proposed.
2. The arrangement of viscoelastic dampers in timber frame structures was found to directly affect the seismic performance of the structure. The peak top displacement, peak bottom shear force and peak story drift were selected as the indicators. Compared to viscoelastic dampers at the top and middle, viscoelastic damper at the bottom was found to be the most advantageous for improving the seismic performance of the structure, which can be given priority consideration in design. However, the improvement of the overall lateral stiffness is limited.

## ACKNOWLEDGMENTS

This work was supported by the Key Research Project of Henan Higher Education Institutions (Grant No. 25A430032).

## REFERENCES CITED

- Akan, A. E., Basok, G. Ç., Er, A., Örmecioglu, H. T., Koçak, S. Z., Cosgun, T., Uzdil, O., and Sayin, B. (2021). "Seismic evaluation of a renovated wooden hypostyle structure: A case study on a mosque designed with the combination of Asian and Byzantine styles in the Seljuk era (14<sup>th</sup> century AD)," *Journal of Building Engineering* 43, article 103112. <https://doi.org/10.1016/j.jobbe.2021.103112>
- BS EN 1995-1-1: 2004+A1: 2008 (2008). "UK National Annex to Eurocode 5: Design of timber structures," BSI, London, UK.
- Chen, H. T., and Peng, L.Y. (2022). "Dynamic analysis of heavily damped structures with supplemental energy dissipation devices," *Journal of Earthquake Engineering* 26(2), 1040-1071. <https://doi.org/10.1080/13632469.2019.1698479>
- Gholamizoj, K., Tremblay-Auclair, J. P., Salenikovitch, A., Homami, P., and Chui, Y. H. (2026). "Seismic performance of a scaled three-story braced timber frame building with elastomeric damper through shake-table tests," *Soil Dynamics and Earthquake Engineering* 206, article 110275. <https://doi.org/10.1016/j.soildyn.2026.110275>
- Kharrazi, M. H. K., Eldeib, S., and Prion, H. G. L. (2008). "Experimental evaluation of an orthotropic, monolithic, modular wooden-dome structural system," *Canadian Journal of Civil Engineering* 35(10), 1163-1176. <https://doi.org/10.1139/L08-060>
- Matteoni, M., Ciurlanti, J., Bianchi, S., and Pampanin, S. (2024). "Fragility functions for low-damage post-tensioned timber frames," *Earthquake Engineering and Structural Dynamics* 53(15), 4741-4762. <https://doi.org/10.1002/eqe.4242>
- Nakamura, Y., and Matsumura, R. (2022). "Cross-story installation of viscous dampers in timber frame houses for earthquake damage reduction," *Frontiers in Built*

- Environment* 8, 1037832. <https://doi.org/10.3389/fbuil.2022.1037832>
- Pei, S. L., Fragiaco, M., Isoda, H., Li, M. H., Xiao, Y., and Smith, T. (2025). "Codification and research for mass timber buildings in selected seismic regions: An overview," *Engineering Structures* 346, article 121685. <https://doi.org/10.1016/j.engstruct.2025.121685>
- Ryan, K. L., and Polanco, J. (2008). "Problems with Rayleigh damping in base-isolated buildings," *Journal of Structural Engineering* 134(11), 1780-1784. [https://doi.org/10.1061/\(ASCE\)0733-9445\(2008\)134:11\(1780\)](https://doi.org/10.1061/(ASCE)0733-9445(2008)134:11(1780))
- Sun, P., Yan, Y., and Yang, H. (2024). "Complex mode superposition method of non-classically damped systems based on hysteretic damping model with frequency-dependent loss factors," *Journal of Sound and Vibration* 571, article 118122. <https://doi.org/10.1016/j.jsv.2023.118122>
- Sun, G. Z., Zhou, G. S., and Sun, P. X. (2026). "Seismic analysis of wood-stone structures with three degree of freedom systems," *BioResources* 21(1), 178-187. <https://doi.org/10.15376/biores.21.1.178-187>
- Xu, J. H., Sun, D. B., Liu, E. Z., Guo, T., and Li, A. Q. (2025). "Optimal design of right-angle viscoelastic dampers for seismic performance of ancient timber structure via improved multi-objective particle swarm algorithm," *Structures* 77, article 109123. <https://doi.org/10.1016/j.istruc.2025.109123>
- Yi, D. H., Yuan, J. C., Pan, Y., and Fan, Y. Q. (2024). "Full-scale seismic performance testing of a predamaged two-story traditional timber frame on a slope reinforced using viscoelastic dampers," *Journal of Structural Engineering* 150(4), article 04024026. <https://doi.org/10.1061/JSENDH.STENG-12843>
- Zhong, W. X., and Cai, Z. Q. (2000). "Precise integration method for LQG optimal measurement feedback control problem," *Applied Mathematics and Mechanics (English Edition)* 21(12), 1417-1422. <https://doi.org/10.1007/BF02459220>

Article submitted: May 16, 2026; Peer review completed: August 1, 2026; Revised version received: August 16, 2026; Accepted: August 17, 2026; Published: September 2, 2026.

DOI: 10.15376/biores.21.4.10269-10275